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Errata to: "On splitting perfect polynomials over $\mathbb{F}_{p^2}$"

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Abstract. Correct versions of Lemmas 2.17 to 2.20 in *Portugal. Math.* **66** (2009), 261–273, are included here below as Lemmas 0.1 to 0.4. All other results of the paper are correct and do not need modifications.

Lemma 2.17 in our paper is not correct, which invalidates also the three lemmas that follow (Lemmas 2.18 through 2.20). Correct versions are provided below by Lemmas 0.1 to 0.4.

Lemma 0.1. *Let* $j \in U \backslash \{0\}$ *and* $u_0, \dots, u_{p-1} \in \mathbb{Q}$ *such that* $\sum_{r \in U} u_r (\omega^j)^r = 0$. *Then*

$$u_r = u_0 \quad \textit{for all } r \in U.$$

Proof. Since $\{1, \omega^j, \dots, (\omega^j)^{p-1}\} = \{1, \omega, \dots, \omega^{p-1}\}$, we may assume that $j = 1$. It suffices to observe that the cyclotomic polynomial $\Phi_p(x) = 1 + \cdots + x^{p-1}$, which is irreducible, is the minimal polynomial of ω. □

Lemma 0.2. *The matrix* S_0 *has rank* $p - 1$.

Proof. By Lemma 2.5, the eigenvalues of the matrix S_0 are

$$v_0 = a_{0,0} + \cdots + a_{0,p-1} = \sum_{(j,i) \in U^2} a_{j,i} = 0,$$
$$v_l = \sum_{r \in U} a_{0,r} (\omega^l)^r \quad \text{for } l \in U \backslash \{0\}.$$

If $v_l = 0$ for some $l \in U \backslash \{0\}$, then by Lemma 0.1, we have

$$a_{0,r} = a_{0,0} \quad \text{for all } r \in U.$$

This is impossible since $a_{0,0} = 1$ and $a_{0,1} = -N$. Thus, S_0 has exactly $p-1$ non-zero eigenvalues. We are done. □

If N does not divide $p-1$, the following two lemmas give the rank of $\tilde{\Delta}_k$ for $k \in U$.

Lemma 0.3. *If N does not divide $p-1$, then the matrix $\tilde{\Delta}_0$ has rank $p-1$.*

Proof. We know from Lemma 2.5 that $\tilde{\Delta}_0$ has the following eigenvalues:

$$\mu_0 = \lambda_{0,0} + \cdots + \lambda_{p-1,0} = \sum_{(j,i) \in U^2} a_{j,i} = 0,$$
$$\mu_l = \sum_{r \in U} \lambda_{r,0}(\omega^l)^r \qquad \text{for } l \in U \backslash \{0\}.$$

If $\mu_l = 0$ for some $l \in U \backslash \{0\}$, then, by Lemma 0.1, we have

$$\lambda_{r,0} = \lambda_{0,0}.$$

We obtain

$$0 = \sum_{(r,s) \in U^2} a_{r,s} = \sum_{r \in U} a_{r,0} + \cdots + \sum_{r \in U} a_{r,p-1} = \sum_{r \in U} \lambda_{r,0} = p\lambda_{0,0}.$$

Hence $\lambda_{0,0} = 0$, and

$$\sum_{r \in U} a_{0,r} = \lambda_{0,0} = 0 = \sum_{(i,j) \in U^2} a_{j,i} = \sum_{r \in U} a_{0,r} + \sum_{(j,r) \in U^2, j \neq 0} a_{j,r}.$$

It follows that $a_{j,r} = 0$ for any $j, r \in U$ such that $j \geq 1$. It is impossible since the matrix S_j is not the zero matrix by Lemma 2.13 ii). □

Lemma 0.4. *If N does not divide $p-1$, then for any $j \in U \backslash \{0\}$, the matrix $\tilde{\Delta}_j$ has rank p.*

Proof. By Lemma 2.5, the matrix $\tilde{\Delta}_j$ has the eigenvalues

$$\mu_{jl} = \sum_{s \in U} \lambda_{s,j}(\omega^l)^s = \sum_{(r,s) \in U^2} a_{s,r}\omega^{rj+sl}, \qquad l \in U.$$

For $t \in U$, we put $U_t = \{(r,s) \in U^2 : sj + rl \equiv t \bmod p\}$. The set U^2 is the disjoint union $U_0 \sqcup \cdots \sqcup U_{p-1}$. So we can write

$$\mu_{jl} = \sum_{(r,s) \in U^2} a_{r,s} \omega^{sj+rl} = \sum_{t \in U} \Big(\sum_{(r,s) \in U_t} a_{r,s} \Big) \omega^t.$$

If $\mu_{jl} = 0$, then, by Lemma 0.1, we have

$$\sum_{(r,s) \in U_t} a_{r,s} = \sum_{(r,s) \in U_0} a_{r,s} \quad \text{for all } t \in U.$$

Thus,

$$0 = \sum_{(r,s) \in U^2} a_{r,s} = \sum_{t \in U} \sum_{(r,s) \in U_t} a_{r,s} = p \sum_{(r,s) \in U_0} a_{r,s}.$$

Hence

$$\sum_{(r,s) \in U_0} a_{r,s} = 0.$$

Furthermore, $a_{r,s} \geq 0$ for any $(r,s) \in U_0$ since $(0,1) \notin U_0$. Thus

$$0 = \sum_{(r,s) \in U_0} a_{r,s} \geq a_{0,0} = 1,$$

which is impossible. □

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