Publ. RIMS, Kyoto Univ.
7 (1971/72), 95-104

Boundary Values of Hyperfunction Solutions of Linear Partial Differential Equations

By

Hikosaburo KOMATSU* and Takahiro KAWAI

Let $P(x, D)$ be a linear partial differential operator with real analytic coefficients in a domain V in $\mathbf{R}^{n+1}$ and let $S \subset V$ be a real analytic hypersurface non-characteristic with respect to $P(x, D)$. The purpose of this paper is to show that every hyperfunction solution u of $P(x, D)u=0$ on one side of $V \setminus S$ has boundary values on S which are hyperfunctions of n variables on S.

This fact has been proved by H. Komatsu [6] and P. Schapira [8] in the case where $P(x, D)$ is elliptic. Their method applies with minor modifications to the general operators.

In §1 we show that the Cauchy-Kowalevsky theorem for the dual equation with the initial values on S is equivalent to a theorem of division of hyperfunctions with supports in S by the differential operator $P(x, D)$.

We define the boundary values in §2 and prove the uniqueness of hyperfunction solutions of the Cauchy problems.

1. Division of Hyperfunctions with Supports in S

Let $P(x, D)$ be a linear differential operator of order m with real analytic coefficients defined on a domain V in $\mathbf{R}^{n+1}$ and let S be an oriented real analytic hypersurface in V non-characteristic with respect to $P(x, D)$.

We denote by $\mathscr{A}$ and $\mathscr{B}$ ($'\mathscr{A}$ and $'\mathscr{B}$) the sheaf of real analytic func-

Received December 15, 1970.
* Department of Mathematics, Faculty of Science, University of Tokyo, Hongo, Bunkyo-ku, Tokyo, Japan. Supported in part by the Matsunaga Science Foundation.

tions and that of hyperfunctions on V (on S respectively). When K is a compact set in V (in S), the space $\mathscr{A}(K)$ ($'\mathscr{A}(K)$) has a natural (DFS)-topology and its dual is identified with the space $\mathscr{B}_K(V)$ ($'\mathscr{B}_K(S)$) of hyperfunctions with supports in K under the inner product

$$<\varphi, f>=\int_V \varphi(x)f(x)dx, \quad \varphi \in \mathscr{A}(K), \ f \in \mathscr{B}_K(V)$$

$$\left(<\varphi, f>=\int_S \varphi(x')f(x')d\omega, \quad \varphi \in '\mathscr{A}(K), \ f \in '\mathscr{B}_K(S)\right),$$

where dx ($d\omega$) denotes the Lebesgue measure on V (on S).

Let $P'(x, D)$ be the formal dual of $P(x, D)$. Then, $P(x, D)$ and $P'(x, D)$ induce sheaf homomorphisms $P(x, D)$: $\mathscr{B} \to \mathscr{B}$ and $P'(x, D)$: $\mathscr{A} \to \mathscr{A}$ respectively. We denote by $\mathscr{B}^P$ and $\mathscr{A}^{P'}$ the kernel sheaves, i.e., the sheaf of solutions of

$$P(x, D)f=0, \quad f \in \mathscr{B}, \tag{1}$$

and that of solutions of

$$P'(x, D)\varphi=0, \quad \varphi \in \mathscr{A} \tag{2}$$

respectively.

Theorem 1. *Let K be a compact set in S. Then, there is no non-trivial solution of* (1) *over V with support in K*:

$$\mathscr{B}_K^P(V)=0. \tag{3}$$

The quotient space $\mathscr{B}_K(V)/P\mathscr{B}_K(V)$ is identified with the dual of the (DFS)-space $\mathscr{A}^{P'}(K)$.

Proof. Consider the complexes:

$$0 \to \mathscr{A}(K) \xrightarrow{P'(x,D)} \mathscr{A}(K) \to \tag{4}$$

$$\qquad \updownarrow \qquad\qquad \updownarrow$$

$$0 \leftarrow \mathscr{B}_K(V) \xleftarrow{P(x,D)} \mathscr{B}_K(V) \leftarrow 0, \tag{5}$$

which are dual to each other in the sense that $\mathscr{A}(K)$ and $\mathscr{B}_K(V)$ with their natural (DFS)-and (FS)-topologies are the strong dual spaces of each other and that $P'(x, D)$ and $P(x, D)$ are continuous linear operators dual to each other.

