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On Cover-Avoiding Subgroups of Sylow Subgroups of Finite Groups

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Dedicated to Professor Shirong Li on the occasion of his seventieth birthday.

ABSTRACT - A subgroup H of a finite group G is called to have the cover-avoidance property in G, H is a CAP subgroup of G in short, if H either covers or avoids every chief factor of G. In the present work, we fix a subgroup D in every Sylow subgroup P of $F^*(G)$ satisfying $1 < |D| < |P|$ and study the structure of G under the assumption that every subgroup H with $|H| = |D|$ has the cover-avoidance property in G. We state our results in the broader context of formation theory.

1. Introduction.

All groups considered in this paper are finite. We use conventional notions and notation, as in Huppert[11]. G always denotes a finite group, $|G|$ the order of G, $\pi(G)$ the set of all primes dividing $|G|$, G_p a Sylow p-subgroup of G for some $p \in \pi(G)$.

A class $\mathcal{F}$ of finite groups is called *a formation* if $G \in \mathcal{F}$ and $N \trianglelefteq G$ then $G/N \in \mathcal{F}$, and if $G/N_i (i = 1, 2) \in \mathcal{F}$ then $G/N_1 \cap N_2 \in \mathcal{F}$. If, in addition, $G/\Phi(G) \in \mathcal{F}$ implies $G \in \mathcal{F}$, we say that $\mathcal{F}$ to be *saturated*. An interesting example of saturated formation is the class of all supersolvable groups, which is denoted by $\mathcal{U}$. $Z_\infty^{\mathcal{U}}(G)$ denotes the $\mathcal{U}$-hypercenter of G, that is, the product of all normal subgroups of G whose G-chief factors are of prime order.

DEFINITION 1.1. *Let L/K be a chief factor of G and H a subgroup of G. We say that*

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i) *H covers* L/K *if* $L \leq HK$;
ii) *H avoids* L/K *if* $L \cap H \leq K$;
iii) *H has the cover-avoidance property in G, H is a CAP subgroup of G in short, if H either covers or avoids every chief factor of G.*

The cover-avoidance property of subgroup was first studied by Gaschütz in [7] to study the solvable groups, later by Gillam [8] and Tomkinson [18]. Clearly normal subgroups are CAP subgroups. Examples of CAP subgroups in the universe of solvable groups are well-known. The most remarkable CAP subgroups of a solvable group are perhaps the Hall subgroups [7]. By a obvious consequence of the definition of supersolvable group every subgroup of supersolvable group is a CAP subgroup. Ezquerro has given the following converse argument (see [[5], Theorem D]): if there exists a normal solvable subgroup E of G such that G/E is supersolvable and all maximal subgroups of any Sylow subgroups of $F(E)$ are CAP subgroups of G, then G is supersolvable. In [1], Asaad said it is possible to extend Ezquerro's result with formation theory. In [14], the authors replaced the Fitting subgroup by the generalized Fitting subgroup so that the hypothesis of the solvability was removed in the result above, that is, Let $\mathcal{F}$ be a saturated formation containing $\mathcal{U}$. Suppose that E is a normal subgroup of G such that $G/E \in \mathcal{F}$. Suppose that all maximal subgroups of the Sylow subgroups of $F^*(E)$, where $F^*(E)$ is the generalized Fitting subgroup of E, are CAP subgroups of G. Then $G \in \mathcal{F}$ ([14], Corollary 4.3). In [21], the authors have given many characterizations of solvable group or p-solvable group or supersolvable group G under assumption that some certain subgroups of G are CAP subgroups of G. In [2], A subgroup H of G is said to be a *strong CAP-subgroup* of G if H is a CAP-subgroup of any subgroup of G containing H. The main result of [2] is as follows: Let $\mathcal{F}$ be a saturated formation containing $\mathcal{U}$ and E a normal subgroup of G such that $G/E \in \mathcal{F}$. Suppose that, for every non-cyclic Sylow subgroup P of $F^*(E)$, P has a subgroup D such that $1 < |D| < |P|$ and all subgroups H of P with order $|H| = |D|$ and with order $|H| = 2|D|$ (if P is a non-abelian 2-group) are strong CAP-subgroups of G. Then $G \in \mathcal{F}$. In this paper, we go further in this direction to give a new result which covers the results mentioned above.

