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A Gauge Theory for the Kadomtsev-Petviashvili System

By

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Abstract

A Lagrangian formalism of scalar fields is considered and a new concept of "connection" is introduced. By this a gauge-theoretic understanding of the Sato theory on the K.-P. system is obtained. Our gauge group $\tilde{G}_-$ is the group consisting of pseudo-differential operators of non-positive orders with certain growth conditions. Then it can be concluded that the space R^* of elements of $\tilde{G}_-$ giving solutions of the K.-P. system defines a flat R^*-connection which we call the K.-P. connection. This connection can be regarded as a special gauge field.

Introduction

It is well known that various soliton equations can be obtained by using the theory of isospectral deformations of linear differential operators. A remarkable unification of soliton equations has been established by M. and Y. Sato [5] in terms of isospectral deformations of $D=d/dx$ in the category of pseudo-differential operators. This unified system of equations is called the Kadomtsev-Petviashvili system (=K.-P. system). They discovered the surprising fact: The space of solutions of the K.-P. system makes the Grassmann manifold of infinite dimension and moreover, any solution of the K.-P. system can be reduced to that of a system of certain linear equations. Several attempts of understandings on the Sato theory and its generalizations have been presented. Some of them are the method of Riemann-Hilbert transforms [10], the method of group-decompositions [4], [7] and the field-theoretic method [1]. The co-adjoint orbit method for the K.-P. system is given by

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using groups of pseudo-differential operators [11]. Out attempt which we present here is a new one, which we call a gauge-theoretic understanding. Although the method in [11] is based on the notion of Hamiltonians rather than connections, the result obtained there is in close relation to our discussion.

In this paper, we see that the K.-P. system can be understood in the view point of Uchiyama's gauge theory [9]. We note that our gauge group is an infinite dimensional Lie group. Hence our gauge theory for soliton equations is contrasted with that of Yang-Mills equations and nonlinear Heisenberg equation in dimensions of their gauge groups [3]. First, we consider the Lagrangian action:

$$\mathcal{L}=\int_{\boldsymbol{R}}\bar{\psi}D\psi dx \qquad (D=d/dx)$$

for scalar fields ψ, $\bar{\psi}$, i.e., wave functions on the real line $\boldsymbol{R}$. We analyse the symmetry of $\mathcal{L}$ and obtain as the gauge group of the first kind a group consisting of invertible pseudo-differential operators with constant coefficients of the form:

$$\cdots+c_nD^n+\cdots+c_1D+c_0+c_{-1}D^{-1}+\cdots+c_{-n}D^{-n}+\cdots.$$

Secondly, we apply the Uchiyama's gauge theory to our Lagrangian formalism. In this case, the gauge group of the second kind becomes a group consisting of invertible pseudo-differential operators with function coefficients of the form:

$$\cdots+u_n(x)D^n+\cdots+u_1(x)D+u_0(x)+u_{-1}(x)D^{-1}+\cdots+u_{-n}(x)D^{-n}+\cdots.$$

Then in order to obtain a new Lagrangian action which is invariant under this group, a connection, i.e., gauge field, necessarily arises in our consideration. It has a worth mentioning that pseudo-differential operators with negative orders, extended from usual differential operators, may be introduced as elements of the gauge group of the first or the second kind.

In Section 1, from a gauge group of pseudo-differential operators we introduce a new concept of "connection". Here we have to pay attention to the fact that our connection has been defined not only for a subgroup but also for a special subset R of the gauge group, although R does not admit a structure of subgroup. We prove that the decomposition law of pseudo-differential operators into the parts of non-negative and negative orders gives rise to the flat connection (Theorem 1). This is our first step to a gauge-theoretic understanding on the K.-P. system. In Section 2, we shall treat the Lagrangian action of scalar fields ψ, $\bar{\psi}$ with infinitely many parameters

$t=(t_1, t_2, \cdots)$:

$$\mathcal{L}_t = \int_{R} \bar{\psi} d\psi dx\,, \quad d = \sum_{n=1}^{\infty} (\partial/\partial t_n) dt_n\,.$$

For this Lagrangian action we consider the gauge groups $\tilde{G}_0$, $\tilde{G}$ of the first and the second kind, and then $\tilde{G}$-connections. Then we can conclude that the space R^* of elements of $\tilde{G}$ giving solutions of the K.-P. system defines the flat R^*-connection which we call the K.-P. connection (Theorem 2).

