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Commutators of Foliation Preserving Homeomorphisms for Certain Compact Foliations

Dedicated to Professor Hiroyasu Ishimoto on his 60th birthday

By

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§1. Introduction

Let M be an n-dimensional closed topological manifold. By $\mathscr{H}(M)$ we denote the group of all homeomorphisms of M which are isotopic to the identity by an isotopy fixed outside a compact set.

In this note we treat certain subgroups of $\mathscr{H}(M)$.

Let (M, N) be a manifold pair, where N is a proper submanifold of M. Let $\mathscr{H}(M, N)$ denote the subgroup of homeomorphisms of $\mathscr{H}(M)$ which are invariant on N.

In §2, we consider the homologies of $\mathscr{H}(M, N)$, that is, the homology groups of the group $\mathscr{H}(M, N)$ and show that the homologies of $\mathscr{H}(\mathbf{R}^n, \mathbf{R}^p)$ $(p>0)$ vanish in all dimension >0. This is a special case of a result of Fukui-Imanishi [F-I] which is a generalization of a result of Mather [Ma] to foliated manifolds. We show in §3 that $\mathscr{H}(M, N)$ is perfect, *i.e.*, is equal to its own commutator subgroup, for a certain manifold pair (M, N).

In §4 and §5, we consider the group of foliation preserving homeomorphisms. We have already discussed in [F-I] about the case of codimension one foliations. We study here the case of compact foliations of codimension greater than one. Let $(M, \mathscr{F})$ be a C^1-foliated manifold and $F(M, \mathscr{F})$ be the group of foliation preserving homeomorphisms of $(M, \mathscr{F})$ isotopic to the identity by a foliation preserving isotopy fixed outside a compact

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set. Using the results in §3 we show in §4 that $F(M, \mathcal{F})$ is perfect for the case that $\mathcal{F}$ is a compact codimension two foliation with no dihedral leaves on a compact manifold M and show in §5 that $F(M, \mathcal{F})$ is perfect for the case that $\mathcal{F}$ is a certain compact Hausdorff codimension three foliation with $\pi_1(L) \cong \mathbb{Z}$ for each leaf L on a compact manifold M.

§2. Homologies of $\mathcal{H}(\mathbb{R}^n, \mathbb{R}^p)$

We recall that if G is any group, then there is a standard chain complex $C(G)$ whose homology is the homology of G.

Let $C_r(G)$ be the free abelian group on the set of all r-tuples $(g_1, \ldots, g_r)$, where $g_i \in G$. The boundary operator $\partial : C_r(G) \to C_{r-1}(G)$ is defined by

$$\partial(g_1, \ldots, g_r) = (g_1^{-1}g_2, \ldots, g_1^{-1}g_r) + \sum_{i=1}^{r} (-1)^i (g_1, \ldots, \hat{g}_i, \ldots, g_r).$$

Then we have $\partial^2 = 0$. The symbol $H_r(G)$ will stand for the r-th homology group of the above chain complex.

Let $\mathbb{R}^n = \{(x_1, \ldots, x_n) \mid x_i \in \mathbb{R}\}$ be an n-dimensional Euclidean space and $\mathbb{R}^p$ the p-dimensional subspace $\{(x_1, \ldots, x_p, 0, \ldots, 0) \mid x_i \in \mathbb{R}\}$ of $\mathbb{R}^n$.

Let U is an open rectangle in $\mathbb{R}^n$ such that $U \cap \mathbb{R}^p \neq \emptyset$. We put $\mathcal{H}_U(\mathbb{R}^n, \mathbb{R}^p) = \{f \in \mathcal{H}(\mathbb{R}^n, \mathbb{R}^p) \mid supp(f) \subset U\}$. Let ι: $\mathcal{H}_U(\mathbb{R}^n, \mathbb{R}^p) \to \mathcal{H}(\mathbb{R}^n, \mathbb{R}^p)$ denote the inclusion map, and let $\iota_*: H_r(\mathcal{H}_U(\mathbb{R}^n, \mathbb{R}^p)) \to H_r(\mathcal{H}(\mathbb{R}^n, \mathbb{R}^p))$ denote the induced homomorphism. By the similar argument as in the proof of Lemma 2.2 of [F-I], we have the following lemma.