MAIN THEOREM. *Let* $\mathcal{F}$ *be a saturated formation containing* $\mathcal{U}$ *and* E *a normal subgroup of* G *such that* $G/E \in \mathcal{F}$. *Suppose that, for every non-cyclic Sylow subgroup* P *of* $F^*(E)$, P *has a subgroup* D *such that*

$1 < |D| < |P|$ *and all subgroups* H *of* P *with order* $|H| = |D|$ *and with order* $|H| = 2|D|$ *(if* P *is a non-abelian* 2*-group and* $|P : D| > 2$*) satisfy the cover-avoidance property in* G*. Then* $G \in \mathcal{F}$*.*

REMARK 1. Our result, together with the main result in [2], was stimulated by the Skiba's results in the nice paper [17], which is analogous to our main result by replacing the cover-avoidance property by weakly s-permutability. There are some generalizations of Skiba's results, such as that in [13], [15]. However it is easy to find solvable groups with CAP subgroups which are not weakly s-permutable subgroup. Conversely, there are also groups with weakly s-permutable subgroup which are not CAP subgroups (ref. [[5], Example 5]).

Recently, Fan, Guo and Shum introduced the semi-cover-avoidance property [6], which is the generalization not only of the cover-avoidance property but also of *c-normality* [19].

DEFINITION 1.2 ([6]). *A subgroup* H *of* G *is said to have the semi-cover-avoidance property in* G *if there exists a chief series of* G*,*

$$1 = G_0 \leq G_1 \leq \cdots \leq G_n = G,$$

such that H *either covers or avoids chief factor* G_{i+1}/G_i *for any* $i \in \{0, 1, \cdots, n-1\}$*. In this case* H *is called a SCAP subgroup of* G*.*

REMARK 2. In [3], SCAP subgroup was named *partial CAP-subgroup.*

For the sake of convenience of statement, we introduce the following notation.

Let P be a p-subgroup of G for some prime p. We say that P satisfies $\diamondsuit_1$ ($\diamondsuit_2$ respectively) in G if

($\diamondsuit_1$): P has a subgroup D such that $1 < |D| < |P|$ and all subgroups H of P with order $|H| = |D| = d_p$ and with order $|H| = 2|D| = 2d_p$(if P is a non-abelian 2-group and $|P : D| > 2$) satisfy the cover-avoidance property in G.

($\diamondsuit_2$): P has a subgroup D such that $1 < |D| < |P|$ and all subgroups H of P with order $|H| = |D| = d_p$ and with order $|H| = 2|D| = 2d_p$(if P is a non-abelian 2-group and $|P : D| > 2$) satisfy the semi-cover-avoidance property in G.

2. Preliminaries.

LEMMA 2.1 ([16]). *Let S be a CAP subgroup of G and N a normal subgroup of G. Then*

(1) *N is a CAP subgroup of G;*
(2) *SN/N is a CAP subgroup of G/N;*
(3) *SN is a CAP subgroup of G;*
(4) *$S \cap N$ is a CAP subgroup of G.*

LEMMA 2.2 ([6], [10]). *Let S be a SCAP subgroup of G. Then*

(1) *Every CAP subgroup of G is a SCAP subgroup of G;*
(2) *If $S \leq K \leq G$, then S is a SCAP subgroup of K;*
(3) *If $N \leq S$ and $N \trianglelefteq G$, then S/N is a SCAP subgroup of G/N;*
(4) *If $N \trianglelefteq G$ and $(|S|, |N|) = 1$, then SN/N is a SCAP subgroup of G/N.*

