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Elemente der Mathematik

Short note F. Commandino, de centro gravitatis solidorum, 1565

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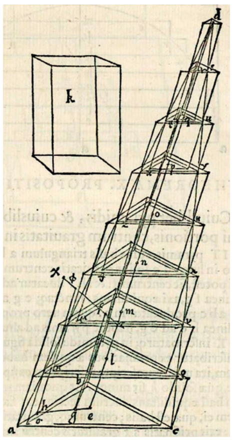
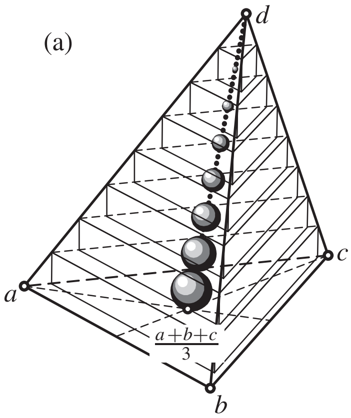
The importance of Federico Commandino (1506–1575) cannot be better described that by quoting E. Rosen[1]: "In the sixteenth century, Western mathematics emerged swiftly from a millennial decline. This rapid ascent was assisted by Apollonius, Archimedes, Aristarchus, Euclid, Eutocius, Heron, Pappus, Ptolemy and Serenus – *as published by Commandino"*.

In addition to all this extraordinary editorial work, Commandino also developped his own research in [1] ($4\frac{1}{2}$ centuries ago), among which the determination of the center of gravity of the Tetrahedron led to one of the first new theorems above the Greek heritage.

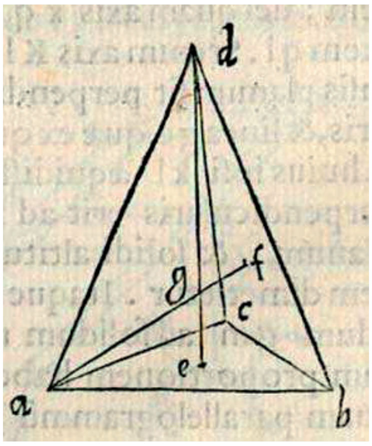


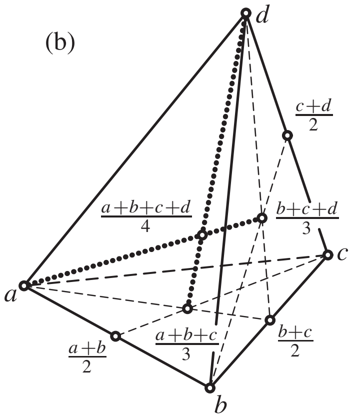


Theorema X. *"pyramidis … centrum grauitatis in axe consistit"* [The barycentre of a tetrahedron lies on the median (i.e., the line connecting a vertex to the barycenter of the opposite triangle)].

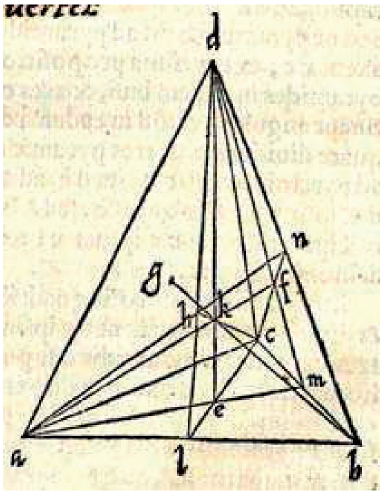
[1] *Biography in Dictionary of Scientific Biography*, New York 1970–1990, also reproduced in www-groups.dcs.st-and.ac.uk/history/Biographies/Commandino.html

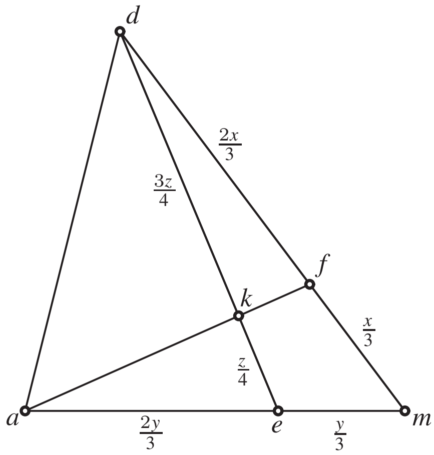
Proof (inspired by Archimedes' *Equilibrium of planes*). Concentrate the masses of slim triangular prisms[2] in their centre of gravity, all lying on the median, and then concentrate *these* masses in one point, which lies on the median, too.

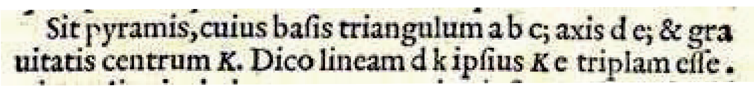




Theorema XIII. *"grauitatis centrum est in puncto, in quo ipsius axes conueniunt"* [The barycentre of a tetrahedron lies on the point where medians meet].






Sit pyramis, cuius basis triangulum a b c; axis d e; & gra uitatis centrum K. Dico lineam d k ipsius K e triplam esse.

Theorema XVIII. *"Dico lineam dk ipsius ke triplam esse"* [The centroid divides the medians in ratio 3 : 1].

[2] If you look closer, you see that Commandino, like Archimedes, distinguished carefully between "upper and lower Riemann sums". Also the method of indirect proofs, by drawing the points g, f and χ, was adopted from Archimedes.

Proof. We consider Commandino's triangle amd, where m is the mid-point of bc (see Figures). We know from Archimedes that f and e, which are the barycenters of two triangular faces, cut *their* medians, of lengths x and y respectively, in ratio 2 : 1.

We reduce Commandino's proof, which extends over 2 pages, to one line by applying Menelaus' Theorem for the triangle dem (see, e.g., [3, pp. 87–88])

$$\frac{ek}{kd} \cdot \frac{df}{fm} \cdot \frac{am}{ae} = 1 \quad \text{or} \quad \frac{ek}{kd} = \frac{1}{2} \cdot \frac{\frac{2}{3}}{1} = \frac{1}{3},$$

which is the stated result. □

Remark. Exactly the same idea, after applying a second time Menelaus' Theorem to the triangle dea and dividing the results, allowed al-Mu'taman ibn Hūd (11th century) to give the first proof of what many centuries later became known as "Ceva's Theorem" (see [2][3]).

Acknowledgement. Thanks to Christian Aebi for suggestions.

References

[1] Federici Commandini Urbinatis, *Liber de centro gravitatis solidorum*, Bononiæ, Ex Officina Alexandri Benacii. MDLXV (Bologna 1565).

[2] J.P. Hogendijk, *The lost geometrical parts of the Istikmāl of Yūsuf al-Mu'taman ibn Hūd* (11*th century*) *in the redaction of Ibn Sartāq* (14*th century*)*: an analytical table of contents*, Arch. Internat. Hist. Sci. 53 (2003) 19–34.

[3] A. Ostermann, G. Wanner, *Geometry by its history*, Springer, 2012.

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[3]The author is grateful to D. Paunić for this reference; see also the Proceedings of ICM 1994, Zürich, p. 1570.