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SOME REMARKS ON MEROMORPHIC FIRST INTEGRALS

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ABSTRACT. A scholium on a paper by Cerveau and Lins Neto.

Our starting point is the following result, recently established by Cerveau and Lins Neto in their paper [CLN]:

THEOREM 1. *Let $\mathcal{F}$ be a germ of holomorphic foliation on* $(\mathbf{C}^2;0)$. *Suppose that there exists a germ of real analytic hypersurface $M \subset (\mathbf{C}^2;0)$ which is invariant by $\mathcal{F}$. Then $\mathcal{F}$ admits a meromorphic first integral.*

Of course, in this statement the hypersurface M may be singular at 0, and this singularity may even be non-isolated. To say that M is *invariant by the foliation* refers to its smooth part M_{reg}.

The proof given in [CLN] is rather involved. There are two cases: the *dicritical* case and the *non-dicritical* one. In the first case, the authors find a first integral by a quite mysterious computation with power series. In the second case, they use delicate dynamical considerations (holonomy group).

Our aim is to give an almost straightforward proof of Theorem 1, which is based only on some general principles of analytic geometry (in the spirit of our previous paper on a closely related subject [Bru]), together with a general (and simple) criterion for the existence of a meromorphic first integral. This relatively new proof will reveal the beautiful geometric structure behind foliations tangent to real analytic hypersurfaces.

Let us also recall that Theorem 1 generalizes to codimension one foliations in higher-dimensional spaces, by a standard sectional argument

†) Les Éditeurs ont appris le décès de Marco Brunella en janvier 2012, après que son article sur les intégrales premières méromorphes eut été accepté pour publication, mais son auteur n'a pas eu le temps d'y apporter d'éventuels derniers changements. Dominique Cerveau a bien voulu rédiger un commentaire, sollicité par les Éditeurs, qui le remercient de sa collaboration.

([M-M], [CLN]); alternatively, our arguments also generalize to higher dimensions with no substantial new difficulty. Another possible generalization concerns foliations defined on singular spaces, instead of $\mathbf{C}^2$.

1. An integrability criterion

Let $\mathcal{F}$ be a holomorphic foliation on a domain $U \subset \mathbf{C}^2$ containing the origin, with $\mathrm{Sing}(\mathcal{F}) = \{0\}$. Set $U^\circ = U \setminus \{0\}$. A *meromorphic first integral* is a nonconstant meromorphic function on U which is constant along the leaves of $\mathcal{F}$.

Proposition 2. *Suppose that there exists an irreducible analytic*[1]) *hypersurface*

$$W \subset U^\circ \times V\,,$$

V being a neighbourhood of 0 in $\mathbf{C}^2$, such that:

(1) *for every $p \in U^\circ$, the fibre*

$$W_p = W \cap (\{p\} \times V) \subset V$$

is a proper analytic curve in V, passing through the origin;

(2) *if $p, q \in U^\circ$ belong to the same leaf of $\mathcal{F}$, then $W_p = W_q$;*

(3) *the projection of W to V is Zariski-dense (i.e., not contained in a curve).*

Then $\mathcal{F}$ admits a meromorphic first integral on U.

The germ-oriented reader should here replace V by its germ at the origin, and W by its germ along $U^\circ \times \{0\}$.

Proof. It can be summarized as follows. We already have, by Assumptions (1) and (2) a "first integral", but, instead of being a meromorphic function, it is a map which takes values into the "space of curves in V through 0". Hence, roughly speaking, we shall give an algebraic structure to such a space of curves, so that the true meromorphic first integral will be obtained by composition of the former "first integral" with a generic meromorphic function on the space of curves. Hypothesis (3) will guarantee that such a first integral is not identically constant. All of this is trivial if, for instance, each W_p is a line:

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[1]) To avoid confusion: 'analytic' without the 'real' attribute means 'complex analytic'.

the space of lines through the origin is the familiar algebraic variety $\mathbf{C}P^1$. The general case only requires some additional blow-ups.