Lemma 2.1. *ι_* is an isomorphism.*

Theorem 2.2. *If $p > 0$, then the homology groups $H_r(\mathcal{H}(\mathbb{R}^n, \mathbb{R}^p)) = 0$ for $r > 0$.*

Proof. We put $U = (1, 2) \times (-1, 1)^{n-1} \subset \mathbb{R}^n$. Then we note $U \cap \mathbb{R}^p = (1, 2) \times (-1, 1)^{p-1}$. Take a homeomorphism $\phi \in \mathcal{H}(\mathbb{R}^n, \mathbb{R}^p)$ given by $\phi(x) = \frac{1}{3}x$ for $x \in B(0, 3) = \{x = (x_1, \ldots, x_n) \in \mathbb{R}^n \mid (x_1)^2 + \cdots + (x_n)^2 < 9\}$. We set $U_j = \phi^j(U) = (\frac{1}{3^j}, \frac{2}{3^j}) \times (-\frac{1}{3^j}, \frac{1}{3^j})^{n-1}$, $(j = 0, 1, 2, \ldots)$. Note that $U_0 = U$.

Then we have that $\bar{U}_j \cap \bar{U}_k = \emptyset$ if $j \neq k$ and $\{\bar{U}_j\}$ shrinks to the origin $0 \in \mathbb{R}^n$ as j goes to ∞.

The rest is proved by the similar way as in the proof of Theorem 2.1 of [F-I].

Corollary 2.3. *$\mathcal{H}_U(\mathbb{R}^n, \mathbb{R}^p)$ and $\mathcal{H}(\mathbb{R}^n, \mathbb{R}^p)$ are perfect groups for $p > 0$.*

Proof. These are immediate consequences of Theorem 2.2 because that

$H_1(G)\cong G/[G, G]$ for any group G.

§3. Commutators of $\mathscr{H}(M, N)$

A locally flat proper manifold pair (M, N) is by definition a pair of topological manifolds such that N is a locally flat submanifold of M, properly imbedded as a closed subset.

In this section, we show that $\mathscr{H}(M, N)$ is perfect for a locally flat manifold pair (M, N) $(\dim N\geq 0)$. For $\dim N>0$, we have the following as a corollary of Corollary 2.3.

Theorem 3.1. *Let* (M, N) *be a locally flat proper manifold pair. Then* $\mathscr{H}(M, N)$ *is perfect for* $\dim N>0$.

Proof. Let $f\in\mathscr{H}(M, N)$. From the relative version of Corollary 1.3 of [E-K], we have $f=f_k\circ f_{k-1}\circ\cdots\circ f_1$, where each f_i is supported either in an open rectangle U_i with $U_i\cap N=\emptyset$ or in an open rectangle U_i with $U_i\cap N\neq\emptyset$.

Hence we can assume that either $f_i\in\mathscr{H}(\mathbf{R}^n)$ or $f_i\in\mathscr{H}(\mathbf{R}^n, \mathbf{R}^p)$.

From the theorem of Mather [Ma] and Corollary 2.3, we have that f is in the commutator subgroup of $\mathscr{H}(M, N)$. Thus $\mathscr{H}(M, N)$ is perfect. This completes the proof.

Corollary 3.2. *Let M be a topological manifold with boundary ∂M. Then* $\mathscr{H}(M, \partial M)$ *is perfect for* $\dim M>1$.

Let $\mathscr{H}(L\times\mathbf{R}, \text{rel } L\times\{-\infty\})$ be the group of homeomorphisms of $L\times\mathbf{R}$ which are the identity on a neighborhood of $L\times\{-\infty\}$ and are isotopic to the identity, where L is a closed manifold. As an immediate consequence of Theorem 1.2 of D. Mcduff [Mc], we have the following.

