Publ. RIMS, Kyoto Univ.
43 (2007), 521–524

Errata : Spherical Functions of the Principal Series Representation of $Sp(2, \mathbb{R})$ as Hypergeometric Functions of C_2 type (Publl. RIMS, Kyoto Univ. 32 (1996), 689-727)

By

Masatoshi Iida*

Abstract

There are some mistakes in the paper cited in the title. By correcting these mistakes, we find that parameters of the spherical function are rational with respect to parameters of the (generalized principal series) representation.

As an additional remark, we see that the shift operator is the Dunkl operator of A_1-type.

§1. Introduction

This is errata of my paper cited in the title.

At the beginning of this study, I used K-types $(k+1, k)$ and $(l+1, l)$ for spherical functions, but later I changed those to $(k, k-1)$ and $(l, l-1)$ respectively. That confused me when I wrote down the paper. As its consequence, we should correct some equations by replacing k, l by $k-1, l-1$, respectively.

There are other mistakes caused by my carelessness.

I will make a list of corrections of those mistakes in this paper.

§2. The list of corrections

- The cited lemma in Proposition 5.3.

  Though I stated "L_0 is the same as in Lemma 4.5." in the last line of Proposition 5.3, 'Lemma 4.5' should be replaced by 'Lemma 5.2'.

Communicated by M. Kashiwara. Received March 28, 2006. Revised October 3, 2006.
2000 Mathematics Subject Classification(s): Primary 33C67; Secondary 33C20.
*Department of Mathematics, Josai University, Keyakidai Sakado-shi Saitamama Japan.
e-mail: `iida@math.josai.ac.jp`

- The equation (8.3) in Lemma 8.2.

  As was stated above, we should use parameters $k-1, l-1$ instead of k, l, respectively. There also exist mistakes of a sign and a suffix. So the equation (8.3) should be replaced by

(8.3)'

$$\begin{aligned}\Bigg[\sum_{i=1}^{2}\partial_{x_i}^2 &+ \{-2(l-1)\coth 2x_1 + 2k\,\mathrm{sh}^{-1}\,2x_1 + 2\,\mathrm{th}\,x_1 + \coth(x_1+x_2)\\ &+ \coth(x_1-x_2)\}\partial_{x_1}\\ &+ \{-2(l-1)\coth 2x_2 + 2k\,\mathrm{sh}^{-1}\,2x_2\\ &+ \coth(x_1+x_2) - \coth(x_1-x_2)\}\partial_{x_2}\\ &+ \mathrm{th}\,x_1(\coth(x_1+x_2)+\coth(x_1-x_2))\\ &- \mathrm{sh}^{-2}(x_1+x_2) - \mathrm{sh}^{-2}(x_1-x_2) + 2l^2 - 8l + 3\Bigg]\psi_{01}\\ &- \{\mathrm{ch}(x_1+x_2)\,\mathrm{sh}^{-2}(x_1+x_2) + \mathrm{ch}(x_1-x_2)\,\mathrm{sh}^{-2}(x_1-x_2)\}\\ &\cdot \mathrm{ch}^{-1}\,x_1\,\mathrm{ch}\,x_2\psi_{10}\\ &= \{\nu_1^2 + (l-1)^2 - 5\}\psi_{01}.\end{aligned}$$

- The equation (8.4) in Lemma 8.2.

  There exists a mistake of a sign in the equation (8.4). So the correct equation is

$$(8.4)' \quad \left\{\partial_{x_2} + \frac{1}{2}(\coth(x_1+x_2) - \coth(x_1-x_2))\right\}\psi_{01} + \frac{1}{2}\,\mathrm{ch}^{-1}\,x_1\,\mathrm{ch}\,x_2\{\mathrm{sh}^{-1}(x_1+x_2) - \mathrm{sh}^{-1}(x_1-x_2)\}\psi_{10} = 0.$$

- The equation (8.5) in Proposition 8.3.

