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Hodge Spectral Sequence on Compact Kähler Spaces

By

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Introduction

Let X be a complex manifold of dimension n and $E_r^{p,q}$ the Hodge spectral sequence on X. The following is fundamental in the study of algebraic varieties.

Theorem (W.V.D. Hodge [8]) *If X is a compact Kähler manifold, then*

$$\text{(H)}\quad \begin{cases} E_1^{p,q} = E_\infty^{p,q} \\ \overline{E_1^{p,q}} \cong E_1^{p,q} \end{cases} \qquad \textit{for any } p \textit{ and } q\,.$$

In 1972, P. Deligne [3] succeeded in generalizing it for an arbitrary quasi-projective variety by analyzing a different spectral sequence. His so called mixed Hodge theory explains how the singular cohomology is composed of the analytic cohomology attached to the variety.

On the other hand, Grauert-Riemenschneider [7] and Fujiki [5] tried to understand the Hodge spectral sequence itself on pseudoconvex manifolds. Inspired by these works, the author [11] could show that (H) is valid for the range $p+q \geq n+r$ on any "very strongly r-convex" Kähler manifold of dimension n. The crucial point was to establish an isomorphism between the ordinary cohomology and the L^2 cohomology with respect to a certain complete Kähler metric on pseudoconvex domains.

Since it has long been known that for any projective variety over $\mathbb{C}$ the complement of the singular locus admits a complete Kähler metric (Grauert [6]), it is natural to ask for a reasonable extension of [11] in such a case.

The purpose of the present paper is to show the following in this spirit.

Theorem 1 *Let X be a compact Kähler space of pure dimension n whose*

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singular points are isolated, and let X^* be the complement of the singular points. Then (H) *holds on X^* for the range $p+q<n-1$.*

Note that the range is optimal since $\dim H^{0,n-1}(X^*)=\infty$ if $X^*\neq X$, where $H^{0,n-1}$ denotes the Dolbeault cohomology of type $(n-1, 0)$.

We shall also give a partial answer to a question of Cheeger-Goreski-MacPherson [2] by showing the following:

Theorem 2 *Under the situation of Theorem* 1,

$$\begin{cases} H^r(X^*) \cong H^r_{(2)}(X^*) & \text{if } r<n-1 \\ H^{p,q}(X^*) \cong H^{p,q}_{(2)}(X^*) & \text{if } p+q<n-1 \end{cases}$$

and

$$H^r_0(X^*) \cong H^r_{(2)}(X^*) \quad \text{if } r>n+1$$

$$H^{p,q}_0(X^*) \cong H^{p,q}_{(2)}(X^*) \quad \text{if } p+q>n+1 .$$

Here, H, H_0 and $H_{(2)}$ denote respectively the ordinary cohomology, the cohomology with compact support, and the L^2 cohomology.

Note that the duality between $H^r_{(2)}$ and $H^{2n-r}_{(2)}$ is not obvious since the metric on X^* is not complete as long as $X^*\neq X$.

Since the intersection cohomology $IH^r(X)$ is isomorphic to $H^r(X^*)$ if $r<n$ and isomorphic to $H^r_0(X^*)$ if $r>n$, Theorem 2 implies the following.

Corollary $IH^r(X) \cong H^r_{(2)}(X^*) \quad$ *if* $r\neq n, n\pm 1$.

Cheeger-Goreski-MacPherson conjectured that the above isomorphism is valid for any degree, and in some special cases it has been verified (cf. [2], [10] and [12]).

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§1. Preliminaries

Let (M, ds^2) be a complete Hermitian manifold of dimension n. We put

$$L_{(2)}\ (=L_{(2)}(M)) \ := \{\text{square integrable complex differential forms on } M\} .$$

$$L^r_{(2)}\ (=L^r_{(2)}(M)) \ := \{f\in L_{(2)};\ \deg f=r\} .$$

$$L_{(2)}^{p,q}\ (=L_{(2)}^{p,q}(M)) := \{f\in L_{(2)}; f \text{ is of type } (p, q)\}\ .$$

The norms and the inner products in $L_{(2)}$ shall be denoted by $\|\ \|$ $(=\|\ \|_M)$ and $(\ ,\ )$ $(=(\ ,\ )_M)$, respectively. The exterior differentiations d, $\bar\partial$ and ∂ are regarded as densely defined closed linear operators on $L_{(2)}$ whose domains of definition are given by

$$\text{Dom } d: = \{f\in L_{(2)};\ df\in L_{(2)}\}\text{, etc}\ .$$

Here the differentiation is in distribution sense.

