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Corrigendum to "Decay of Solutions of Wave-type Pseudo-differential Equations over $p-$adic Fields"

By

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1. The proof of Theorem 4 in [7] contains a gap. This theorem deals with the asymptotic estimation of exponential sums depending on several parameters (or oscillatory integrals depending on several parameters). By using a result due to Cluckers [3, Theorem 6.1], a more general version of Theorem 4 can be proved easily, however, the decay rate obtained is not optimal. With the notation given in [7], the statement and the proof of Theorem 4 should be corrected as follows.

Theorem 4. *Let $\phi(x) \in R_K[x]$, $x = (x_1, \ldots, x_{n-1})$, be a non-constant polynomial such that $C_\phi(K) = \{0\} \subset K^{n-1}$. Let $d_j(\phi)$ be the degree of ϕ with respect the variable x_j, and let $\beta_\phi := \max_j d_j(\phi)$. Let Θ_S be the characteristic function of a compact open set S, let*

$$Y = \{x \in K^n \mid x_n = \phi(x_1, \ldots, x_{n-1})\},$$

and let $d\mu_Y = \Theta_S d\sigma_Y$. Then

$$\left|\widehat{d\mu_Y(\xi)}\right| \leq C \|\xi\|_K^{-\beta}, \tag{1}$$

for $0 \leq \beta \leq \beta_\phi - \epsilon$, with $\epsilon > 0$.

Proof. By passing to a sufficiently fine covering we may suppose that

$$\widehat{d\mu_Y(\xi)} = \int_{(z_0+\pi^{e_0}R_K)^{n-1}} \Psi(-\xi_n\phi(x) - [x, \xi']) \, |dx|.$$

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By applying Theorem 6.1 of [3], we have

$$\left|\widehat{d\mu_Y(\xi)}\right| \leq C\left(\log_q \|\xi\|_K\right)^{n-1} \|\xi\|_K^{-\beta_\phi},$$

and then

$$\left|\widehat{d\mu_Y(\xi)}\right| \leq C\|\xi\|_K^{-\beta}, \text{ for } 0 \leq \beta \leq \beta_\phi - \epsilon,\ \epsilon > 0.$$

It is important to mention that Cluckers' Theorem 6.1 is established only for $\mathbb{Q}_p$, however this result is valid for any p-adic field. Indeed, the proof of this result is based on a result of Chubarikov [2, Lemma 3] whose proof uses inductively an estimation for one-dimensional exponential sums due to I. M. Vinogradov (see e.g. [1, Theorem 2.1]). The proof of this last estimation as given in [1] can be adapted to the case of p-adic fields easily using the notion of dilation as in [8]. □

The Cluckers' result does not give an optimal decay rate, and then β_ϕ is not optimal (see also [9]).

2. The following remark should be added after Theorem 4.

Remark 1. If $\phi(x) = \sum_{i=1}^{n-1} a_i x_i^2$, then the phase of $\widehat{d\mu_Y(\xi)}$ around any critical point has the form $\sum_{i=1}^{n-1} a_i' x_i^2$. By using [7, Theorem 3], one verifies that the decay rate around any critical point is $\frac{n-1}{2}$, therefore Theorem 4 holds for $0 \leq \beta \leq \frac{n-1}{2} = \beta_\phi$. Note that by the principle of stationary phase the contribution of the non-critical points can be neglected (see [7, Theorem 1]). If $n = 1$ and $\phi(x) = x^d$, $d > 1$, the phase of $\widehat{d\mu_Y(\xi)}$ around a critical point can take the form $x^f p(x)$, $2 \leq f \leq d$, $p(x) \neq 0$ locally. By using the fact the real parts of the possible poles of the corresponding local zeta functions have the form $\frac{-1}{f}$, $2 \leq f \leq d$, and Theorem 8.4.2 in [4], one verifies that Theorem 4 holds for $0 \leq \beta \leq \frac{1}{d} = \beta_\phi$.

