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Lie Ideals and Jordan Triple Derivations in Rings

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ABSTRACT - In this paper we prove that on a 2-torsion free semiprime ring R every Jordan triple (resp. generalized Jordan triple) derivation on Lie ideal L is a derivation on L(resp. generalized derivation on L) .

1. Introduction.

This paper is motivated by the work of Jing and Lu [13]. Throughout the present paper, R will denote an associative ring with center $Z(R)$. A ring R is said to be 2-torsion free, if whenever $2x = 0$ with $x \in R$, implies $x = 0$. Recall that R is prime if for any $a, b \in R$, $aRb = \{0\}$ implies that $a = 0$ or $b = 0$, and R is semiprime if for all $a \in R$, $aRa = \{0\}$ implies $a = 0$. An additive subgroup L of R is said to be a Lie ideal of R if $[L, R] \subseteq L$. A Lie ideal L is said to be square-closed if $a^2 \in L$ for all $a \in L$. An additive mapping $\delta : R \longrightarrow R$ is called a derivation (resp. a Jordan derivation) if $(xy)^\delta = x^\delta y + xy^\delta$(resp. $(x^2)^\delta = x^\delta x + xx^\delta$) holds for all $x, y \in R$. A famous result due to Herstein [11, Theorem 3.3] states that a Jordan derivation of a prime ring of characteristic not equal to 2 must be a derivation. A brief proof of Herstein's result can also be found in [8]. This result was extended to 2-torsion free semiprime rings by Cusack [10] and subsequently, by Bresar [7] .

Following [6], an additive mapping $\delta : R \longrightarrow R$ is called a Jordan triple derivation if $(xyx)^\delta = x^\delta yx + xy^\delta x + xyx^\delta$ holds for all $x, y \in R$. One can

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easily prove that any Jordan derivation on an arbitrary 2-torsion free ring is a Jordan triple derivation (see for example [8] where further references can be found). Bresar has proved the following result.

THEOREM 1.1 (6, Theorem 4.3). *Let R be a 2-torsion free semiprime ring and let $\delta : R \to R$ be a Jordan triple derivation. In this case δ is a derivation.*

An additive mapping $F : R \longrightarrow R$ is said to be a generalized derivation (resp. a generalized Jordan derivation) on R if there exists a derivation $\delta : R \longrightarrow R$ such that $(xy)^F = x^F y + xy^\delta$(resp. $(x^2)^F = x^F x + xx^\delta$) holds for all $x, y \in R$. An additive mapping $F : R \longrightarrow R$ is said to be a generalized Jordan triple derivation on R if there exists a Jordan triple derivation $\delta : R \longrightarrow R$ such that $(xyx)^F = x^F yx + xy^\delta x + xyx^\delta$ holds for all $x, y \in R$. Motivated by the above result Jing and Lu have proved the following generalization of Theorem 1.1.

THEOREM 1.2 (13, Theorem 3.5). *Let R be a 2-torsion free prime ring, then every generalized Jordan triple derivation on R is a generalized derivation.*

Very recently, Vukman [18] extended the result mentioned above for 2-torsion free semiprime rings. In the present paper, our objective is to generalize Theorem 1.1 and 1.2 on Lie ideal of R.

2. Jordan Triple Derivations.

It is obvious to see that evey derivation is a Jordan triple derivation, but the converse need not to be true in general. Motivated by the Theorem 1.1, in the present section, it is shown that on a 2-torsion free semiprime ring R every Jordan triple derivation on Lie ideal L is a derivation on L. More precisely, we prove the following:

THEOREM 2.1. *Let R be a 2-torsion free semiprime ring and $L \not\subseteq Z(R)$ be a square-closed Lie ideal of R. If an additive mapping $\delta : R \longrightarrow R$ satisfies*

$$(aba)^\delta = a^\delta ba + ab^\delta a + aba^\delta \text{ for all } a, b \in L$$

and $L^\delta \subseteq L$, then δ is a derivation on L.