Definition

$$H_{(2)}\ (=H_{(2)}(M)) := \text{Ker } d/\text{Im } d\ .$$
$$H_{(2)}^{r}\ (=H_{(2)}^{r}(M)) := \text{Ker } d\cap L_{(2)}^{r}/\text{Im } d\cap L_{(2)}^{r}\ .$$
$$H_{(2)}^{p,q}\ (=H_{(2)}^{p,q}(M)) := \text{Ker } \bar\partial\cap L_{(2)}^{p,q}/\text{Im } \bar\partial\cap L_{(2)}^{p,q}\ .$$

We denote by d^* and $\bar\partial^*$ the adjoints of d and $\bar\partial$, respectively. Note that $H_{(2)}\cong\text{Ker } d\cap\text{Ker } d^*$ (resp. $H_{(2)}^{p,q}\cong\text{Ker } \bar\partial\cap\text{Ker } \bar\partial^*\cap L_{(2)}^{p,q}$) if and only if Im d is closed (resp. Im $\bar\partial\cap L_{(2)}^{p,q}$ is closed). $H_{(2)}$ are called L^2 cohomologies of M. For any family of supports Φ, L^2 cohomologies with supports in Φ are also defined similarly as above.

The following is first due to H. Donnelly and C. Fefferman, but the proof below is different from theirs.

Theorem 1.1 (cf. [4]) *Suppose that there exists a C^∞ real valued function F on M such that the fundamental form of ds^2 is $i\partial\bar\partial F$ and that $|\partial F|_\infty$ $(:=\sup|\partial F|)<\infty$. Then, for any $u\in\text{Ker } d\cap L_{(2)}^{r}$ with $r\neq n$, there exists a $v\in\text{Dom } d\cap L_{(2)}^{r-1}$ such that $dv=u$ and $\|v\|\leq 2|\partial F|_\infty\|u\|$. Similarly, if $p+q\neq n$, then for any $u\in\text{Ker } \bar\partial\cap L_{(2)}^{p,q}$ there exists a $v\in\text{Dom } \bar\partial\cap L_{(2)}^{p,q-1}$ such that $\bar\partial v=u$ and $\|v\|\leq(1+\sqrt{2})|\partial F|_\infty\|u\|$. In particular,*

$$\begin{cases} H_{(2)}^{r}=0 & \text{if } r\neq n\ ,\\ H_{(2)}^{p,q}=0 & \text{if } p+q\neq n\ .\end{cases}\tag{1}$$

Proof. The assertions are equivalent to that

$$\|u\|\leq 2|\partial F|_\infty\|d^*u\|\ ,\qquad \text{for any}\quad u\in\text{Ker } d\cap\text{Dom } d^*\cap L_{(2)}^{r}$$

and

$$\|u\|\leq(1+\sqrt{2})|\partial F|_\infty\|\bar\partial^*u\|\ ,\qquad \text{for any}\quad u\in\text{Ker } \bar\partial\cap\text{Dom } \bar\partial^*\cap L_{(2)}^{p,q}\ ,$$

respectively (cf. [9]). They are proved as follows:

For any differential form θ, let $e(\theta)$ be the multiplication by θ from the left. Then we have the following formula.

$$[\bar{\partial}, e(\bar{\partial}F)^*]+[\partial^*, e(\partial F)] = [e(i\partial\bar{\partial}F), e(i\partial\bar{\partial}F)^*].$$

Here [,] denotes the commutator with weight (i.e., $[S, T] := S\circ T-(-1)^{\deg S \deg T} T\circ S$) and * denotes the adjoint.