Our discussions show that the space of solutions of soliton equations determines a special gauge field. Hence, we may expect to extend our discussions to the Yang-Mills equation and nonlinear Heisenberg equation by a gauge-theoretic version of the Sato theory on the Minkowski space-time [3].

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§ 1. A Lagrangian Formalism and R-connections

We consider complex valued functions defined on the real line and a collection of pseudo-differential operators. A pseudo-differential operator is called an operator simply. Let ψ and $\bar{\psi}$ denote two functions. Here $\bar{\psi}$ may *not* be the complex conjugate of ψ.

First, we deal with a Lagrangian action for ψ and $\bar{\psi}$ given by

$$\int_{R} \bar{\psi} D\psi dx\,, \quad D = d/dx\,. \tag{1.1}$$

For a function ψ and an operator $\bar{\psi}$ $(=\bar{\psi}\circ 1)$, identified with the function $\bar{\psi}$, we act an operator W on the pair as

$$\psi \to \psi' = W\psi\,, \quad \bar{\psi} \to \bar{\psi}' = \bar{\psi} W^{-1}\,. \tag{1.2}$$

Under this action the function $\bar{\psi}\psi$ is invariant. We are interested in a set of invertible operators W which makes a group G_0 and preserves $\bar{\psi}D\psi$ invariant, equivalently satisfies $WD=DW$. Choices of such groups are not unique. One of possible groups can be obtained by

$$G_0 = \{W \mid W = \textstyle\sum_{n=-\infty}^{n=+\infty} c_n D^n \text{ with constant coefficients}\}\,. \tag{1.3}$$

For an invertible operator W we put

$$\psi_W = W\psi\,, \quad \bar{\psi}^W = \bar{\psi} W^{-1}\,. \tag{1.4}$$

Proposition (1.5). *The Lagrangian action*

(1.6) $$\mathcal{L}_0 = \int_R \bar{\psi}^W D\psi_W \, dx\,, \qquad W \in G_0$$

is invariant under the action of the group G_0.

Proof. We choose arbitrary elements W and W' of G_0 and set ϕ by $W=\phi W'$, namely, $\phi = WW'^{-1}$. Since

(1.7) $$\psi_W = \phi\psi_{W'}\,, \quad \bar{\psi}^W = \bar{\psi}^{W'}\phi^{-1}\,,$$

we obtain $\bar{\psi}^W D\psi_W = \bar{\psi}^{W'}\phi^{-1}D\phi\psi_{W'} = \bar{\psi}^{W'}D\psi_{W'}$.

The group G_0 is called *the gauge group of the first kind.* Next we proceed to a group

(1.8) $$G = \{W \mid W = \textstyle\sum_{n=-\infty}^{n=+\infty} u_n(x)D^n \text{ with function coefficients}\}\,.$$

We call an element of G a formal pseudo-differential operator [5]. G is called *the gauge group of the second kind.* In order to obtain exact mathematical meanings, we have to restrict our considerations to special groups. For example, we may choose a group G consisting of elements W with the following condition: Every $u_n(x)$ is analytic function and there exists an integer n_0 such that ord $u_n(x) \geq n - n_0$ for any sufficiently large n ([4], [7], [8]). For a complex valued analytic function u with the Taylor expansion

$$u = c_n x^n + c_{n+1}x^{n+1} + \cdots \qquad (c_n \neq 0)\,,$$

the order of u is defined by ord $u = n$. We have to pay attention to the fact that the Lagrangian action $\mathcal{L}_0$ is not invariant under G, because the commutator $[D, W] = DW - WD$ does not vanish identically. Hence we note that the following equalities hold:

(1.9) $$[D, W] = \textstyle\sum (Du_n(x))D^n \qquad \text{for} \quad W = \textstyle\sum u_n(x)D^n$$

and

(1.10) $$WDW^{-1} = -[D, W]W^{-1} + D \qquad \text{for} \quad W \in G\,.$$

The Uchiyama gauge theory [9] says that in order to get a new Lagrangian action which is invariant under the group of the second kind, a connection, i.e., a gauge field, has to be introduced. Then we can make the following definition:

Definition (1.11). *Let* G *be a group of operators described in* (1.8) *and* *let R be a subset of G. A collection $\{\Omega(W) \mid W \in R\}$ of operators is called an R-connection if* (1) *there exists a pair (G_1, ρ) constituted with an injective set-map $\rho\colon G_1 \to G$ of a group G_1 to G such that $R=\rho(G_1)$ and* (2) *$L_\Omega(W) \equiv D-\Omega(W)$ satisfies*

$$L_\Omega(W) = \phi L_\Omega(W')\phi^{-1} \quad \text{for} \quad W, W' \in R \quad \text{where} \quad W = \phi W'. \tag{1.12}$$

In particular, we call it a G-connection if in addition ρ is a group-isomorphism.

The following are examples of G-connections:

Examples (1) $\Omega(W)=D$. (2) $\Omega(W)=[D, W]W^{-1}$, in this case

$$L(W) \equiv L_\Omega(W) = WDW^{-1}. \tag{1.13}$$

(3) Let G' be a subgroup of G and $\iota\colon G' \to G$ be the natural inclusion mapping. If $\Omega(W)$ $(W \in G)$ is a G-connection, then $\Omega(W)$ $(W \in G')$ becomes a G'-connection.

Immediately from (1.12) we see that if $\Omega_1(W)$ and $\Omega_2(W)$ are R-connections, then the relation

$$\Omega_1(W)-\Omega_2(W) = \phi(\Omega_1(W')-\Omega_2(W'))\phi^{-1} \tag{1.14}$$

holds for $W, W' \in R$ where $W=\phi W'$. This fact and Example (2) show that operators $\hat{\Omega}(W)$ given by

$$\hat{\Omega}(W) = W^{-1}([D, W]W^{-1}-\Omega(W))W \quad \text{for} \quad W \in R \tag{1.15}$$

satisfy the condition $\hat{\Omega}(W)=\hat{\Omega}(W')$ for any pair of W and W' of R, namely $\hat{\Omega}(W)$ does not depend on a choice of $W \in R$. Therefore, we may write as $\hat{\Omega}=\hat{\Omega}(W)$. We call $\hat{\Omega}$ the *connection form* determined by $\Omega(W)$. An R-connection is called to be *flat* if its connection form vanishes identically, namely $\Omega(W)=[D, W]W^{-1}$.

By an application of Uchiyama theory to the Lagrangian action (1.6), we obtain

Proposition (1.16). *Let $\Omega(W)$ be a G-connection. The Lagrangian action*

$$\mathcal{L} = \int_R \bar{\psi}^W(D-\Omega(W))\psi_W dx \qquad W \in G \tag{1.17}$$

is invariant under the group G.

Proof. For arbitrary elements W and W' where $W=\phi W'$ in G we have

$$\bar{\psi}^W L(W)\psi_W = \bar{\psi}^{W'}\phi^{-1}(\phi L(W')\phi^{-1})\phi\psi_{W'} = \bar{\psi}'^W L(W')\psi_{W'},$$

which implies the invariance of $\mathcal{L}$ under G.

The following group is important for a study on the K.-P. system. We put

$$G_- = \{\sum_{n=0}^{\infty} v_n(x)D^{-n} \in G \mid v_0(x) = 1\}. \tag{1.18}$$

Further we make the following definition:

Definition (1.19).

$$\begin{aligned} \mathfrak{g} &= \{\sum_{n=-\infty}^{\infty} u_n(x)D^n\}, \\ \mathfrak{g}_+ &= \{\sum_{n=0}^{\infty} u_n(x)D^n\} \quad \textit{and} \quad \mathfrak{g}_- = \{\sum_{n=1}^{\infty} u_{-n}(x)D^{-n}\}. \end{aligned} \tag{1.20}$$