Given a sequence of ℓ blow-ups

$$\pi\colon \widetilde{V} \to V$$

over the origin, denote by $D = \bigcup_{j=1}^{\ell} D_j$ the exceptional divisor $\pi^{-1}(0)$, and set

$$\Pi = id \times \pi\colon U^\circ \times \widetilde{V} \to U^\circ \times V.$$

Denote by $\widetilde{W}$ the *strict transform* of W, i.e. the closure of the inverse image by Π of $W \smallsetminus (W \cap (U^\circ \times \{0\}))$. The trace of $\widetilde{W}$ on $U^\circ \times D$ is a hypersurface (of dimension 2), and we shall denote by Z the union of those irreducible components whose projections to U° are dominant (the other components project to curves). Thus, for $p \in U^\circ$ generic, the fibre

$$Z_p = Z \cap (\{p\} \times D)$$

is a finite subset of D, which actually coincides with the trace on D of the strict transform of W_p (here we have to exclude not only those points p such that Z_p contains some component of D, but also those points which belong to the projection of the non-dominant components of the trace of $\widetilde{W}$: these are precisely the conditions ensuring that the fibre of $\widetilde{W}$ over p is equal to the strict transform of W_p).

Now, hypothesis (3) implies the following: there exists a sequence of blow-ups $\pi\colon \widetilde{V} \to V$ over the origin such that Z is *not* of the type $U^\circ \times \{\text{finite set}\}$. Indeed, in the opposite case the generic curves W_p would all be unseparable by any sequence of blow-ups, i.e. they would be all equal, and this contradicts the Zariski-density of the projection $W \to V$ (here we use the irreducibility of W, and also the fact that every W_p passes through the origin).

In this way, we get an irreducible component of D (say, D_ℓ) such that the part of Z inside $U^\circ \times D_\ell$ (call it Z_ℓ) is dominant over U° and Zariski-dense over D_ℓ.

If k is the degree of $Z_\ell \to U^\circ$, then Z_ℓ defines a meromorphic map I from U° to $D_\ell^{(k)}$, the k-fold symmetric product of D_ℓ. Such a map is not constant, but it is constant along the leaves of $\mathcal{F}$, by hypothesis (2). Since $D_\ell^{(k)}$ is an algebraic variety, we can find $F \in \mathcal{M}\big(D_\ell^{(k)}\big)$ such that $f = F \circ I$ is a nonconstant meromorphic function, constant along the leaves. Finally, f extends from U° to U by Levi's theorem. $\square$

REMARK 3. Consider a foliation $\mathcal{F}$ on $U \subset \mathbf{C}^2$, $\mathrm{Sing}(\mathcal{F}) = \{0\}$, such that every leaf L is a so-called *separatrix* at 0 : $L \cup \{0\}$ is a proper analytic curve in U. This occurs if $\mathcal{F}$ has a meromorphic first integral having 0 as indeterminacy point, but, as is well known, the converse implication is far from being true, see for instance [Mou] and references therein. We have a naturally defined subset S of $U^\circ \times U$: its fibre over p is, by definition, the curve $\overline{L_p} = L_p \cup \{0\}$. However, generally speaking this subset S is *not* an analytic subset, since it may be not closed.

Of course, we may take the Zariski-closure $\widehat{S}$ of S, which however could be the full $U^\circ \times U$. If it is not the case, i.e. if $\dim \widehat{S} = 3$, then by Proposition 2 we get a meromorphic first integral, and the converse is also true by an easy argument. Note, however, that in this special case our Proposition 2 is closely related to old results by B. Kaup and Suzuki [Suz, §5], relating the existence of first integrals with the analyticity of the graph of the foliation.

Let us stress that, even when a first integral exists, the subset S is typically not an analytic subset, that is its Zariski-closure $\widehat{S}$ may be much larger than S. Indeed, the fibre of $\widehat{S}$ over p may contain, besides $\overline{L_p}$, other components $\overline{L_{p_1}}, \dots, \overline{L_{p_n}}$. These additional separatrices are precisely the ones which cannot be separated from $\overline{L_p}$ by meromorphic first integrals. In other words, while S represents the (nonanalytic) equivalence relation generated by the leaves, $\widehat{S}$ represents the (analytic) equivalence relation generated by level sets of meromorphic first integrals.

There is a variant of Proposition 2 in which the hypothesis that every W_p is a curve passing through the origin of V is replaced by a similar asymptotic hypothesis over the singular point of the foliation. First we observe that if $W \subset U^\circ \times V$ is as in Proposition 2, then, by standard extension theorems, W can be prolonged to an irreducible analytic hypersurface in $U \times V$. However, it may happen that the fibre over 0 of this extension is not a curve, but the full V; this is precisely the case in which the meromorphic first integral has an indeterminacy point at 0.