To facilitate our discussion, we shall begin with the following lemmas:

LEMMA 2.1 (4, Lemma 4). *Let $L \not\subseteq Z(R)$ be a Lie ideal of a 2-torsion free prime ring R and $a, b \in R$. If $aLb = \{0\}$, then $a = 0$ or $b = 0$.*

LEMMA 2.2 (17, Lemma 2.4). *Let R be a 2-torsion free semiprime ring, L be a Lie ideal of R such that $L \not\subseteq Z(R)$ and let $a \in L$. If $aLa = \{0\}$, then $a^2 = 0$ and there exisits a nonzero ideal $K = R[L, L]R$ of R generated by $[L, L]$ such that $[K, R] \subseteq L$ and $Ka = aK = \{0\}$.*

COROLLARY 2.1. *Let R be a 2-torsion free semiprime ring, L be a Lie ideal of R such that $L \not\subseteq Z(R)$ and let $a, b \in L$.*

(1) *If $aLa = \{0\}$, then $a = 0$.*
(2) *If $aL = \{0\}$ (or $La = \{0\}$), then $a = 0$.*
(3) *If L is square-closed, and $aLb = \{0\}$, then $ab = 0$ and $ba = 0$.*

PROOF. (1) Since $Ka = R[L, L]Ra = \{0\}$ and $a^2 = 0$ by Lemma 2.2, we have $0 = [[a, x], a]ya = axaya$ for all $x, y \in R$, and so, we have $axayaxa = 0$. Since R is semiprime, we get $axa = 0$ for all $x \in R$. Moreover, by semiprimeness of R, we get $a = 0$.

(2) It is clear by (1).
(3) If $aLb = \{0\}$, then we have $baLba = \{0\}$ obviously, and so we have $ba = 0$ by (1). Moreover, since $abLab \subseteq aLb = \{0\}$, we get $ab = 0$.

LEMMA 2.3 (17, Lemma 2.5). *Let R be a 2-torsion free ring, L be a Lie ideal of R and let $a, b \in L$. If $aub + bua = 0$ for all $u \in L$, then $aubLaub = \{0\}$.*

LEMMA 2.4 (17, Lemma 2.7). *Let $G_1, G_2, \cdots, G_n$ be additive groups, R a 2-torsion free semiprime ring and $L \not\subseteq Z(R)$ be a Lie ideal of R. Suppose that mappings $S : G_1 \times G_2 \times \cdots \times G_n \longrightarrow R$ and $T : G_1 \times G_2 \times \cdots \times G_n \longrightarrow R$ are addtitive in each argument. If $S(a_1, a_2, \cdots, a_n)xT(a_1, a_2, \cdots, a_n) = 0$ for all $x \in L$, $a_i \in G_i$ $i = 1, 2, \cdots n$, then $S(a_1, a_2, \cdots, a_n)xT(b_1, b_2, \cdots, b_n) = 0$ for all $x \in L$, $a_i, b_i \in G_i$ $i = 1, 2, \cdots n$.*

LEMMA 2.5. *Let δ be a Jordan triple derivation and L be a Lie ideal of R. For arbitrary $a, b, c \in L$, we have*

$$(abc + cba)^{\delta} = a^{\delta}bc + ab^{\delta}c + abc^{\delta} + c^{\delta}ba + cb^{\delta}a + cba^{\delta}.$$

PROOF. We have

$$(aba)^\delta = a^\delta ba + ab^\delta a + aba^\delta \quad \text{for all } a, b \in L. \tag{2.1}$$

We compute $W = \{(a+c)b(a+c)\}^\delta$ in two different ways. On one hand, we find that $W = (a+c)^\delta b(a+c) + (a+c)b^\delta(a+c) + (a+c)b(a+c)^\delta$, and on the other hand $W = (aba)^\delta + (abc+cba)^\delta + (cbc)^\delta$. Comparing two expressions, we obtain the required result.

REMARK 2.1. It is easy to see that every Jordan derivation of a 2-torsion free ring satisfies (2.1) (see [1] for reference).