In fact, with respect to the operator $\Lambda := e(i\partial\bar{\partial}F)^*$ we have $[\bar{\partial}, \Lambda]=i\partial^*$ and $[e(\partial F), \Lambda]=ie(\bar{\partial}F)^*$ (cf. [12]). Therefore

$$\begin{aligned}
[\bar{\partial}, e(\bar{\partial}F)^*] &= \bar{\partial}e(\bar{\partial}F)^*+e(\bar{\partial}F)^*\bar{\partial} \\
&= -i\bar{\partial}[e(\partial F), \Lambda]-i[e(\partial F), \Lambda]\,\bar{\partial} \\
&= [e(i\partial\bar{\partial}F), \Lambda]+ie(\partial F)\,[\bar{\partial}, \Lambda]+i[\bar{\partial}, \Lambda]\,e(\partial F) \\
&= [e(i\partial\bar{\partial}F), \Lambda]-[e(\partial F), \partial^*].
\end{aligned}$$

Hence, for any compactly supported C^∞ r-form u,

$$\begin{aligned}
&([e(i\partial\bar{\partial}F), \Lambda]\, u, u) \\
&\leq |\partial F|_\infty ||u||(||\bar{\partial}u||+||\bar{\partial}^* u||+||\partial u||+||\partial^* u||).
\end{aligned}$$

Since the metric is Kählerian, we have

$$\begin{aligned}
||\bar{\partial}u||^2+||\bar{\partial}^*u||^2 &= ||\partial u||^2+||\partial^*u||^2 \\
&= \frac{1}{2}(||du||^2+||d^*u||^2) \quad \text{(cf. [14])}.
\end{aligned}$$

On the other hand, $[e(i\partial\bar{\partial}F), \Lambda]\, u=(r-n)u$. Thus we obtain

$$||u||\leq 2|\partial F|_\infty(||du||^2+||d^*u||^2)^{1/2}$$

and

$$||u||\leq(1+\sqrt{2})\,|\partial F|_\infty(||\bar{\partial}u||+||\bar{\partial}^*u||),$$

if $r\neq n$.

Since the metric ds^2 is complete, the required estimate follows from the above (cf. [13]).

§2. A Poincaré-Dolbeault Lemma

Let X be a complex analytic space of pure dimension n. In what follows the nonsingular part of X will be denoted by X^*. Suppose that o is an isolated singular point of X. Then we have a holomorphic embedding of the germ $(X, \underline{o}) \hookrightarrow (\mathbf{C}^N, O)$. We fix in the followings a holomorphic coordinate $z\,(=(z_1, \cdots, z_N))$ of $\mathbf{C}^N$ and the euclidean norm $|z|$ of z. We put $B_c^* := \{z; 0<|z|<c\}$ and $X_c^*\ (=X_{\underline{o},c}^*) := X \cap B_c^*$ (c sufficiently small). As a candidate of the potential F in Theorem 1.1, we put

$$F_c(z)\ (=F_{c,\underline{o}}(z)) := -\log\log(c/|z|)\,. \tag{2}$$

Proposition 2.1 *The length of $\partial(F_c|X_c^*)$ with respect to the metric $2\partial\bar\partial(F_c|X_c^*)$ is bounded.*

Proof. On B_c^* we have

$$\partial F_c = \frac{-\partial \log|z|}{\log(c/|z|)}$$

and

$$\partial\bar\partial F_c \geq \frac{\partial \log|z|\,\bar\partial \log|z|}{\log^2(c/|z|)}\,. \tag{3}$$

Hence $|\partial(F_c|X_c^*)| \leq 1$.

In what follows we fix c and regard X_b^* for $b \leq c$ as a Kähler manifold with metric $2\partial\bar\partial(F_b|X_b^*)$. Moreover c is fixed so that $\partial \bar X_b^*$ is compact for all $b \leq c$. It is clear from (3) that X_b^* are then complete Kähler manifolds.

Combining (1) in Theorem 1.1 and Proposition 2.1 we obtain the following:

Proposition 2.2 *For any $b \leq c$,*

$$\begin{cases} H_{(2)}^r(X_b^*) = 0 & \text{if } r \neq n\,, \\ H_{(2)}^{p,q}(X_b^*) = 0 & \text{if } p+q \neq n\,. \end{cases}$$

The following observation was already made in [10], but we shall repeat the proof because of the completeness.

