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Regelungstheorie

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Abstract. The workshop "Regelungstheorie" (control theory) covered a broad variety of topics that were either concerned with fundamental mathematical aspects of control or with its strong impact in various fields of engineering.

Mathematics Subject Classification (2000): 93-xx, 49-xx.

Introduction by the Organisers

Control theory is an interdisciplinary field that is located at the crossroads of pure and applied mathematics with systems engineering. It therefore covers a wide variety of topics, ranging from fundamental mathematical aspects to real world engineering applications of industrial relevance. In particular, it has deep connections to different branches of pure and applied mathematics, including e.g. operator theory, real and complex analysis, probability theory, commutative algebra, as well as algebraic and differential geometry.

The Oberwolfach workshop "Regelungstheorie" has the traditional goal of bringing active researchers with both a mathematical and an engineering background together in order to stimulate a fruitful interaction between these communities. This diversity of our field was once again nicely reflected in the expertise of the 42 participants of the 2005 workshop from all over the world. A particular effort has been devoted to inviting newcomers from the somewhat younger generation which had the refreshing side-effect of an increase in the number of female participants.

The particular mission of the workshop was as well reflected in the various themes that have been addressed in the 29 stimulating presentations. The program has been clustered into rather coherent sessions of three lectures each revolving around

- Observer and estimation theory
- Robust and fault-tolerant control
- Behaviors
- Optimal control
- Model reduction
- System Identification
- Tracking and path-following
- Infinite dimensional systems

In addition to regular session talks of thirty minutes, each day was initialized with a somewhat longer presentation that was followed by a five-minute discussant statement. This novel feature provided an extra stimulus for lively general discussions during the subsequent coffee breaks and beyond.

As a particular highlight, Vincent Blondel and Jan Willems organized and chaired a session on open problems in systems and control on Tuesday evening. Eight contributors had the opportunity to present concrete research questions in seven minutes, followed by a brief two minutes discussion with the audience. Six of the corresponding abstracts related to the open problems have been collected at the end of this report and should, in view of the raised challenges, stimulate lots of new research initiatives.

In addition to the excellent scientific program most participants could as well enjoy the traditional Wednesday afternoon walk to St. Roman, with beautiful weather and in a wonderful winter landscape. The day was concluded by an impressive musical evening with excerpts from Schubert's "Winterreise" presented by P. Hippe (bariton) and P. Lohmann (piano).

Workshop: Regelungstheorie

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Abstracts

Issues in Model Reduction of Large-Scale Systems

Athanasios C. Antoulas

There are two main approaches to model reduction of large-scale systems. The first is known as the SVD approach and includes approximation methods like balanced truncation and Hankel norm approximation. The second is known as the Krylov approach and consists of moment matching methods. The latter include parameters (interpolation points) which need to be appropriately chosen. The purpose of this talk was to discuss the choice of these parameters so that certain goals are achieved. In particular we first discussed the choice of interpolation points so that passivity is preserved, that is, the reduced system is passive provided that the original is passive, and subsequently we investigated the choice of interpolation points so as to achieve reduction which is (sub)optimal in the h2 norm. The main feature of both choices is the fact that the points turn out to be mirror images of system poles or zeros, with respect to the imaginary axis.

A New Test for Passivity of Descriptor Systems

Peter Benner
(joint work with Delin Chu)

Passivity is an important concept in circuit and control theory [1]. A linear system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \qquad A \in \mathbb{R}^{n\times n},\ B \in \mathbb{R}^{n\times m}, \\ y(t) &= Cx(t) + Du(t), \qquad C \in \mathbb{R}^{p\times n},\ B \in \mathbb{R}^{p\times m}, \end{aligned}$$

is passive if and only if its transfer function

$$G(s) = C(sI_n - A)^{-1}B + D$$

is positive real. The positive real lemma (or Kalman–Yakubovich–Popov–Anderson lemma) states that for minimal systems this is the case if and only if the *linear matrix inequality (LMI)*

$$\begin{bmatrix} A^TX + XA & XB - C^T \\ B^TX - C & -(D + D^T) \end{bmatrix} \leq 0. \tag{1}$$

has a positive semidefinite solution $X \in \mathbb{R}^{n\times n}$. Moreover, if $D + D^T > 0$, then $G(s)$ is (strictly) positive real if the *algebraic Riccati equation (ARE)*

$$A^TX + XA + (XB - C^T)(D + D^T)^{-1}(B^TX - C) = 0,$$

has a stabilizing solution X.