For the purpose of this section, we shall write; $\Delta(abc) = (abc)^\delta - a^\delta bc - ab^\delta c - abc^\delta$, and $\Lambda(abc) = abc - cba$. We list a few elementary properties of δ and Λ:

(i) $\Delta(abc) + \Delta(cba) = 0$
(ii) $\Delta((a+b)cd) = \Delta(acd) + \Delta(bcd)$ and $\Lambda((a+b)cd) = \Lambda(acd) + \Lambda(bcd)$
(iii) $\Delta(a(b+c)d) = \Delta(abd) + \Delta(acd)$ and $\Lambda(a(b+c)d) = \Lambda(abd) + \Lambda(acd)$
(iv) $\Delta(ab(c+d)) = \Delta(abc) + \Delta(abd)$ and $\Lambda(ab(c+d)) = \Lambda(abc) + \Lambda(abd)$

PROPOSITION 2.2. *Let R be a 2-torsion free semiprime ring and $L \not\subseteq Z(R)$ be a square closed Lie ideal of R. If $\Delta(abc) = 0$ holds for all $a, b, c \in L$, then δ is a derivation of L.*

PROOF. Let assume $\Delta(abc) = 0$ for all $a, b, c \in L$, that is,

$$(abc)^\delta = a^\delta bc + ab^\delta c + abc^\delta.$$

Let $M = abxab$. We have

$$\begin{aligned} M^\delta = \{a(bxa)b\}^\delta &= (a)^\delta(bxa)b + ab^\delta xab + abx^\delta ab \\ &+ (abx)a^\delta b + (abxa)b^\delta \quad \text{for all } x, a, b \in L. \end{aligned} \tag{2.2}$$

On the other hand,

$$M^\delta = \{(ab)x(ab)\}^\delta = (ab)^\delta(xab) + (ab)x^\delta(ab) + (abx)(ab)^\delta. \tag{2.3}$$

Comparing (2.3) with (2.2) we get

$$\{(ab)^\delta - a^\delta b - ab^\delta\}(xab) + (abx)\{(ab)^\delta - a^\delta b - ab^\delta\} = 0$$

that is, $a^bxab + abxa^b = 0$, where a^b stands for $(ab)^\delta - a^\delta b - ab^\delta$. Thus, by Lemma 2.3 we find that $(a^bxab)y(a^bxab) = 0$ for all $a, b, x, y \in L$. Now, by

Corollary 2.1, we obtain $a^b xab = 0$ for all $a, b, x \in L$. Now by Lemma 2.4, we obtain $a^b xcd = 0$ for all $a, b, c, d, x \in L$. Hence, by using Corollary 2.1, we get $a^b = 0$ for all $a, b \in L$ that is, δ is a derivation on L.

LEMMA 2.6. *Let L be a Lie ideal of R. For any $a, b, c, x \in L$, we have*

$$\Delta(abc)x\Lambda(abc) + \Lambda(abc)x\Delta(abc) = 0.$$

PROOF. For any $a, b, c, x \in L$, suppose that $N = abcxcba + cbaxabc$. Now we find

$$\begin{aligned} N^\delta &= \{a(bcxcb)a + c(baxab)c\}^\delta = \{a(bcxcb)a\}^\delta + \{(c(baxab)c\}^\delta \\ &= a^\delta bcxcba + ab^\delta cxcba + abc^\delta xcba \\ &\quad + abcx^\delta cba + abcxc^\delta ba + abcxcb^\delta a \\ &\quad + abcxcba^\delta + c^\delta baxabc + cb^\delta axabc \\ &\quad + cba^\delta xabc + cbax^\delta abc + cbaxa^\delta bc \\ &\quad + cbaxab^\delta c + cbaxabc^\delta. \end{aligned}$$

On the other hand, we have

$$\begin{aligned} N^\delta &= \{(abc)x(cba) + (cba)x(abc)\}^\delta \\ &= (abc)^\delta x(cba) + (abc)x^\delta(cba) + (abcx)(cba)^\delta \\ &\quad + (cba)^\delta(xabc) + (cba)x^\delta(abc) + (cbax)(abc)^\delta. \end{aligned}$$

By comparing last two expressions, we get

$$-\Delta(cba)(xcba) + \Delta(cba)(xabc) + (abcx)\Delta(cba) - (cbax)\Delta(cba) = 0.$$

This implies that $\Delta(abc)x\Lambda(abc) + \Lambda(abc)x\Delta(abc) = 0$ for all $a, b, c \in L$.

