Finiteness of Numbers of Curves on a
Minimal Surface with $\kappa=1$

By

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§ 0. Introduction

In [4], Namikawa proposed the following problem.

Problem. *Let S be a minimal compact analytic surface. For an arbitrary curve (i.e. a reduced, irreducible effective divisor) C on S, let us define the arithmetic genus of C by*

$$\pi(C)=\frac{1}{2}C(C+K_S)+1, \quad \text{where } K_S \text{ is the canonical divisor on } S.$$

If we fix a non-negative integer g, how many algebraic families of curves of genus g are there on S?

And he proved the finiteness of numbers of algebraic families of curves with the fixed arithmetic genus modulo Aut(S) except the case where $\kappa(S)=1$. The purpose of this note is to prove that the above finiteness also holds in the case where $\kappa(S)=1$. We state our main theorem.

Theorem A. *Let S be a minimal analytic surface with $\kappa(S)=1$ and g be a fixed non-negative integer. Then the number of algebraic families of curves on S with the arithmetic genus g is finite modulo Aut(S). In particular, the number of non-singular rational curves on S is finite modulo Aut(S).*

Notations and Convention

By a surface, we mean a compact complex manifold of dimension two. For a compact connected complex manifold X, we use the following notation.

K_X: the canonical bundle of X.

$\kappa(X)$: the Kodaira dimension of X.

Pic(X):$=H^1(X, \mathcal{O}_X^*)$: the Picard group of X, which has the natural structure of

a commutative complex Lie group.

NS(X): the Néron-Severi group of X, i.e. the group of algebraic equivalence
classes of divisors on X.

$\mathcal{O}_X$: the structure sheaf of X.

$\rho(X):=\mathrm{rank}_{\mathbf{Z}} NS(X)$: the Picard number of X.

$\chi(\mathcal{O}_X)$: the Euler-Poincaré characteristic.

The symbol $\approx$ (resp. $\sim$) indicates algebraic equivalence (resp. linear equivalence)
of divisors.

§ 1. Preliminaries

In this section, we shall prove Theorem A in the case where S has the
structure of the Jacobian fibration (i.e. elliptic fibration with a section). The
following theorem is fundamental.

Theorem 1.1. (Miyaoka, Umezu [3]) *Let S be a surface with $\kappa(S)\geqq 0$ and
fix a non-negative integer g. Then there exists a positive integer $N=N(S, g)$
which is determined only by S and g such that $C \cdot K_S < N$ for any curve C on S
with the arithmetic genus g.*

Proposition 1.2. *Let $f : S \to \Delta$ be a relatively minimal elliptic surface over
a non-singular complete curve Δ with the zero section (o) and at least one singular
fiber. Assume that $\kappa(S)=1$ and fix a non-negative integer g. Then the number
of algebraic families of curves on S with the arithmetic genus g is finite modulo
$\mathrm{Aut}(S)$.*

Corollary 1.3.   *Under the same conditions as in Proposition 1.2, if the
Mordell-Weil rank of S vanishes, the number of algebraic families of curves on
S with the fixed arithmetic genus g is finite.*

Proof of Proposition 1.2. Fix a non-negative integer g and take an arbitrary
algebraic curve C on S with the arithmetic genus g. Since all sections are
mapped to the zero section (o) by an automorphism of S and each irreducible
component of a reducible singular fiber is a non-singular rational curve with
self-intersection number -2, we may assume that C is a multi-section, that is,
$f|_C : C \to \Delta$ is a finite covering of degree $m > 1$.

Now let r be the rank of the Mordell-Weil group $\mathcal{E}(K)$ and take r generators
$s_1, \cdots, s_r$ of $\mathcal{E}(K)$ modulo the torsion group $\mathcal{E}(K)_{tor}$. $\mathcal{E}(K)_{tor}$ is generated by at
most two elements t_1, t_2 of order e_1, e_2 with $1 \leqq e_2, e_2 | e_1$; $|\mathcal{E}(K)_{tor}| = e_1 e_2$. Let
$\{F_\lambda\}_{\lambda \in \Delta}$ (resp. F) be the set of all singular fibers of $f : S \to \Delta$ (resp. generic fiber
of f) and for each $\lambda \in \Delta$ we denote by $\Theta_{\lambda, i}$ $(0 \leqq i \leqq m_\lambda - 1)$ the irreducible com-
ponent of F_λ, m_λ being the number of irreducible components. Then we have