Recently, a similar LMI-based criterion for testing positive realness (and thereby passivity) of descriptor systems

$$(2) \qquad \begin{array}{rcll} E\dot{x}(t) & = & Ax(t)+Bu(t), & A,E\in\mathbb{R}^{n\times n},\ B\in\mathbb{R}^{n\times m},\\ y(t) & = & Cx(t)+Du(t), & C\in\mathbb{R}^{p\times n},\ B\in\mathbb{R}^{p\times m}, \end{array}$$

was proposed by Freund and Jarre in [3]. It states that the descriptor system (2) is positive real if the LMI (1) has a solution satisfying $E^TX = X^TE \geq 0$. A Riccati equation based test is not known in the case that E is singular.

The task of checking passivity of descriptor systems arises, e.g., when validating models of passive devices generated by automatic modeling tools. It also plays an important role in model order reduction techniques for the large-scale dynamical systems that arise in the simulation of VLSI circuits. Most of the methods in use for this purpose do not compute a reduced-order model that can be guaranteed to be passive. Unfortunately, the complexity of solving the semidefinite program related to the LMI arising in the positive real lemma for descriptor systems makes this test infeasible for many of the aforementioned applications. For single-input single-output systems, a positive realness test exclusively relying on eigenvalue computations is proposed in [2], but this does not extend to the general situation.

Here, we investigate a numerical method for testing whether a given general rational matrix is positive real. The main features of our method are:

- it exclusively relies on orthogonal restricted system equivalence transformations;
- it has the acceptable computational complexity of order n^3;
- it can be implemented in a numerically reliable manner.

The main contribution is that the positive realness test for an arbitrary rational matrix function is reduced to testing positive realness of a proper rational matrix function in a special format. Employing this special format, we can use the positive real lemma for standard systems by employing a recursive reduction procedure along the lines of the method proposed in [4].

The following lemma is the main step needed for the reduction to the case of a proper rational function.

Lemma 1. *For any regular pencil $A-\lambda E$ there exist orthogonal matrices $U,V\in\mathbb{R}^{n\times n}$ such that*

$$U(A-\lambda E)V = \begin{array}{c} \begin{array}{cccc} n_1 & n_2 & n_3 & n_4 \end{array} \\ \left[\begin{array}{cccc} A_{11}-\lambda E_{11} & A_{12}-\lambda E_{12} & A_{13}-\lambda E_{13} & A_{14}-\lambda E_{14} \\ 0 & A_{22} & A_{23}-\lambda E_{23} & A_{24}-\lambda E_{24} \\ 0 & 0 & A_{33} & A_{34} \\ 0 & 0 & 0 & A_{44} \end{array}\right] \begin{array}{l} \}n_1 \\ \}n_3 \\ \}n_2 \\ \}n_4 \end{array} \end{array},$$

where $\operatorname{rank}(E_{11}) = n_1$, $\operatorname{rank}(E_{23}) = n_3$, $\operatorname{rank}(A_{44}) = n_4$, *and*

$$\operatorname{rank}\left(\left[\begin{array}{cc} A_{22} & A_{23}-\lambda E_{23} \\ 0 & A_{33} \end{array}\right]\right) = n_2+n_3 \quad \forall\lambda\in\mathbb{C}.$$

The proof of this lemma is constructive and yields an algorithm to compute the given form. The algorithm requires a sequence of orthogonal decompositions

including URV and QR factorizations and the computation of a generalized Schur form.

Our positive realness test makes use of the fact that the transfer function of (2) has an expansion at $s = \infty$ of the form

$$G(s) = C(sE - A)^{-1}B + D = \sum_{k=-\infty}^{q} s^k M_k,$$

where $M_k \in \mathbb{R}^{m\times m}$ are the *Markov parameters* of G. Positive realness can be related to the Markov parameters as follows, see [1, 3].

Proposition 1. *Given a rational matrix-valued function*

$$G(s) = G_p(s) + sM_1 + \sum_{k=2}^{q} s^k M_k,$$

where G_p is the proper part of G, then $G(s)$ is positive real if and only if

(1) *$G_p(s)$ is positive real,*
(2) *$M_1 \geq 0$,*
(3) *$M_k = 0$, $k = 2, 3, \ldots, q$.*

Applying the restricted system equivalence

$$(E, A, B, C, D) \mapsto (UEV, UAV, UB, CV, D)$$

induced by the matrices U, V from Lemma 1, we can prove the following result.

