COMMISSION INTERNATIONALE DE L'ENSEIGNEMENT MATHÉMATIQUE (THE INTERNATIONAL COMMISSION ON MATHEMATICAL INSTRUCTION)

Encyclopedia of Mathematics Education An invitation to read*

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The *Encyclopedia of Mathematics Education*, written under the direction of Stephen Lerman, has been published in 2014 as a traditional book and simultaneously as a dynamic online resource on the Springer Reference website.

This article does not claim to be a complete review of this Encyclopedia – I did not yet read all the 163 articles! Its aim is more to present the perspective retained by the Editorial board, and give a flavor of the Encyclopedia content, as an invitation to the potential reader.

A comprehensive resource for a large audience. The Encyclopedia project was to write a "comprehensive reference text, covering every topic in the field of mathematics education research" (Lerman, Preface, vii). Moreover, this text should be informative, taking into account the very last results of research, but also accessible to anyone who has an interest in mathematics education: not only researchers in mathematics education, but also mathematicians, teachers, students or policy makers. For this purpose, the Editorial board has gathered a team of 174 authors from more than 30 countries, specialists of different aspects of mathematics education. The entries can concern specific mathematic topics (and the related learning and teaching issues), like "Algebra teaching and learning" (by Carolyn Kieran); "Calculus Teaching and Learning" (by Ivy Kidron) or more transversal issues, like "Inquiry-based mathematics education" (by Jean-Luc Dorier and Katia Mass); "Problem solving in mathematics education" (by Manuel Santos-Trigo) or "Instrumentation in mathematics education" (by Luc Trouche). They can also present theories or concepts used in mathematics education: "Activity theory in mathematics education" (by Wolff-Michael Roth); "Didactic contract in mathematics education" (by Guy Brousseau). The content of the Encyclopedia represents actually the major results obtained in mathematics education for more than 40 years, with a variety of perspectives (epistemological, cognitive, socio-cultural etc.) developed by its international group of authors.

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Visiting the letter "M". An Encyclopedia can be a useful tool for answering a precise question. It can also be viewed as a place to wander... Let's try a brief random walk, within the letter "M" of the encyclopedia – one of the richest letters, with 25 entries! Not surprising, since Mathematics is the central focus here. The first entry is entitled: "Manipulatives in Mathematics Education" (by Maria G. Bartoloni Bussi and Francesca Martignone). This article mentions various kinds of manipulatives, ranging from historical ones like Napier bones to very recent digital tools, like the Bee-bot floor robot; it discusses the differences between concrete and virtual manipulatives, from an educational perspective. It identifies critical issues, linked with the use of manipulatives, like students' autonomy or their age (why are high school teachers often reluctant to use manipulatives in class?); it also presents a theoretical approach, the semiotic mediation, especially relevant to study the learning-teaching processes when manipulatives are involved.

After the "Manipulatives" entry, starts a long list of "Mathematical ...". "Mathematical ability", "Mathematical approaches", "Mathematical functions teaching and learning" ... Let's read this one (by Mogens Niss), the first article we meet in the list with a focus on a specific mathematical theme. Within the huge body of research on this topic, the author retains a focus on students' difficulties. Functions can have diverse representations: algebraic, graphical, tabular etc.; this causes several specific difficulties that have been clearly identified. It led to the design of teaching interventions using special software supporting the articulation of several representations. Another dimension of complexity is that functions have different aspects: a simple correspondence linking "every element in a given domain to one and only one element in another domain" is for the learner very different from a tool intervening in the modeling of extramathematical situations, for example. "Mathematical Modeling and Applications in Education" and "Mathematical representations" are other entries under the letter M, which can usefully complement the article about functions. "Mathematical Proof, Argumentation and Reasoning" faces also the challenge of synthesizing multiple research works on the subject. In this article, Gila Hanna recalls that "a proof is much more than a sequence of logical steps that justifies an assertion", it can play various roles in the mathematics practice, like establishing connections, suggesting new hypotheses. It can also take different forms, remaining informal but providing a high level of reliability. Teachers have to introduce students to these different kinds of proofs, and at the same time, teach them the rules of reasoning as well as present patterns of argumentation. A delicate task, indicating the need for an adequate teacher education (pre-service or in-service) - this connects us directly with the "Mathematics Teacher Education Organization" entry, a few steps further in the Encyclopedia... This article (by Jarmila Novotná, Hana Moraová and Maria Tatto) offers an international view of the multiple existing organizations for teacher education, but also discusses the skills, abilities, knowledge and attitudes, that student graduating from teacher preparation programs should master. The reader interested by teacher education can go on and read the "Models of In-service Mathematics Teacher Education" and "Models of Preservice Mathematics Teacher Education" entries, and can naturally also switch to the letter T, where we will find, for example, "Teacher Education Development Study - Mathematics (TEDS-M)". The challenge here might be to stop reading the Encyclopedia!

Final word (or not). Let us go back to the foreword of the Encyclopedia, written by Jeremy Kilpatrick:

This encyclopedia represents a major step forward in the field of mathematics education, bringing to everyone with a professional interest in mathematics education access to the latest and best thinking in the field. (Kilpatrick, Foreword, vi)

I naturally fully support this enthusiastic statement. Moreover, this major step is not a final step, since the online version should permit regular updates and discussions between authors and readers. For all your questions about research in mathematics education, you will find elements of answer in the *Encyclopedia of Mathematics Education*; and you can contribute with your comments to a continuous improvement of its content!

S. LERMAN (Ed.) (2014). *Encyclopedia of Mathematics Education*. New York: Springer. http://www.springerreference.com (search by book title)