COINCIDENCE SITUATIONS FOR ABSOLUTELY SUMMING NON-LINEAR MAPPINGS

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Recommended by Eric Carlen

Abstract: Situations where every linear operator between certain Banach spaces is absolutely $(p;q)$-summing are very useful in linear functional analysis. In this paper we investigate situations of this type for three classes (two new and one well known) of nonlinear mappings which are generalizations of the notion of absolutely summing linear operators.

1 – Introduction

By $\mathcal{L}(E;F)$ we denote the Banach space of all bounded linear operators between the Banach spaces E and F, and by $\mathcal{L}_{as(p;q)}(E;F)$ the subspace of all absolutely $(p;q)$-summing operators ($1 \leq q \leq p < \infty$). The striking success of the theory of absolutely summing operators is due, among other reasons, to the fact that the class of absolutely summing operators is neither very small (in the sense that $\mathcal{L}_{as(p;q)}(E;F) = \mathcal{L}(E;F)$ in some important cases) nor very large (in the sense that $\mathcal{L}_{as(p;q)}(E;F) \neq \mathcal{L}(E;F)$ in most of the cases). Illustrative examples of such situations are Grothendieck's theorem ($\mathcal{L}_{as(1;1)}(l_1;l_2) = \mathcal{L}(l_1;l_2)$) and the (weak)-Dvoretzky–Rogers theorem ($\mathcal{L}_{as(p;p)}(E;E) \neq \mathcal{L}(E;E)$ for every $1 \leq p < \infty$ and every infinite-dimensional Banach space E).

The notion of absolutely summing operator has already been generalized to multilinear mappings and homogeneous polynomials in several different ways

(for example: mappings which are absolutely summing [1, 4], semi-integral [1, 22], dominated [4, 8, 18], multiple or fully summing [19, 29], strongly summing [13] and absolutely summing at every point [20, 24, 26]). Similarly to the linear theory, coincidence and non-coincidence theorems are largely studied in the nonlinear setting, and several results are known for the classes already investigated. Furthermore, whenever a class of polynomials forms a holomorphy type, we can investigate coincidence situations for the spaces of holomorphic mappings that are of that type.

In this paper we study coincidence situations in two new classes of absolutely summing multilinear mappings, homogeneous polynomials and holomorphic mappings we introduce in sections 4 and 5 (strongly check mappings and strongly fully mappings). In section 2 we prove some new coincidence results for a well studied class of absolutely summing mappings. Our basic point of view is that the more we know about coincidence situations with respect to a certain class of absolutely summing nonlinear mappings, the more we are able to say how good the class is as a generalization of the linear theory.

2 – Basic concepts and terminology

$E, E_1, ..., E_n$ and F will represent Banach spaces and the scalar field $\mathbb{K}$ can be either $\mathbb{R}$ or $\mathbb{C}$. n will always be a positive integer. The space of all continuous n-linear mappings $A\colon E_1\times \cdots \times E_n \to F$ will be denoted by $\mathcal{L}(E_1, ..., E_n; F)$. It becomes a Banach space with the natural norm

$$\|A\| \,=\, \sup\Big\{\|A(x_1, ..., x_n)\|\colon \ \|x_j\| \leq 1, \ j = 1, ..., n\Big\} \ .$$

If $E_1 = \cdots = E_n = E$ we write $\mathcal{L}(^nE; F)$. Given a continuous n-linear mapping $A \in \mathcal{L}(^nE; F)$, the map

$$P\colon E \to F\colon \quad P(x) = A(x, x, ..., x) \ \text{ for every } \ x \in E \ ,$$

is called a *continuous n-homogeneous polynomial*. The space of all continuous n-homogeneous polynomials from E to F will be denoted by $\mathcal{P}(^nE; F)$, and it becomes a Banach space under the norm

$$\|P\| \,=\, \sup\Big\{\|P(x)\|\colon \|x\| \leq 1\Big\} \,=\, \inf\Big\{C\colon \|P(x)\| \leq C \cdot \|x\|^n, \ \forall x \in E\Big\} \ .$$

The fact that P is the polynomial generated by A, that is $P(x) = A(x, x, ..., x)$, will be denoted by $\hat{A} = P$. An n-linear mapping $A \in \mathcal{L}(^nE; F)$ is *symmetric* if

$A(x_1, ..., x_n) = A(x_{\sigma(1)}, ..., x_{\sigma(n)})$ for every $x_1, ..., x_n \in E$ and every permutation σ of the set $\{1, ..., n\}$. For each polynomial $P \in \mathcal{P}(^nE; F)$ there is a unique symmetric n-linear mapping $\overset{\vee}{P} \in \mathcal{L}(^nE; F)$ which generates P. For convenience, we identify $\mathcal{L}(^0E; F)$ and $\mathcal{P}(^0E; F)$ with F (constant functions). A mapping $f \colon E \to F$ is *holomorphic* if, corresponding to every $x \in E$ there is a (unique) sequence of polynomials $(P_m)_{m=0}^\infty$, $P_m \in \mathcal{P}(^mE; F)$ for every m, and $\rho > 0$ such that

$$f(a) = \sum_{m=0}^\infty P_m(a - x) \qquad \text{uniformly for} \quad \|a - x\| < \rho .$$

As usual we set the notations

$$d^m f(x) = m! \, \overset{\vee}{P_m} \quad \text{and} \quad \overset{\wedge}{d^m} f(x) = m! \, P_m ,$$

and denote the linear space of all holomorphic mappings from E to F by $\mathcal{H}(E; F)$. For the theory of polynomial/multilinear/holomorphic mappings we refer to [14, 21]. If F is the scalar field we shall use the simplified notations $\mathcal{L}(E_1, ..., E_n)$, $\mathcal{L}(^nE)$, $\mathcal{P}(^nE)$ and $\mathcal{H}(E)$.

Now we state the concept of global holomorphy type [6], adapted from Nachbin's original concept of holomorphy types [21].

A **global holomorphy type** $\mathcal{P}_\theta$ is a subclass of the class of continuous homogeneous polynomials between Banach spaces such that for every natural n and every Banach spaces E and F, the component $\mathcal{P}_\theta(^mE; F) := \mathcal{P}(^mE; F) \cap \mathcal{P}_\theta$ is a linear subspace of $\mathcal{P}(^mE; F)$, which is a Banach space when endowed with a norm $\|\cdot\|_\theta$ and

   (1) $\mathcal{P}_\theta(^0E; F) = F$, as a normed linear space for all E and F.

   (2) There exists a real number $\sigma \geq 1$ such that: given any $l, m \in \mathbb{N}$, $l \leq m$, Banach spaces E and F, $x \in E$ and $P \in \mathcal{P}_\theta(^mE; F)$, we have $\overset{\wedge}{d^l} P(x) \in \mathcal{P}_\theta(^lE; F)$ and

$$\left\| \frac{1}{l!} \overset{\wedge}{d^l} P(x) \right\|_\theta \leq \sigma^m \|P\|_\theta \|x\|^{m-l} .$$

In this case, a mapping $f \in \mathcal{H}(E; F)$ is of θ-holomorphy type at $x \in E$ if

   (1) $\overset{\wedge}{d^m} f(x) \in \mathcal{P}_\theta(^mE; F)$ for every natural m;

   (2) There exist real numbers $C \geq 0$ and $c \geq 0$ such that

$$\left\| \frac{1}{m!} \overset{\wedge}{d^m} f(x) \right\|_\theta \leq C c^m, \qquad \text{for every natural} \quad m .$$

When $f \in \mathcal{H}(E; F)$ is of θ-holomorphy type at every $x \in E$, we write $f \in \mathcal{H}_{\mathcal{P}_\theta}(E; F)$.