The 0-th cohomology group of (4) is $\mathscr{A}^{P'}(K)$ and the 1-st cohomology group of (4) vanishes by the Cauchy-Kowalevsky theorem. In particular, $P'(x, D)$ has a closed range. Thus it follows from Serre's lemma (see e.g. [5] Theorem 19) that $P(x, D)$ has a closed range and that the cohomology groups of (4) and (5) are the strong dual spaces of each other. Therefore, $\mathscr{B}_K(V)/P\mathscr{B}_K(V)$ is the dual of $\mathscr{A}^{P'}(K)$ and ker $P(x, D)=\mathscr{B}_K^P(V)$ vanishes.

Let $C_j(x, D)$, $j=1, 2,\dots, m$, be linear differential operators of order $m-j$ with real analytic coefficients on a neighborhood of S for which S is ron-characteristic (e.g. $C_j(x, D)=(\partial/\partial n)^{m-j}$). Then the Cauchy-Kowalevsky theorem yields the topological isomorphism

$$\rho:\ \mathscr{A}^{P'}(K)\approx{}'\mathscr{A}(K)^m \tag{6}$$

defined by

$$\rho(\varphi)=(C_j(x, D)\varphi|_S),\ \varphi\in\mathscr{A}^{P'}(K). \tag{7}$$

we have, therefore, the dual isomorphism

$$\rho':\ {}'\mathscr{B}_K(S)^m\approx\mathscr{B}_K(V)/P\mathscr{B}_K(V). \tag{8}$$

Obviously ρ can be extended by (7) to a continuous linear operator $\tilde{\rho}:\ \mathscr{A}(K)\to{}'\mathscr{A}(K)^m$. Since the open mapping theorem holds for (DFS)-spaces, the exact sequence

$$0\to{}'\mathscr{A}(K)^m \xrightarrow{\rho^{-1}}\mathscr{A}(K) \xrightarrow{P'(x,D)}\mathscr{A}(K)\to 0 \tag{9}$$

splits topologically and we have the topological isomorphism:

$$\mathscr{A}(K)\approx{}'\mathscr{A}(K)^m\oplus\mathscr{A}(K) \tag{10}$$

defined by

$$\varphi\mapsto(C_j(x, D)\varphi|_S)\oplus P'(x, D)\varphi. \tag{11}$$

Correspondingly the dual exact sequence

$$0\leftarrow{}'\mathscr{B}_K(S)^m\xleftarrow{(\rho^{-1})'}\mathscr{B}_K(V)\xleftarrow{P(x,D)}\mathscr{B}_K(V)\leftarrow 0 \tag{12}$$

splits topologically.

Since $\tilde{\rho}$ is the composite of the differential operators $(C_j(x, D))$ and the restriction to S, the dual $\tilde{\rho}'$: $'\mathscr{B}_K(S)^m \to \mathscr{B}_K(V)$ is the mapping $(f_j) \mapsto \sum_{j=1}^{m} C_j'(x, D)(f_j \otimes \delta_S)$, where $C_j'(x, D)$ is the formal dual of $C_j(x, D)$ and $f_j \otimes \delta_S$ is the hyperfunction on V defined by

$$(13) \qquad < f_j \otimes \delta_S, \varphi > = \int_S f_j(x')\varphi(x')d\omega, \ \varphi \in \mathscr{A}(K).$$

Consequently, each $f \in \mathscr{B}_K(V)$ is uniquely decomposed as

$$(14) \qquad f = \sum_{j=1}^{m} C_j'(x, D)(f_j \otimes \delta_S) + P(x, D)g,$$

where $f_j \in {}'\mathscr{B}_K(S)$ and $g \in \mathscr{B}_K(V)$. Under this correspondence we have a topological isomorphism

$$(15) \qquad \mathscr{B}_K(V) \approx {}'\mathscr{B}_K(S)^m \oplus \mathscr{B}_K(V).$$

In particular, the inverse $(\rho')^{-1}$: $\mathscr{B}_K(V)/P\mathscr{B}_K(V) \approx {}'\mathscr{B}_K(S)^m$ of isomorphism (8) is the mapping which takes the class of f to (f_j) in the decomposition (14). Obviously f_j depend on the choice of $C_j(x, D)$. However, the sum $\sum C_j'(x, D)(f_j \otimes \delta_S)$ and $P(x, D)g$ do not depend on $C_j(x, D)$ because neither $\operatorname{im} \rho^{-1} = \mathscr{A}^{P'}(K)$ nor $\ker \tilde{\rho}$ depends on $C_j(x, D)$.

