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Corrigendum to "Positive integers divisible by the product of their nonzero digits", *Portugaliae Math.* 64 (2007), 1: 75–85

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Abstract. In this paper, we correct a numerical error from [1].

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Let $\mathcal{N}_0(x)$ be the set of positive integers $n \le x$ divisible by the product of their base 10 nonzero digits and let $\mathcal{N}(x)$ be the set of all $n \le x$ divisible by the product of their digits. Theorem 1 in [1] states that inequalities

$$x^{.495} < \#\mathcal{N}_0(x) < x^{.654} \quad \text{and} \quad x^{.122} < \#\mathcal{N}(x) < x^{.618}$$

hold for $x > x_0$. Recently, Tomohiro Yamada pointed out to us that there is a flaw in the proof of the above upper bounds. Indeed, on Page 81, Line -2, the formula of δ versus β should be

$$\delta = 1/\log 10 - \beta(1/\log 10 + 1/\log 2)$$

instead of

$$\delta = 1 - \beta(1/\log 10 + 1/\log 2).$$

Inserting this formula for δ into the equation at Lines -2 and -1 of Page 81, we get $\beta = 0.0999683\ldots$, leading to $\#\mathcal{N}_0(x) \le x^{0.901}$ for $x > x_0$. A similar mistake (with a similar correction) appears on Page 82, Lines -5, -4 and -3 of Section 2. Inserting the correct value of δ into the equation relating β and δ at this paragraph we get $\beta = 0.137991\ldots$, leading to $\#\mathcal{N}(x) \le x^{0.863}$.

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Reference

[1] J.-M. De Koninck and F. Luca, 'Positive integers divisible by the product of their nonzero digits', Portugaliae Math. **64** (2007), 75–85.

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