Ideals of multilinear functionals were introduced by Pietsch [31] and ideals of polynomials were first considered by Braunss [10] (see also [5, 6, 7, 15, 16]).

Definition 2.1. An **ideal of multilinear mappings** $\mathcal{M}$ is a subclass of the class of all continuous multilinear mappings between Banach spaces such that for all n and $E_1, ..., E_n, F$ the components $\mathcal{M}(E_1, ..., E_n; F) := \mathcal{L}(E_1, ..., E_n; F) \cap \mathcal{M}$ satisfy:

 (**i**) $\mathcal{M}(E_1, ..., E_n; F)$ is a linear subspace of $\mathcal{L}(E_1, ..., E_n; F)$ which contains the n-linear mappings of finite type.

 (**ii**) If $A \in \mathcal{M}(E_1, ..., E_n; F)$, $u_j \in \mathcal{L}(G_j; E_j)$ for $j = 1, ..., n$ and $\varphi \in \mathcal{L}(F; H)$, then $\varphi \circ A \circ (u_1, ..., u_n) \in \mathcal{M}(G_1, ..., G_n; H)$.

When there exists $\|\cdot\|_{\mathcal{M}} \colon \mathcal{M} \to [0, \infty[$ satisfying

 (**i′**) $\|\cdot\|_{\mathcal{M}}$ restricted to $\mathcal{M}(E_1, ..., E_n; F)$ is a norm (resp. quasi-norm) for all $E_1, ..., E_n, F$ and all natural numbers n,

 (**ii′**) $\big\|A \colon \mathbb{K}^n \to \mathbb{K}; \ A(x_1, ..., x_n) = x_1 ... x_n\big\|_{\mathcal{M}} = 1$ for all n,

 (**iii′**) If $A \in \mathcal{M}(E_1, ..., E_n; F)$, $u_j \in \mathcal{L}(G_j; E_j)$ for $j = 1, ..., n$ and $\varphi \in \mathcal{L}(F; H)$, then

$$\big\|\varphi \circ A \circ (u_1, ..., u_n)\big\|_{\mathcal{M}} \leq \|\varphi\| \, \|A\|_{\mathcal{M}} \, \|u_1\| ... \|u_n\| \ ,$$

$\mathcal{M}$ is called a **normed** (resp. **quasi-normed**) **ideal of multilinear mappings**.

An **ideal of (homogeneous) polynomials** $\mathcal{Q}$ is a subclass of the class of all continuous homogeneous polynomials between Banach spaces such that for all $n \in \mathbb{N}$ and all Banach spaces E, F, the components $\mathcal{Q}(^nE; F) := \mathcal{P}(^nE; F) \cap \mathcal{Q}$ satisfy:

 (**i**) $\mathcal{Q}(^nE; F)$ is a linear subspace of $\mathcal{P}(^nE; F)$ which contains the polynomials of finite type.

 (**ii**) If $P \in \mathcal{Q}(^nE; F)$, $\varphi_1 \in \mathcal{L}(G; E)$ and $\varphi_2 \in \mathcal{L}(F; H)$, then $\varphi_2 \circ P \circ \varphi_1 \in \mathcal{Q}(^nG; H)$.

When there exists $\|\cdot\|_{\mathcal{Q}} \colon \mathcal{Q} \to [0, \infty[$ satisfying

 (**i′**) $\|\cdot\|_{\mathcal{Q}}$ restricted to $\mathcal{Q}(^nE; F)$ is a norm (resp. quasi-norm) for all Banach spaces E and F and all n,

 (**ii′**) $\big\|P \colon \mathbb{K} \to \mathbb{K}; \ P(x) = x^n\big\|_{\mathcal{Q}} = 1$ for all n,

(iii′) If $P \in \mathcal{Q}(^nE; F)$, $u \in \mathcal{L}(G; E)$ and $\varphi \in \mathcal{L}(F; H)$, then $\|\varphi \circ P \circ u\|_{\mathcal{Q}} \leq$ $\|\varphi\| \, \|P\|_{\mathcal{Q}} \, \|u\|^n$, $\mathcal{Q}$ is called a **normed** (resp. **quasi-normed**) **ideal of polynomials.**

In the case of quasi-normed ideals, the quasi-norm constants depend (eventually) only on n, not depending on the underlying spaces $E_1, ..., E_n, E, F$. □

For $n = 1$, we write $\mathcal{M}(E; F)$ and $\mathcal{Q}(E; F)$ instead of $\mathcal{M}(^1E; F)$ and $\mathcal{Q}(^1E; F)$. Given a normed ideal of multilinear mappings $\mathcal{M}$, defining

$$\mathcal{P}_{\mathcal{M}} := \left\{ P \, ; \, \overset{\vee}{P} \in \mathcal{M} \right\} , \qquad \|P\|_{\mathcal{P}_{\mathcal{M}}} := \|\overset{\vee}{P}\|_{\mathcal{M}} ,$$

we obtain an ideal of polynomials, called the ideal of polynomials generated by $\mathcal{M}$.

As we have mentioned, the ideal of absolutely $(p; q)$-summing operators is represented by $\mathcal{L}_{as(p;q)}$. The $(p; q)$-summing norm on $\mathcal{L}_{as(p;q)}$ will be denoted by $\|\cdot\|_{as(p;q)}$ (see [12]). Since we are interested in coincidence situations, it seems to be interesting to consider ideals of multilinear mappings which reproduce, to polynomials and holomorphic mappings, exactly the linear coincidence situations. More precisely, given $1 \leq q \leq p$, it may be interesting to consider ideals of multilinear mappings $\mathcal{M}_{p,q}$ such that:

(i) $\mathcal{P}_{\mathcal{M}_{p,q}}$ is a global holomorphy type;

(ii) $\mathcal{M}_{p,q}(E; F) = \mathcal{L}_{as(p;q)}(E; F)$ for every Banach spaces E, F;

(iii) Regardless of the $n \in \mathbb{N}$ and Banach spaces E, F, we have

(1)
$$\begin{aligned}
\mathcal{P}_{\mathcal{M}_{p,q}}(^nE; F) = \mathcal{P}(^nE; F) &\iff \mathcal{L}_{as(p;q)}(E; F) = \mathcal{L}(E; F) \\
&\iff \mathcal{H}_{\mathcal{P}_{\mathcal{M}_{p,q}}}(E; F) = \mathcal{H}(E; F).
\end{aligned}$$

Next example shows that, though interesting *a priori*, this approach may be misleading.

