

Erratum to: “A categorification of quantum $\mathfrak{sl}(n)$ ”

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We thank Marco Mackaay, Marko Stošić and Pedro Vaz for pointing out a sign error in Lemma 6.4. It should read as follows:

Lemma 6.4. *For $i, j \in I$ with $i \neq j$*

$$\Gamma \left(\begin{array}{c} i \quad j \\ \downarrow \quad \downarrow \\ \text{Diagram 1} \\ \downarrow \quad \downarrow \\ j \quad i \end{array} \right) = \begin{cases} H_{\underline{k}+j} \otimes_{H_{\underline{k}}} H_{\underline{k}+i} \rightarrow H_{+i+j\underline{k}+i} \otimes_{H_{+i+j\underline{k}}} H_{+i\underline{k}+j}, \\ \xi_j^{\alpha_1} \otimes \xi_i^{\alpha_2} \mapsto \xi_i^{\alpha_2} \otimes \xi_j^{\alpha_1}, \end{cases}$$

$$\Gamma \left(\begin{array}{c} \lambda + i_X \\ \downarrow \quad \downarrow \\ \text{Diagram 2} \\ \downarrow \quad \downarrow \\ j \quad i \end{array} \right) = \begin{cases} H_{+j+i+i\underline{k}} \otimes_{H_{+j+i\underline{k}}} H_{+j\underline{k}+i} \rightarrow H_{\underline{k}+i} \otimes_{H_{\underline{k}}} H_{\underline{k}+j}, \\ \xi_j^{\alpha_1} \otimes \xi_i^{\alpha_2} \mapsto \begin{cases} -\xi_i^{\alpha_2} \otimes \xi_j^{\alpha_1} & \text{if } \overset{j}{\circ} \longrightarrow \overset{i}{\circ}, \\ \xi_i^{\alpha_2} \otimes \xi_j^{\alpha_1} & \text{otherwise.} \end{cases} \end{cases}$$

These bimodule maps have degree zero for all $i, j \in I$ and all weights λ .

In the proof of Lemma 6.4 the sentence “The case when $i = j$ appears in [21] so we will omit this case here” should be removed.

Definition 4.1 in Section 4.2, p. 58–59, should be changed to:

Definition 4.1. $\mathcal{U}_{\rightarrow}(\mathfrak{sl}_n)$ is a additive \mathbb{k} -linear 2-category with translation. The 2-category $\mathcal{U}_{\rightarrow}(\mathfrak{sl}_n)$ has objects, morphisms, and generating 2-morphisms as described in Definition 3.1, but some of the relations on 2-morphisms are modified.

- The \mathfrak{sl}_2 relations and the shift isomorphism relations are the same as before, see equations (3.1)–(3.9).
- Generating 2-morphisms are cyclic with respect to the bidual structure, see (3.3) and (3.10), except when $i \cdot j = -1$ we have

$$\text{Diagram 1} = - \text{Diagram 2}$$

- Sideways crossings are defined by the equations

$$\begin{array}{c} j \\ \nearrow \\ \searrow \\ i \end{array} \lambda := \begin{array}{c} j \\ \downarrow \\ \nearrow \\ i \end{array} \lambda, \quad \lambda \begin{array}{c} \nearrow \\ j \\ \searrow \\ i \end{array} := \lambda \begin{array}{c} \downarrow \\ j \\ \nearrow \\ i \end{array}.$$

Then the relations (3.13) for $i \neq j$ become

$$\begin{array}{c} \nearrow \\ \searrow \\ \nearrow \\ \searrow \\ i \quad j \end{array} \lambda = \begin{cases} \begin{array}{c} \downarrow \\ i \\ \downarrow \\ j \end{array} \lambda & \text{if } i \cdot j = 0, \\ (i-j) \begin{array}{c} \downarrow \\ i \\ \downarrow \\ j \end{array} \lambda & \text{if } i \cdot j = -1, \end{cases}$$

$$\begin{array}{c} \searrow \\ \nearrow \\ \searrow \\ \nearrow \\ i \quad j \end{array} \lambda = \begin{cases} \begin{array}{c} \downarrow \\ i \\ \downarrow \\ j \end{array} \lambda & \text{if } i \cdot j = 0, \\ (j-i) \begin{array}{c} \downarrow \\ i \\ \downarrow \\ j \end{array} \lambda & \text{if } i \cdot j = -1. \end{cases}$$

- The signed $R(\nu)$ relations are:

(a) For $i \neq j$, the relations

$$\begin{array}{c} \nearrow \\ \searrow \\ \nearrow \\ \searrow \\ i \quad j \end{array} \lambda = \begin{cases} \begin{array}{c} \downarrow \\ i \\ \downarrow \\ j \end{array} \lambda & \text{if } i \cdot j = 0, \\ (i-j) \left(\begin{array}{c} \bullet \\ \downarrow \\ i \\ \downarrow \\ j \end{array} \lambda - \begin{array}{c} \downarrow \\ i \\ \bullet \\ \downarrow \\ j \end{array} \lambda \right) & \text{if } i \cdot j = -1. \end{cases}$$

(b) For $i \neq j$, the relations

$$\begin{array}{c} \nearrow \\ \bullet \\ \searrow \\ j \end{array} \lambda = \begin{array}{c} \nearrow \\ \bullet \\ \searrow \\ j \end{array} \lambda, \quad \begin{array}{c} \bullet \\ \nearrow \\ \searrow \\ j \end{array} \lambda = \begin{array}{c} \bullet \\ \nearrow \\ \searrow \\ j \end{array} \lambda.$$

for all λ .

(c) Unless $i = k$ and $j = i \pm 1$

$$\begin{array}{c} \nearrow \\ \searrow \\ \nearrow \\ \searrow \\ i \quad j \quad k \end{array} \lambda = \begin{array}{c} \nearrow \\ \searrow \\ \nearrow \\ \searrow \\ i \quad j \quad k \end{array} \lambda.$$

For $j = i \pm 1$

$$\begin{array}{c} \downarrow \\ i \\ \downarrow \\ j \\ \downarrow \\ i \end{array} \lambda = (i-j) \left(\begin{array}{c} \nearrow \\ \searrow \\ \nearrow \\ \searrow \\ i \quad j \quad i \end{array} \lambda - \begin{array}{c} \searrow \\ \nearrow \\ \searrow \\ \nearrow \\ i \quad j \quad i \end{array} \lambda \right).$$

The rightmost term in the statement of Proposition 6.5 should have a minus sign when $i \cdot j = -1$. In the statement of Proposition 6.6 when $i \cdot j = -1$ the first equality should have the sign $(i - j)$ and the second equality should have the sign $(j - i)$.

The 2-category $\mathcal{U}_{\rightarrow}(\mathfrak{sl}_n)$ described above with modified relations is isomorphic to the 2-category $\mathcal{U}(\mathfrak{sl}_n)$. The only part of the isomorphism $\Sigma: \mathcal{U} \rightarrow \mathcal{U}_{\rightarrow}$ that needs to be modified is the image of caps and cups. The rescaling of these caps and cups also appears in a work in progress of the second author with Sabin Cautis.

Let $d_i = (-1)^i$. Then Σ is defined as before except that caps and cups are given as follows:

$$\Sigma\left(\begin{array}{c} i \\ \frown \\ \lambda \end{array}\right) = \begin{cases} d_i^{\lambda_i-1} \begin{array}{c} i \\ \frown \\ \lambda \end{array} & \lambda_i = 2\ell, 2\ell + 1, -2\ell, -(2\ell + 3) \\ & \text{for } \ell \in 2\mathbb{Z}_{\geq 0}, \\ \begin{array}{c} i \\ \frown \\ \lambda \end{array} & \text{otherwise,} \end{cases}$$

$$\Sigma\left(\begin{array}{c} i \\ \smile \\ \lambda \end{array}\right) = \begin{cases} d_i^{\lambda_i-1} \begin{array}{c} i \\ \smile \\ \lambda \end{array} & \lambda_i = 2\ell, 2\ell + 1, -2\ell, -(2\ell + 1) \\ & \text{for } \ell \in 2\mathbb{Z}_{\geq 0}, \\ \begin{array}{c} i \\ \smile \\ \lambda \end{array} & \text{otherwise,} \end{cases}$$

$$\Sigma\left(\begin{array}{c} \lambda \\ \smile \\ i \end{array}\right) = \begin{cases} d_i^{\lambda_i-1} \begin{array}{c} \lambda \\ \smile \\ i \end{array} & \lambda_i = 2\ell + 2, 2\ell + 3, -(2\ell + 1), -(2\ell + 2) \\ & \text{for } \ell \in 2\mathbb{Z}_{\geq 0}, \\ \begin{array}{c} \lambda \\ \smile \\ i \end{array} & \text{otherwise,} \end{cases}$$

$$\Sigma\left(\begin{array}{c} \lambda \\ \frown \\ i \end{array}\right) = \begin{cases} d_i^{\lambda_i-1} \begin{array}{c} \lambda \\ \frown \\ i \end{array} & \lambda_i = 2\ell + 2, 2\ell + 3, -(2\ell + 1), -(2\ell + 2) \\ & \text{for } \ell \in 2\mathbb{Z}_{\geq 0}, \\ \begin{array}{c} \lambda \\ \frown \\ i \end{array} & \text{otherwise.} \end{cases}$$

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