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Special issue on computational algebra

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Computational algebra is a subject which in little over half a century has become a vital tool for many aspects of research in algebra. Some of its early successes were in finite group theory, where the construction of most of the sporadic simple groups found in the 20th century depended on computation, and the groups of order 64 were determined by Hall and Senior.

In a sense, the subject is older, since both in group theory and in linear algebra, the basic algorithms (Gaussian elimination and the Schreier–Reidemeister and Todd–Coxeter algorithms) were known long before there were machines to implement them.

One major change in the past half century has been the development of general purpose computer algebra packages. No longer is it necessary for someone wishing to use computer algebra to program their own implementation of the Todd–Coxeter algorithm; the two leading systems, Magma and GAP include implementations of this and many other standard algorithms.

Another change is the establishment of on-line databases which are closely integrated with the computer algebra packages. These include the on-line Atlas of Finite Groups and John Cremona's tables of elliptic curves. From the first, for example, a user with Magma or GAP and an internet connection can seamlessly download generators for huge almost simple groups and then study their properties.

A further change is underway, the rise of parallel processing. It is very important to examine existing algorithms and see which ones lend themselves to parallelisation, and which could be re-written in suitable form to take advantage of this.

In the present collection, the paper by Jonusas, Mitchell, and Pfeiffer considers computations in finite semigroups; these are traditionally more difficult than similar computations in groups, and the authors present algorithms which will take advantage of parallel computing.

Hulpke's paper presents an algorithm for a problem which is of interest to lattice theorists as well as group theorists: finding the intermediate subgroups between two given subgroups of a finite group. The algorithm is designed for the

GAP system [1]. Soicher gives a nice example of how a combination of group theory and computation can have dramatic results in certain searches, and illustrates with a classification of the maximal partial spreads in projective 3-space over fields of order 7 and 8 admitting an automorphism of order 5.

More general mathematical packages such as Maple and Mathematica do not have the capabilities of GAP and Magma for handling abstract algebraic structures such as groups, but can be extremely useful for more traditional algebra such as dealing with complicated polynomials. The paper by Aljohani, Bamberg, and Cameron gives an example of this, applied to a question on synchronizing automata.

At the other end, there are more specialised packages, such as those for dealing with and visualising geometry, such as GeoGebra [2], used by Breda and Dos Santos to construct new monohedral spherical tilings with pentagonal faces; and those for investigating identities in algebraic structures, such as Prover9 [3], used by Araújo and Kinyon for studying commutativity theorems in finite groups.

References

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