LEMMA 2.3. *Every minimal normal subgroup of G is a minimal CAP subgroup of G.*

PROOF. Suppose that N_1 is a proper subgroup of N which is a CAP subgroup of G. Then N_1 either covers or avoids G-chief factor $N/1$. If N_1 covers $N/1$, then $N_1 = N$, a contradiction. If N_1 avoids $N/1$, then $N_1 = N_1 \cap N = 1$. Therefore N is a minimal CAP subgroup of G. □

LEMMA 2.4. *Let P be a normal p-subgroup of G and $\Phi(P) = 1$ for some prime p. If P satisfies $\diamondsuit_1$ in G, then every G-chief factor of P is a cyclic group of prime order, in another word, $P \leq Z_\infty^{\mathcal{U}}(G)$.*

PROOF. Suppose that N is a minimal normal subgroup of G contained in P. Let U be a complement of N in P. It follows from Lemma 2.3 that $|N| \leq |D| = d_p$. So we can pick a maximal subgroup M of N and a subgroup L of U such that $|LM| = d_p$. Hence LM is a CAP subgroup of G by the hypotheses. Thus $M = LM \cap N$ is a CAP subgroup of G by Lemma 2.1 (4). Lemma 2.3 implies that $M = 1$. Therefore $|N| = p$. So $N \leq Z_\infty^{\mathcal{U}}(G)$.

Consider the factor group G/N. If $d_p > p$, then $1 < D/N < P/N$. It follows from Lemma 2.1 (2) that P/N satisfies $\diamondsuit_1$ in G/N. Hence G/N satisfies the hypotheses of the lemma. Therefore $P/N \leq Z_\infty^{\mathcal{U}}(G/N) = Z_\infty^{\mathcal{U}}(G)/N$ by induction. Hence $P \leq Z_\infty^{\mathcal{U}}(G)$, as desired. Now assume that $d_p = p$. Take an arbitrary subgroup T/N of P/N of order p. Since $\Phi(P) = 1$, $T = T_1N$ where T_1 is a subgroup of T with prime order. Hence

T_1 is a CAP subgroup of G by the hypotheses. Thus $T_1N/N = T/N$ is a CAP subgroup of G/N by Lemma 2.1 (2). So P/N satisfies $\diamondsuit_1$ in G/N. Hence $P \le Z_\infty^{\mathcal{U}}(G)$ by induction. □

The generalized Fitting subgroup $F^*(G)$ of G is the unique maximal normal quasinilpotent subgroup of G. Its definition and important properties can be found in ([12], X, 13). We would like to give the following basic facts we will use in our proof.

LEMMA 2.5. *Let G be a group and N a subgroup of G.*

(1) *If N is normal in G, then $F^*(N) \le F^*(G)$;*
(2) $F^*(G) \ne 1$ *if* $G \ne 1$; *in fact,* $F^*(G)/F(G) = Soc(F(G)C_G(F(G))/F(G))$;
(3) $F^*(F^*(G)) = F^*(G) \ge F(G)$; *if $F^*(G)$ is solvable, then $F^*(G) = F(G)$;*
(4) $C_G(F^*(G)) \le F(G)$;
(5) *Let $N = Z(E(G))\Phi(F(G))$. Then $F^*(G/N) = F^*(G)/N$, where $E(G)$ is the layer of G;*
(6) *Suppose that P is a normal subgroup of G contained in $O_p(G)$. Then $F^*(G/\Phi(P)) = F^*(G)/\Phi(P)$.*

PROOF. (1)-(4) please see ([12], X, 13); (5) is ([9], Proposition 4.10). (6) is a corollary of (5). □

LEMMA 2.6. *Let p be the smallest prime dividing the order of $F^*(G)$ and P a Sylow p-subgroup of $F^*(G)$. Suppose that P satisfies $\diamondsuit_2$ in G. Then $F^*(G)$ is solvable.*

PROOF. Assume that the lemma is false and choose G to be a counterexample of the smallest order. By the well-known Odd Order Feit-Thompson Theorem, we have $p = 2$.

