# Special issue on the occasion of Fritz Grunewald's 60th birthday

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# Fritz Grunewald 1949–2010

Fritz Grunewald was 60 on 28 March 2009. The birthday conference held the following week in Oxford to mark this occasion attracted over 100 participants, from several continents and diverse areas of mathematics. This special issue of GGD was supposed to be an additional, more permanent, celebration of Fritz at 60. His sudden death just one week short of his 61st birthday has come as a total shock to his numerous friends and admirers; he was aware that this issue was in production, but sadly has not lived to see the result.

Fritz was a widely loved and inspirational mathematician. At heart he was an explorer rather than a theory-builder; his many profound insights grew out of extended and painstaking calculation, by hand and by computer. The breadth of his knowledge was remarkable, and is reflected (though by no means fully represented) in the diverse range of areas where he made important contributions.

Mathematics for him was a communal enterprise: all but two of his numerous publications were joint work. He brought a unique vision to these collaborations, and following up his ideas will keep many mathematicians busy for a long time to come.

His main work was in group theory – approached with the vision of a number-theorist and geometer; in number theory – particularly as it relates to Bianchi groups and hyperbolic geometry; and in algebraic geometry – particularly in connection with fundamental groups. This is not the place for a comprehensive account of his results, which include the solution of well-known problems and the creation of several new fields of research. We mention a few highlights; most of these are sample results from broader theories which are rich with ideas for the future.

Three themes running through much of Fritz's *œuvre* are *arithmetic groups*, in one form or another, properties of *profinite completions*, and *effective decidability*.

In three 1980 *Annals* papers, he established (1) finiteness of genus for polycyclic groups (with Pickel and Segal), (2) decidability of the 'orbit problem' for arithmetic groups (with Segal), and (3) decidability of the isomorphism problem for nilpotent groups (with Segal). An earlier and more explicit case of (2) was his solution of the conjugacy problem in $\mathrm{GL}_n(\mathbb{Z})$ (*Word Problems II* (1976)). A later application of material from (2) was the solvability of quadratic Diophantine equations in positive integers (*Crelle's J.* (2004), with Segal).

An important series of works with Platonov explores the phenomenon of rigidity and clarifies the nature of finite extensions of arithmetic groups; a key result here is that $H^1(G, \Delta)$ is finite for every arithmetic group $\Delta$ and any finite group $G$ acting on $\Delta$ (*Duke J.* (1999)). A further application of these methods to the automorphism groups of polycyclic groups led to the proof that $\mathrm{Out}(G)$ is arithmetic for every polycyclic group $G$ (*IHES Publ. Math.* (2006), with Baues).

The 2009 *GAFA* paper (with Lubotzky) initiates the study of arithmetic images of $\mathrm{Aut}(F_n)$; among many suggestive results and conjectures is the theorem that $\mathrm{Aut}(F_3)$ is 'large'. The 2004 *Annals* paper (with Bridson) presents the negative solution of Grothendieck's problem on profinite completions of finitely presented groups.

A major project with Kunyavskii, Plotkin and several other collaborators initiates a theory of 'dynamics of word-maps' in finite simple groups; this arose in the construction of sequences of two-variable identities characterizing solubility in finite groups (*Compositio Math.* (2006)). In related work, Fritz characterizes the soluble radical of a finite group as the set of elements $y$ such that any four conjugates of $y$ generate a soluble subgroup (a best-possible result) (*J. Pure Appl. Algebra* (2009), with Gordeev, Kunyavskii and Plotkin).

*Zeta functions associated to groups and rings.* The 1988 *Inventiones* paper (with Segal and Smith) introduced the definition of 'subgroup-growth' zeta functions for nilpotent groups and for rings; the main result is that the 'local factors' are rational functions. This generated much activity by many mathematicians; a highlight was Fritz's 2000 *Annals* paper (with du Sautoy) which establishes the rationality of the abscissa of convergence and analytic continuation. This was the topic of their address at the 2006 International Congress.

