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Erratum to "The cyclic homology of the group rings"

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Abstract. This erratum corrects the statements of Propositions II and IIp of [Comment. Math. Helv. 60, 354–365 (1985)].

In the paper [1, Propositions II and IIp], both consequences of the main result are not entirely true as stated. The statements become correct provided that the cyclic homology, respectively, periodic cyclic homology are replaced by their reduced versions and the ring R is a $\mathbb{Q}$-algebra, for instance, a field of characteristic zero.[1] Also in [1, line 1, p. 363], in order to make the statement correct, one shall replace "$P_x \neq \{x\}$" by "$N_x \neq \mathbb{Z}_{k(x)}$" with $\mathbb{Z}_k$ denoting the finite cyclic group of order k, and $k(x)$ the largest integer k such that $x = y^k$.

For an arbitrary commutative ring with unit R, [1, Propositions II and IIp] should be corrected as follows:

(1) In [1, Proposition II], HC_* has to be replaced by the reduced version $\widetilde{HC}_*$, and the sentence

"*$R_{\hat{\alpha}} = R$ regarded as a graded module concentrated in the degree zero*",

by

"$R_{\hat{\alpha}} = H_*(B(\mathbb{Z}_{k(x)}); R)$, $x \in \hat{\alpha}$".

(2) In [1, Proposition IIp], PHC_* has to be replaced by its reduced version $P\widetilde{HC}_*$, and in the right-hand side of the equality one should add $\bigoplus_{\hat{x} \in U} T_*(\hat{x}; R)$ with

$$U = \{\hat{x} \in \langle H * G\rangle \mid \hat{x} \cap (e_H * G) = \emptyset, \hat{x} \cap (H * e_G) = \emptyset\},$$

where $\langle \Gamma \rangle$ denotes the set of conjugacy classes of elements of the group Γ and e_Γ the neuter element of Γ.

[1] The reduced cyclic, resp. periodic cyclic, homology of $R[G]$ is quotient of the obvious split injective maps $i_G\colon HC_*(R) \to HC_*(R[G])$, resp. $i_G\colon PHC_*(R) \to PHC(R[G])$, induced by the inclusion $e_G \in G$.

Note that for $x \in \hat{x} \in U$, the group-subgroup pair $(G_x, \{x\})$ is isomorphic to the pair $(\mathbb{Z}, k(x)\mathbb{Z})$, hence $N_x = \mathbb{Z}_{k(x)}$, and

$$T_*(\hat{x}; R) = \begin{cases} H_1(B(\mathbb{Z}_{k(x)}); R) & \text{for } * \text{ odd,} \\ H_2(B(\mathbb{Z}_{k(x)}); R) & \text{for } * \text{ even.} \end{cases}$$

Recall that

$$\begin{aligned} T_*(\hat{x}; R) &\simeq T_*(x; R) \\ &:= \varprojlim_n \big(\cdots \to H_{*+2n}(B(\mathbb{Z}_{k(x)}); R) \xrightarrow{\Sigma} H_{*+2n-2}(B(\mathbb{Z}_{k(x)}); R) \cdots\big), \end{aligned}$$

where Σ is the isomorphism in the homology Gysin sequence of the fibration

$$S^1 = B(k\mathbb{Z}) \to B(Z) \to B(Z_k),$$

which vanishes when R is a $\mathbb{Q}$-algebra.

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References

[1] D. Burghelea, The cyclic homology of the group rings. *Comment. Math. Helv.* **60** (1985), no. 3, 354–365 Zbl 0595.16022 MR 814144

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