LEMMA 2.7. *Let R be a semiprime ring and $L \not\subseteq Z(R)$ be a square closed Lie ideal of R. Then $\Delta(abc)x\Lambda(rst) = 0$ holds for all $a, b, c, r, s, t, x \in L$.*

PROOF. By Lemma 2.6 we have $\Delta(abc)x\Lambda(abc) + \Lambda(abc)x\Delta(abc) = 0$ for all $a, b, c, x \in L$. Thus, we get $\Delta(abc)x\Lambda(abc)L\Delta(abc)x\Lambda(abc) = \{0\}$ by Lemma 2.3, and hence we obtain $\Delta(abc)x\Lambda(abc) = 0$, for all $a, b, c, x \in L$ by Corollary 2.1. Now, we find that $\Delta(abc)x\Lambda(rst) = 0$, for all $a, b, c, r, s, t, x \in L$ by Lemma 2.4.

LEMMA 2.8. *Let R be a 2-torsion free semiprime ring and L be a square closed Lie ideal of R. If $\Lambda(abc) = 0$ for all $a, b, c \in L$, then $L \subseteq Z(R)$.*

PROOF. Assume that $L \not\subseteq Z(R)$. We have $\Lambda(abc) = 0$ for all $a, b, c \in L$, that is, $abc = cba$. Replacing b by $2tb$, we get $2atbc = 2ctba$ for all

$a, b, c, t \in L$. Again replacing t by $2tw$ and using the fact that R is 2-torsion free, we get $atwbc = ctwba$ and hence $a(twb)c = a(bwt)c = a(bc)tw = awtbc$. Thus, we find that $a[t, w]bc = 0$ for all $a, b, c, t, w \in L$. By Corollaray 2.1, we get $[t, w] = 0$ for all $t, w \in L$, that is, L is a commutative Lie ideal of R. And so, we have $[a, [a, t]] = 0$ for all $t \in R$ and hence by Sublemma on page 5 of [11], $a \in Z(R)$. Hence $L \subseteq Z(R)$, a contradiction. This completes the proof of the Lemma.

Proof of Theorem 2.1. Suppose $\delta : R \longrightarrow L$ is a Jordan triple derivation on L. Our goal will be to show that δ is a derivation of associative triple systems. We know that $\Delta(abc) = -\Delta(cba)$. Hence

$$\begin{aligned} 2\Delta(abc)x\Delta(abc) &= \Delta(abc)x\{\Delta(abc) - \Delta(cba)\} \\ &= \Delta(abc)x\{\Lambda(abc)^{\delta} + \Lambda(c^{\delta}ba) \\ &\quad + \Lambda(cb^{\delta}a) + \Lambda(cba^{\delta})\}. \end{aligned}$$

By Lemma 2.7, the above relation reduces to

$$2\Delta(abc)x\Delta(abc) = \Delta(abc)x\Lambda(abc)^{\delta} \tag{2.4}$$

Similarly we obtain

$$2\Delta(abc)x\Delta(abc) = \Lambda(abc)^{\delta}x\Delta(abc) \tag{2.5}$$

Now we have

$$\begin{aligned} 0 &= \{\Delta(abc)x\Lambda(abc) + \Lambda(abc)x\Delta(abc)\}^{\delta} \\ &= \Delta(abc)^{\delta}x\Lambda(abc) + \Delta(abc)x^{\delta}\Lambda(abc) \\ &\quad + \Delta(abc)x\Lambda(abc)^{\delta} + \Lambda(abc)^{\delta}x\Delta(abc) \\ &\quad + \Lambda(abc)x^{\delta}\Delta(abc) + \Lambda(abc)x\Delta(abc)^{\delta} \end{aligned}$$

and according to Lemma 2.6 and (2.4), and (2.5), we get

$$0 = 4\Delta(abc)x\Delta(abc) + \Delta(abc)^{\delta}x\Lambda(abc) + \Lambda(abc)x\Delta(abc)^{\delta}.$$