PROPOSITION 4. *Suppose that there exists an irreducible analytic hypersurface $W \subset U \times V$, where V is a neighbourhood of 0 in $\mathbf{C}^2$, such that:*

(1) *for every $p \in U$, the fibre $W_p = W \cap (\{p\} \times V) \subset V$ is a proper analytic curve in V, passing through the origin when $p = 0$;*
(2) *if $p, q \in U^\circ$ belong to the same leaf of $\mathcal{F}$, then $W_p = W_q$;*
(3) *the projection of W to V is Zariski-dense.*

Then $\mathcal{F}$ *admits a holomorphic first integral on some (possibly smaller) neighbourhood of* 0.

Proof. It is even simpler than the previous one; in some sense, it is the "no blow-up case".

Take a (possibly singular) disk $D \subset V$ passing through 0 and intersecting W_0 only at 0. Take the trace Z of W on $U \times D$. Then, up to shrinking U, Z is a hypersurface in $U \times D$ and the projection $Z \to U$ is proper, say of degree k. We thus obtain, as before, a first integral with values in $D^{(k)}$. This last space admits a lot of holomorphic functions, and so we get a holomorphic first integral. Thanks to hypothesis (3), and by a suitable choice of D, this first integral will not be identically constant (it is sufficient to choose D highly tangent to a branch of W_0). □

REMARK 5. Consider a foliation $\mathcal{F}$ on $U \subset \mathbf{C}^2$, $\mathrm{Sing}(\mathcal{F}) = \{0\}$, such that there is a finite number of separatrices and any other leaf is a proper analytic curve in U. Then, on a possibly smaller $U' \subset U$, the foliation admits a holomorphic first integral [M-M]. This result can be recast into Proposition 4, but one needs some further work. The idea is to look again at the subset $S \subset U^\circ \times U$ of Remark 3, and to show that the *topological* closure $\bar{S}$ in $U \times U$ is an analytic hypersurface, which cuts the fibre over 0 along a curve passing through 0. This last curve will be the union of the separatrices (plus the origin).

This indispensable further work can be found in [Mou]. Let $\Sigma \subset U$ be the union of the separatrices and the origin. We may assume that the closure of each separatrix is a (singular) disk passing through 0 and transverse to the boundary of U. According to [Mou, Lemme 1], if p is suffciently close to 0, and outside Σ, then L_p is a curve transverse to the boundary of U. Using the finiteness of the holonomy of L_p (which is an elementary fact) and Reeb stability, it is then easy to see that the restriction of S to $(U' \setminus \Sigma') \times U$ is an analytic hypersurface, where U' is a sufficiently small neighbourhood of 0 and $\Sigma' = \Sigma \cap U'$. Take now the topological closure $\bar{S}$ of S in $U' \times U$. By standard results (Remmert-Stein), if $\bar{S}$ is not an analytic hypersurface, then it must contain an irreducible component of $\Sigma' \times U$; this is however impossible, again by [Mou, Lemme 1] (which implies that the $\mathcal{F}$-saturation of U' cannot be the full U). Hence, $\bar{S}$ is an analytic hypersurface in $U' \times U$. For the same reason, its fibre over 0 cannot be the full U, and therefore it must coincide with Σ. We can now apply Proposition 4.

It is also worth observing that the fibre of $\bar{S}$ over a point $p \in U' \setminus \Sigma'$ is the single leaf L_p. This corresponds to the fact that the leaves outside the separatrices can be separated by holomorphic first integrals.

2. Complexification of real hypersurfaces

Consider now the setting of Theorem 1: $\mathcal{F}$ is a foliation on $U \subset \mathbf{C}^2$, singular at $0 \in U$, and M is a real analytic hypersurface passing through the origin and invariant by $\mathcal{F}$. We denote by $M_{\text{reg}} \subset M$ the (open) subset of *regular points*, i.e. the points where M is a real analytic submanifold of dimension 3. We assume that $0 \in \overline{M_{\text{reg}}}$ (otherwise, the germ of M at 0 would not be a germ of *hypersurface*, as prescribed by Theorem 1). Without loss of generality, we may also assume that M is irreducible, and even that the germ of M at 0 is irreducible.