Lemma 2. *Let (E, A, B, C, D) be a minimal realization of the descriptor system (2). Then:*

a) *If the descriptor system is positive real, then $n_2 = n_3$.*
b) *If $n_2 = n_3$, then $M_k \geq 0$ for all $k \geq 2$.*

With this lemma, the positive realness test of minimal descriptor systems is reduced to checking $M_1 \geq 0$ and positive realness of the proper part of G.

We can now distinguish two cases.

Case 1: $n_2 = n_3 = 0$**:** in this case, it is easy to see that in the new coordinates induced by Lemma 1, $M_1 = 0$ and the proper part of G can be transformed via another orthogonal restricted system equivalence to

$$G_p(s) = \begin{bmatrix} \mathcal{C}_1 & \mathcal{C}_2 \end{bmatrix} \left(s \begin{bmatrix} \mathcal{E}_{11} & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} \\ \mathcal{A}_{21} & \mathcal{A}_{22} \end{bmatrix} \right)^{-1} \begin{bmatrix} \mathcal{B}_1 \\ \mathcal{B}_2 \end{bmatrix} + \mathcal{D}, \tag{3}$$

with $\mathcal{E}_{11}, \mathcal{A}_{22}$ nonsingular.

Case 2: $n_2 = n_3 \neq 0$**:** this case is slightly more involved, but using the structure imposed by Lemma 1, we obtain a reliable test for $M_1 \geq 0$ and we can show that $G_p(s)$ can be transformed to the same representation as in (3) using again only orthogonal transformations.

Thus, in both cases, we have reduced the passivity test for descriptor systems to testing positive realness of a proper transfer function which can be done using the Riccati equation-based criterion resulting from the standard positive real lemma

together with a specially adapted version of the recursive reduction procedure given in [4].

References

[1] B.D.O. Anderson and S. Vongpanitlerd, *Network Analysis and Synthesis*, Prentice-Hall, Englewood Cliffs, NJ (1973).
[2] Z. Bai and R.W. Freund, *Eigenvalue-based characterization and test for positive realness of scale transfer functions*, IEEE Trans. Automat. Control **AC-45** (2000), 2396–2401.
[3] R.W. Freund and F. Jarre, *An extension of the positive real lemma to descriptor systems*, Optim. Meth. Softw. **18** (2004), 69–87.
[4] H. Weiss, Q. Wang and J.L. Speyer. *System characterization of positive real conditions*, IEEE Trans. Automat. Control **AC-39** (1994), 541–544.

Stable sets of matrices and efficient approximations of the joint spectral radius

Vincent D. Blondel
(joint work with Yurii Nesterov)

The joint spectral radius of a set of matrices is a measure of the maximal asymptotic growth rate that can be obtained by forming long products of matrices taken from the set. This quantity appears in a number of application contexts but is notoriously difficult to compute and to approximate. We describe a procedure for approximating the joint spectral radius of a finite set of matrices with arbitrary high accuracy. Our approximation procedure is polynomial in the size of the matrices once the number of matrices and the desired accuracy are fixed.

Let $\{A_1, \ldots, A_m\}$ be some set of square matrices with real entries. To the finite sequence $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_k) \in \{1, \ldots, m\}^k$ we associate the corresponding matrix product

$$A_\sigma = A_{\sigma_k} \cdots A_{\sigma_2} A_{\sigma_1}.$$

With this notation, the joint spectral radius is defined by

$$\rho(A_1, \ldots, A_m) = \limsup_{k \to +\infty} \max_{\sigma \in \{1,\ldots,m\}^k} \|A_\sigma\|^{1/k}.$$

Our approximation procedures for the joint spectral radius provide approximations of relative accuracy $1 - \epsilon$ in time polynomial in $n^{(\ln m)/\epsilon}$, where m is the number of matrices and n is their size. These bounds are close from optimality since we show that, unless P=NP, no approximation algorithm is possible that provides a relative accuracy of $1 - \epsilon$ and runs in time polynomial in n and $1/\epsilon$.