In the case of real numbers the results described in the previous remark are well-known (see e.g. [6]).

3. The hypothesis "let $\phi(x) \in K[x]$, $x = (x_1, \ldots, x_{n-1})$, *be a non-degenerate polynomial with respect to its Newton polyhedron* $\Gamma(\phi)$" should be replace by "let $\phi(x) \in R_K[x]$, $x = (x_1, \ldots, x_{n-1})$, *be a non-constant polynomial such that* $C_\phi(K) = \{0\} \subset K^{n-1}$" in the Theorems 5 and 6. The proofs do not need any modification. The new versions are as follows.

Theorem 5. *Let* $\phi(x) \in R_K[x]$, $x = (x_1, \ldots, x_{n-1})$, *be a non-constant polynomial such that* $C_\phi(K) = \{0\} \subset K^{n-1}$. *Let*

$$Y = \{x \in K^n \mid x_n = \phi(x_1, \ldots, x_{n-1})\}$$

with the measure $d\mu_{Y,S} = \Theta_S d\sigma_Y$, *where* Θ_S *is the characteristic function of a compact open subset* S *of* K^n. *Then*

$$\left(\int\limits_{Y} |\mathcal{F}g(\xi)|_K^2 \, d\mu_Y(\xi) \right)^{\frac{1}{2}} \leq C(Y) \|g\|_{L^\rho},$$

holds for each $1 \leq \rho < \frac{2(1+\beta_\phi)}{2+\beta_\phi}$.

Theorem 6 (Main Result). *Let* $\phi(\xi) \in R_K[\xi]$, $\xi = (\xi_1, \ldots, \xi_n)$, *be a non-constant polynomial such that* $C_\phi(K) = \{0\} \subset K^n$. *Let*

$$(H\Phi)(t,x) = \mathcal{F}^{-1}_{(\tau,\xi)\to(t,x)} \left(|\tau - \phi(\xi)|_K \mathcal{F}_{(t,x)\to(\tau,\xi)} \Phi \right), \Phi \in \mathbb{S}(K^{n+1}),$$

be a pseudo-differential operator with symbol $|\tau - \phi(\xi)|_K$. *Let* $u(x,t)$ *be the solution of the following initial value problem*:

$$\begin{cases} (Hu)(x,t) = 0, \ x \in K^n, \ t \in K, \\ u(x,0) = f_0(x), \end{cases}$$

where $f_0(x) \in \mathbb{S}(K^n)$. *Then*

$$\|u(x,t)\|_{L^\sigma(K^{n+1})} \leq A \|f_0(x)\|_{L^2(K^n)},$$

for $\frac{2(1+\beta_\phi)}{\beta_\phi} < \sigma \leq \infty$.

4. The last subsection (Wave-type Equations with Quasi-homogeneous Symbols) should be rewritten as follows.

§ 5.2 Wave-type Equations with Homogeneous Symbols

In the cases $\phi(\xi) = a_1\xi_1^2 + \cdots + a_n\xi_n^2$ and $n = 1$, $\phi(\xi) = \xi^d$ by using Remark 1, we have the following estimations for the solution of Cauchy problem (1).

Theorem 7. *If* $\phi(\xi) = a_1\xi_1^2 + \cdots + a_n\xi_n^2$, *then*

$$\|u(x,t)\|_{L^{\frac{2(2+n)}{n}}(K^{n+1})} \leq C \|f_0(x)\|_{L^2(K^n)}.$$

Theorem 8. *If* $n = 1$ *and* $\phi(\xi) = \xi^d$, *then*

$$\|u(x,t)\|_{L^{2(d+1)}(K^2)} \leq C \|f_0(x)\|_{L^2(K)}.$$

In particular if $d = 3$, *then*

$$\|u(x,t)\|_{L^8(K^2)} \le C \|f_0(x)\|_{L^2(K)}.$$

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