The uniqueness of the decomposition shows that the components f_j and g are independent of the compact set K which contains the support of f. Namely we have an isomorphism

$$(16) \qquad \Gamma_*(\mathscr{H}_S^0(\mathscr{B})|_S, S) \approx \Gamma_*('\mathscr{B}, S)^m \oplus \Gamma_*(\mathscr{H}_S^0(\mathscr{B})|_S, S)$$

which preserves the support, where Γ_* denotes the space of sections with compact supports and $\mathscr{H}_S^0(\mathscr{B})|_S$ the restriction to S of the sheaf of sections of $\mathscr{B}$ with supports in S.

Let us denote $\mathscr{H}_S^0(\mathscr{B})|_S$ by $\mathscr{B}_S$ for short. Since $\mathscr{B}_S$ and $'\mathscr{B}$ are flabby, it follows that the isomorphism is extended to a sheaf isomorphism (see e.g. [4] Lemma 2.3). Thus we have proved the following theorem.

Theorem 2. *If $C_j'(x, D)$, $j=1, \cdots, m$, are linear differential operators of order $m-j$ with real analytic coefficients on a neighborhood of S for which S is non-characteristic, then we have a sheaf isomorphism*

(17) $$\mathscr{B}_S \approx {}'\mathscr{B}^m \oplus \mathscr{B}_S$$

defined by

(18) $$f = \sum_{j=1}^{m} C'_j(x, D)(f_j \otimes \delta_S) + P(x, D)g,$$

where $f \in \mathscr{B}_S$, $f_j \in {}'\mathscr{B}$ *and* $g \in \mathscr{B}_S$. *The last component* g *does not depend on the choice of* $C'_j(x, D)$.

In particular, there is no non-trivial solution $g \in \mathscr{B}^P(V)$ *with support in* S:

(19) $$\mathscr{B}^P_S(V) = 0.$$

This theorem means that on division by $P(x, D)$ each $f \in \mathscr{B}_S$ has a unique quotient $g \in \mathscr{B}_S$ and a remainder $\sum C'_j(x, D)(f_j \otimes \delta_S)$ with $f_j \in {}'\mathscr{B}$. We have derived this from the Cauchy-Kowalevsky theorem via the duality of $\mathscr{A}(K)$ and $\mathscr{B}_K(V)$ and that of ${}'\mathscr{A}(K)$ and ${}'\mathscr{B}_K(S)$. Conversely Theorem 2 implies the exactness of (12) and hence that of (9). Thus Theorem 2 of division is equivalent to the Cauchy-Kowalevsky theorem.

2. Boundary Values of Hyperfunction Solutions

Let W be an open subset of V. We have the following commutative diagram:

(20) $$\begin{array}{ccccccccc} & & & & 0 & & 0 & & \\ & & & & \downarrow & & \downarrow & & \\ & & 0 & \longrightarrow & \mathscr{B}^P(W) & \longrightarrow & \mathscr{B}^P(W \setminus S) & \longrightarrow & 0 \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 0 & \rightarrow & \mathscr{B}_{S\cap W}(W) & \longrightarrow & \mathscr{B}(W) & \longrightarrow & \mathscr{B}(W \setminus S) & \longrightarrow & 0 \\ & & \downarrow {\scriptstyle P(x,D)} & & \downarrow {\scriptstyle P(x,D)} & & \downarrow {\scriptstyle P(x,D)} & & \\ 0 & \rightarrow & \mathscr{B}_{S\cap W}(W) & \longrightarrow & \mathscr{B}(W) & \longrightarrow & \mathscr{B}(W \setminus S) & \longrightarrow & 0, \\ & & \downarrow & & & & & & \\ & & 0 & & & & & & \end{array}$$

where $\mathscr{B}_{S\cap W}(W)$ denotes the space of hyperfunctions on W with supports in $S \cap W$. Since $\mathscr{B}$ is flabby, the last two rows are exact; the last two columns are exact by the definition; the 0-th cohomology group of the first row and that of the first column vanish since there is no non-trivial solution with support in $S \cap W$.