$$F_\lambda = \Theta_{\lambda,0} + \sum_\lambda \sum_{i \geq 1} \mu_{\lambda,i} \Theta_{\lambda,i}, \qquad \mu_{\lambda,i} \geq 0,$$

where $\Theta_{\lambda,0}$ is the unique component of F_λ intersecting the zero section (o). By a theorem of Shioda [5], C can be written uniquely in the following way:

$$C \approx \alpha(o) + \beta F + \sum_{i=1}^{r} a_i s_i + \sum_{j=1}^{2} b_j t_j + \sum_\lambda \sum_{k=1}^{m_\lambda - 1} d_{\lambda,k} \Theta_{\lambda,k},$$

$0 \leq b_j < e_j$, where α, β, a_i, b_j and $d_{\lambda,k}$ are all rational integers.

Claim 1. $m := (C, F)$ *is bounded above by a constant* $A = A(S, g)$, *which is determined only by* S *and* g.

Proof of Claim 1. By the canonical bundle formula of Kodaira [2], we have $K_S \approx (2\pi(\Delta) - 2 + \chi(\mathcal{O}_S))F$. Since $\kappa(S) = 1$, we have $2\pi(\Delta) - 2 + \chi(\mathcal{O}_S) > 0$. Moreover by Theorem 1.1, there exists a constant $N = N(S, g)$ depending only on S and g such that $C \cdot K_S < N$. Put $A(S, g) = \dfrac{N(S, g)}{2\pi(\Delta) - 2 + \chi(\mathcal{O}_S)}$. Since $C \cdot F = m$, we have $m < A$.   q. e. d.

Claim 2. *In the above situation, by translating* C *by a suitable automorphism* Φ *of* S, *we may assume that* $0 \leq a_i < m$ *for all* $1 \leq i \leq r$.

Proof of Claim 2. We can consider S as a one-dimensional abelian variety $\mathcal{E}$ over the function field K of Δ, given with a K-rational point o. Then each section s_i $(1 \leq i \leq r)$ defines a K-rational point $\tilde{s}_i$ and let σ_i be a birational mapping of S induced from a translation of $\mathcal{E}$ by $\tilde{s}_i$ over K. Since S is relatively minimal, σ_i is an analytic automorphism of S. We choose integers c_i and δ_i with $a_i = mc_i + \delta_i$, $0 \leq \delta_i < m$ and define an automorphism Φ of S by $\Phi = (\sigma_1^{-1})^{c_{1_0}} \cdots \circ (\sigma_r^{-1})^{c_r}$. Put $\bar{C} = \Phi(C)$. Clearly $\bar{C}$ is also a m-section of S, where we have

$$m = C \cdot F = \alpha + \sum_{i=1}^{r} a_i + \sum_{j=1}^{2} b_j.$$

Put $C' := \left(\alpha + m \sum_{i=1}^{r} c_i\right)(o) + \beta' F + \sum_{i=1}^{r} \delta_i s_i + \sum_{j=1}^{2} b_j t_j + \sum_\lambda \sum_{k=1}^{m_\lambda - 1} d_{\lambda,k} \Theta_{\lambda,k}$. The divisor $\bar{C}$ (resp. C') cuts out on the generic fiber F divisors of degree m and the sum $S(\bar{C})$ (resp. $S(C')$) of points in $\bar{C}$ (resp. C') under the group operation on the abelian group F gives a K-rational point. By our construction, we get $S(\bar{C}) = S(C')$. With the aid of Abel's theorem on an elliptic curve, the divisor $\bar{C}|_F$ is linearly equivalent to $C'|_F$ on F. Hence the divisor $\bar{C} - C'$ is linearly equivalent to a divisor contained in fibers of f and $\bar{C}$ can be written in the form:

$$\bar{C} \approx \left(\alpha + m \sum_{i=1}^{r} c_i\right)(o) + \bar{\beta} F + \sum_{i=1}^{r} \delta_i s_i + \sum_{j=1}^{2} b_j t_j + \sum_\lambda \sum_{k \geq 1} d'_{\lambda,k} \Theta_{\lambda,k},$$

where $0 \leq \delta_i < m$ for all $1 \leq i \leq r$.   q. e. d.