References

[1] V. D. Blondel and J. N. Tsitsiklis. A survey of computational complexity results in systems and control. *Automatica*, 36:9, pp. 1249-1274, 2000.
[2] G.-C. Rota and W. G. Strang. A note on the joint spectral radius. *Indag. Math.*, 22, pp. 379-381, 1960.

[3] J. N. Tsitsiklis, V. D. Blondel The Lyapunov exponent and joint spectral radius of pairs of matrices are hard – when not impossible – to compute and to approximate. Mathematics of Control, Signals, and Systems, 10, pp. 31-40, 1997.

Controllability for nonlinear behaviors

Fritz Colonius
(joint work with Wolfgang Kliemann)

The analysis of differential equations and, more generally, dynamical systems, via the time shift on a space of trajectories is a classical approach going at least back to the work of Bebutov [1] in 1940 and has fostered the development of topological dynamics, compare Sell [3]. In control theory, the analysis of input- and output-functions has a long tradition. A new paradigm, called the behavioral approach to control, has been introduced by Willems [4] considering systems interacting with the environment without making a difference between inputs and outputs. However, so far, this latter theory has essentially been restricted to an algebraic framework. The present talk aims at the analysis of behaviors via topological dynamics of the time shift. It turns out that the basic notion of controllability does not directly lend itself to such an analysis. However, a weakened version, chain controllability, is intimately related to the classical notion of chain transitivity in topological dynamics (see, e.g., Robinson [2]). Then we provide a sufficient condition which allows us to show that, generically, chain controllability implies controllability.

References

[1] M.V. Bebutov, *Dynamical systems in the space of continuous functions*, Dokl. Akad. Nauk SSSR **27** (1940), 904–906.
[2] C. Robinson, *Dynamical Systems. Stability, Symbolic Dynamics, and Chaos*, CRC Press Inc., 1995.
[3] G.R. Sell, *Topological Dynamics and Ordinary Differential Equations*, Van Nostrand Reinhold, 1971.
[4] J. Willems, *Models for dynamics*, Dynamics Reported **2** (1989), 171–269.

Fault-Tolerant Control for Uncertain Nonlinear Systems

Michel Fliess
(joint work with Cédric Join, Hebertt Sira-Ramírez)

1. Introduction

We are further developing recent works on closed-loop fault detection and isolation for linear [6] and nonlinear [5, 12] systems, which may contain uncertain parameters. This important subject which is attracting more and more attention (see, *e.g.*, [1] and the references therein) is treated in the nonlinear case like in [5, 12], *i.e.*, via *differential algebra* and the estimation techniques of [9].

Introducing on-line *accommodation*, or *fault-tolerant control*, *i.e.*, the possibility of still controlling a nonlinear system if a fault does occur, is the main novelty of this extended abstract (see [7] for a more complete exposition). We are therefore achieving in the context of diagnosis one of the fundamental aims of nonlinear control, *i.e.* we are able to combine on-line parameter estimation, and closed-loop fault-tolerant control. Our control design moreover is robust with respect to a large variety of noises, without any necessity of knowing their statistical properties.

Acknowledgement. Two authors (MF & CJ) belong to the *équipe ALIEN* of *INRIA Futurs*, which is partially supporting them.

2. Estimation of the derivatives of a noisy signal

2.1. **Importance of derivatives.** According to [2, 3] a system is *observable* if, and only if, any system variable, a state component for instance, is a *differential function* of the control and output variables, *i.e.*, it is a function of the control and output variables and of their time derivatives up to some finite order. A parameter is *identifiable* [2, 3] if, and only if, it is a differential function of the control and output variables. A fault variable is *isolable* if, and only if, it satisfies a *parity equation* where the coefficients are differential functions of the control and output variables. This parity equation yields fault indicators, *i.e.*, *residuals*.

2.2. **Estimation**[1]**.** Consider a real-valued time function $x(t)$ which is assumed to be analytic on some interval $t_1 \leq t \leq t_2$. Assume for simplicity's sake that $x(t)$ is analytic around $t = 0$ and introduce its truncated Taylor expansion

$$x(t) = \sum_{\nu=0}^{N} x^{(\nu)}(0)\frac{t^\nu}{\nu!} + O(t^{\nu+1})$$

[1]See [9] and [4] for more details and related references.