Lemma 2.3 *Let $r>n$ and $u \in L_{(2)}^r(X_c^*)$. Then, $u|X_b^* \in L_{(2)}^r(X_b^*)$, for any $b \leq c$.*

Proof. Since

$$\partial\bar\partial F_b = \frac{\partial\bar\partial \log|z|}{\log(b/|z|)} + \frac{\partial \log|z|\,\bar\partial \log|z|}{\log^2(b/|z|)}\,,$$

for any b, the eigenvalues $\lambda_1 \leq \cdots \leq \lambda_N$ of $\partial\bar\partial F_b$ measured by $\partial\bar\partial F_c$ are given by

$$\lambda_j = \frac{\log(c/|z|)}{\log(b/|z|)}\,, \qquad \text{for} \quad 1 \leq j \leq N-1$$

and

$$\lambda_N = \frac{\log^2(c/|z|)}{\log^2(b/|z|)}.$$

Let $\mu_1 \leq \cdots \leq \mu_n$ be the eigenvalues of $\partial\bar{\partial}(F_b | X_b^*)$ measured by $\partial\bar{\partial}(F_c | X_c^*)$. Then, by Courant's minimax principle,

$$\mu_j = \frac{\log(c/|z|)}{\log(b/|z|)}, \qquad \text{for} \quad 1 \leq j \leq n-1$$

and

$$\frac{\log(c/|z|)}{\log(b/|z|)} \leq \mu_n \leq \frac{\log^2(c/|z|)}{\log^2(b/|z|)}.$$

Now it is easy to see that $||u | X_b^*||_{X^*} \leq ||u||_{X_c^*}$, for any $u \in L_{(2)}^r(X_c^*)$ with $r > n$.

On the opposite side $r < n$ we have the following, which will be used to prove Theorem 2.

Lemma 2.4 *Let $b < c$ and $u \in L_{(2)}^r(X_b^*)$ with $r < n$. Let $\tilde{u}$ be a form in $L_{(2)}^r(X_c^*)$ defined by $\tilde{u} := u$ on X_b^* and $\tilde{u} := 0$ on $X_c^* \setminus X_b^*$. Then $\tilde{u} \in L_{(2)}^r(X_c^*)$. Moreover if $r < n-1$, then $\tilde{u} \in \mathrm{Dom}\, d$ (resp. $\tilde{u} \in \mathrm{Dom}\, \bar{\partial}$) if $u \in \mathrm{Dom}\, d$ (resp. $u \in \mathrm{Dom}\, \bar{\partial}$).*

Proof. The first part is proved similarly as in the proof of Lemma 2.3. The latter part follows from the first part and (3). In fact, let $\chi: \mathbb{R} \to \mathbb{R}$ be a C^∞ function such that $\chi \equiv 1$ on $(-\infty, -2)$ and $\chi \equiv 0$ on $(-1, \infty)$. Then, if $u \in \mathrm{Dom}\, d$ (resp. $\mathrm{Dom}\, \bar{\partial}$), the sequence $\{\chi(k(|z|-b))\, u\}_{k=1}^\infty$ is convergent on X_c^* with respect to the graph norm of d (resp. the graph norm of $\bar{\partial}$), if $r < n-1$.

Let Φ be a family of closed subsets of X_c^* defined by $\Phi := \{K \subset X_c^*;\ K \cap \overline{X_{c/2}^*}$ is compact$\}$.

Then the following is an immediate consequence of Proposition 2.2 and Lemma 2.4.

Theorem 2.5 *The images of the following natural homomorphisms are zero.*

$$H_0^r(X_c^*) \to H_\Phi^r(X_c^*), \qquad \textit{for} \quad r < n,$$

$$H_0^{p,q}(X_c^*) \to H_\Phi^{p,q}(X_c^*), \qquad \textit{for} \quad p+q < n.$$

Here H_Φ denotes the cohomology with supports in Φ.

Theorem 2.5 is not used to prove Theorem 1 and Theorem 2, but it may have some application in the theory of isolated singularities.

§3. Proof of Theorem 1

Let X be a complex space of dimension n. For any isolated singular point $\underline{o}\in X$ we shall freely use the notations X_c, F_c, etc. in §2. X is called a Kähler space if there exist an open covering $\mathcal{U}=\{U_j\}_{j\in J}$ of X and a system of C^∞ strictly plurisubharmonic functions φ_j, each φ_j being defined on $U_j\in\mathcal{U}$, such that $\varphi_j-\varphi_k$ is pluriharmonic on $U_j\cap U_k$. Given such a system of functions, $\{\partial\bar\partial\varphi_j\}_{j\in J}$ defines a Kähler metric on X^*. Clearly, this metric is locally quasi-isometric to those induced from the euclidean one by embedding X locally into $\mathbb{C}^N$.