Now, we continue the proof of Proposition 1.2. Since $m = C \cdot F = \alpha + \sum_{i=1}^{r} a_i + \sum_{j=1}^{2} b_j$, it follows from Claim 1 and Claim 2 that α, $a_i (1 \leq i \leq r)$ and $b_j (1 \leq j \leq 2)$ are all bounded by constants determined only by S and g. Since C is a multi-section, we have $0 \leq C \cdot \Theta_{\lambda, k} \leq C \cdot F_\lambda = C \cdot F = m < A(S, g)$ for each λ and $1 \leq k \leq m_\lambda - 1$. We have

$$(*) \qquad (C, \Theta_{\lambda, p}) = \sum_{i=1}^{r} a_i(s_i, \Theta_{\lambda, p}) + \sum_{j=1}^{2} b_j(t_j, \Theta_{\lambda, p}) + \sum_{k=1}^{m_\lambda - 1} d_{\lambda, k}(\Theta_{\lambda, k}, \Theta_{\lambda, p}).$$

Since the intersection matrix $((\Theta_{\lambda, k}, \Theta_{\lambda, p}))_{1 \leq k, p \leq m_\lambda - 1}$ is negative definite and $(C, \Theta_{\lambda, p})$, a_i and b_j are bounded by constants determined only by S and g, it follows from $(*)$ that $d_{\lambda, k}$'s are uniquely determined and bounded by constants depending on S and g. Setting $\Gamma = \alpha(o) + \sum_{i=1}^{r} a_i s_i + \sum_{j=1}^{2} b_j t_j + \sum_\lambda \sum_{k \geq 1} d_{\lambda, k} \Theta_{\lambda, k}$, we have $(\Gamma, F) = m$ and $C^2 = (\Gamma + \beta F)^2 = \Gamma^2 + 2\beta m$. Since (C, K_S) and Γ^2 are bounded by constants determined by S and g, we infer from the above equality that β is also bounded by constants determined only by S and g. q.e.d.

§2. Proof of Theorem A

In this section, we shall prove Theorem A in the case where S has the structure of an elliptic surface with multiple fibers. First, we need the following lemma.

Lemma 2.1. *Let $g : S \to C$ be a relatively minimal algebraic elliptic surface with multiple fibers. Then there exists a curve s_0 with $g(s_0) = C$ such that (s_0, F) divides (D, F) for general fiber F of g and for all curve D with $g(D) = C$.*

Proof. Put $m_0 = \min \{D \cdot F \mid D$ is a curve on S with $g(D) = C\}$ and fix a curve s_0 with $s_0 \cdot F = m_0$ and $g(s_0) = C$ arbitrarily. For any curve D on S with $g(D) = C$, we choose integers t, r with $D \cdot F = m_0 t + r$, $0 \leq r < m_0$. Then $0 \leq (D - m_0 s_0, F) = r < m_0$ and $g_* \mathcal{O}_S(D - m_0 s_0)$ is a coherent $\mathcal{O}_C$-module of rank r. By Serre's theorem, we have $H^0(C, g_* \mathcal{O}_S(D - m_0 s_0) \otimes \mathcal{O}_C(np)) \neq 0$, $p \in C$ and hence $H^0(S, \mathcal{O}_S(D - m_0 s_0 + n S_p)) \neq 0$, $S_p = g^{-1}(p)$ for a sufficiently positive integer n. A general member $\Delta \in |D - m_0 s_0 + n S_p|$ can be written in the form: $\Delta = H + \Gamma$, where H and Γ are effective divisors with $g(\text{supp}(H)) = C$ and $\text{supp}(\Gamma)$ contained in fibers of g. Then $0 \leq r = (\Delta, F) = (H, F) < m_0$ and from the minimality of (s_0, F), we have $H = 0$ and $r = 0$. q.e.d.

Now, we are ready to prove our main Theorem A.