Let us recall a few facts concerning *complexification*, see also [Bru, §3].

Denote by U^* the complex manifold conjugate to U: it is the same differentiable manifold, but with the opposite complex structure; equivalently, holomorphic functions on U^* are the same as antiholomorphic functions on U. Remark that if A is an analytic subset of U, then it is analytic also as a subset of U^*. As such, it will be denoted by A^*. In particular, every point $p \in U$ has a "mirror" point $p^* \in U^*$. Similarly, if $\mathcal{F}$ is a holomorphic foliation on U, then it is holomorphic also as a foliation on U^*, and as such it will be denoted by $\mathcal{F}^*$. Remark that, generally speaking, the two foliations $\mathcal{F}$ and $\mathcal{F}^*$ are *different* as holomorphic foliations: the identity map $U \to U^*$ obviously conjugates $\mathcal{F}$ to $\mathcal{F}^*$, but such a map is *anti*holomorphic, and not holomorphic. For example, if $\gamma \subset L \in \mathcal{F}$ is a loop with linear holonomy λ, then the same loop $\gamma \subset L^* \in \mathcal{F}^*$ has linear holonomy $\bar{\lambda}$.

In the product space $U \times U^*$ (with the product complex structure) we have the involution

$$\jmath\colon U \times U^* \to U \times U^*$$
$$\jmath(p, q^*) = (q, p^*)\,.$$

It is antiholomorphic. Its fixed point set is the diagonal Δ, and it is a totally real submanifold.

It is convenient to look at our real analytic hypersurface M in U as a subset of the diagonal:

$$M \subset \Delta \subset U \times U^*\,.$$

Then, M can be complexified: there exists a neighbourhood $\widehat{U} \subset U \times U^*$ of the diagonal and an irreducible complex analytic hypersurface $M^{\mathbf{C}}$ in $\widehat{U}$ such that

$$M^{\mathbf{C}} \cap \Delta = M\,.$$

Up to restricting U around the origin, we may assume that $\widehat{U} = U \times U^*$. Remark that

$$\jmath(M^{\mathbf{C}}) = M^{\mathbf{C}} \quad \text{and} \quad \mathrm{Fix}(\jmath|_{M^{\mathbf{C}}}) = M\,.$$

Actually, this complexification can be done on any real analytic subset. In particular, we can start with a complex analytic subset $A \subset U$ and look at it as a subset of Δ, thus forgetting its complex analytic structure and retaining only its real analytic one. Its complexification is then simply the product $A \times A^*$ (which could be pompously called "complexification of the decomplexification of A").

Consider now the projection

$$pr\colon M^{\mathbf{C}} \to U$$

to the first factor, and for every $p \in U$ set

$$M_p^{\mathbf{C}} = pr^{-1}(p)\,.$$

It is an analytic subset of U^*.

LEMMA 6. *Up to shrinking U around the origin, we have: for every $p \in U^\circ$, $M_p^{\mathbf{C}}$ is a (nonempty) curve in U^*.*

Proof. The irreducibility of $M^{\mathbf{C}}$ implies that the set of points of U over which the fibre is two-dimensional (i.e., the full U^*) is discrete. Hence, up to shrinking U, we get that $M_p^{\mathbf{C}}$ is at most one-dimensional for every $p \in U^\circ$ (note that a shrinking of U implies a simultaneous shrinking of U^*, but this is not a problem).

Obviously $M_p^{\mathbf{C}}$ cannot contain isolated points, because $M^{\mathbf{C}}$ is a hypersurface. Therefore, it remains to show that it is not empty. Of course $M_0^{\mathbf{C}}$ is not empty, for any choice of U, and we can distinguish two cases:

(a) $M_0^{\mathbf{C}} = U^*$, i.e. $M^{\mathbf{C}}$ contains $\{0\} \times U^*$. Because $M^{\mathbf{C}}$ is $\jmath$-invariant, this means that also the horizontal fibre $U \times \{0^*\}$ is fully contained in $M^{\mathbf{C}}$. As a consequence, every $M_p^{\mathbf{C}}$, $p \in U^\circ$, is a curve which, moreover, passes through the origin.