Then the following decomposition holds:

$$\mathfrak{g} = \mathfrak{g}_+ + \mathfrak{g}_-, \tag{1.21}$$

which implies that any element S of $\mathfrak{g}$ has the decomposition: $S=(S)_+ + (S)_-$ for $(S)_+ \in \mathfrak{g}_+$ and $(S)_- \in \mathfrak{g}_-$. Then we can prove

Theorem 1. *$\omega(W)$ ($W \in G_-$) is the flat G_--connection if and only if*

$$\omega(W) = -(L(W))_- \quad \textit{for} \quad W \in G_-. \tag{1.22}$$

Proof. For W, $W' \in G_-$, where $W=\phi W'$, it holds that

$$\begin{aligned} (L(W))_- &= (\phi L(W')\phi^{-1})_- = (\phi(L(W'))_+\phi^{-1})_- + (\phi(L(W'))_-\phi^{-1})_- \\ &= (\phi D\phi^{-1})_- + \phi(L(W'))_-\phi^{-1} \\ &= (-[D, \phi]\phi^{-1} + D)_- + \phi(L(W'))_-\phi^{-1} \qquad \text{(by (1.10))} \\ &= -D + \phi D\phi^{-1} + \phi(L(W'))_-\phi^{-1}, \end{aligned}$$

which implies $D-\omega(W)=\phi(D-\omega(W'))\phi^{-1}$. Hence $\omega(W)$ is a G_--connection. Comparing the non-positive orders of the both sides of (1.10), we obtain $\omega(W) = -(L(W))_- = [D, W]W^{-1}$, i.e., $\omega(W)$ is flat. Conversely, if $\omega(W)$ ($W \in G_-$) is the flat G_--connection, then $\omega(W)$ reduces to $\omega(W)=[D, W]W^{-1} = -(L(W))_-$ by (1.10).

§ 2. A Gauge Theory for the K.-P. System

We consider a Lagrangian formalism for scalar fields, $\psi=\psi(x, t)$ and

$\bar{\psi}=\bar{\psi}(x, t)$ defined on the real line $(x\in)\mathbf{R}$ with infinitely many parameters

$$t=(t_1, t_2, \cdots),$$

and for some collections of operators including $D=d/dx$ and $D_n=\partial/\partial t_n$. The total differential operator with respect to the parameters is denoted by

$$d=\sum_{n=1}^{\infty} D_n dt_n. \tag{2.1}$$

The Lagrangian action which we treat here is given by

$$\mathcal{L}(t)=\int_{\mathbf{R}} \bar{\psi}(x, t) d\psi(x, t) dx \tag{2.2}$$

for functions ψ and $\bar{\psi}$. We proceed to our discussions analogous to the one done in the previous section. We are interested in invertible operators $W=W(x, t)$, considering together with the action law for ψ and $\bar{\psi}$:

$$\psi \rightarrow \psi'=W\psi\ (=\psi_W), \quad \bar{\psi} \rightarrow \bar{\psi}'=\bar{\psi} W^{-1}\ (=\bar{\psi}^W). \tag{2.3}$$

Hence, the function $\bar{\psi}\psi$ is invariant under this action.

First, we consider a group

$$\tilde{G}_0=\{W \mid W=\textstyle\sum_{n=-\infty}^{n=+\infty} c_n(x) D^n\}. \tag{2.4}$$

In this case, we observe that coefficients $c_n(x)$ are constant with respect to t. Immediately, from $Wd=dW$ we have

Proposition (2.5). *The Lagrangian*

$$\mathcal{L}_0=\int_{\mathbf{R}} \bar{\psi}^W d\psi_W dx, \qquad W\in\tilde{G}_0, \tag{2.6}$$

possesses the symmetry of the group $\tilde{G}_0$.