Proposition 3.3. $\mathscr{H}(L\times\mathbf{R}, \text{rel } L\times\{-\infty\})$ *is perfect.*

Proposition 3.4. $\mathscr{H}(\mathbf{R}^n, 0)$ *is perfect.*

Proof. Let $f\in\mathscr{H}(\mathbf{R}^n, 0)$. We denote by the same latter f the restriction of f to $\mathbf{R}^n-\{0\}$. Since $\mathbf{R}^n-\{0\}$ is homeomorphic to the product $S^{n-1}\times\mathbf{R}$, we can naturally regard that f is in $\mathscr{H}(S^{n-1}\times\mathbf{R}, \text{rel } S^{n-1}\times\{-\infty\})$.

From Proposition 3.3, there exist g_i, h_i in $\mathscr{H}(S^{n-1}\times\mathbf{R}, \text{rel } S^{n-1}\times\{-\infty\})$ $(i=1,2, \ldots, k)$ such that $f=\Pi_{i=1}^{k}[g_i, h_i]$.

Since $\mathbf{R}^n$ is homeomorphic to the one point compactification of $S^{n-1}\times\mathbf{R}$, $S^{n-1}\times\mathbf{R}/S^{n-1}\times\{\infty\}$, we see that g_i and h_i: $S^{n-1}\times\mathbf{R}\rightarrow S^{n-1}\times\mathbf{R}$ $(i=1,2, \ldots, k)$ can be extended to homeomorphisms of $\mathbf{R}^n$ with compact support by mapping the origin to the origin.

Thus f is in the commutator subgroup of $\mathscr{H}(\mathbb{R}^n, 0)$. This completes the proof.

Remark 3.5. Let $Diff^{\infty}(\mathbb{R}^n, 0)$ be the group of C^{∞}-orientation preserving diffeomorphisms of $(\mathbb{R}^n, 0)$ isotopic to the identity. Then Theorem 1 of [F1] and Corollary 1.3 of [Mc] imply that $H_1(Diff^{\infty}(\mathbb{R}^n, 0)) \cong \mathbb{R}$.

Theorem 3.6. *Let N be a zero dimensional submanifold of M. Then $\mathscr{H}(M, N)$ is perfect.*

Proof. Let $f \in \mathscr{H}(M, N)$. From the relative version of Corollary 1.3 of [E-K], we have $f = f_k \circ f_{k-1} \circ \cdots \circ f_1$, where each f_i is supported either in an open rectangle U_i with $U_i \cap N = \emptyset$ or in an open rectangle U_i with $U_i \cap N = \{one\ point\}$.

Hence we can assume that either $f_i \in \mathscr{H}(\mathbb{R}^n)$ or $f_i \in \mathscr{H}(\mathbb{R}^n, 0)$.

From the theorem of Mather [Ma] and Proposition 3.4, we have that f is in the commutator subgroup of $\mathscr{H}(M, N)$. Thus $\mathscr{H}(M, N)$ is perfect. This completes the proof.

The following is a corollary of Theorem 3.6.

Corollary 3.7 (cf. Lemma 4.4 of [F-I]). *Let $\mathscr{H}([0, 1])$ be the group of orientation preserving homeomorphisms of the interval $[0, 1]$. Then $\mathscr{H}([0, 1])$ is perfect.*

§4. Commutators of $F(M, \mathscr{F})$, Case of Codimension Two

Let M be a compact C^1-manifold without boundary and $\mathscr{F}$ a compact Hausdorff codimension q C^1-foliation of M, where $\mathscr{F}$ is said to be *Hausdorff* if the leaf space $M/\mathscr{F}$ is Hausdorff. Then we have a nice picture of the local behavior of $\mathscr{F}$ as follows.