Approximate $x(t)$ by the polynomial $x_N(t) = \sum_{\nu=0}^{N} x^{(\nu)}(0)\frac{t^\nu}{\nu!}$ in the interval $(0, \varepsilon)$, $\varepsilon > 0$. The usual rules of symbolic calculus in Schwartz's distributions theory yield

$$x_N^{(N+1)}(t) = x(0)\delta^{(N)} + \dot{x}(0)\delta^{(N-1)} + \cdots + x^{(N)}(0)\delta$$

where δ is the Dirac measure at 0. From $t\delta = 0$, $t\delta^{(\alpha)} = -\alpha\delta^{(\alpha-1)}$, $\alpha \geq 1$, we obtain the following triangular system of linear equations for determining estimated values $[x^{(\nu)}(0)]_e$ of the derivatives $x^{(\nu)}(0)$:

$$t^\alpha x^{(N+1)}(t) = t^\alpha \left([x(0)]_e\delta^{(N)} + [\dot{x}(0)]_e\delta^{(N-1)} + \cdots + [x^{(N)}(0)]_e\delta\right)$$
$$\alpha = 0, \ldots, N$$

The time derivatives of $x(t)$, the Dirac measure and its derivatives are removed by integrating with respect to time both sides of the above equation at least N times:

$$\int^{(\nu)} \tau_1^\alpha x^{(N+1)}(\tau_1) = \int^{(\nu)} \tau_1^\alpha \left([x(0)]_e\delta^{(N)} + [\dot{x}(0)]_e\delta^{(N-1)} + \cdots + [x^{(N)}(0)]_e\delta\right)$$
$$\nu \geq N,\ \alpha = 0, \ldots, N$$

where $\int^{(\nu)} = \int_0^t \int_0^{\tau_{\nu-1}} \ldots \int_0^{\tau_1}$. A quite accurate value of the estimates may be obtained with a small time window $(0, t)$.

Remark 2.1. *The derivative estimations need to be reset after a short time interval (see, e.g.,* [15] *for further details).*

Remark 2.2. *Those iterated integrals are moreover low pass filters. They are attenuating high frequency noises, which are usually dealt with in a statistical setting.*

3. A concrete case-study

The three-tank system, which is among the most popular case-studies in the fault-diagnosis community (see, *e.g.* [1]), is a *flat hybrid* system, *i.e.*, it is *flat* in each one of four subsets defined by physical inequalities. The fault variables are either actuator or sensor faults. The viscosity coefficients are poorly known. A nonlinear extension of a classic PI controller (see [10, 11]) is utilized around a flatness-based reference trajectory. Our numerical simulations, where noises have been taken into account, may be, to the best of our knowledge, favorably compared to recent publications on this subject where only off-line fault diagnosis was obtained.

Remark 3.1. *See* [4] *for applications to signal processing.*

4. Conclusion

Those simple solutions of long-standing problems in nonlinear control, which may be quite easily implemented in real time, were made possible by a complete change of viewpoint:

- The utilization of flatness-based control which is already playing a crucial rôle in many concrete and industrial applications (see, *e.g.*, [13, 14] and the references therein).

- The algebraic approach to the estimation of unknown quantities (*cf.* [8]), such as parameters, state variables and fault indicators, which does not necessitate any asymptotic techniques nor any probabilistic tools.

Further studies will demonstrate the possibility of controlling nonlinear systems with poorly known models, *i.e.*, not only with uncertain parameters.

References

[1] M. Blanke, M. Kinnaert, J. Lunze, M. Staroswiecki, *Diagnosis and Fault-Tolerant Control*, Springer, 2003.

[2] S. Diop, M. Fliess, On nonlinear observability, *Proc.* 1^{st} *Europ. Control Conf.*, Hermès, 1991, pp. 152-157.

[3] S. Diop, M. Fliess, Nonlinear observability, identifiability and persistent trajectories, *Proc.* 36^{th} *IEEE Conf. Decision Control*, Brighton, 1991, pp. 714-719.

[4] M. Fliess, C. Join, M. Mboup, H. Sira-Ramírez, *Compression différentielle de transitoires bruités*, C.R. Acad. Sci. Paris série I **339** (2004), 821–826.

[5] M. Fliess, C. Join, H. Mounier, An introduction to nonlinear fault diagnosis with an application to a congested internet router, in *Advances in Communication Control Networks*, S. Tabouriech, C.T. Abdallah, J. Chiasson (Eds), Lect. Notes Control Inform. Sci. **308**, Springer, 2005, pp. 327-344.