Example 2.2. Given $r \in [1, \infty)$, we denote by $l_r^w(E)$ the Banach space of all weakly r-summable sequences $(x_j)_{j=1}^\infty$ in E with the norm $\|(x_j)_{j=1}^\infty\|_{w,r} = \sup_{\varphi \in B_{E'}} \left(\sum_{j=1}^\infty |\varphi(x_j)|^r \right)^{\frac{1}{r}}$. Given $p \geq q \geq 1$, a continuous n-linear mapping $T : E_1 \times \cdots \times E_n \to F$ is said to belong to $\mathcal{L}_{s(p;q)}(E_1, ..., E_n; F)$ if there exist $C \geq 0$

such that

$$(2) \qquad \left(\sum_{j=1}^{\infty} \|T(x_j, a_2, ..., a_n)\|^p \right)^{\frac{1}{p}} \le C \left\| (x_j)_{j=1}^{\infty} \right\|_{w,q} \|a_2\| ... \|a_n\| \, ,$$

for each $a_k \in E_k$, $k = 2, .., n$ and every $(x_j)_{j=1}^{\infty}$ in $l_q^w(E_1)$. Defining $\|T\|_{s(p;q)} = \inf\{C;$ inequality (2) holds$\}$ for every $T \in \mathcal{L}_{s(p;q)}(E_1, ..., E_n; F)$, it is easy to see that $\mathcal{L}_{s(p;q)}$ is a normed ideal of multilinear mappings for which conditions (i) to (iii) above are satisfied. Nevertheless, this class is quite artificial, offering no practical interest, being actually the ideal of absolutely $(p; q)$-summing linear operators in an non-linear disguise. Our purpose in this paper is to show that more interesting classes also present a good behavior concerning coincidence situations. □

3 – Absolutely summing multilinear mappings

In this section we prove new coincidence theorems for the multilinear generalization of absolutely summing linear operators introduced by R. Alencar and M. Matos [1].

By $l_p(F)$ we mean the space of absolutely p-summable sequences in F endowed with its natural norm. A continuous multilinear mapping $T \colon E_1 \times \cdots \times E_n \to F$ is **absolutely** $(p; q_1, ..., q_n)$**-summing** (or $(p; q_1, ..., q_n)$-summing) if $\left(T\big(x_j^{(1)}, ..., x_j^{(n)}\big) \right)_{j=1}^{\infty} \in l_p(F)$ for all $(x_j^{(s)})_{j=1}^{\infty} \in l_{q_s}^w(E_s)$, $s = 1, ..., n$. The space of absolutely $(p; q_1, ..., q_n)$-summing n-linear mappings from $E_1 \times \cdots \times E_n$ into F is denoted by $\mathcal{L}_{as(p;q_1,...,q_n)}(E_1, ..., E_n; F)$. When $q_1 = ... = q_n = q$, we write $\mathcal{L}_{as(p;q)}(E_1, ..., E_n; F)$. For the characterization of absolutely summing multilinear mappings by inequalities and the corresponding ideal norm $\|\cdot\|_{as(p;q_1,...,q_n)}$ on $\mathcal{L}_{as(p;q_1,...,q_n)}(E_1, ..., E_n; F)$ we refer to [4, 18]. Following the lines of [2, Theorem 4] and [25, Theorem 3], next we show that these results can be pushed further.

Theorem 3.1. Let $A \in \mathcal{L}(E_1, ..., E_n; F)$ and suppose that there exist $1 \le r < n$ and $C > 0$ so that for any $x_1 \in E_1$, ..., $x_r \in E_r$, the s-linear $(s = n - r)$ mapping $A_{x_1....x_r}(x_{r+1}, ..., x_n) = A(x_1, ..., x_n)$ is absolutely $(p; q_1, ..., q_s)$-summing and $\|A_{x_1....x_r}\|_{as(p;q_1,...,q_s)} \le C \|A\| \|x_1\| ... \|x_r\|$. Then A is absolutely $(p; 1, ..., 1, q_1, ..., q_s)$-summing.

Proof: Given $m \in \mathbb{N}$ and $x_1^{(1)}, ..., x_1^{(m)} \in E_1,\ ...,\ x_n^{(1)}, ..., x_n^{(m)} \in E_n$, let us consider $\varphi_j \in B_{F'}$ such that $\left\| A\big(x_1^{(j)}, ..., x_n^{(j)}\big) \right\| = \varphi_j\big(A\big(x_1^{(j)}, ..., x_n^{(j)}\big)\big)$ for every $j = 1, ..., m$. Fix $b_1, ..., b_m \in \mathbb{K}$ so that $\sum\limits_{j=1}^{m} |b_j|^q = 1$, where $\frac{1}{p} + \frac{1}{q} = 1$, and

$$
\left(\sum_{j=1}^{m} \left\| A\big(x_1^{(j)}, ..., x_n^{(j)}\big) \right\|^p \right)^{\frac{1}{p}} = \left\| \left(\left\| A\big(x_1^{(j)}, ..., x_n^{(j)}\big) \right\| \right)_{j=1}^{m} \right\|_p
$$

$$
= \sum_{j=1}^{m} b_j \left\| A\big(x_1^{(j)}, ..., x_n^{(j)}\big) \right\| .
$$

If (r_j) are the Rademacher functions and λ is the Lebesgue measure on $I = [0,1]^r$, we have

$$
\int_I \sum_{j=1}^{m} \left(\prod_{l=1}^{r} r_j(t_l) \right) b_j\, \varphi_j\, A\left(\sum_{j_1=1}^{m} r_{j_1}(t_1) x_1^{(j_1)}, ..., \sum_{j_r=1}^{m} r_{j_r}(t_r) x_r^{(j_r)}, x_{r+1}^{(j)}, ..., x_n^{(j)} \right) d\lambda =
$$

$$
= \sum_{j,j_1,...j_r=1}^{m} b_j\, \varphi_j\, A\left(x_1^{(j_1)}, ..., x_r^{(j_r)}, x_{r+1}^{(j)}, ..., x_n^{(j)} \right) \int_0^1 r_j(t_1) r_{j_1}(t_1)\, dt_1 \cdots \int_0^1 r_j(t_r) r_{j_r}(t_r)\, dt_r
$$

$$
= \sum_{j=1}^{m} \sum_{j_1=1}^{m} \cdots \sum_{j_r=1}^{m} b_j\, \varphi_j\, A\left(x_1^{(j_1)}, ..., x_r^{(j_r)}, x_{r+1}^{(j)}, ..., x_n^{(j)} \right) \delta_{jj_1} \cdots \delta_{jj_r}
$$

$$
= \sum_{j=1}^{m} b_j\, \varphi_j\, A\left(x_1^{(j)}, ..., x_n^{(j)} \right) .
$$