Proposition 4.1 ([E2]). *There is a generic leaf L_0 with property that there is an open dense subset of M, where the leaves have all trivial holonomy and are all diffeomorphic to L_0. Given a leaf L, we can describe a neighborhood $U(L)$ of L, together with the foliation on the neighborhood as follows. There is a finite subgroup $G(L)$ of $O(q)$ such that $G(L)$ acts freely on L_0 on the right and $L_0/G(L) \cong L$. Let D^q be the unit disk. We foliate $L_0 \times D^q$ with leaves of the form $L_0 \times \{pt\}$. This foliation is preserved by the diagonal action of $G(L)$, defined by $g(x, y) = (x \cdot g^{-1}, g \cdot y)$ for $g \in G(L)$, $x \in L_0$ and $y \in D^q$. So we have a foliation induced on $U = L_0 \times_{G(L)} D^q$. The leaf corresponding to $y = 0$ is $L_0/G(L)$. Then there is a*

C^1-imbedding φ: $U \to M$ with $\varphi(U) = U(L)$, which preserves leaves and $\varphi(L_0/G(L)) = L$.

We consider here compact codimension two foliations. In this case, by the results of D. Epstein [E1] and R. Edwards, K. Millett and D. Sullivan [E-M-S], we have the following: There is an upper bound on the volumes of the leaves of $\mathscr{F}$. So every compact codimension two C^1-foliation is Hausdorff.

Since $G(L)$ is a finite subgroup of $O(2)$, $G(L)$ is either a group of k rotations which is isomorphic to $\mathbf{Z}_k$ or a group of l rotations and l reflections which is isomorphic to $\mathbf{D}_l = \{u, v; u^l = v^2 = (uv)^2 = 1\}$. We regard $\mathbf{Z}_k$ as a fixed group of rotations of D^2. Note that $\mathbf{D}_1$ which is generated by one reflection is isomorphic to $\mathbf{Z}_2$ but it is different from $\mathbf{Z}_2$.

Definition 4.2. A leaf L is *singular* if $G(L)$ is not trivial. The order of $G(L)$ is called the order of holonomy of L. Such an L is called *a rotation leaf*, *a reflection leaf* or *a dihedral leaf* according to whether $G(L)$ is isomorphic to $\mathbf{Z}_k$ $(k>1)$, $\mathbf{D}_1$ or $\mathbf{D}_l\,(l>1)$.

From now on we assume that $\mathscr{F}$ has no dihedral leaves. From Proposition 4.1, there are finitely many rotation leaves in $\mathscr{F}$ because of the compactness of M. Let S be the union of all reflection leaves of $\mathscr{F}$ and $L_1, \dots, L_r$ all rotation leaves of $\mathscr{F}$. We denote by B the leaf space $M/\mathscr{F}$ which is a compact V-manifold of dimension two and the quotient map $p: M \to B$ is a V-bundle (see I. Satake [S] for definitions). B is also a topological manifold. Put $a_i = p(L_i)$ $(i = 1, 2, \dots, r)$ and $\bar{S} = p(S)$. $\bar{S}$ is the boundary of B if S is non-empty. Then we note that p: $M - S \cup L_1 \cup \cdots \cup L_r \to B - \bar{S} \cup \{a_1, \dots, a_r\}$ is a fibration with generic leaf L as fibre.

Theorem 4.3. *Let M be a compact C^1-manifold without boundary and $\mathscr{F}$ a compact codimension two C^1-foliation of M. We assume that $\mathscr{F}$ has no dihedral leaves. Then $F(M, \mathscr{F})$ is perfect.*

Proof. Every foliation preserving homeomorphism $f: M \to M$ induces a homeomorphism $\bar{f}$ of the leaf space B such that the diagram commutes

$$\begin{array}{ccc} M & \xrightarrow{\ f\ } & M \\ {\scriptstyle p}\downarrow & & \downarrow{\scriptstyle p} \\ B & \xrightarrow{\ \bar{f}\ } & B. \end{array}$$

Let $L_1, \dots, L_r$ be all rotation leaves of $\mathscr{F}$ and $a_i = p(L_i)$ $(i = 1, 2, \dots, r)$. In this case we have the natural homomorphism π: $F(M, \mathscr{F}) \to \mathscr{H}(B, \bar{S} \cup \{a_1, \dots, a_r\})$

defined by $\pi(f)=\bar{f}$.