(b) $M_0^{\mathbf{C}}$ is a curve in U^*. Then, by a standard result (Remmert's Rank Theorem), the map pr is open, and hence surjective for a suitable choice of U. $\square$

We shall see that case (a) corresponds to the dicritical case, and case (b) to the non-dicritical one.

Recall now that we have a holomorphic foliation $\mathcal{F}$ on U, which leaves M invariant.

LEMMA 7. *For every $p \in U^\circ$, the curve $M_p^{\mathbf{C}} \subset U^*$ is invariant by $\mathcal{F}^*$. Moreover, if p and q belong to the same leaf, then $M_p^{\mathbf{C}} = M_q^{\mathbf{C}}$.*

Proof. This is basically [Bru, Lemma 3.1], but let us explain it in a slightly different manner.

On $U \times U^*$ we have the foliation (of dimension 2) $\mathcal{F} \times \mathcal{F}^*$. It is nonsingular on $U \times U^{\circ *}$, and its leaf through (p, q^*) is

$$L_{p,q^*} = L_p \times L^*_{q^*} ,$$

where the first factor is the leaf of $\mathcal{F}$ through p and the second factor is the leaf of $\mathcal{F}^*$ through q^*. In particular, if $(p, p^*) \in \Delta$, then $L_{p,p^*} = L_p \times L^*_{p^*}$, and this is also the complexification of $L_p \subset \Delta$ (here L_p is not yet properly embedded, but it does not matter for the next arguments). It follows that if we take a leaf L_p contained in M, then the leaf L_{p,p^*} is contained in $M^{\mathbf{C}}$. Thus we have found a continuum of leaves of $\mathcal{F} \times \mathcal{F}^*$ which are contained in $M^{\mathbf{C}}$, and therefore we get that $M^{\mathbf{C}}$ is *invariant* by $\mathcal{F} \times \mathcal{F}^*$ (this is not a surprise, for this last foliation can be understood as the complexification of the decomplexification of $\mathcal{F}$, which leaves M invariant).

As a consequence of this, if L is any leaf of $\mathcal{F}$, its preimage $pr^{-1}(L) \subset M^{\mathbf{C}}$ is a union of leaves of $\mathcal{F} \times \mathcal{F}^*$ (plus possibly some singular point on $U^\circ \times \{0^*\}$), i.e. it is of the form $L \times (L_1^* \cup \ldots \cup L_n^*)$ for suitable leaves L_j^* of $\mathcal{F}^*$ (plus possibly some singular point). But this is precisely the assertion of the lemma. $\square$

REMARK 8. Without assuming the existence of $\mathcal{F}$, the same argument shows the following: if $M \subset U$ is any real analytic Levi-flat hypersurface, then on $M^{\mathbf{C}}$ we have a two-dimensional foliation whose leaves are products of horizontal and vertical fibres of $M^{\mathbf{C}}$. Here the essential point is that if we take a horizontal fibre and a vertical fibre of $M^{\mathbf{C}}$, passing through the same point of $M^{\mathbf{C}}$, then their product is still contained in $M^{\mathbf{C}}$. This is a remarkable symmetry property of $M^{\mathbf{C}}$, and of course it is a manifestation of the Levi-flatness of M. This foliation appears also in [CLN], as complexification of the Levi foliation, but the authors obtain the properness of leaves only after a long tour.

REMARK 9. The fact that $M_p^{\mathbf{C}}$ may contain several leaves of $\mathcal{F}^*$ should be compared with the phenomenon described in Remark 3. Note also that on a neighbourhood of M_{reg} we have a *Schwarz reflection* at the level of the leaf space [Bru, p. 669]. If p is close to M_{reg}, then $M_p^{\mathbf{C}}$ contains the Schwarz reflection of L_p (which must be understood as a leaf of $\mathcal{F}^*$). The fact that $M_p^{\mathbf{C}}$ is defined for every p could be interpreted as a sort of "globalization" of that Schwarz reflection, and the fact that $M_p^{\mathbf{C}}$ contains several leaves suggests that "reflection" should be replaced by "correspondence".

For example, suppose that $\mathcal{F}$ is the radial foliation $(z\,dw - w\,dz = 0)$, so that M corresponds to a real algebraic curve $\gamma \subset \mathbf{C}P^1$ (= the space of leaves of $\mathcal{F}$). The complexification of γ is a complex algebraic curve $\gamma^{\mathbf{C}} \subset \mathbf{C}P^1 \times \mathbf{C}P^{1*}$, which gives an antiholomorphic correspondence of $\mathbf{C}P^1$ with itself [2]).