Proof of Theorem A. Clearly, we may assume that S is algebraic. Since $\kappa(S) = 1$, S has the unique structure of an elliptic surface $f : S \to C$ with multiple fibers $m_i E_i$ of multiplicity m_i at $p_i \in C$. $(1 \leq i \leq l)$. Then by Kodaira [2], S can

be obtained from the basic member $g: B \to C$ by twisting and successive logarithmic transformations, that is, $S = L_{p_1} \cdots L_{p_l}(B^\eta)$, where B^η is obtained by twisting $g: B \to C$ by $\eta \in H^1(C, \mathcal{O}(B_0^\#))$.

First, we assume throughout that $g: B \to C$ is not smooth, i.e. there is at least one singular fiber.

Let r be the rank of the Mordell-Weil group $MW(B)$ of $g: B \to C$ and take r generators $s_1, \cdots, s_r$ of $MW(B)$ modulo the torsion group $MW(B)_{tor}$. $MW(B)_{tor}$ is generated by at most two elements t_1, t_2 of order e_1, e_2 with $e_2 | e_1$; $|MW(B)_{tor}| = e_1 e_2$.

By considering B as an one-dimensional abelian variety $\mathcal{E}$ over the function field K of C, each section s_i (resp. t_j) defines a K-rational point $\tilde{s}_i$ (resp. $\tilde{t}_j$). Since S is isomorphic to B^η outside the multiple fibers, $\tilde{s}_i$ (resp. $\tilde{t}_j$) induces an automorphism of $S|_{C \setminus \bigcup_i p_i}$ by locally translating B^η by $\tilde{s}_i$ (resp. $\tilde{t}_j$). And from the definition of logarithmic transformations, we see easily that it can be extended to an automorphism g_i (resp. h_j) of S. We fix a multi-section D_0 of $f: S \to C$ which enjoys the properties as in Lemma 2.1 and put $D_i = g_i(D_0)$, $T_j = h_j(D_0)$. Clearly D_i's and T_j's also enjoys the property in Lemma 2.1. Let $\{F_\lambda\}_{\lambda \in C}$ (resp. F) be the set of all singular fibers of $f: S \to C$ (resp. general fiber of f). For each non-multiple singular fiber F_λ (resp. multiple singular fiber $m_\lambda E_\lambda$), choose an irreducible component $\Theta_{\lambda,0}$ of F_λ (resp. E_λ) of multiplicity one arbitrarily and fix them. Then we have

$$F_\lambda = \Theta_{\lambda,0} + \sum_\lambda \sum_{i \geq 1} \mu_{\lambda,i} \Theta_{\lambda,i}, \ \mu_{\lambda,i} > 0 \qquad (\text{resp. } E_\lambda = \Theta_{\lambda,0} + \sum_{i \geq 1} \Theta_{\lambda,i}),$$

where we denote by $\Theta_{\lambda,i}$ ($0 \leq i \leq n_\lambda - 1$) the irreducible component of F_λ (resp. E_λ), n_λ being the number of the irreducible components of F_λ (resp. E_λ).

Claim 1. $[D_0]$, $[F]$, $[D_i]$ $(1 \leq i \leq r)$ *and* $[\Theta_{\lambda,i}]$*'s* $(\lambda \in C, 1 \leq i \leq n_\lambda - 1)$ *are basis of* $NS(S) \underset{\mathbb{Z}}{\otimes} \mathbb{Q}$.

Proof of Claim 1. For an arbitrary divisor D on S, put $d = (D, F)$, $d_0 = (D_0, F)$. By Lemma 2.1, there exists a positive integer k with $d = d_0 k$. Since $(D - kD_0, F) = 0$, the divisor $D - kD_0$ cuts out on the generic fiber F a divisor δ of degree 0. Since S is isomorphic to B^η outside the multiple fibers, the sum $S(\delta)$ of points in δ under the group operation on the abelian group F gives a section of $g: B|_{C \setminus \bigcup_i p_i} \to C \setminus \bigcup_i p_i$. By Kodaira [3], it can be extended to a holomorphic section s of $g: B \to C$ over C, since B is a basic member. By the Mordell-Weil theorem, we can write:

$$s = \sum_{i=1}^r a_i' s_i + \sum_{j=1}^2 b_j' t_j, \qquad \text{where } a_i \text{ and } 0 \leq b_j < e_j \text{ are integers.}$$