[6] M. Fliess, C. Join, H. Sira-Ramírez, *Robust residual generation for linear fault diagnosis: an algebraic setting with examples*, Internat. J. Control **77** (2004), 1223–1242.

[7] M. Fliess, C. Join, H. Sira-Ramírez, Closed-loop fault-tolerant control for uncertain nonlinear systems, in *Symp. Nonlinear Control and Observer Design: From Theory to Applications*, Stuttgart, Lect. Notes Control Informat. Sci., Springer, 2005.

[8] M. Fliess, H. Sira-Ramírez, *An algebraic framework for linear identification*, ESAIM Control Optim. Calc. Variat. **9** (2003), 151–168.

[9] M. Fliess, H. Sira-Ramírez, Control via state estimations of some nonlinear systems, *Proc.* 6^{th} *IFAC Symp. Nonlinear Control Syst. (NOLCOS)*, Stuttgart, 2004.

[10] V. Hagenmeyer, E. Delaleau, *Exact feedforward linearization based on differential flatness*, Internat. J. Control **76** (2003), 537-556.

[11] V. Hagenmeyer, E. Delaleau, *Robustness analysis of exact feedforward linearization based on differential flatness*, Automatica **39** (2003), 1941-1946.

[12] C. Join, H. Sira-Ramírez, M. Fliess, Control of an uncertain three-tank-system via on-line parameter identification and fault detection, *Proc.* 16^{th} *IFAC World Congress on Automatic Control*, Prague, 2005.

[13] J. Rudolph, *Beiträge zur flacheitsbasierten Folgeregelung linearer und nichtlinearer Syteme endlicher und undendlicher Dimension*, Shaker Verlag, 2003.

[14] H. Sira-Ramírez, S. Agrawal, *Differentially Flat Systems*, Marcel Dekker, 2004.

[15] H. Sira-Ramírez, M. Fliess, On the output feedback control of a synchronous generator, *Proc.* 43^{th} *IEEE Conf. Decision Control*, Atlantis, Bahamas, 2004.

Geometric control and behaviors

Paul A. Fuhrmann

Our object in this presentation is to point out some connections between geometric control and the theory of behaviors, see Polderman and Willems [1997] and the references within. This is part of a larger program trying to develop a unifying

approach to linear systems, an approach which is based on functional representations. This is an intermediate level of abstraction between an abstract module theoretic approach and that of first order, i.e. state space based, representations.

That there is a close connection between geometric control and behaviors is intuitively clear for the reason that geometric control is basically a study of zeros of systems, hence, considering matrix fraction representations, of rectangular polynomial matrices. Behaviors on the other hand can be characterized by having autoregressive (AR) representations, with the defining polynomial matrix being in general rectangular. What is striking is, the long overlooked fact, that even in problems based on an input/output approach, i.e. very far from the behavioral philosophy based on trajectories, behaviors enter into the picture in a very natural way.

Shift invariance is at the root of the unification program for linear system theory. One of the roots of the use of shift invariant spaces is the theorem of Rota on the universality of the backward shift operator. The other root stems from the Kalman's realization that linear systems are modules over the ring of polynomials. This led to abstract realization theory where the state spaces are either of the form $\mathbb{F}[z]^m/\operatorname{Ker} f$ or of the form $\operatorname{Im} f$, where f is the restricted input/output map. In turn f can be identified with the Hankel operator H_G, G being the transfer function of the system. Using left and right matrix fraction representations for G, we are led directly to the introduction of polynomial and rational models, see Fuhrmann [1976]. The roots of duality between polynomial submodules and behaviors can be traced to the duality between polynomial and rational models. Once we have introduced these two classes of models, it is of primary interest to develop a theory of equivalence of models. This is split naturally into two parts, the first being the characterization of all $\mathbb{F}[z]$-module homomorphisms, while the second is the study of the invertibility properties of these homomorphisms. We note that a rational model X^D is at the same time the autonomous behavior $\operatorname{Ker} D(\sigma)$. This indicates that the theory of homomorphisms of polynomial and rational models can be extended to the behavioral context, see Fuhrmann [2002] for the details.