For the remaining cohomology groups we have a natural homomorphism

$$b:\ \mathscr{B}^P(W\setminus S)/\mathscr{B}^P(W)\to\mathscr{B}_{S\cap W}(W)/P\mathscr{B}_{S\cap W}(W).$$

Let $u\in\mathscr{B}^P(W\setminus S)$ and let $\tilde{u}$ be an extension in $\mathscr{B}(W)$. Since $P(x, D)\tilde{u}=0$ on $W\setminus S$, $P\tilde{u}$ belongs to $\mathscr{B}_{S\cap W}(W)$. If $\tilde{u}_1$ is another extension of u, $\tilde{u}-\tilde{u}_1$ belongs to $\mathscr{B}_{S\cap W}(W)$. Therefore the class of $P\tilde{u}$ in $\mathscr{B}_{S\cap W}(W)/P\mathscr{B}_{S\cap W}(W)$ is determined uniquely by u. If u is the restriction to $W\setminus S$ of a $\tilde{u}\in\mathscr{B}^P(W)$, we have $P\tilde{u}=0$. Thus we can define a homomorphism b which assigns for the class of $u\in\mathscr{B}^P(W\setminus S)$ the class of $P\tilde{u}\in\mathscr{B}_{S\cap W}(W)$.

Theorem 3. *The homomorphism*

$$b:\ \mathscr{B}^P(W\setminus S)/\mathscr{B}^P(W)\to\mathscr{B}_{S\cap W}(W)/P\mathscr{B}_{S\cap W}(W) \tag{21}$$

is injective for any open set W in V and commutes with restrictions. b is surjective if and only if

$$P(x, D)\mathscr{B}(W)\supset\mathscr{B}_{S\cap W}(W). \tag{22}$$

Proof. By the definition it is clear that b commutes with restrictions. To prove the injectivity, let $P\tilde{u}=Pu_1$ for a $u_1\in\mathscr{B}_{S\cap W}(W)$. Since $\tilde{u}-u_1\in\mathscr{B}^P(W)$ and its restriction to $W\setminus S$ is equal to u, the class of u is zero.

Let b be surjective. Then, for each $g\in\mathscr{B}_{S\cap W}(W)$ there exist $h\in\mathscr{B}_{S\cap W}(W)$ and $\tilde{u}\in\mathscr{B}(W)$ such that $g+Ph=P\tilde{u}$. Thus $\mathscr{B}_{S\cap W}(W)\subset P\mathscr{B}(W)$.

Conversely suppose that for each $g\in\mathscr{B}_{S\cap W}(W)$ there is a $\tilde{u}\in\mathscr{B}(W)$ such that $g=P\tilde{u}$. Then, the restriction u of $\tilde{u}$ belongs to $\mathscr{B}^P(W\setminus S)$. Therefore b is surjective.

It is known that (22) holds if the coefficients of $P(x, D)$ are constants or if $P(x, D)$ is elliptic.

Now, let ω be an open set of S and let $W\supset W'$ be two open sets in V with $S\cap W=S\cap W'=\omega$. The restriction $\mathscr{B}^P(W\setminus S)\to\mathscr{B}^P(W'\setminus S)$ induces a homomorphism

$$r:\ \mathscr{B}^P(W\setminus S)/\mathscr{B}^P(W)\to\mathscr{B}^P(W'\setminus S)/\mathscr{B}^P(W'). \tag{23}$$

Since $\mathscr{B}_{S\cap W}(W)/P\mathscr{B}_{S\cap W}(W)=\mathscr{B}_{S\cap W'}(W')/P\mathscr{B}_{S\cap W'}(W')$ and since the injections b_W and $b_{W'}$ commute with r, it follows that r is injective.

r is surjective if and only if $\mathscr{B}^P(W'\setminus S)=\mathscr{B}^P(W\setminus S)|_{W'\setminus S}+\mathscr{B}^P(W')|_{W'\setminus S}$ and this holds if

$$H^1(W, \mathscr{B}^P)=0 \tag{24}$$

by the Mayer-Vietoris theorem.

It is also known that (24) holds for any open set W if the coefficients of $P(x, D)$ are constants or if $P(x, D)$ is elliptic.