*Number theory.* Starting with his early probing paper (*Algebra i Logika* (1978), with Helling and Mennicke) on automorphic forms over imaginary quadratic number fields, Fritz made fundamental contributions to the theory of automorphic forms associated with hyperbolic manifolds. That paper was the first to explicate the theory in terms of the cohomology of congruence subgroups of Bianchi groups, and they used their description to investigate these questions numerically. Their results, which pointed to the analogue of Eichler–Shimura theory in this context, set in motion many works and this investigation remains very active. In particular some explicit cases of their discoveries/conjectures have been settled (Harris–Soudry–Taylor *Invent. Math.* (1993), and Berger–Harcos *IMRN* (2007)).

In order to investigate these and related arithmetical problems, Fritz and his collaborators, most notably Elstrodt and Mennicke, developed the analytic and arithmetic theory of automorphic forms for hyperbolic manifolds in 3 and higher dimensions. The tools used range from topological and cohomological ones to spectral,analytic as well as arithmetical algebro-geometric ones. Among their many important results are ones giving bounds towards the generalized Ramanujan/Selberg conjectures for the spectra of such hyperbolic manifolds (*Invent. Math.* (1990)). Their book *Groups acting on hyperbolic space: harmonic analysis and number theory* (1998) is by now the classical text in the subject. It gives a complete treatment of the hyperbolic geometry and related topology, spectral theory of the Laplacian and related Hecke operators, as well as the number theory that is needed to study these manifolds.

His recent paper in *Experimental Math.* (2010), with Finis and Tirao, gives a comprehensive account of what is known today about the cohomology of arithmetic and some nonarithmetic hyperbolic three manifolds, and again through experimentation it points to a number of new phenomena and problems which will continue to drive research in this area for years to come.

*Algebraic geometry*. The actions of infinite and finite groups have ever since the birth of geometry, and especially in the last 150 years, been an important subject of study, and a source for the construction of new algebraic varieties, especially exotic ones presenting 'pathological' behaviours. Fritz has been collaborating with Ingrid Bauer and Fabrizio Catanese in several related projects along these lines.

His first important contribution to the field of algebraic surfaces, in an appendix to Catanese's article in *J. Differential Geom.* (2007), was the construction of what are now called *Kuga–Shavel–Grunewald* surfaces. These are compact quotients of the bidisk by a discrete subgroup such that every commensurable subgroup acts freely: here Fritz used his knowledge of quaternion algebras to show the existence of these rigid surfaces, which constitute countably many QED classes.

Other works deal with the classification and construction of surfaces in the difficult region where the geometric genus is zero. Especially interesting here is the explicit determination of the fundamental groups, for which it is easy to write a presentation, but hard to obtain a precise description. An important structure theorem is based on results in the 2008 *Duke* paper by Grunewald, Jaikin-Zapirain and Zalesski on the profinite completions of Bianchi groups.

A major ongoing project (*Mediterranean J. Math.* (2006), arXiv:0706.1466) concerns the action of the absolute Galois group on varieties defined over number fields, and the corresponding change in the fundamental groups (which preserves profinite completions).

Another project transforms into group theoretical questions the investigation of certain rigid algebraic surfaces that generalize a construction due to Beauville. Given a group $G$, one seeks equivalence classes, called *Beauville structures*, of generating triples $a, b, c$ and $x, y, z$ satisfying $abc = 1$, $xyz = 1$ and some other more complicated properties. Which finite groups admit a Beauville structure? First answers were given by the three authors. The question was taken up by several other mathematicians; their conjecture (in *Geometric methods in algebra and number theory* (2005)) that all non-abelian finite simple groups except for $A_5$ admit a Beauville structure has recently been proved by R. Guralnick and G. Malle.

*Legacy.* Fritz in his 61st year was at the height of his creative powers. We don't know how many projects he was actively engaged in: they probably can't be counted on the fingers of two hands. We hope that his bereft collaborators will in due course bring to completion the many existing preprints and drafts that represent the current state of his work.

*The editors*