Suppose that $F^*(G) < G$. Since P satisfies $\diamondsuit_2$ in G, P satisfies $\diamondsuit_2$ in $F^*(G)$ by Lemma 2.2 (2). The minimal choice of G implies that $F^*(F^*(G)) = F^*(G)$ is solvable, a contradiction. Therefore $F^*(G) = G$. This is to say that G is a quasinilpotent group. If $F(G) \ne 1$, then take a minimal normal subgroup N of G, such that $N \le F(G)$. Then N is an elementary abelian q-group for some prime q. Since the class of quasinilpotent groups is a formation (see Remark IX 2.7 in [4]), then factor group G/N is also quasinilpotent. This is to say that $F^*(G/N) = G/N = F^*(G)/N$. Now if q is odd, then PN/N satisfies $\diamondsuit_2$ in G/N by Lemma 2.2(4) and then G/N is solvable by the minimal choice of G. This implies that G is solvable, a contradiction. Hence $q = 2$. If $d_2 > 2$, then P/N satisfies $\diamondsuit_2$ in G/N by Lemma 2.2(3). The minimal choice of G implies that G/N is solvable and so is G, again a contradiction. Hence $d_2 = 2$. In this case G is 2-nilpotent by Theorem 3.8 of [10]. Since 2-nilpotent groups are solvable by the Odd-Order Feit-Thompson Theorem, we have again a contradiction. Therefore $F(G) = 1$. By Theorem X.13.18 of [12], the group G is a direct product of non-abelian simple group and therefore every chief factor of G is a non-abelian simple group. No 2-subgroup can cover a non-abelian chief factor of G. Thus, if H is a subgroup of P of order d_2, then H avoids every chief factor of some chief series of G. This is to say that $d_2 = 1$. This is the final contradiction. □

3. The Proof of Main Theorem.

Assume that the theorem is false and choose G to be a counterexample of the smallest order.

(1) $F^*(E)$ is solvable and $F^*(E) = F(E)$.

Let p be the smallest prime dividing the order of $F^*(E)$ and P a Sylow p-subgroup of $F^*(E)$. If P is cyclic, then $F^*(G)$ is p-nilpotent by ([11], IV, Satz 2.8). So $F^*(E)$ is solvable and $F^*(E) = F(E)$. Hence assume that P is non-cyclic. By the hypotheses, P satisfies $\diamondsuit_1$ in G. Hence P satisfies $\diamondsuit_2$ in G by Lemma 2.2(1). Applying Lemma 2.6, we have $F^*(E)$ is solvable and $F^*(E) = F(E)$.

(2) There exists a prime $q \in \pi(F^*(E))$ such that some G-chief factors L/K of $O_q(E)$ are not of prime order. Furthermore, $O_q(E)$ is not cyclic and $\Phi(O_q(E)) \neq 1$.

If otherwise, then we have $F^*(E) = F(E) \leq Z_{\infty}^{\mathcal{U}}(G)$. Then $G/C_G(F^*(E)) \in$ $\in \mathcal{F}$ by Theorem 6.10 of Chapter IV in [4]. It follows that $G/C_E(F^*(E)) =$ $= G/E \cap C_G(F^*(E)) \in \mathcal{F}$. Thus $G/F(E) \in \mathcal{F}$ by Lemma 2.5 (4). Since every G-chief factor of $F(E)$ is of prime order and $\mathcal{F}$ contains the class of all supersolvable groups, $G \in \mathcal{F}$ which is a contradiction.

By Lemma 2.4, we have $\Phi(O_q(E)) \neq 1$.

(3) Suppose that $\Phi(O_q(E)) \neq 1$ for some $q \in \pi(F^*(E))$. Take a minimal normal subgroup N of G contained in $\Phi(O_q(E))$. Then $|N| = |D| = d_q$.