Taking the inductive limit with respect to the open neighborhoods of ω, we have the injection

$$b: \quad (\mathscr{B}^P_+(\omega)\oplus\mathscr{B}^P_-(\omega))/\mathscr{B}^P(\omega)\to\mathscr{B}_S(\omega)/P\mathscr{B}_S(\omega), \tag{25}$$

where $\mathscr{B}^P_+(\omega)$ $(\mathscr{B}^P_-(\omega))$ denotes the space of germs of solutions on $W\setminus S$ which vanish on the negative (positive) side of S. $\mathscr{B}^P_\pm$ are sheaves over S which describe the boundary behavior of solutions outside S.

It follows from Theorem 3 that b in (25) is surjective if and only if

$$P(x, D)\mathscr{B}(\omega)\supset\mathscr{B}_S(\omega). \tag{26}$$

Furthermore, noticing that the sheaf associated with the presheaf $(\mathscr{B}^P_+(\omega)\oplus\mathscr{B}^P_-(\omega))/\mathscr{B}^P(\omega)$ is the restriction $\mathscr{H}^1_S(\mathscr{B}^P)|_S$ to S of the first derived sheaf with support in S (see [4]), we have the injection

$$b: \quad \mathscr{H}^1_S(\mathscr{B}^P)|_S\to\mathscr{B}_S/P\mathscr{B}_S \tag{27}$$

which is surjective if and only if

$$P(x, D)\mathscr{B}|_S\supset\mathscr{B}_S. \tag{28}$$

Obviously (28) is satisfied if $P(x, D)$ is locally solvable on S, i.e. if

$$P(x, D): \quad \mathscr{B}(x)\to\mathscr{B}(x) \text{ is surjective for } x\in S. \tag{29}$$

This is known for operators with constant coefficients or of elliptic type. Moreover, T. Kawai [3] proves the existence of local elementary solutions and hence the local solvability of operators $P(x, D)$ of simple characteristics with real principal parts. Thus (27) is an isomorphism for such operators. Combining this with the isomorphism $\mathscr{B}_S/P\mathscr{B}_S\approx'\mathscr{B}^m$

given in Theorem 2, we have an isomorphism

$$\mathscr{H}_S^1(\mathscr{B}^P)|_S \approx {}'\mathscr{B}^m. \tag{30}$$

Definition. Let W be an open set in V, let $\omega = S \cap W$ and let W_+ be the positive part of $W \setminus S$. For each solution $u \in \mathscr{B}^P(W_+)$ we define its *boundary values* $(f_j) \in {}'\mathscr{B}(\omega)^m$ on S to be the image of u under the composite of mappings $\mathscr{B}^P(W_+) \to \mathscr{B}_+^P(\omega) \to (\mathscr{B}_+^P(\omega) \oplus \mathscr{B}_-^P(\omega))/\mathscr{B}^P(\omega) \xrightarrow{b} \mathscr{B}_S(\omega)/P\mathscr{B}_S(\omega) \to {}'\mathscr{B}(\omega)^m$, where the last mapping is the isomorphism obtained in Theorem 2 as an extension of $(\rho')^{-1}$ in (8). In other words, $(f_j) \in {}'\mathscr{B}(\omega)^m$ is the unique m-tuple of hyperfunctions on ω which satisfy

$$P(x, D)\tilde{u} = \sum_{j=1}^{m} C_j'(x, D)(f_j \otimes \delta_S) \tag{31}$$

for an extension $\tilde{u} \in \mathscr{B}(W)$ vanishing on the negative side of $W \setminus S$.

As we remarked earlier, the extension $\tilde{u}$ which satisfies (31) is uniquely determined by u and does not depend on the choice of $C_j(x, D)$, so that we call $\tilde{u}$ the *canonical extension* of u.

Let θ_S be the characteristic function of W_+ in W. Then, there are unique linear differential operators $B_j(x, D)$, $j = 1, \dots, m$, of order $j-1$ with real analytic coefficients in a neighborhood of S such that S is non-characteristic and that

$$\begin{aligned} &P(x, D)(\theta_S(x)u(x)) - \theta_S(x)(P(x, D)u(x)) \\ &= \sum_{j=1}^{m} C_j'(x, D)(B_j(x, D)u(x)(1 \otimes \delta_S)) \\ &= \sum_{j=1}^{m} C_j'(x, D)((B_j(x, D)u(x))|_S \otimes \delta_S) \end{aligned} \tag{32}$$

for any $u \in \mathscr{A}(W)$ or more generally for any $u \in \mathscr{B}(W)$ which is real analytic in the normal direction on S (see [1] for the real analyticity in parameter and the restrictions of hyperfunctions to submanifolds).