Let $\bar{f}\in\mathscr{H}(B,\bar{S}\cup\{a_1,\dots,a_r\})$. From the relative version of Corollary 1.3 of [E-K], we have $\bar{f}=\bar{f}_k\circ\bar{f}_{k-1}\circ\cdots\circ\bar{f}_1$, where each $\bar{f}_i$ is supported either in a small open rectangle U_i with $U_i\cap(\bar{S}\cup\{a_1,\dots,a_r\})=\emptyset$ or in a small open rectangle U_i with $U_i\cap\bar{S}\neq\emptyset$ and $U_i\cap\{a_1,\dots,a_r\}=\emptyset$ or in a small open rectangle U_i with $U_i\cap\bar{S}=\emptyset$ and $U_i\cap\{a_1,\dots,a_r\}=\{\textit{one point}\}$. Thus each $\bar{f}_i$ can be lifted to f_i in $F(M,\mathscr{F})$. Hence π is onto.

Let $f\in F(M,\mathscr{F})$. We may assume that f is close to the identity. It is clear that $\pi(f)=\bar{f}(\in\mathscr{H}(B,\bar{S}\cup\{a_1,\dots,a_r\}))$ is also close to the identity. From Corollary 3.2 and Theorem 3.6, $\mathscr{H}(B,\bar{S}\cup\{a_1,\dots,a_r\})$ is perfect. Thus $\bar{f}$ is in the commutator subgroup of $\mathscr{H}(B,\bar{S}\cup\{a_1,\dots,a_r\})$, that is, $\bar{f}=\prod_{i=1}^{s}[\bar{g}_i,\bar{h}_i]$ $(\bar{g}_i,\bar{h}_i\in\mathscr{H}(B,\bar{S}\cup\{a_1,\dots,a_r\}))$. Here $\bar{g}_i$ and $\bar{h}_i\,(i=1,2,\dots,k)$ can be supported in small neighborhoods in B. By lifting $\bar{g}_i$ and $\bar{h}_i$ to g_i and h_i in $F(M,\mathscr{F})$, we have $f'=\prod_{i=1}^{s}[g_i,h_i]$. Since $\pi(f\circ(f')^{-1})=id$, $f\circ(f')^{-1}$ is contained in $L(M,\mathscr{F})$, where $L(M,\mathscr{F})$ is the group of leaf preserving homeomorphisms of $(M,\mathscr{F})$ which are isotopic to the identity. From Theorem 3.2 of [F-I], $f\circ(f')^{-1}$ is in the commutator subgroup of $L(M,\mathscr{F})$. Hence f is in the commutator subgroup of $F(M,\mathscr{F})$. Thus $F(M,\mathscr{F})$ is perfect. This completes the proof.

Corollary 4.4. *Let M be a compact C^1-manifold without boundary and $\mathscr{F}$ a compact codimension two C^1-foliation of M. We assume that $\pi_1(L)\cong\mathbb{Z}$ for every leaf L of $\mathscr{F}$. Then $F(M,\mathscr{F})$ is perfect.*

Proof. Take a singular leaf L of $\mathscr{F}$. Then $G(L)$ is isomorphic to a finite cyclic group since $\pi_1(L)\cong Z$ for every leaf L of $\mathscr{F}$. Thus any dihedral leaves do not appear in $\mathscr{F}$ and hence the corollary follows from Theorem 4.3.

Corollary 4.5. *Let M be a compact 4-dimensional C^1-manifold without boundary and $\mathscr{F}$ a C^1-foliation of M by orientable surfaces. If the genus of a generic leaf is even, then $F(M,\mathscr{F})$ is perfect.*

Proof. Take a generic leaf L_0 and a singular leaf L of $\mathscr{F}$. Then we have a regular covering $\pi\colon L_0\to L$ with structure group $G(L)$. Thus if the genus of L_0 is even, then the order of $G(L)$ is not even, hence $G(L)$ can not be a dihedral group. Hence the corollary follows from Theorem 4.3.

§5. Commutators of $F(M,\mathscr{F})$, Case of Codimension Three

In this section we consider the group of foliation preserving homeomorphisms for compact Hausdorff codimension three foliations.