By Lemma 2.3 we have $|N| \leq |D|$. If $|D| > |N|$, then consider the factor group G/N. Then $F^*(E/N) = F^*(E)/N$ by Lemma 2.5 (6). Now Lemma 2.1

(2) implies that $(G/N, E/N)$ satisfy the hypotheses of the theorem. It follows that $G/N \in \mathcal{F}$ by the minimal choice of G. So $G \in \mathcal{F}$, a contradiction. Therefore $|N| = |D| = d_q$.

(4) If $\Phi(O_q(E)) \neq 1$, then $|D| = d_q = q$.

Take a minimal normal subgroup N of G contained in $\Phi(O_q(E))$. Then $|N| = |D| = d_q$ by (3). Now we claim that $|N| = d_q = q$.

Pick a maximal subgroup M of N such that $M \trianglelefteq O_q(E)$. If there is another minimal subgroup T/M of $O_q(E)/M$ different to N/M. Then $|T| = |N| = d_q$. Hence T is a CAP subgroup of G by hypotheses. Thus $M = T \cap N$ is also a CAP subgroup of G by Lemma 2.1 (4). Hence $|N| = q$ by Lemma 2.3, as desired. So we assume that N/M is the unique minimal subgroup of $O_q(E)/M$. Take a minimal normal subgroup L/N of $O_q(E)/N$. Then $|L/M| = q^2$ and N/M is also the unique minimal subgroup of L/M. Therefore L/M is a cyclic group of order q^2. Denote $L/M = \langle aM \rangle$. Since $a^q \in \Phi(L) \leq N$ and $exp(N) = q$, we have $L = M\langle a \rangle$ and $o(a) = q^2$. If $\Phi(L) = N$, then $L = M\langle a \rangle = \langle a \rangle$. Hence $N = \langle a^q \rangle$ is of prime order. So we assume that $\Phi(L) < N$. Since L is normal in $O_q(E)$, $\Phi(L)$ is normal in $O_q(E)$. Hence we can pick a maximal subgroup M_1 of N containing $\Phi(L)$ such that $M_1 \trianglelefteq O_q(E)$. Since $a^q \in \Phi(L) \leq M_1$, we have $|M_1\langle a \rangle| = |N| = d_q$. By the hypotheses, $M_1\langle a \rangle$ is a CAP subgroup of G. Then $M_1 = N \cap M_1\langle a \rangle$ is a CAP subgroup of G by Lemma 2.1 (4). It follows from Lemma 2.3 that $|N| = q$.

(5) Final contradiction.

By (2) we know that there exists a prime $q \in \pi(F^*(E))$ such that some G-chief factors L/K of $O_q(E)$ are not of prime order. We may choose L/K satisfying that every G-chief factor T/W is of prime order with $T < L$.

If there is some element $x \in L \backslash K$ with $o(x) = q$ or $o(x) = 4$, then $\langle x \rangle$ is a CAP subgroup of G by the hypotheses of the theorem and (4). It follows from Lemma 2.1 (2) that $\langle x \rangle K/K$ is a CAP subgroup of G/K. By Lemma 2.3, we have $L/K = \langle x \rangle K/K$ which contradicts with the choice of L/K. Therefore all cyclic subgroups of order q or order 4 of L are contained in K. Suppose that we have a G-chief series of $O_q(E)$ as follows

$$1 = U_0 \trianglelefteq U_1 \trianglelefteq \cdots \trianglelefteq U_n = K \trianglelefteq L.$$

Then $|U_i/U_{i-1}| = q$ for $i \in \{1, 2, \cdots, n\}$ by the choice of L/K. Thus $G/C_G(U_i/U_{i-1})$ is an Abelian group of exponent dividing $q-1$. Denote

$$X = \bigcap_{i=1}^{n} C_G(U_i/U_{i-1}).$$

Then G/X is an Abelian group of exponent dividing $q-1$. Let R be a q'-subgroup of X. Then R centralizes K and so R centralizes all elements of L of order q and order 4. Then R centralizes L by Satz IV.5.12 of [11]. Thus $X/C_X(L/K)$ is a q-group. Then $XC_G(L/K)/C_G(L/K)$ is a normal q-subgroup of $G/C_G(L/K)$. Hence $XC_G(L/K)/C_G(L/K) = 1$ by Corollary 6.4 of [20] (page 221). So $X \leq C_G(L/K)$. This implies that G/X acts irreducibly on L/K. So $|L/K| = q$ by Lemma 1.3 of [20] (page 3), the final contradiction.