Following the Uchiyama theory, next we deal with a group

$$\tilde{G}=\{W \mid W=\textstyle\sum_{n=-\infty}^{n=+\infty} u_n(x, t) D^n \text{ with the property } (*)\} \tag{2.7}$$

(*) $u_n(x, t)$ $(n=0, \pm 1, \pm 2, \cdots)$ are analytic functions of x and t satisfying the following growth condition: There exists an integer n_0 such that $\operatorname{ord} u_n(x, t) \geqq n-n_0$ for any sufficiently large n

(see [4], [7], [8]). The Lagrangian action (2.6) gives rise to a gauge group $\tilde{G}_0$ of the first kind and a gauge group $\tilde{G}$ of the second kind respectively. $\mathcal{L}_0$ is not invariant under $\tilde{G}$, since commutators

$$[D_m, W] = \sum (D_m u_n(x, t))D^n \qquad (m = 1, 2, \cdots)$$

for $W=\sum u_n(x, t)D^n$, do not vanish identically, i.e., $[d, W]\neq 0$. Hence we have to make

Definition (2.8). *Let $\tilde{R}$ be a subset of the group $\tilde{G}$ described in* (2.7). *A set $\{\Omega(W)\mid W\in\tilde{R}\}$ of operators is called an $\tilde{R}$-multiconnection (or, simply $\tilde{R}$-connection) if $\Omega(W)$ has the form $\Omega(W)=\sum_n \Omega_n(W)dt_n$ whose $\Omega_n(W)$ is a connection with a range $\tilde{R}$ with respect to D_n:*

$$D_n-\Omega_n(W) = \phi(D_n-\Omega_n(W'))\phi^{-1}$$

for W and $W'\in\tilde{R}$, where $W=\phi W'$ ($\phi\in\tilde{G}$). $\Omega_n(W)$ is called the partial connection of $\Omega(W)$.

We note that an $\tilde{R}$-multiconnection $\Omega(W)$ implies

$$d-\Omega(W) = \sum_n \phi(D_n-\Omega_n(W'))\phi^{-1} = \phi(d-\Omega(W'))\phi^{-1}$$

for W, $W'\in\tilde{R}$ with $W=\phi W'$.

By use of Uchiyama's theory, we obtain

Proposition (2.9). *Let $\Omega(W)$ be a $\tilde{G}$-connection. The Lagrangian*

$$\mathcal{L} = \int_R \bar{\psi}^W(d-\Omega(W))\psi_W\, dx \qquad \textit{for} \quad W\in\tilde{G}$$

is invariant under the group $\tilde{G}$.

We set

$$(2.10)\quad \tilde{G}_+ = \{\textstyle\sum_{n=0}^{n=+\infty} u_n D^n\in\tilde{G}\mid u_0\not\equiv 0\}\,,\quad \tilde{G}_- = \{\textstyle\sum_{n=0}^{n=+\infty} u_{-n} D^{-n}\in G\mid u_0\equiv 1\}\,.$$

Corresponding to $\tilde{G}$, $\tilde{G}_+$ and $\tilde{G}_-$, we consider the spaces of operators $\tilde{\mathfrak{g}}=\{\sum_{n=-\infty}^{n=+\infty} u_n D^n\}$, and its complementary subspaces

$$(2.11)\qquad \tilde{\mathfrak{g}}_+ = \{\textstyle\sum_{n=0}^{n=+\infty} u_n(x, t)D^n\}\,,\quad \tilde{\mathfrak{g}}_- = \{\textstyle\sum_{n=1}^{n=\infty} u_{-n}(x, t)D^{-n}\}\,,$$

that is the direct sum $\tilde{\mathfrak{g}}=\tilde{\mathfrak{g}}_+ +\tilde{\mathfrak{g}}_-$. Hence, any element $X\in\tilde{\mathfrak{g}}$ is written as $X=(X)_+ +(X)_-$ for $(X)_+\in\tilde{\mathfrak{g}}_+$ and $(X)_-\in\tilde{\mathfrak{g}}_-$.

Here we recall the K.-P. system. The operator $L=WDW^{-1}$ for $W\in G_-$ derived from the flat connection implies that $L^n=WD^nW^{-1}$ and its decomposition $L^n=(L^n)_+ +(L^n)_-$. In this case, $(L^n)_+$ is the n-th order differential operator. The *K.-P. system* is a system of equations defined by

$$(2.12)\qquad \partial L/\partial t_n = [(L^n)_+, L] \qquad (n=1, 2, \cdots)\,.$$

