Elem. Math. 62 (2007) 121 – 121
0013-6018/07/030121-1

Elemente der Mathematik

A short proof of Morley's theorem

Yoshitake Hashimoto

Yoshitake Hashimoto received his D.Sc. from University of Tokyo in 1990. He then had a postdoctoral position at University of Tokyo. Since 1994 he has a position at Osaka City University, where he is now associate professor in the Department of Mathematics. His main fields of research are topology and differential geometry.

We present a proof of the following:

Morley's theorem (1899) *In any triangle, the three points of intersection of the adjacent angle trisectors form an equilateral triangle.*

Proof. Let α, β, γ be arbitrary positive angles with $\alpha+\beta+\gamma=60^\circ$. For any angle η we put $\eta' := \eta + 60^\circ$.

Let $\triangle DEF$ be an equilateral triangle, and A [resp. B, C] be the point lying opposite to D [resp. E, F] with respect to EF [resp. FD, DE] and satisfying $\angle AFE = \beta'$, $\angle AEF = \gamma'$ [resp. $\angle BDF = \gamma'$, $\angle BFD = \alpha'$; $\angle CED = \alpha'$, $\angle CDE = \beta'$]. Then $\angle EAF = 180^\circ - (\beta' + \gamma') = \alpha$, and similarly $\angle FBD = \beta$, $\angle DCE = \gamma$. By symmetry it is enough to show that $\angle BAF = \alpha$ and $\angle ABF = \beta$ as well.

The perpendiculars from F to AE and BD have the same length s. If the perpendicular from F to AB has length $h < s$, then $\angle BAF < \alpha$ and $\angle ABF < \beta$. If, on the other hand, $h > s$, then $\angle BAF > \alpha$ and $\angle ABF > \beta$. Since

$$\angle BAF + \angle ABF = \alpha' + \beta' + 60^\circ - 180^\circ = \alpha + \beta,$$

we see that necessarily $h = s$ and $\angle BAF = \alpha$, $\angle ABF = \beta$. □

Yoshitake Hashimoto
Department of Mathematics
Graduate School of Science
Osaka City University
3-3-138, Sugimoto
Sumiyoshi-ku
Osaka, 558-8585 Japan
e-mail hashimot@sci.osaka-cu.ac.jp