We multiply the relation above from left by $\Delta(abc)x\Delta(abc)y$ for all a, b, c, x, $y \in L$ and by Lemma 2.7 we obtain $4\Delta(abc)x\Delta(abc)y\Delta(abc)x\Delta(abc) = 0$ for all $a, b, c, x, y \in L$. Since R is a 2-torsion free, it follows that $\Delta(abc)x\Delta(abc)y\Delta(abc)x\Delta(abc) = 0$, that is, $\Delta(abc)x\Delta(abc)L\Delta(abc)x\Delta(abc) = \{0\}$. By Corollary 2.1, we find that $\Delta(abc)x\Delta(abc) = 0$ and again using Corollary 2.1, we find that $\Delta(abc) = 0$ for all $a, b, c \in L$ and hence by Proposion 2.1, we get the required result.

THEOREM 2.2. *Let R be a 2-torsion free prime ring and $L \not\subseteq Z(R)$ be a nonzero square-closed Lie ideal of R. If an additive mapping $\delta : R \longrightarrow L$ satisfies that*

$$(aba)^{\delta} = a^{\delta}ba + ab^{\delta}a + aba^{\delta} \text{ for all } a, b \in L,$$

then δ is a derivation on L.

PROOF. By Lemma 2.7, we have $\Delta(abc)x\Delta(rst) = 0$ for all $a, b, c, r, s, t, x \in L$, that is, $\Delta(abc)L\Delta(rst) = \{0\}$. Now, either $\Delta(abc) = 0$ or $\Delta(rst) = 0$ by Lemma 2.1. If $\Delta(rst) = 0$ for all $r, s, t \in L$, then we get $L \subseteq Z(R)$ by Lemma 2.8, a contradiction. On the other hand if $\Delta(abc) = 0$ for all $a, b, c \in L$, then we get the required result by Proposition 2.1.

3. Generalized Jordan Triple Derivations.

An additive mapping $\mu : R \to R$ is called a Jordan triple left centralizer on L if $(aba)^{\mu} = a^{\mu}ba$ for all $a, b \in L$, and called a Jordan left centralizer on L if $(a^2)^{\mu} = a^{\mu}a$ for all $a \in L$.

THEOREM 3.1. *Let R be a 2-torsion free semiprime ring and $L \not\subseteq Z(R)$ be a square-closed Lie ideal. If $\mu : R \to R$ is a Jordan triple left centralizer on L, then μ is a Jordan left centralizer on L.*

PROOF. By the hypothesis, we have

$$(aba)^{\mu} = a^{\mu}ba \text{ for all } a, b \in L. \tag{3.1}$$

Replacing a by $a + c$ in (3.1), we find that

$$\{(a + c)b(a + c)\}^{\mu} = a^{\mu}ba + c^{\mu}ba + a^{\mu}bc + c^{\mu}bc \text{ for all } a, b, c \in L.$$

On the other hand, we obtain

$$\{(a + c)b(a + c)\}^{\mu} = (abc + cba)^{\mu} + a^{\mu}bc + c^{\mu}ba \text{ for all } a, b, c \in L.$$

Combining last two expressions, we get

$$(abc + cba)^{\mu} = a^{\mu}bc + c^{\mu}ba \text{ for all } a, b, c \in L. \tag{3.2}$$

