

COMMISSION INTERNATIONALE
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(THE INTERNATIONAL COMMISSION
ON MATHEMATICAL INSTRUCTION)

**Discussion Document for the Twenty-third ICMI Study
(short version)**

Primary mathematics Study on whole numbers

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A new Study will be conducted by the International Commission on Mathematical Instruction. This Study, the twenty-third led by ICMI, addresses for the first time mathematics teaching and learning in the primary school (and pre-school), taking into account inclusive international perspectives, socio-cultural diversity and institutional constraints. The broad area of *Whole Number Arithmetic* (WNA) including operations and relations and arithmetic word problems form the core content of all primary mathematics curricula. The Study of this core content area is often regarded as foundational for later mathematics learning. However, the principles and main goals of instruction in the foundational concepts and skills in WNA are far from universally agreed upon, and practice varies substantially from country to country. An ICMI Study that provides a meta-level analysis and synthesis of what is known about WNA would provide a useful base from which to gauge gaps and silences and an opportunity to learn from the practice of different countries and contexts.

Whole numbers are part of everyday language in most cultures, but there are different views on the most appropriate age at which to introduce whole numbers in the school context. Whole numbers, in some countries, are introduced in the pre-school, where the majority of children attend before the age of 6 years. In some countries, primary schooling includes Grades 1–6; in others it includes Grades 1–5. Thus the entrance age of students for primary school may vary from country to country. For these reasons, this Study addresses teaching and learning WNA from the early grades, i.e., the periods in which WNA is systematically approached in the formal school, and in some contexts this includes the pre-school.

Primary schooling is compulsory in most countries (in all Western countries), although there is considerable variation in the facilities, resources and opportunities for students. This is the uneven context where mathematics teaching and learning takes place. Mathematics is a central feature of early education and the content, quality and delivery of the curriculum is of critical importance in view of the kinds of citizens each country seeks to produce.

In the international literature there are many contributions about primary school mathematics. In many cases, especially in the West, early processes of mathematical thinking, often observed in early childhood (i.e., 3–8 year-old children), are also investigated by cognitive and developmental psychologists. They sometimes study the emergence of these processes in clinical settings, where children are stimulated by suitable models so as to observe the emergence of aspects such as one-to-one correspondences, counting, measuring and other processes). In several countries, Piaget's theory has been very influential despite criticism. Neuroscientists have also been studying for some years the emergence of "number sense". However, recent perspectives highlight that what is still needed is serious and deep interdisciplinary work with experts in mathematics education.

ICMI has acknowledged that it is timely to launch, for the first time in its history, an international Study that especially focuses on early mathematics education, that is both basic and fundamental mathematically. When foundational processes are concerned, a strong epistemological basis is needed. This is where the involvement of ICMI adds value with respect to analyses carried out in other fields. Such epistemological analysis was part of classical works of professional mathematicians (e.g., Klein, Smith, and Freudenthal) who played a major role in the history of ICMI and considered mathematics teaching as a whole.

The ICMI Study will be organized around five themes that provide complementary perspectives on approaches to early WNA in mathematics teaching and learning. Contributions to the separate themes will be distinguished by the theme's specific foci and questions, although it is expected that interconnections between themes will emerge and warrant attention.

The five themes are:

1. the why and what of WNA;
2. whole number thinking, learning, and development;
3. aspects that affect whole number learning;
4. how to teach and assess WNA;
5. whole numbers and connections with other parts of mathematics.

Themes 1 and 2 address foundational aspects from the cultural-historic-epistemological perspective and from the (neuro)cognitive perspective. What is especially needed are reports about the impact of foundational aspects on practices (both at the micro-level of students and classrooms and at the macro-level of curricular choices).

Themes 3 and 4 address learning and teaching respectively, although it is quite difficult, sometimes, to separate the two aspects, because for example in some languages and cultures (e.g., Chinese, Japanese, Russian) the two words collapse into only one.

Theme 5 addresses the usefulness (or the need) to consider WNA in connection with (or as the basis for) the transition to other kinds of numbers (e.g., rational numbers) or with other areas of mathematics, traditionally separated from arithmetic (e.g., algebra, geometry, modelling).

ICMI Study 23 is designed to enable teachers, teacher educators, researchers and policy makers around the world to share research, practices, projects and analyses. Although reports will form part of the program, substantial time will also be allocated for collective work on significant problems in the field, that will eventually form part of the Study volume. As in every ICMI Study, the ICMI Study 23 is built around an International Conference and directed

towards the preparation of a published volume. The Study Conference will take place in Macau, China and will be hosted by the University of Macau (*June 3–7, 2015*).

As is the usual practice for ICMI studies, participation in the Study Conference will be by invitation only for the authors of submitted contributions, which are accepted. Contributions have to be submitted before *September 15, 2014*, they will be reviewed and a selection will be made according to the quality of the work, the potential to contribute to the advancement of the Study, with explicit links to the themes contained in the Discussion Document and the need to ensure diversity among the perspectives. The number of invited participants will be limited to approximately 100 people.

The *first product* of the ICMI Study 23 is an electronic volume of Proceedings, to be made available first on the Conference website and later in the ICMI website: it will contain all the accepted papers as reviewed papers in a Conference Proceedings (with ISBN number).

The *second product* is a gallery of commented video-clips about practices in WNA, to be hosted in the Conference website and, possibly, later, in the ICMI website.

The *third product* is the ICMI Study volume. The volume will be informed by the papers, the video-clips and the discussions at the Study Conference as well as its outcomes.

The IPC for ICMI Study 23 invites submissions of contributions of several kinds: theoretical or cultural–historic–epistemological essays (with deep connection with classroom practice, curricula or teacher education programs); position papers discussing policy and practice issues; discussion papers related to curriculum issues; reports on empirical studies; video-clips on explicit classroom or teacher education practice. To ensure a rich and varied discussion, participation from countries with different economic capacity or with different cultural heritage and practices is encouraged.

The ICMI Study 23 website is opened at the address:

<http://www.umac.mo/fed/ICMI23/>

The website contains a longer version of this discussion document and will be regularly updated with information about submission procedure and the Study Conference and will be used for sharing the contributions of those invited to the Conference in the form of Conference pre-proceedings. Further information may be asked at the following address:

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