The proof of the theorem is now complete. □

4. Final Remarks.

Many known results are special cases of our Main Theorem and one also can obtain several new results from our Main Theorem.

COROLLARY 4.1. *Let $\mathcal{F}$ be a saturated formation containing $\mathcal{U}$ and E a normal subgroup of G such that $G/E \in \mathcal{F}$. Suppose for any every non-cyclic Sylow subgroup P of $F^*(E)$ at least one of the following holds:*

(1) *every maximal subgroup of P is a CAP subgroup of G;*
(2) *every cyclic subgroup of P of prime order or order* 4 *(if P is a* 2-*group) is a CAP subgroup of G.*

Then $G \in \mathcal{F}$.

COROLLARY 4.2 ([14]). *Let $\mathcal{F}$ be a saturated formation containing $\mathcal{U}$ and E a normal subgroup of G such that $G/E \in \mathcal{F}$. Suppose for any every non-cyclic Sylow subgroup P of $F^*(E)$, every maximal subgroup of P is a CAP subgroup of G. Then $G \in \mathcal{F}$.*

COROLLARY 4.3. *Let $\mathcal{F}$ be a saturated formation containing $\mathcal{U}$ and E a normal subgroup of G such that $G/E \in \mathcal{F}$. Suppose for any every non-cyclic Sylow subgroup P of $F^*(E)$, every cyclic subgroup of P of prime order or order* 4 *(if P is a* 2-*group) is a CAP subgroup of G. Then $G \in \mathcal{F}$.*

COROLLARY 4.4 ([1]). *Let $\mathcal{F}$ be a saturated formation containing $\mathcal{U}$ and E a solvable normal subgroup of G such that $G/E \in \mathcal{F}$. Suppose that all maximal subgroups of non-cyclic Sylow subgroups of $F(E)$ are CAP subgroups of G. Then $G \in \mathcal{F}$.*

COROLLARY 4.5. *Let $\mathcal{F}$ be a saturated formation containing $\mathcal{U}$ and E a solvable normal subgroup of G such that $G/E \in \mathcal{F}$. Suppose that all minimal subgroups and cyclic subgroup of order 4 of non-cyclic Sylow subgroups of $F(E)$ are CAP subgroups of G. Then $G \in \mathcal{F}$.*

COROLLARY 4.6 ([5]). *Let E be a normal subgroup of G such that G/E is supersolvable. Then G is supersolvable if E is solvable and all maximal subgroups of the Sylow subgroups of $F(E)$ are CAP subgroups of G.*

COROLLARY 4.7 ([2], Theorem A). *Let $\mathcal{F}$ be a saturated formation containing $\mathcal{U}$ and E a normal subgroup of G such that $G/E \in \mathcal{F}$. Suppose that, for every non-cyclic Sylow subgroup P of $F^*(E)$, P has a subgroup D such that $1 < |D| < |P|$ and all subgroups H of P with order $|H| = |D|$ and with order $|H| = 2|D|$(if P is a non-abelian 2-group) are strong CAP-subgroups of G. Then $G \in \mathcal{F}$.*

Naturally we have following question:

QUESTION. *Let $\mathcal{F}$ be a saturated formation containing $\mathcal{U}$ and E a normal subgroup of G such that $G/E \in \mathcal{F}$. Suppose that every non-cyclic Sylow subgroup P of $F^*(E)$ satisfies $\diamondsuit_2$ in G. Is then G in $\mathcal{F}$?*

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