Conversely if $B_j(x, D)$, $j = 1, \dots, m$, are linear differential operators of order $j-1$ with real analytic coefficients for which S is non-characteristic, we can find linear differential operators $C_j(x, D)$ of order $m-j$ such that S is non-characteristic and that (32) holds. This is only a local formulation of Green's formula.

Therefore, if u is the restriction to W_+ of a solution $u_1 \in \mathscr{A}^P(W)$ we have

(33) $$f_j = B_j(x, D)u_1|_S, \ j=1,\cdots, m.$$

This holds also for the restriction u of a solution $u_1 \in \mathscr{B}^P(W)$, because u_1 is real analytic in the normal direction on S by Sato's fundamental theorem of analyticity (see [1]).

Taking this into account we will write the boundary values

(34) $$f_j = B_j(x, D)u|_{S_+}, \quad j=1,\cdots, m.$$

Similarly we can define the boundary values $B_j(x, D)u|_{S_-}$ of solutions u on the negative side of $W \setminus S$. The following is clear from the definition.

Theorem 4. *A solution $u \in \mathscr{B}^P(W \setminus S)$ is extended to a solution $u \in \mathscr{B}^P(W)$ if and only if*

(35) $$B_j(x, D)u|_{S_+} = B_j(x, D)u|_{S_-}, \ j=1,\cdots, m.$$

This may be regarded as a generalization of the classical Painlevé theorem.

If the operator $P(x, D)$ is locally solvable on S or if (28) holds, then the isomorphism (30) shows that the Plemelj problem

(36) $$B_j(x, D)u|_{S_+} - B_j(x, D)u|_{S_-} = f_j, \ j=1,\cdots, m$$

has a local solution $u \in \mathscr{B}^P_+(x) \oplus \mathscr{B}^P_-(x)$ for any $f_j \in {}'\mathscr{B}(x)$ on S.

Lastly the Holmgren theorem by T. Kawai [2] and P. Schapira [7] asserts that

(37) $$\mathscr{B}^P_+(\omega) \cap \mathscr{B}^P(\omega) = \{0\} \ \text{and} \ \mathscr{B}^P_-(\omega) \cap \mathscr{B}^P(\omega) = \{0\}.$$

Therefore the mapping $\mathscr{B}^P_+(\omega) \to {}'\mathscr{B}(\omega)^m$ is injective. Thus we have

Theorem 5. *A solution $u \in \mathscr{B}^P(W_+)$ on the positive side of $W \setminus S$ vanishes in a neighborhood of $\omega = W \cap S$ if and only if the boundary values $B_j(x, D)u|_S$ vanish for all $j = 1,\cdots, m$.*

References

[1] Kashiwara, M., On the structure of hyperfunctions (following M. Sato), *Sûgaku no Ayumi*, **15** (1970), 9-72 (Japanese).

[2] Kawai, T., The theory of Fourier transforms in the theory of hyperfunctions and its applications, Master's thesis, Univ. of Tokyo, 1970, partly reprinted in *Surikaiseki Kenkyûsho Kôkyûroku*, No. 108 Kyoto Univ. pp. 84-288 (Japanese).

[3] ———, Construction of a local elementary solution for linear partial differential operators (I), *Proc. Japan Acad.* **47** (1971), 19-23.

[4] Komatsu, H., Relative cohomology of sheaves of solutions of differential equations, *Séminaire Lions-Schwartz*, 1966-67, reprinted in *Symposium on Algebraic Geometry Expecting Applications to Hyperfunctions*, Katata, 1969.

[5] ———, Projective and injective limits of weakly compact sequences of locally convex spaces, *J. Math. Soc. Japan*, **19** (1967), 366-383.

[6] ———, Boundary values for solutions of elliptic equations, *Proc. Intern. Conf. Functional Analysis and Related Topics*, 1969, Univ. Tokyo Press, 1970, pp. 107-121.

[7] Schapira, P., Théorème d'unicité de Holmgren et opérateurs hyperboliques dans l'espace de hyperfonctions, to appear in *Anais Acad. Brasil. Sci.*

[8] ———, Hyperfonctions et problèmes aux limites elliptiques, to appear in *Ann. Inst. Fourier*.