Proposition 3.1 *Let X be a compact Kähler space with isolated singularities. Then X^* admits a complete Kähler metric which is quasi-isometric to $\partial\bar\partial F_{\underline{o},c}$ on $X^*_{\underline{o},c/2}$ for each singular point $\underline{o}$.*

Proof. Let $\underline{o}_\nu$ ($\nu=1, \cdots, m$) be the singular points of X. We choose c so that $X^*_{\underline{o}_\nu,c}$ are mutually disjoint regarded as subsets of X. Let ρ_ν be C^∞ functions on X^* such that $\rho_\nu=1$ on $X^*_{\underline{o}_\nu,c/2}$ and $\rho_\nu=0$ on $X^*\backslash X^*_{\underline{o}_\nu,c}$. Then, for $A\gg 0$,

$$\sum_{\nu=1}^{m}\partial\bar\partial(\rho_\nu F_{\underline{o}_\nu,c})+A\,\partial\bar\partial\varphi_j$$

gives a complete Kähler metric with the required property.

Proof of Theorem 1 Once for all we regard X^* as a complete Kähler manifold with a metric such as in Proposition 3.1. Since $H^r_{(2)}(X^*)=\bigoplus_{p+q=r} H^{p,q}_{(2)}(X^*)$ and $H^{p,q}_{(2)}(X^*)=\overline{H^{q,p}_{(2)}(X^*)}$, it suffices to show that

$$(4)\qquad \begin{cases} H^r_{(2)}(X^*)\cong H^r(X^*) & \text{if } r<n-1\,, \\ H^{p,q}_{(2)}(X^*)\cong H^{p,q}(X^*) & \text{if } p+q<n-1\,. \end{cases}$$

Since dim $H^r(X^*)$ and dim $H^{p,q}(X^*)$ are finite on the above ranges (cf. [1]), by Serre's duality (4) is equivalent to that

$$(5)\qquad \begin{cases} H^r_{(2)}(X^*)\cong H^r_0(X^*) & \text{if } r>n+1 \\ H^{p,q}_{(2)}(X^*)\cong H^{p,q}_0(X^*) & \text{if } p+q>n+1\,. \end{cases}$$

But (5) is immediate from Proposition 2.2 and Lemma 2.3. In fact, to show that the natural homomorphism from $H^r_0(X^*)$ to $H^r_{(2)}(X^*)$, say α, is surjective, one has only to know that square integrable forms on X^* are in $L^r_{(2)}(X^*_c)$ around each singular point, is already assured for $r>n$ by Lemma 2.3. To show that α is injective, let u be in $L^r_{(2)}(X^*)$ and compactly supported, such that there exists a $v\in L^{r-1}_{(2)}(X^*)$ with $dv=u$. Since $dv=0$ near the singularity, by the same reason as above one can replace v by a compactly supported form in $L^{r-1}_{(2)}(X^*)$. The other isomorphism is proved similarly.

Remark It is also easy to prove (4) directly from Proposition 2.2 by using Lemma 2.4 instead of Lemma 2.3.

§4. Proof of Theorem 2

Let X be a compact Kähler space of pure dimension n. Now we need to distinguish two metrics on X^*, i.e. the original Kähler metric and a complete Kähler metric given in Proposition 3.1. Let us denote the original metric by ds^2 and make the distinction by $H^r_{(2)}(X^*)_{ds^2}$, etc.

While Theorem 1 was a consequence of Proposition 2.2, the proof of Theorem 2 is clearly reduced to the following local cohomology vanishing.