Hence, defining $z_l = \sum\limits_{j=1}^{m} r_j(t_l) x_l^{(j)}$, $l = 1, ..., r$, we obtain

$$
\left(\sum_{j=1}^{m} \left\| A\big(x_1^{(j)}, ..., x_n^{(j)}\big) \right\|^p \right)^{\frac{1}{p}} =
$$

$$
= \sum_{j=1}^{m} b_j\, \varphi_j\, A\left(x_1^{(j)}, ..., x_n^{(j)} \right)
$$

$$
\leq \int_I \left| \sum_{j=1}^{m} \left(\prod_{l=1}^{r} r_j(t_l) \right) b_j\, \varphi_j\, A\left(\sum_{j_1=1}^{m} r_{j_1}(t_1) x_1^{(j_1)}, ..., \sum_{j_r=1}^{m} r_{j_r}(t_r) x_r^{(j_r)}, x_{r+1}^{(j)}, ..., x_n^{(j)} \right) \right| d\lambda
$$

$$
\leq \int_I \sum_{j=1}^{m} |b_j| \left\| A\left(\sum_{j_1=1}^{m} r_{j_1}(t_1) x_1^{(j_1)}, ..., \sum_{j_r=1}^{m} r_{j_r}(t_r) x_r^{(j_r)}, x_{r+1}^{(j)}, ..., x_n^{(j)} \right) \right\| d\lambda \ \leq
$$

$$\leq \sup_{t_l \in [0,1], l=1,...,r} \sum_{j=1}^m |b_j| \left\| A\left(\sum_{j_1=1}^m r_{j_1}(t_1) x_1^{(j_1)}, ..., \sum_{j_r=1}^m r_{j_r}(t_r) x_r^{(j_r)}, x_{r+1}^{(j)}, ..., x_n^{(j)} \right) \right\|$$

$$\leq \sup_{t_l \in [0,1], l=1,...,r} \left(\sum_{j=1}^m \left\| A\left(\sum_{j_1=1}^m r_{j_1}(t_1) x_1^{(j_1)}, ..., \sum_{j_r=1}^m r_{j_r}(t_r) x_r^{(j_r)}, x_{r+1}^{(j)}, ..., x_n^{(j)} \right) \right\|^p \right)^{\frac{1}{p}}$$

$$\leq \sup_{t_l \in [0,1], l=1,...,r} \left\| A_{z_1...z_r} \right\|_{as(p;q_1,...,q_s)} \left\| (x_{r+1}^{(j)})_{j=1}^m \right\|_{w,q_1} \cdots \left\| (x_n^{(j)})_{j=1}^m \right\|_{w,q_s}$$

$$\leq \sup_{t_l \in [0,1], l=1,...,r} C \left\| A \right\| \left\| z_1 \right\| \cdots \left\| z_r \right\| \left\| (x_{r+1}^{(j)})_{j=1}^m \right\|_{w,q_1} \cdots \left\| (x_n^{(j)})_{j=1}^m \right\|_{w,q_s}$$

$$\leq C \left\| A \right\| \left(\prod_{l=1}^r \left\| (x_l^{(j)})_{j=1}^m \right\|_{w,1} \right) \left(\prod_{l=r+1}^n \left\| (x_l^{(j)})_{j=1}^m \right\|_{w,q_l} \right) . \blacksquare$$

Corollary 3.2. *If $\mathcal{L}(E_1,...,E_m;F) = \mathcal{L}_{as(p;q_1,...,q_m)}(E_1,...,E_m;F)$ then, for any Banach spaces $E_{m+1},...,E_n$, we have $\mathcal{L}(E_1,...,E_n;F) = \mathcal{L}_{as(p;q_1,...,q_m,1,...,1)}(E_1,...,E_n;F)$. In particular, for $p \geq 1$ and any Banach spaces $E_1,...,E_m$, we have $\mathcal{L}(E_1,...,E_m) = \mathcal{L}_{as(p;p,1,...,1)}(E_1,...,E_m)$.* $\blacksquare$

The proof of Theorem 3.1 provides good estimates for absolutely summing norms:

Corollary 3.3. *If, for some E and F, $\mathcal{L}(E;F) = \mathcal{L}_{as(p;q)}(E;F)$ and $\|T\|_{as(p;q)} \leq C\|T\|$ for every $T \in \mathcal{L}(E;F)$, then $\mathcal{L}(E, E_2,...,E_m;F) = \mathcal{L}_{as(p;q,1,...,1)}(E, E_2,...,E_m;F)$ and $\|A\|_{as(p;q,1,...,1)} \leq C\|A\|$ for every $A \in \mathcal{L}(E, E_2,...,E_m;F)$ and every $E_2,...,E_m$.* $\blacksquare$

Remark 3.4.

(a) Corollary 3.3 improves [4, Theorems 2.2(ii) and 2.5(ii)], results which were reobtained in [11, Theorem 6].

(b) It is well-known that $\mathcal{L}(^n l_1; l_2) = \mathcal{L}_{as(1;1)}(^n l_1; l_2)$ for every n, but the optimal constant C (for $n \geq 2$) such that $\|\cdot\|_{as(1;1)} \leq C\|\cdot\|$ seems to be unknown. Note that from Grothendieck's theorem, Corollary 3.3 shows that, for every n, $\|\cdot\|_{as(1;1)} \leq K_G \|\cdot\|$, where K_G is Grothendieck's constant. Let us see that the constant K_G is optimal: given $\varepsilon > 0$, choose a linear operator $T \colon l_1 \to l_2$ so that $\|T\|_{as(1;1)} > (K_G - \varepsilon) \|T\|$. Define $A \colon l_1 \times l_1 \to l_2$ by $A(x, y) = y_1 T(x)$. It is not difficult to see that $\|A\| = \|T\|$ and $\|A\|_{as(1,1)} = \|T\|_{as(1,1)}$. $\square$

4 – Strongly check summing polynomials

The concept of strongly p-summing multilinear mappings, which is due to V. Dimant [13], is adapted to strongly $(p;q)$-summing multilinear mappings as follows:

Definition 4.1. An n-linear mapping $T \in \mathcal{L}(E_1, ..., E_n; F)$ is **strongly** $(p;q)$-summing if there exists a constant $C \geq 0$ such that for every $m \in \mathbb{N}$, $x_1^{(i)}, ..., x_m^{(i)} \in E_i$, $i = 1, ..., n$, we have

$$(3) \quad \left(\sum_{j=1}^{m} \left\| T\big(x_j^{(1)}, ..., x_j^{(n)}\big) \right\|^p \right)^{1/p} \leq C \left(\sup_{\phi \in B_{\mathcal{L}(E_1, ..., E_n)}} \sum_{j=1}^{m} \left| \phi\big(x_j^{(1)}, ..., x_j^{(n)}\big) \right|^q \right)^{1/q} .$$

In this case we write $T \in \mathcal{L}_{ss(p;q)}(E_1, ..., E_n; F)$. The infimum of the C such that (3) holds defines a norm on $\mathcal{L}_{ss(p;q)}$, represented by $\|\cdot\|_{ss(p;q)}$. $\square$