Put $D' = \sum_{i=1}^r a_i'(D_i - D_0) + \sum_{j=1}^2 b_j'(T_j - T_0)$. Since $S((D_i - D_0) \cdot F) = d_0 s_i$ (resp. $S((T_j -$

$D_0) \cdot F) = d_0 t_j$, we see that $S(d_0(D - kD_0) \cdot F) = S(D' \cdot F)$ (resp. $S(e_j(T_j - D_0) \cdot F) = 0$) for a generic fiber F of f.

By Abel's theorem on an elliptic curve, the divisor $d_0(D - kD_0)|_F$ (resp. $e_j(T_j - D_0)|_F$) is linearly equivalent to $D'|_F$ (resp. 0) on F. Hence the divisor $d_0(D - kD_0) - D'$ (resp. $e_j(T_j - D_0)$) is linearly equivalent to a divisor contained in fibers of f. Hence we have:

$$(*) \qquad \mathrm{med}_0 D \approx \mathrm{med}\, D_0 + \beta F + \sum_{i=1}^{r} a_i (D_i - D_0) + \sum_{\lambda} \sum_{k=1}^{n_\lambda - 1} d_{\lambda, k} \Theta_{\lambda, k},$$

where m (resp. e) is the least common multiple of $m_1, \cdots, m_l$ (resp. e_1, e_2) and β, a_i's and $d_{\lambda, k}$'s are all integers. And it is easy to see that D_0, F, D_i's and $\Theta_{\lambda, k}$'s are linearly independent in $\mathrm{NS}(S) \underset{\mathbf{Z}}{\otimes} \mathbf{Q}$. Hence the claim follows.   q. e. d.

Now, let D be an arbitrary curve on S with the fixed arithmetic genus g. We may assume that $f(D) = C$.

Claim 2. $d := (D, F) \in \mathbf{Z}_{\geq 0}$ is bounded above by a constant $A = A(S, g)$, which is determined only by S and g.

Proof of Claim 2. By the canonical bundle formula of Kodaira [2],

$$K_S \approx (2\pi(C) - 2 + \chi(\mathcal{O}_S)) F + \sum_{j=1}^{l} (m_j - 1) E_j.$$

Since $\kappa(S) = 1$.

$$M(S) := 2\pi(C) - 2 + \chi(\mathcal{O}_S) + \sum_{j=1}^{l} \frac{m_j - 1}{m_j} > 0.$$

By Theorem 1.1, there exists a constant $N = N(S, g)$ depending only on S and g with $D \cdot K_S < N$. If we put $A = A(S, g) = \dfrac{N(S, g)}{M(S)}$, we have $d < A$.   q. e. d.

Claim 3. By translating D by a suitable automorphism ρ of S, we may assume that $0 \leq a_i < \mathrm{med}$ for all i in $(*)$, Claim 1.

Proof of Claim 3. By the proof of Claim 2, we have:

$$d_0 D \approx d D_0 + \beta' F + \sum_{i=1}^{l} p_i E_i + \sum_{i=1}^{r} a_i' (D_i - D_0) + \sum_{j=1}^{2} b_j' (T_j - D_0) + \sum_{\lambda} \sum_{k=1}^{n_\lambda - 1} d_{\lambda, k}' \Theta_{\lambda, k},$$

where β, p_i, a_i', b_j' and $d_{\lambda, k}'$ are all integers and $0 \leq p_i < m_i$. We choose integers c_i and δ_i with $a_i' = dc_i + \delta_i$, $0 \leq \delta_i < d$, and define an automorphism ρ of S by $\rho := (g_1^{-1})^{c_1} \cdots (g_r^{-1})^{c_r}$. Put $\bar{D} = \rho(D)$. Then we have $(\bar{D}, F) = d$.