Proposition 4.1 *For each singular point $\underline{o}\in X$,*

$$(6)\qquad \begin{cases} \varinjlim H^r_{(2)}(X^*_c)_{ds^2} = 0 & \text{if } r>n \\ \varinjlim H^{p,q}_{(2)}(X^*_c)_{ds^2} = 0 & \text{if } p+q>n \end{cases}$$

and

$$(7)\qquad \begin{cases} \varprojlim H^r_{(2)}(X^*_c)^{(\Phi)}_{ds^2} = 0 & \text{if } r<n \\ \varprojlim H^{p,q}_{(2)}(X^*_c)^{(\Phi)}_{ds^2} = 0 & \text{if } p+q<n\,. \end{cases}$$

Here $H_{(2)}(X_c)^{(\Phi)}_{ds^2}$ denote the L^2 cohomologies with supports in Φ and the limits are taken by letting $c\to 0$.

Proof. We put $F_\varepsilon(z):=-\log((c^2-|z|^2)\log^\varepsilon(c/|z|))$ for any $\varepsilon\geq 0$. Then $\partial\bar\partial F_\varepsilon>0$ on X^*_c and $\partial\bar\partial F_\varepsilon$ converges to $-\partial\bar\partial\log(c^2-|z|^2)$ on the compact subsets of X^*_c.

We have

$$\begin{aligned} &\partial\bar\partial F_\varepsilon \\ &=\frac{\partial\bar\partial|z|^2}{c^2-|z|^2}+\frac{\partial|z|^2\bar\partial|z|^2}{(c^2-|z|^2)^2} \\ &\quad+\varepsilon\left(\frac{\partial\bar\partial\log|z|}{\log(c/|z|)}+\frac{\partial\log|z|\bar\partial\log|z|}{\log^2(c/|z|)}\right) \end{aligned}$$

$$\geq \partial \log(c^2-|z|^2)\,\bar{\partial} \log(c^2-|z|^2)$$
$$+\varepsilon^{-1}\,\partial \log \log^{\varepsilon}(c/|z|)\,\bar{\partial} \log \log^{\varepsilon}(c/|z|)\,.$$

From the above inequality it is clear that $\partial\underline{\varrho}F_{\varepsilon}|X_c$ is a complete Kähler metric on X_c^* and $|\partial F_{\varepsilon}|_{\varepsilon}\leq 2$ if $0\leq\varepsilon<1$. Here $|\ \ |_{\varepsilon}$ denotes the length with respect to $\partial\bar{\partial}F_{\varepsilon}$.

From the above, the eigenvalues $\xi_1, \cdots, \xi_N$ of $\partial\bar{\partial}F_{\varepsilon}$ measured by the euclidean metric $\partial\bar{\partial}|z|^2$ are given by

$$\xi_j=\frac{1}{c^2-|z|^2}+\frac{\varepsilon}{|z|^2\log(c/|z|)}, \quad 1\leq j\leq N-1\,,$$
$$\xi_N=\frac{c^2}{(c^2-|z|^2)^2}+\frac{\varepsilon}{|z|^2\log^2(c/|z|)}\,.$$

Thus, similarly as in Lemma 2.3, one can find a constant A such that

$$||u||_{\varepsilon}\leq A||u||_{ds^2} \quad \text{for any} \quad u\in L^r_{(2)}(X_c^*)_{ds^2}\,,$$

if $0\leq\varepsilon<1$. Here $||\ \ ||_{\varepsilon}$ denotes the L^2-norm with respect to $\partial\bar{\partial}F_{\varepsilon}|X_c^*$. If $r>n$ and $du=0$, then by Theorem 1.1, there exist $v_{\varepsilon}\in L^{r-1}_{(2)}(X_c^*)$ such that $dv_{\varepsilon}=u$ and $||v_{\varepsilon}||_{\varepsilon}\leq 4A||u||_{ds^2}$ if $0<\varepsilon<1$. Let $\{v_{\varepsilon_\nu}\}_{\nu=1}^{\infty}$ be a subsequence of $\{v_{\varepsilon}\}_{0<\varepsilon<1}$ which converges weakly on each compact subset of X_c^*, and let v be the limit on X_c^*. Then $||v||_0\leq 4A||u||_{ds^2}$ and $dv=u$. Since $\partial\bar{\partial}F_0$ is quasi-isometric near $\underline{o}$ to ds^2, this proves that $\varinjlim H^r_{(2)}(X_c^*)=0$ for $r>n$. The proofs of the other vanishings are similar except that for the vanishing with supports in Φ one should use Lemma 2.4. This is a slight change and we shall not repeat the whole argument. The detail is left to the reader.

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