Now replacing c by a^2 in (3.2), we get

$$(aba^2 + a^2ba)^{\mu} = a^{\mu}ba^2 + (a^2)^{\mu}ba. \tag{3.3}$$

Again replacing b by $ab + ba$ in (3.1), we get

$$(aba^2 + a^2ba)^\mu = a^\mu aba + a^\mu ba^2. \tag{3.4}$$

Combining (3.3) and (3.4), we obtain

$$\{(a^2)^\mu - a^\mu a\}ba = 0 \text{ for all } a, b \in L.$$

By setting $\Omega^a = (a^2)^\mu - a^\mu a$, we have

$$\Omega^a ba = 0 \text{ for all } a, b \in L. \tag{3.5}$$

Thus, by Corollary 2.1, we obtain

$$\Omega^a a = 0 = a\Omega^a \text{ for all } a \in L. \tag{3.6}$$

Linearizing (3.6), we get

$$\Omega^{a+b} a + \Omega^{a+b} b = 0 \text{ for all } a, b \in L. \tag{3.7}$$

Now, we compute $\Omega^{a+b} = \{(ab + ba)^\mu - a^\mu b - b^\mu a\} + (a^2)^\mu - a^\mu a + + (b^2)^\mu - b^\mu b$ and hence we have

$$\Omega^{a+b} = \Phi(a, b) + a^\mu + b^\mu \text{ for all } a, b \in L, \tag{3.8}$$

where $\Phi(a, b) = (ab + ba)^\mu - a^\mu b - b^\mu a$. Thus, in view of (3.8), the expression (3.7) implies that

$$\Omega^a b + \Phi(a, b)a + \Omega^b a + \Phi(a, b)b = 0. \tag{3.9}$$

Again, replacing a by $-a$ in (3.9), we get

$$\Omega^a b + \Phi(a, b)a - \Omega^b a - \Phi(a, b)b = 0. \tag{3.10}$$

Adding (3.9) with (3.10), we find that

$$\Omega^a b + \Phi(a, b)a = 0. \tag{3.11}$$

On right multiplication of (3.11) by Ω^a, we obtain

$$0 = \Omega^a b\Omega^a + \Phi(a, b)a\Omega^a = \Omega^a b\Omega^a.$$

Thus, we obtain $\Omega^a = 0$ for all $a \in L$ by Corollary 2.1. This gives that μ is a Jordan left centralizer.

THEOREM 3.2. *Let R be a 2-torsion free semiprime ring and $L \not\subseteq Z(R)$ be a square-closed Lie ideal. If $F : R \to R$ is a generalized Jordan triple derivation on L with a Jordan triple derivation δ such that $L^\delta \subseteq L$, then F is a generalized Jordan derivation on L.*

PROOF. Since F is a generalized Jordan triple derivation on L. Therefore we have

$$(aba)^F = a^F ba + ab^\delta a + aba^\delta \text{ for all } a, b \in L. \tag{3.12}$$

In (3.12), we take δ as a Jordan triple derivation on L. Since R is a 2-torsion free semiprime ring, so in view of Theorem 2.1 δ is a derivation on L. Now we write $\Gamma = F - \delta$. Then we have

$$\begin{aligned}(aba)^\Gamma &= (aba)^{F-\delta}\\ &= (aba)^F - (aba)^\delta\\ &= (a^F - a^\delta)ba = a^{F-\delta}ba = a^\Gamma ba \text{ for all } a, b \in L.\end{aligned}$$

And so, we have $(aba)^\Gamma = a^\Gamma ba$ for all $a, b \in L$. In otherwords, Γ is a Jordan triple left centralizer on L. Since R is a 2-torsion free semiprime ring, one can conclude that Γ is a Jordan left centralizer by Theorem 3.1. Hence F is of the form $F = \Gamma + \delta$, where δ is a derivation and Γ is a Jordan left centralizer on L. Hence F is a generalized Jordan derivation on L.

THEOREM 3.3. *Let R be a 2-torsion free prime ring and $L \not\subseteq Z(R)$ be a square-closed Lie ideal. If $F : R \to R$ is a generalized Jordan triple derivation on L, then F is a generalized derivation on L.*

PROOF. By Theorem 3.2 and Theorem of [2] we get the required result.

In conclusion, it is tempting to conjecture as follows:

CONJECTURE 3.1. *Let R be a 2-torsion free semiprime ring and $L \not\subseteq Z(R)$ be a Lie ideal of R. If $F : R \to R$ is a generalized Jordan triple derivation on L, then F is a generalized derivation on L.*

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