Let $\mathscr{F}$ be a compact Hausdorff codimension three C^1-foliation of a compact manifold M. We assume that $\pi_1(L)\cong\mathbf{Z}$ for every leaf L of $\mathcal{F}$. Then $G(L)$ in Proposition 4.1 is isomorphic to a finite cyclic subgroup of $O(3)$. Then we have the following.

Proposition 5.1 (cf. [F2]). *Each cyclic subgroup G of $O(3)$ is classified as follows*:

(i) *Type I* $(G\subset SO(3))$: *G is isomorphic to $\mathbf{Z}/n\mathbf{Z}$ which is generated by*

$$A=\begin{pmatrix}\cos 2\pi/n & -\sin 2\pi/n & 0\\ \sin 2\pi/n & \cos 2\pi/n & 0\\ 0 & 0 & 1\end{pmatrix}.$$

(ii) *Type II* $(G\not\subset SO(3),\ G\ni J=-E_3)$: *$G$ is isomorphic to $\mathbf{Z}/2\mathbf{Z}$ which is generated by J.*

(iii) *Type III* $(G\not\subset SO(3),\ G\not\ni J)$: *$G$ is isomorphic to $\mathbf{Z}/n\mathbf{Z}$ (n: even) which is generated by JA.*

Definition 5.2. We say such a singular leaf L is of type Ⅰ, type Ⅱ, type Ⅲ_1 or type Ⅲ_2 if $G(L)$ is of Type Ⅰ, Type Ⅱ, Type Ⅲ and $n=2$, or Type Ⅲ and $n\geq 4$.

We can clarify the local structure of the leaf space $M/\mathcal{F}$ using Proposition 5.1 as follows.

Let $U(L)$ be a saturated neighborhood of L as in Proposition 4.1.

(1) In case L is of type Ⅰ, $U(L)/\mathcal{F}$ is homeomorphic to $C\times(-1,1)$, where C is the quotient space of D^2 by a linear action of $\mathbf{Z}/n\mathbf{Z}(\subset SO(2))$. The union of leaves of type Ⅰ corresponds to $\{(0,0)\}\times(-1,1)$. Therefore $U(L)/\mathcal{F}$ is still a topological manifold.

(2) In case L is of type Ⅱ, $U(L)/\mathcal{F}$ is homeomorphic to the suspension of the projective plane P^2.

(3) In case L is of type Ⅲ_1, $U(L)/\mathcal{F}$ is homeomorphic to $D^2\times[0,1)$. The union of leaves of type Ⅲ_1 corresponds to $D^2\times\{0\}$. Therefore the leaves of type Ⅲ_1 correspond to the points of the boundary of $M/\mathcal{F}$.

(4) In case L is of type Ⅲ_2, $U(L)/\mathcal{F}$ is homeomorphic to $C\times[0,1)$, where C is that in (1). The point $\{(0,0)\}\times\{0\}$ corresponds to the leaf of type Ⅲ_2, the points in $\{(0,0)\}\times(0,1)$ correspond to the leaves of type Ⅰ and the points in $(C-(0,0))\times\{0\}$ correspond to the leaves of type Ⅲ_1.

In case L is of type Ⅱ, $U(L)/\mathcal{F}$ is homeomorphic to the one point compactification of the product $P^2\times\mathbf{R}$, $P^2\times\mathbf{R}/P^2\times\{\infty\}$, which is denoted by $S(P^2)$. $S(P^2)$ is not a manifold but has a manifold structure except for the infinity point $P^2\times\{\infty\}=\infty$.

Let $\mathscr{H}(S(P^2), \infty)$ be the group of homeomorphisms of $S(P^2)$ which are isotopic to the identity by an isotopy fixed outside a compact set and leave ∞ fixed.