Next, put $D' = d D_0 + \beta' F + \sum_{i=1}^{r} \delta_i (D_i - D_0) + \sum_{j=1}^{2} b_j' (T_j - T_0)$. The divisor $D' - dD_0$ (resp. $d_0(\bar{D} - kD_0)$) cuts out on the generic fiber F a divisor G' (resp. $\bar{G}$) of degree 0 and the sum $S(G')$ (resp. $S(\bar{G})$) of points in G' (resp. $\bar{G}$) under the

group operation gives a section of $g: B \to C$. Since $S(D_i - D_0) = d_0 s_i$, we have:

$$S(\bar{G}) = \sum_{i=1}^{r} a_i'(d_0 s_i) - \sum_{i=1}^{r} c_i(d_0 d s_i) = d_0 \sum_{i=1}^{r} (a_i' - c_i d) s_i$$

$$= \sum_{i=1}^{r} \delta_i(d_0 s_i)$$

$$= S(G').$$

Hence the divisor $d_0 \bar{D} - D' = (d_0 \bar{D} - d D_0) - (D' - d D_0)$ is algebraically equivalent to a linear combination of F, E_i's and $\Theta_{\lambda, k}$'s over $\mathbf{Z}$. Since $0 \leq \delta_i < d$, Claim 3 follows immediately by the same argument as in the proof of Claim 1.   q. e. d.

Now, we continue the proof of Theorem A. We use the same notation as before. By Claim 2 and Claim 3, we have

$$0 \leq a_i < me A(S, g) \qquad \text{for all } 1 \leq i \leq r.$$

Since D_0 and D are multi-sections of f, we have:

$$0 \leq (D, \Theta_{\lambda, p}) \leq (D, F_\lambda) = (D, F) = d < A(S, g) \qquad \text{and}$$

$$0 \leq (D_0, \Theta_{\lambda, p}) \leq (D_0, F_\lambda) = (D_0, F) = d_0$$

for each λ and $1 \leq k \leq n_\lambda - 1$. We have:

$$(**) \quad \text{med}_0 (D, \Theta_{\lambda, p}) = \text{med}(D_0, \Theta_{\lambda, p}) + \sum_{i=1}^{r} a_i(D_i - D_0, \Theta_{\lambda, p}) + \sum_{k=1}^{n_\lambda - 1} d_{\lambda, k}(\Theta_{\lambda, k}, \Theta_{\lambda, p}),$$

since the intersection matrix $(\Theta_{\lambda, k}, \Theta_{\lambda, p})_{1 \leq k, p \leq n_\lambda - 1}$ is negative definite and $(D, \Theta_{\lambda, p})$, $(D_0, \Theta_{\lambda, p})$ and a_i's $(1 \leq i \leq r)$ are all bounded by constants determined only by S and g, $d_{\lambda, k}$'s are also bounded by constants determined by S and g. Setting $\Gamma = \text{med} D_0 + \sum_{i=1}^{r} a_i(D_i - D_0) + \sum_{\lambda} \sum_{k=1}^{n_\lambda - 1} d_{\lambda, k} \Theta_{\lambda, k}$, we have $(\text{med}_0)^2 D^2 = (\Gamma + \beta F)^2$ $= \Gamma^2 + 2\beta \, \text{med}_0$. Since (D, K_S) and Γ^2 are all bounded by constants determined by S and g, we infer from the above equality that β is also bounded by constants determined only by S and g. Hence if we express D as a linear combination of D_0, F, D_i $(1 \leq i \leq r)$ and $\Theta_{\lambda, k}$ $(1 \leq k \leq n_\lambda - 1)$ in $NS(S) \underset{\mathbf{Z}}{\otimes} \mathbf{Q}$, the number of the possible values for each coefficient is bounded by S and g and Theorem A has been proved.

Next, we consider the case where $f: S \to C$ is a Seifert fiber space, that is, S is an elliptic surface with constant moduli which has at most multiple singular fibers. Let D be an arbitrary curve with the fixed arithmetic genus g. We may assume that $f(D) = C$. Then by completely the same method as above, we can show that by translating D by a suitable automorphism ρ of S, we may assume that:

$$mD \approx \alpha D_0 + \beta F, \quad \text{where } m \text{ is the least common multiple of } m_i \text{'s and } \alpha \text{ and } \beta$$

are integers.   Hence we can easily show the boundedness of α and β by Miyaoka's Theorem 1.1.

   Thus we have finished the proof of Theorem A.

References

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