Strongly summing polynomials are defined in [13] by a related inequality. It is obvious that multilinear forms and scalar-valued polynomials are strongly p-summing, and non-trivial coincidence results are proved in [13, Proposition 3.3]. The ideal of strongly p-summing polynomials is not suitable for our purposes because it fails to be a (global) holomorphy type. In this section we work with the ideal of polynomials $\mathcal{P}_{\mathcal{L}_{ss(p;q)}}$ generated by the ideal of strongly $(p;q)$-summing multilinear mappings, which happens to be a global holomorphy type (this is an easy consequence of [6, Theorem 3.2]) and enjoys nice properties related to coincidence situations. For example, it is easy to see that

$$(4) \quad \mathcal{P}_{\mathcal{L}_{ss(p;q)}}(^nE; F) = \mathcal{P}(^nE; F) \text{ for some } n \implies \mathcal{L}_{as(p;q)}(E; F) = \mathcal{L}(E; F) .$$

Polynomials belonging to $\mathcal{P}_{\mathcal{L}_{ss(p;q)}}$ are called **strongly check** summing polynomials. The following two coincidences are also easily justified:

- In [13] it is stated that $\mathcal{L}_{ss(1;1)}(^nl_1; l_2) = \mathcal{L}(^nl_1; l_2)$ for every n. It follows immediately that $\mathcal{P}_{\mathcal{L}_{ss(1;1)}}(^nl_1; l_2) = \mathcal{P}(^nl_1; l_2)$ for every n.
- $\mathcal{H}_{\mathcal{P}_{\mathcal{L}_{ss(p;p)}}}(E) = \mathcal{H}(E)$ for every E because

$$\|P\| \leq \|P\|_{\mathcal{P}_{\mathcal{L}_{ss(p;p)}}} = \|\overset{\vee}{P}\|_{\mathcal{L}_{ss(p;p)}} = \|\overset{\vee}{P}\| \leq e^n \|P\|$$

for every n and every $P \in \mathcal{P}(^nE)$.

The following simple Lemma will be useful later. Given an ideal of multilinear mappings $\mathcal{M}$, by $\mathcal{M}_S$ we denote the set of all symmetric multilinear mappings belonging to $\mathcal{M}$.

Lemma 4.2.   *Let $\mathcal{M}$ be a Banach ideal of multilinear mappings such that $\mathcal{P}_{\mathcal{M}}$ is a global holomorphy type. If E and F are Banach spaces such that*

(i) $\mathcal{M}_S(^nE;F) = \mathcal{L}_S(^nE;F)$ *for every n;*

(ii) *There are $C, c > 0$ such that $\|A\|_{\mathcal{M}} \le C\, c^n \|A\|$, for every n and every $A \in \mathcal{L}_S(^nE;F)$;*

then $\mathcal{H}_{\mathcal{P}_{\mathcal{M}}}(E;F) = \mathcal{H}(E;F)$.

Proof: Given $f \in \mathcal{H}(E;F)$ and $x \in E$, let c_1 and C_1 be such that $\left\|\frac{1}{n!}\, \widehat{d^n}f(x)\right\| \le C_1 c_1^n$ for every n. We have that $d^n f(x)$ is a symmetric n-linear mapping, so by (ii), $d^n f(x) \in \mathcal{M}_S(^nE;F)$. It follows that $\widehat{d^n}f(x) \in \mathcal{P}_{\mathcal{M}}(^nE;F)$. $f \in \mathcal{H}_{\mathcal{P}_{\mathcal{M}}}(E;F)$ because

$$\left\|\frac{1}{n!}\, \widehat{d^n}f(x)\right\|_{\mathcal{P}_{\mathcal{M}}} = \left\|\frac{1}{n!}\, d^n f(x)\right\|_{\mathcal{M}} \le C\,c^n \left\|\frac{1}{n!}\, d^n f(x)\right\| \le C\,c^n e^n \left\|\frac{1}{n!}\, \widehat{d^n}f(x)\right\|$$

$$\le C\,c^n e^n C_1\, c_1^n = C C_1 (c\,c_1\,e)^n \ . \ \blacksquare$$

If F has cotype q, it is well known that $\mathcal{L}_{as(q;1)}(E;F) = \mathcal{L}(E;F)$ for every E. Next we lift this result to strongly (check) $(q;1)$-summing polynomials and holomorphic mappings.

Proposition 4.3.   *If F has cotype q, then $\mathcal{H}_{\mathcal{P}_{\mathcal{L}_{ss(q;1)}}}(E;F) = \mathcal{H}(E;F)$ for every E.*

Proof: Consider $T \in \mathcal{L}(^nE;F)$. Since F has cotype q, $id \colon F \to F$ is $(q;1)$-summing and $\|id\|_{as(q;1)} \le C_q(F)$, where $C_q(F)$ is the cotype q constant of F. Hence

$$\left(\sum_{j=1}^m \left\|T\big(x_j^{(1)}, ..., x_j^{(n)}\big)\right\|^q\right)^{\frac{1}{q}} \le C_q(F) \left\|\left(T\big(x_j^{(1)}, ..., x_j^{(n)}\big)\right)_{j=1}^m\right\|_{w,1}$$

$$\le C_q(F) \|T\| \sup_{\Phi \in B_{\mathcal{L}(^nE)}} \sum_{j=1}^m \left|\Phi\big(x_j^{(1)}, ..., x_j^{(n)}\big)\right|$$

and $\|T\|_{ss(q;1)} \le C_q(F) \|T\|$. Thus, Lemma 4.2 completes the proof. $\blacksquare$

Next result shows that, in special situations, coincidence results for strongly check polynomials and holomorphic mappings are exactly the ones that hold in the linear case. From now on we denote $\cot E = \inf\{q;\ E \text{ has cotype } q\}$.

Theorem 4.4. *Let E be an infinite-dimensional Banach space with cotype $q = \cot E < \infty$. The following assertions are equivalent:*

(a) $p \geq q$.

(b) $\mathcal{L}_{as(p;1)}(E;E) = \mathcal{L}(E;E)$.

(c) $\mathcal{P}_{\mathcal{L}_{ss(p;1)}}(^nE;E) = \mathcal{P}(^nE;E)$ *for some n.*

(d) $\mathcal{P}_{\mathcal{L}_{ss(p;1)}}(^nE;E) = \mathcal{P}(^nE;E)$ *for every n.*

(e) $\mathcal{H}_{\mathcal{P}_{\mathcal{L}_{ss(p;1)}}}(E;E) = \mathcal{H}(E;E)$.