  The equation (8.5) was obtained from equations (8.3) and (8.4), which have been stated to be incorrect. So the correct version of equation (8.5) is

(8.5)'

$$\begin{aligned}&\left[\sum_{i=1}^{2} y_i(y_i-1)\partial_{y_i}^2 + \left\{(3-l)y_1 - 1 - \frac{k-l}{2} + \frac{y_1(y_1-1)}{y_1-y_2}\right\}\partial_{y_1}\right.\\ &\left.+ \left\{(1-l)y_2 - \frac{k-l}{2} - 3\frac{y_2(y_2-1)}{y_1-y_2}\right\}\partial_{y_2} - \frac{1}{4}\{\nu_1^2 - (l-3)^2\}\right]\psi_{01} = 0.\end{aligned}$$

- The equation in the proof of Proposition 8.3.

  This equation in the proof of Proposition 8.3 was obtained from the incorrect equation (8.4), so we have the correct one from (8.4)' as

$$\left\{\partial_{x_1} + \frac{1}{2}(\coth(x_1+x_2) + \coth(x_1-x_2))\right\}\psi_{10} + \frac{1}{2}\operatorname{ch} x_1 \operatorname{ch}^{-1} x_2 \{\operatorname{sh}^{-1}(x_1+x_2) + \operatorname{sh}^{-1}(x_1-x_2)\}\psi_{01} = 0.$$

- The parameters $\mu_\pm$ in Theorem 8.9.

  The parameters $\mu_\pm$ in the equation (8.9) should be read

$$\mu_\pm = -\frac{l-3\pm\nu_1}{2}.$$

- The parameters A and λ in the proof of Theorem 8.9.

  The parameters A and λ in the proof of Theorem 8.9 should be changed as

$$A = -l+1, \lambda = \frac{\nu_1^2 - (l-3)^2}{4}.$$

- Remark 8.10.

  Remark 8.10 should be corrected as:

  In case $\nu_1 = \mp(l+1)$ (this means $\mu_\pm = 2$), the solution given above is Appell's hypergeometric function

$$F_1(-l+1, 3/2, 1/2, (3+k-l)/2; y_1, y_2).$$

- The condition on the parameter B in Theorem 9.2.

  The condition $B \notin \mathbb{Z}$ should be replaced by

$$B \notin \{0, -1, -2, ...\},$$

  since we only ask $(B)_n$, which appeared in denominators of the Maclaurin coefficients of the functions F_{10} and F_2 in equation (9.8), not to be zero for $\forall n \in \mathbb{N} \cup \{0\}$.

§3. A Remark

I take this opportunity to state an additional remark to Lemma 8.2.

Remark 1. By changing variables as $y_i = -\operatorname{sh} x_i^2$ $(i = 1, 2)$ (same as the original paper), we obtain the differential equation

$$\left(\partial_{y_2} + \frac{1}{2} \cdot \frac{1}{y_2 - y_1}\right) \psi_{01}(y_1, y_2) + \frac{1}{2} \cdot \frac{1}{y_2 - y_1} \psi_{10}(y_1, y_2) = 0$$

from the equation (8.4)' in Lemma 8.2.

From Lemma 4.3 (ii), we see that there is a relationship between ψ_{01} and ψ_{10}:

$$(\sigma\psi_{01})(y_1, y_2) = -\psi_{10}(y_1, y_2),$$

where σ is the generator of the symmetric group S_2 acting on functions of two variables as $(\sigma\psi)(y_1, y_2) = \psi(y_2, y_1)$. Therefore the above equation will be written as the differential-difference equation of ψ_{01}:

$$\left(\partial_{y_2} + \frac{1}{2} \cdot \frac{1}{y_2 - y_1}(1 - \sigma)\right) \psi_{01}(y_1, y_2) = 0.$$

The operator in this equation is the Dunkl operator of A_1 type.

Acknowledgement

The author would like to thank Professor Takayuki Oda for pointing out mistakes on parameters k, l and B of the original paper. He would like to express gratitude to the referee for pointing out some other mistakes. He is also grateful to Professor Hiroyuki Ochiai for his comments on this paper.