Proposition 5.3. *$\mathscr{H}(S(P^2),\infty)$ is perfect.*

Proof. Let $f \in \mathscr{H}(S(P^2), \infty)$. We denote by the same letter f the restriction of f to $P^2 \times \mathbb{R}$. By Proposition 3.3, we have

$$f=\prod_{i=1}^{k} [g_i,h_i],\ g_i,h_i \in \mathscr{H}(P^2 \times \mathbb{R}, \text{ rel } P^2 \times \{-\infty\})\ (i=1, 2, \dots, k).$$

Then we see that g_i and h_i: $P^2 \times \mathbb{R} \to P^2 \times \mathbb{R}$ $(i=1, 2, \dots, k)$ can be extended to homeomorphisms of $S(P^2)$ with compact support by mapping the infinity point to the infinity point.

Thus f is in the commutator subgroup of $\mathscr{H}(S(P^2), \infty)$. This completes the proof.

Theorem 5.4. *Let M be a compact C^1-manifold without boundary and $\mathscr{F}$ a compact Hausdorff codimension three C^1-foliation of M with leaf space B. We assume that $\pi_1(L) \cong \mathbb{Z}$ for every leaf L of $\mathscr{F}$ and $\mathscr{F}$ has no leaves of type III_2. Then $F(M, \mathscr{F})$ is perfect.*

Proof. From the assumption, B has a manifold structure except for points corresponding to leaves of type II. Since M is compact, such points are finite. We denote them by $a_1, \dots, a_r$. Let N be the submanifold of B corresponding to the union of leaves of type I and type III_1. Then we have the natural homomorphism π: $F(M, \mathscr{F}) \to \mathscr{H}(B, N \cup \{a_1, \dots, a_r\})$ defined by $\pi(f) = \bar{f}$ as in the proof of Theorem 4.3..

Let $\bar{f} \in \mathscr{H}(B, N \cup \{a_1, \dots, a_r\})$. From the relative version of Corollary 1.3 of [E-K], we have $\bar{f}=\bar{f}_k \circ \bar{f}_{k-1} \circ \cdots \circ \bar{f}_1$, where each $\bar{f}_i$ is supported either in a small open neighborhood U_i with $U_i \cap (N \cup \{a_1,\dots,a_r\}) = \emptyset$ or in a small open neighborhood U_i with $U_i \cap N \neq \emptyset$ and $U_i \cap \{a_1, \dots, a_r\} = \emptyset$ or in a small open neighborhood U_i with $U_i \cap N = \emptyset$ and $U_i \cap \{a_1, \dots, a_r\} = \{\textit{one point}\}$. Thus each $\bar{f}_i$ can be lifted to f_i in $F(M,\mathscr{F})$. Hence π is onto.

Let $f \in F(M,\mathscr{F})$. We may assume that f is close to the identity. It is clear that $\pi(f) = \bar{f}\ (\in \mathscr{H}(B, N \cup \{a_1, \dots, a_r\}))$ is also close to the identity. From Theorem 3.1 and Proposition 5.3, $\mathscr{H}(B, N \cup \{a_1,\dots,a_r\})$ is perfect. Thus $\bar{f}$ is in the commutator subgroup of $\mathscr{H}(B, N \cup \{a_1,\dots,a_r\})$, that is, $\bar{f}=\prod_{i=1}^{s} [\bar{g}_i, \bar{h}_i]$ $(\bar{g}_i, \bar{h}_i \in \mathscr{H}(B, N \cup \{a_1, \dots, a_r\}))$. Here $\bar{g}_i$ and $\bar{h}_i$ $(i=1, 2, \dots, k)$ can be supported in small neighborhoods in B. By lifting $\bar{g}_i$ and $\bar{h}_i$ to g_i and h_i in $F(M,\mathscr{F})$, we have $f'=\prod_{i=1}^{s} [g_i,h_i]$. Since $\pi(f \circ (f')^{-1}) = id$, $f \circ (f')^{-1}$ is contained in $L(M,\mathscr{F})$. From Theorem 3.2 of [F-I], $f \circ (f')^{-1}$ is in the commutator subgroup of $L(M,\mathscr{F})$.

Hence f is in the commutator subgroup of $F(M,\mathscr{F})$. Thus $F(M,\mathscr{F})$ is perfect. This completes the proof.

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