We can now immediately complete the proof of Theorem 1. First we note that, by Lemmas 6 and 7, every leaf of $\mathcal{F}$ is properly embedded in U°. If $M_0^{\mathbf{C}}$ is the full U^* then, as observed in the proof of Lemma 6, every $M_p^{\mathbf{C}}$, $p \neq 0$, is a curve through the origin, and so we can apply Proposition 2 to get a meromorphic first integral. If $M_0^{\mathbf{C}}$ is a curve, then it is a curve through the origin, by symmetry, and so we can apply Proposition 4 (actually, in that proposition the requirement that W_0 passes through 0 can obviously be replaced by $W_0 \neq \varnothing$).

Let us conclude with a question. In the setting of Theorem 1, consider first the case where we have a (primitive) holomorphic first integral f, with $f(0) = 0$. It is then easy to see that $M = f^{-1}(\gamma)$, with $\gamma \subset \mathbf{C}$ a real analytic curve passing through the origin. Indeed, we obviously have $M = \widehat{f}^{-1}(\widehat{\gamma})$ where $\widehat{f}$ is the projection to the space of leaves Σ (a *non-Hausdorff Riemann surface* [Mou] [Suz]) and $\widehat{\gamma} \subset \Sigma$ is a real analytic curve. Moreover, $f = e \circ \widehat{f}$, where $e \colon \Sigma \to V \subset \mathbf{C}$ is the map which collapses nonseparated points. However, due to the special structure of Σ in this case (there is only a finite set of nonseparated points, all sent to 0 by e) we certainly have $\widehat{\gamma} = e^{-1}(\gamma)$ for some $\gamma \subset V$, and so $M = f^{-1}(\gamma)$. Now, consider the dicritical case, where the first integral f is only meromorphic, and 0 is an indeterminacy point. Can we find a real algebraic curve $\gamma \subset \mathbf{C}P^1$ such that $M = f^{-1}(\gamma)$? The problem here is that the collapsing map $e \colon \Sigma \to \mathbf{C}P^1$ is much more complicated, and in principle the curve $\widehat{\gamma} \subset \Sigma$ could be not of the form $e^{-1}(\gamma)$, i.e. there could exist two unseparable leaves L, L' with L in M but not L'. Of course, we

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[2]) We say that a real analytic curve $\gamma \subset \mathbf{C}P^1$ is *real algebraic* if its complexification $\gamma^{\mathbf{C}}$, which is in principle defined only on a neighbourhood of the diagonal, extends to the full $\mathbf{C}P^1 \times \mathbf{C}P^{1*}$. With this definition, it is easy to see that a radial Levi-flat hypersurface M is analytic at 0 if and only if the corresponding γ is real algebraic; the complex curve $\gamma^{\mathbf{C}}$ is then the trace of $\widetilde{M}^{\mathbf{C}}$ on $\mathbf{C}P^1 \times \mathbf{C}P^{1*} \subset \widetilde{U} \times \widetilde{U}^*$, where $\widetilde{M}$ is the strict transform of M in $\widetilde{U} =$ the blow-up of U at 0.

can set $\gamma = e(\widehat{\gamma})$ and take $f^{-1}(\gamma)$, but this last one could be reducible, and our initial M could be only one irreducible component of it.

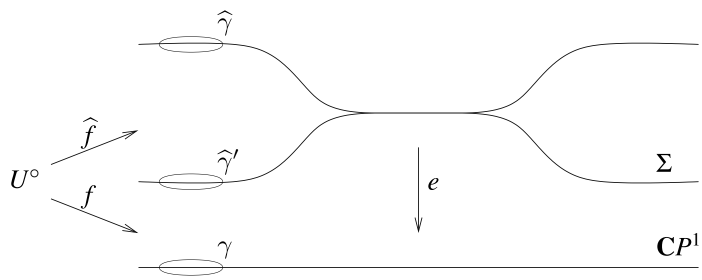

FIGURE 1
Can $M = \widehat{f}^{-1}(\widehat{\gamma})$ and $M' = \widehat{f}^{-1}(\widehat{\gamma}')$ be irreducible components of $f^{-1}(\gamma)$?

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