When W $(\in \tilde{G}_-)$ is an element described in the solution $L = WDW^{-1}$ of the K.-P. system, we shall say that W gives a solution of the K.-P. system. It is known ([1], [5], [6]) that *an element* W *of* $\tilde{G}_-$ *gives a solution of the K.-P. system if and only if* W *satisfies*

$$\partial W/\partial t_n + (L^n(W))_- W = 0 \qquad (n = 1, 2, \cdots). \tag{2.13}$$

The following theorem is our main result:

Theorem 2. *Let* R^* *be the space of all elements of* $\tilde{G}_-$ *each of which gives a solution of the K.-P. system. Then the set* $\{\Omega_{K.P}(W) \mid W \in R^*\}$ *defined by*

$$\Omega_{K.P}(W) = \sum_n \Omega_n(W) dt_n, \quad \Omega_n(W) = -(L^n(W))_- \tag{2.14}$$

becomes the flat R^**-connection (say, the K.-P. connection).*

Remark. (1) The K.-P. connection is a direct generalization of the connection given in Theorem 1, when we identify t_1 with x and set $t_n = 0$ $(n = 2, 3, \cdots)$. (2) The flatness of the K.-P. connection is well known as the Zakharov-Shabat equation.

For the proof of this theorem we need the following two lemmas:

Lemma 1 (Mulase's decomposition theorem [4]). *The group* $\tilde{G}$ *described in* (2.7) *can be decomposed into*

$$\tilde{G} = \tilde{G}_- \circ \tilde{G}_+,$$

in a sense that any element $g \in \tilde{G}$ *determines the unique pair of elements* $g_1 \in \tilde{G}_-$ *and* $g_2 \in \tilde{G}_+$ *such that* $g = g_1 \circ g_2$.

Lemma 2 ([4], [6]). *There exists a one-to-one correspondence between the space* R^* *and the space* Q *of solutions* U *of the initial value problem*:

$$\partial U/\partial t_n = [D^n, U], \quad U|_{t=0} = U_0 \in G_-, \tag{2.15}$$

where G_- *is given in* (1.18). *The exact correspondence is described in the following manner*: *A solution* U *of* (2.15) *determines an element* W *of* $\tilde{G}_-$ *by the decomposition* $U = W^{-1}V$ *in Lemma* 1. *Then* $L(W) = WDW^{-1}$ *gives a solution of* (2.12). *Conversely, for a solution* W *of* (2.12), *we can find a unique element* V *of* $\tilde{G}_+$ *such that* $V|_{t=0} =$ *identity and* $U = W^{-1}V$ *gives a solution of* (2.15).

The proof of Theorem 2. Let U_0 be any element of G_-. U_0 determines a unique solution U $(\in \tilde{G}_-)$ of (2.15) by Lemma 2. U can be decomposed uniquely as $U = W^{-1}V$ with $W \in \tilde{G}_-$ and $V \in \tilde{G}_+$ by Lemma 1. This gives rise to a mapping ρ: $G_{-}\to\tilde{G}_{-}$ which maps U_0 to W. This mapping ρ is injective ([4], [6]). Then we see that $R^*=\rho(G_{-})$. Next we show that $\Omega_{K.P}(W)$ becomes an R^*-connection. Let W and W' be elements of R^* and set ϕ ($\phi\in\tilde{G}_{-}$) by $W=\phi W'$. It follows from

$$\partial W/\partial t_n = (\partial\phi/\partial t_n)W' + \phi(\partial W'/\partial t_n)$$

and from (2.13) that

$$-(L^n(W))_{-}W = (\partial\phi/\partial t_n)W' - \phi(L^n(W'))_{-}W' .$$

Hence

$$\omega_n(W) = (\partial\phi/\partial t_n)\phi^{-1} + \phi\omega_n(W')\phi^{-1}$$

holds, which implies that $\omega_n(W)$ ($W\in R^*$) is a partial R^*-connection. Therefore, $\Omega_{K.P}(W)$ ($W\in R^*$) is an R^*-connection. The flatness of the connection follows from (2.13):

$$0 = \textstyle\sum_n (\partial W/\partial t_n + (L^n(W))_{-}W)dt_n = \sum_n (\partial W/\partial t_n - \omega_n(W)W)dt_n$$
$$= [d, W] - \Omega_{K.P}(W)W .$$

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