Proof: The equivalence between (a) and (b) is well known from the linear theory. Since E has cotype $q = \cot E$, Proposition 4.3 gives $\mathcal{H}_{\mathcal{P}_{\mathcal{L}_{ss(q;1)}}}(E;E) = \mathcal{H}(E;E)$. Hence, if $p \geq q$, we have

$$\mathcal{H}_{\mathcal{P}_{\mathcal{L}_{ss(p;1)}}}(E;E) = \mathcal{H}(E;E) \ , \qquad \mathcal{P}_{\mathcal{L}_{ss(p;1)}}(^nE;E) = \mathcal{P}(^nE;E) \ \ (\forall n)$$

$$\text{and} \quad \mathcal{L}_{as(p;1)}(E;E) = \mathcal{L}(E;E) \ .$$

Therefore (a) (or (b)) implies all the other assertions. Since (e)$\Rightarrow$(b) and (d)$\Rightarrow$(b) are obvious, we just need to show that (c)$\Rightarrow$(b). But this is consequence of (4). ∎

5 – Strongly fully summing mappings

It is well known that $\mathcal{L}(E;F) = \mathcal{L}_{as(q;1)}(E;F)$ whenever E has cotype q. We do not know if a related coincidence holds true for strongly check summing polynomials. This question motivated us to introduce, in this section, another nonlinear generalization of absolutely summing linear operators — a class which present better coincidence results — as follows: the inequality

$$(5) \qquad \left(\sum_{j=1}^{m} \|u(x_j)\|^p \right)^{1/p} \leq C \sup_{\phi \in B_{E'}} \left(\sum_{j=1}^{m} |\phi(x_j)|^q \right)^{1/q},$$

where u is a $(p;q)$-summing linear operator, make possible several different non-linear generalizations. The replacement of the linear operator u by a multilinear mapping or a homogeneous polynomial and the corresponding product of the

weak l_q-norms in the right hand side leads to the Alencar–Matos concept we studied in section 2. Replacing linear functionals by multilinear forms in the right hand side of the inequality we obtain the Dimant concept of strongly summing mappings we treated in section 3. Summing in multiple indexes instead of in only one index in the left hand side of the inequality we obtain the concept of fully (or multiple) summing mappings, which was introduced by M. Matos [19] and developed in [3, 27, 28, 29, 30, 32], among others. More precisely, we say that an n-linear mapping $T \in \mathcal{L}(E_1, ..., E_n; F)$ is fully $(p; q)$-summing if there exists $C \geq 0$ such that

$$(6) \qquad \left(\sum_{j_1,...,j_n=1}^m \left\| T\big(x_{j_1}^{(1)}, ..., x_{j_n}^{(n)}\big) \right\|^p \right)^{1/p} \leq C \prod_{r=1}^n \left\| \big(x_j^{(r)}\big)_{j=1}^m \right\|_{w,q} ,$$

for every $m \in \mathbb{N}$ and $x_j^{(r)} \in E_r$, $j = 1, ..., m$, $r = 1, ..., n$. In this case we write $T \in \mathcal{L}_{fas(p;q)}(E_1, ..., E_n; F)$.

Combining the summation in multiple indexes with the consideration of multilinear forms instead of linear functionals we obtain the notion of strongly fully summing mappings:

Definition 5.1. Given $p \geq q \geq 1$, a multilinear mapping $T \in \mathcal{L}(E_1, ..., E_n; F)$ is said to be strongly fully $(p; q)$-summing if there exists $C \geq 0$ such that

$$(7)$$
$$\left(\sum_{j_1,...,j_n=1}^m \left\| T\big(x_{j_1}^{(1)}, ..., x_{j_n}^{(n)}\big) \right\|^p \right)^{1/p} \leq C \left(\sup_{\phi \in B_{\mathcal{L}(E_1,...,E_n)}} \sum_{j_1,...,j_n=1}^m \left| \phi\big(x_{j_1}^{(1)}, ..., x_{j_n}^{(n)}\big) \right|^q \right)^{1/q}$$

for every $m \in \mathbb{N}$, $x_{j_l}^{(l)} \in E_l$ with $l = 1, ..., n$ and $j_l = 1, ..., m$.

The space composed by all strongly fully $(p; q)$-summing n-linear mappings from $E_1 \times \cdots \times E_n$ into F will be represented by $\mathcal{L}_{sf(p;q)}(E_1, ..., E_n; F)$ and the infimum of the C for which the inequality always holds defines a norm $\| \cdot \|_{sf(p;q)}$ on $\mathcal{L}_{sf(p;q)}(E_1, ..., E_n; F)$. Under this norm, $\mathcal{L}_{sf(p;q)}(E_1, ..., E_n; F)$ is complete. When $q = p$ we simply write $\mathcal{L}_{sf,p}(E_1, ..., E_n; F)$ and $\| \cdot \|_{sf,p}$. □

One can easily verify the following properties:

Proposition 5.2. *Given $p \geq q \geq 1$, $n \in \mathbb{N}$ and Banach spaces $E, E_1, ..., E_n, F$:*

(i) $\mathcal{L}_{fas(p;q)}(E_1, ..., E_n; F) \subset \mathcal{L}_{sf(p;q)}(E_1, ..., E_n; F)$.

(ii) $\mathcal{L}_{ss(p;q)}(E_1, ..., E_n; F) \subset \mathcal{L}_{sf(p;q)}(E_1, ..., E_n; F)$.

(**iii**) If $\mathcal{L}_{sf(p;q)}(E_1, ..., E_n; F) = \mathcal{L}(E_1, ..., E_n; F)$,   then $\mathcal{L}(E_{j_1}, ..., E_{j_k}; F) = \mathcal{L}_{sf(p;q)}(E_{j_1}, ..., E_{j_k}; F)$, for every $k = 1, ..., n$ and every $j_1, ..., j_k$ in $\{1, ..., n\}$ with $j_r \neq j_s$ if $r \neq s$.

(**iv**) (*Dvoretzky–Rogers type Theorem*)
$\mathcal{L}_{sf,p}(^n E; E) = \mathcal{L}(^n E; E) \iff \dim E < \infty.$ ∎

Since $\left(\mathcal{L}_{fas,p} \cup \mathcal{L}_{ss,p} \right) \subseteq \mathcal{L}_{sf,p}$, every coincidence result for strongly p-summing and/or fully p-summing multilinear mappings still holds for strongly fully summing mappings. On the other hand, property (iii) above shows that coincidence results of the form $\mathcal{L}_{sf,p}(E_1, ..., E_n; F) = \mathcal{L}(E_1, ..., E_n; F)$ are not so common. For example, since $\mathcal{L}(^n l_1; l_2) = \mathcal{L}_{ss,1}(^n l_1; l_2)$ (see [13]), by (ii), (iii) and [17, Theorem 4.2] it follows that if E has unconditional Schauder basis and F is an infinite dimensional Banach space, then $\mathcal{L}_{sf,1}(^n E; F) = \mathcal{L}(^n E; F)$ if and only if E is isomorphic to l_1 and F is a Hilbert space.

Now we consider the ideal of polynomials $\mathcal{P}_{\mathcal{L}_{sf(p;q)}}$, which can be proved to be a global holomorphy type using the results of [6], and the class of holomorphic mappings $\mathcal{H}_{\mathcal{P}_{\mathcal{L}_{sf(p;q)}}}$.

In the scalar-valued case it is obvious that $\mathcal{P}_{\mathcal{L}_{sf(p;q)}}(^n E) = \mathcal{P}(^n E)$ for all n and all E. Proceeding as in Section 4 we have $\mathcal{H}_{\mathcal{P}_{\mathcal{L}_{sf(p;q)}}}(E) = \mathcal{H}(E)$ for every E. Next results show that strongly fully summing mappings have a good behavior concerning coincidence situations. For simplicity, we will write $\mathcal{H}_{sf(p;q)}$ and $\mathcal{P}_{sf(p;q)}$ instead of $\mathcal{H}_{\mathcal{P}_{\mathcal{L}_{sf(p;q)}}}$ and $\mathcal{P}_{\mathcal{L}_{sf(p;q)}}$.

