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**Commentarii Mathematici Helvetici**

# Addendum to Projective and Hilbert modules over group algebras, and finitely dominated spaces

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The following two remarks came to the attention of the author after the paper had appeared. They do not affect the validity of the results but they simplify some of the statements. Terminology, Sections, notations etc. refer to the above paper.

The remarks concern the Bass conjecture for a group $G$, as described in Section 2. We here just recall that the Hattori-Stallings rank $r_P$ of a finitely generated projective $\mathbb{Z}G-$module $P$ is a $\mathbb{Z}-$valued function of the conjugacy classes in $G$. The (strong) Bass conjecture (SB) claims that $r_P(x) = 0$ for $x \neq 1 \in G$ and thus $r_P(1) = rk\,P = dim_{\mathbb{R}} \mathbb{R} \otimes_G P$.

**Remark A1.** *Residually finite groups fulfill the strong Bass conjecture (SB).*

Residually finite groups should therefore be added to the list in Section 2.2. The separate treatment of that class of groups is thus unnecessary; it seems, however, to have its own interest, especially in the proof of Proposition 2 (direct reduction to Swan's Theorem).

It was observed by *Guido Mislin* that the proof of **A1** above can be deduced directly from Linnell's Lemma 4.1 (ii) in [L]: If $r_P(x) \neq 0$ on $x \in G,\ x \neq 1$ then $G$ contains a subgroup isomorphic to the additive group of rationals $\mathbb{Q}^+$. But $\mathbb{Q}^+$, and thus $G$, is not residually finite. – On the other hand *I.Emmanouil* (preprint, University of Michigan) has recently exhibited a class of groups containing properly the residually finite ones, fulfilling (SB). His method using cyclic homology (cf. [E]) is entirely different from Linnell's.

**Remark A2.** *The Hattori-Stallings rank $r_P$ always vanishes on finite conjugacy classes* $\neq 1$.

The rôle of finite conjugacy classes in Proposition 2 and 3 remains unchanged. But no special assumption is needed. For the main result however, $\ell_2 G \otimes_G P = \ell_2 G^{rkP}$, the validity of the Bass conjecture is needed.

**A2** follows from the proof of Linnell's Lemma 4.1 (ii). It is shown there that for an element $x \in G$ of infinite order with $r_P(x) \neq 0$ infinitely many powers of $x$ are conjugate to $x$.

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