Proposition 5.3. *If E has cotype q, then $\mathcal{H}_{sf(q;1)}(E; F) = \mathcal{H}(E; F)$ for every F.*

Proof: Let us consider $T \in \mathcal{L}(^n E; F)$. Since E has cotype q, $id \colon E \to E$ is $(q; 1)$-summing and $\|id\|_{as(q;1)} \leq C_q(E)$. Hence

$$\left(\sum_{j_1, ..., j_n = 1}^{m} \left\| T\big(x_{j_1}^{(1)}, ..., x_{j_n}^{(n)}\big) \right\|^q \right)^{\frac{1}{q}} \leq \|T\| \left(\sum_{j=1}^{m} \left\| x_j^{(1)} \right\|^q \right)^{\frac{1}{q}} \cdots \left(\sum_{j=1}^{m} \left\| x_j^{(n)} \right\|^q \right)^{\frac{1}{q}}$$

$$\leq C_q(E)^n \, \|T\| \sup_{\Phi \in B_{\mathcal{L}(^n E)}} \sum_{j_1, ..., j_n = 1}^{m} \left| \Phi\big(x_{j_1}^{(1)}, ..., x_{j_n}^{(n)}\big) \right|,$$

and $\|T\|_{sf(q;1)} \leq C_q(E)^n \, \|T\|$. Lemma 4.2 completes the proof. ∎

Combining the argument used in the proof of Proposition 4.3 with Lemma 4.2 we obtain:

Proposition 5.4. *If F has cotype q, then $\mathcal{H}_{sf(q;1)}(E;F) = \mathcal{H}(E;F)$ for every E.* ∎

Theorem 5.5. *Let E be an infinite-dimensional $\mathcal{L}_\infty$ space and F has cotype $q = \cot F < \infty$. The following assertions are equivalent:*

(a) $p \geq q$.

(b) $\mathcal{L}_{as(p;1)}(E;F) = \mathcal{L}(E;F)$.

(c) $\mathcal{P}_{sf(p;1)}(^nE;F) = \mathcal{P}(^nE;F)$ *for some* n.

(d) $\mathcal{P}_{sf(p;1)}(^nE;F) = \mathcal{P}(^nE;F)$ *for every* n.

(e) $\mathcal{H}_{sf(p;1)}(E;F) = \mathcal{H}(E;F)$.

Proof: (a)⇒(e) follows from Proposition 5.4.

(e)⇒(d)⇒(c) are obvious.

(c)⇒(b) By assumption we have that every symmetric n-linear mapping from E^n to F is strongly fully $(p;1)$-summing. Using a property similar to Proposition 5.2 (iii), considering only symmetric multilinear mappings, we obtain (b).

(b)⇒(a) follows from [23, Corollary 4], or, more precisely, [9, Example 2.1]. ∎

Remark 5.6. It is not difficult to see that a version of Theorem 4.4 for strongly fully polynomials is also valid. □

Our next aim is to compare the classes of fully summing and strongly fully summing mappings. From the definitions it is easy to see that

$$\mathcal{L}_{fas(p;q)}(^nE) = \mathcal{L}(^nE) \implies \mathcal{L}_{fas(p;q)}(^nE;F) = \mathcal{L}_{sf(p;q)}(^nE;F) \text{ for every } F.$$

From [28] we know that, for $1 \leq p \leq 2$, we have $\mathcal{L}_{fas(p;p)}(^nl_1) = \mathcal{L}(^nl_1)$. Hence $\mathcal{L}_{fas(p;p)}(^nl_1;F) = \mathcal{L}_{sf(p;p)}(^nl_1;F)$ for every F. However, in general these classes are not the same, for example $\mathcal{L}_{fas(2;2)}(^2l_2) \neq \mathcal{L}(^2l_2) = \mathcal{L}_{sf(2;2)}(^2l_2)$.

When $\mathcal{L}_{fas(p;p)}(^nE) \neq \mathcal{L}(^nE)$, consider $T \in \mathcal{L}(^nE) - \mathcal{L}_{fas(p;p)}(^nE)$. Let $F \neq \{0\}$ be a Banach space and $0 \neq v \in F$. Define $R \in \mathcal{L}(^nE;F)$ by $R(x_1,....,x_n) = T(x_1,...,x_n)v$.

Then $R \in \mathcal{L}_{sf(p;p)}(^nE; F)$, because

$$\left(\sum_{j_1,\dots,j_n=1}^{m} \left\| R(x_{j_1}^{(1)}, \dots, x_{j_n}^{(n)}) \right\|^p \right)^{\frac{1}{p}} = \|v\| \left(\sum_{j_1,\dots,j_n=1}^{m} \left| T(x_{j_1}^{(1)}, \dots, x_{j_n}^{(n)}) \right|^p \right)^{\frac{1}{p}}$$

$$\leq \|v\| \|T\| \left(\sup_{\Phi \in B_{\mathcal{L}(^nE)}} \sum_{j_1,\dots,j_n=1}^{m} \left| \Phi(x_{j_1}^{(1)}, \dots, x_{j_n}^{(n)}) \right|^p \right)^{\frac{1}{p}}.$$

Moreover, $R \notin \mathcal{L}_{fas(p;p)}(^nE; F)$. In fact,

$$\left(\sum_{j_1,\dots,j_n=1}^{m} \left\| R(x_{j_1}^{(1)}, \dots, x_{j_n}^{(n)}) \right\|^p \right)^{\frac{1}{p}} = \|v\| \left(\sum_{j_1,\dots,j_n=1}^{m} \left| T(x_{j_1}^{(1)}, \dots, x_{j_n}^{(n)}) \right|^p \right)^{\frac{1}{p}},$$

and since $T \notin \mathcal{L}_{fas(p;p)}(^nE)$, it follows that $R \notin \mathcal{L}_{fas(p;p)}(^nE; F)$.

We thus can state the following result:

Theorem 5.7.   *The following assertions are equivalent for a given Banach space E:*

   (a) $\mathcal{L}_{fas(p;p)}(^nE) = \mathcal{L}(^nE)$.
   (b) $\mathcal{L}_{fas(p;p)}(^nE; F) = \mathcal{L}_{sf(p;p)}(^nE; F)$ *for some* $F \neq \{0\}$.
   (c) $\mathcal{L}_{fas(p;p)}(^nE; F) = \mathcal{L}_{sf(p;p)}(^nE; F)$ *for every* F. ∎

Example 5.8. It is easy to prove that $\mathcal{L}_{fas(2;2)}(^2l_2) \neq \mathcal{L}(^2l_2)$. From this inequality and Theorem 5.7 we have $\mathcal{L}_{fas(2;2)}(^2l_2; F) \neq \mathcal{L}_{sf(2;2)}(^2l_2; F)$ for every Banach space $F \neq \{0\}$. □

Example 5.9. In [3] it is shown that if E is c_0 or a *GT* space of cotype 2, then $\mathcal{L}_{fas(2;2)}(^nE) = \mathcal{L}(^nE)$. By combining this result and Theorem 5.7 we obtain $\mathcal{L}_{fas(2;2)}(^nE; F) = \mathcal{L}_{sf(2;2)}(^nE; F)$ for every Banach space F. □

We also have some inclusions between these classes:

Proposition 5.10. *Let E and F be Banach spaces, $p \geq 2$ and $n \in \mathbb{N}$. Then*

$$\mathcal{L}_{fas(p;\frac{2n}{n+1})}(^nE; F) \subset \mathcal{L}_{sf(p;\frac{2n}{n+1})}(^nE; F) \subset \mathcal{L}_{fas(p;1)}(^nE; F) .$$

In particular,

$$\mathcal{L}_{fas(p;2)}(^nE; F) \subset \mathcal{L}_{sf(p;2)}(^nE; F) \subset \mathcal{L}_{fas(p;1)}(^nE; F) .$$

Proof: In [30] it is shown that $\mathcal{L}_{fas(\frac{2n}{n+1};1)}(^nE) = \mathcal{L}(^nE)$ and it suffices to combine this result with the definition of strongly fully mappings. ∎

REFERENCES

[1] ALENCAR, R. and MATOS, M.C. – *Some classes of multilinear mappings between Banach spaces*, Publicaciones Departamento Análisis Matematico, Universidad Complutense Madrid, Section 1, Number 12 (1989).

[2] ARON, R.; LACRUZ, M.; RYAN, R. and TONGE, A. – The generalized Rademacher functions, *Note Mat.,* 12 (1992), 15–25.

[3] BOMBAL, F.; PÉREZ-GARCÍA, D. and VILLANUEVA, I. – Multilinear extensions of a Grothendieck's theorem, *Q. J. Math.,* 55 (2004), 441–450.

[4] BOTELHO, G. – Cotype and absolutely summing multilinear mappings and homogeneous polynomials, *Proc. Roy. Irish Acad.,* 97 (1997), 145–153.

[5] BOTELHO, G. – Ideals of polynomials generated by weakly compact operators, *Note Mat.,* 25 (2005), 69–102.

[6] BOTELHO, G.; BRAUNSS, H.-A.; JUNEK, H. and PELLEGRINO, D. – Holomorphy types and ideals of multilinear mappings, *Studia Math.,* 177 (2006), 43–65.

[7] BOTELHO, G. and PELLEGRINO, D. – Two new properties of ideals of polynomials and applications, *Indag. Math.,* 16 (2005), 157–169.

[8] BOTELHO, G. and PELLEGRINO, D. – Scalar-valued dominated polynomials on Banach spaces, *Proc. Amer. Math. Soc.,* 134 (2006), 1743–1751.

[9] BOTELHO, G. and PELLEGRINO, D. – Absolutely summing polynomials on Banach spaces with unconditional basis, *J. Math. Anal. Appl.,* 321 (2006), 50–58.

[10] BRAUNSS, H.-A. – *Ideale multilinear Abbildungen und Räume holomorfer Funktionen*, Dissertation (A), Pädagogische Hoschschule 'Karl Liebknecht', Potsdam, 1984.

[11] CHOI, Y.; KIM, S.; MELÉNDEZ, Y. and TONGE, A. – Estimates for absolutely summing norms for polynomials and multilinear maps, *Q. J. Math.,* 52 (2001), 1–12.

[12] DIESTEL, J.; JARCHOW, H. and TONGE, A. – *Absolutely Summing Operators*, Cambridge Stud. Adv. Math., 43, Cambridge University Press, Cambridge 1995.

[13] DIMANT, V. – Strongly p-summing multilinear mappings, *J. Math. Anal. Appl.,* 278 (2003), 182–193.

[14] DINEEN, S. – *Complex Analysis on Infinite Dimensional Spaces*, Springer Verlag, London, 1999.

[15] FLORET, K. and HUNFELD, S. – Ultrastability of ideals of homogeneous polynomials and multilinear mappings on Banach spaces, *Proc. Amer. Math. Soc.,* 130 (2002), 1425–1435.

[16] FLORET, K. and GARCÍA, D. – On ideals of polynomials and multilinear mappings between Banach spaces, *Arch. Math.,* 81 (2003), 300–308.

[**17**] LINDENSTRAUSS, J. and PEŁCZYŃSKI, A. – Absolutely summing operators in $\mathcal{L}_p$ spaces and their applications, *Studia Math.,* 29 (1968), 275–326.

[**18**] MATOS, M.C. – Absolutely summing holomorphic mappings, *An. Acad. Bras. Ci.,* 68 (1996), 1–13.

[**19**] MATOS, M.C. – Fully absolutely summing mappings and Hilbert Schmidt operators, *Collect. Math.,* 54 (2003), 111–136.

[**20**] MATOS, M.C. – Nonlinear absolutely summing multilinear mappings between Banach spaces, *Math. Nachr.,* 258 (2003), 111–136.

[**21**] NACHBIN, L. – *Topology on spaces of holomorphic mappings*, Ergeb. Math. Grenzgeb., 47, Springer Verlag, Berlin, 1969.

[**22**] PELLEGRINO, D. – *Aplicações entre espaços de Banach relacionadas à convergência de séries*, Doctoral thesis, Universidade Estadual de Campinas, UNICAMP, 2002.

[**23**] PELLEGRINO, D. – Cotype and absolutely summing homogeneous polynomials in $\mathcal{L}_p$ spaces, *Studia Math.,* 157 (2003), 121–131.

[**24**] PELLEGRINO, D. – Almost summing mappings, *Arch. Math.,* 83 (2004), 68–80.

[**25**] PELLEGRINO, D. – On scalar-valued nonlinear absolutely summing mappings, *Ann. Polon. Math.,* 83 (2004), 281–288.

[**26**] PELLEGRINO, D. – Cotype and nonlinear absolutely summing mappings, *Math. Proc. R. Ir. Acad.,* 105A (2005), 75–92.

[**27**] PELLEGRINO, D. and SOUZA, M.L.V. – Fully summing multilinear and holomorphic mappings into Hilbert spaces, *Math. Nachr.,* 278 (2005), 877–887.

[**28**] PÉREZ-GARCÍA, D. – The inclusion theorem for multiple summing operators, *Studia Math.,* 165 (2004), 275–290.

[**29**] PÉREZ-GARCÍA, D. and VILLANUEVA, I. – Multiple summing operators on Banach spaces, *J. Math. Anal. Appl.,* 285 (2003), 86–96.

[**30**] PÉREZ-GARCÍA, D. and VILLANUEVA, I. – Multiple summing operators in $C(K)$-spaces, *Ark. Mat.,* 42 (2004), 153–171.

[**31**] PIETSCH, A. – *Ideals of multilinear functionals*, in "Proceedings of the Second International Conference on Operator Algebras, Ideals and their Applications in Theoretical Physics", Teubner-Texte, Leipzig, pp. 185–199, 1983.

[**32**] SOUZA, M.L.V. – *Aplicações multilineares completamente absolutamente somantes*, Doctoral Thesis, Universidade Estadual de Campinas, UNICAMP, 2003.

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