To see how behaviors enter the classical picture, assume we have a proper rational function having the coprime factorizations $G = T^{-1}V = \overline{V}\overline{T}^{-1}$. As is well known, these can be embedded in a doubly coprime factorization

$$\begin{pmatrix} -V & T \\ Y & -X \end{pmatrix}\begin{pmatrix} \overline{X} & \overline{T} \\ \overline{Y} & \overline{V} \end{pmatrix} = \begin{pmatrix} \overline{X} & \overline{T} \\ \overline{Y} & \overline{V} \end{pmatrix}\begin{pmatrix} -V & T \\ Y & -X \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}.$$

From the coprime factorizations we have the intertwining identities $V\overline{T} = T\overline{V}$ and $\overline{X}T = \overline{T}X$. These, and the corresponding Bezout identities, lead to the $\mathbb{F}[z]$-module homomorphisms $\phi : X_{\overline{T}} \longrightarrow X_T$, defined by $\phi f = \pi_T V f$ and its inverse $\psi : X^T \longrightarrow X^{\overline{T}}$ defined by $\psi g = -\pi_{\overline{T}}\overline{X}g$. Using the isomorphism of polynomial and rational models, we have the isomorphism $\overline{V}(\sigma) : X^{\overline{T}} \longrightarrow X^T$, with $\overline{V}(\sigma)$ defined, for $h \in z^{-1}\mathbb{F}[[z^{-1}]]^m$, by $\overline{V}(\sigma)h = \pi_-\overline{V}h$. This interpretation of the

identity $V\overline{T} = T\overline{V}$ assumes X_T and $X_{\overline{T}}$, or the corresponding rational models, as the primary objects and the polynomial matrices $V, \overline{V}$ as the secondary ones. However, and this took quite some time to realize, one can reverse the point of view. In fact, if we take the behaviors $X^V = \operatorname{Ker} V(\sigma)$ and $X^{\overline{V}} = \operatorname{Ker} \overline{V}(\sigma)$ as the primary objects, then the same equations as before lead to the behavior isomorphism $Y(\sigma) : X^V \longrightarrow X^{\overline{V}}$ and its inverse $\overline{T}(\sigma) : X^{\overline{V}} \longrightarrow X^V$. This is only an indication of how much information is encoded in the doubly coprime factorization.

This is also the entry point of geometric control into our story. Of course, geometric control in the style of Basile, Marro, Wonham and Morse is a state space based theory. So the first step is to bridge the gap between external representations of systems and internal ones. This is the role of realization theory and, in particular for our purposes, the shift realization. For a representation $G = VT^{-1}U + W$ of a proper, $p \times m$ rational function, we define a system, in the state space X_T, by

$$\begin{cases} A = S_T & B\xi = \pi_T U\xi \\ Cf = (VT^{-1}f)_{-1} & D = G(\infty). \end{cases}$$

If we use the right matrix fraction $G = \overline{VT}^{-1}$, then, with respect to the shift realization, a subspace $\mathcal{V} \subset X^{\overline{T}}$ is controlled invariant if and only if it has a representation $\mathcal{V} = \pi^{\overline{T}} X^{\overline{V}}$, for some behavior $X^{\overline{V}} = \operatorname{Ker} \overline{V}(\sigma) \subset z^{-1}\mathbb{F}[[z^{-1}]]^m$. Similarly, starting from the left matrix fraction $G = T^{-1}V$, then, with respect to the corresponding shift realization, a subspace $\mathcal{V} \subset X_T$ is conditioned invariant if and only if it has a representation $\mathcal{V} = X_T \cap \mathcal{M}$ for some submodule $\mathcal{M}$ of $\mathbb{F}[z]^p$. As rank k submodules of $\mathbb{F}[z]^p$ are in a bijective correspondence with behaviors in $z^{-1}\mathbb{F}[[z^{-1}]]^k$, this opens up the possibility of developing an appropriate duality theory that would encompass geometric control and behaviors. For a beginning of such a theory, see Fuhrmann [1981,2005b]. Neither of the two representations, i.e. those of controlled and conditioned invariant subspaces is unique. They can be made essentially unique if we impose constraints of minimality (in the case of conditioned invariant subspaces) and maximality (in the case of controlled invariant subspaces). In general, we end up with rectangular polynomial matrices. Spectral assignment problems, by state feedback or output injection as is appropriate, are reduced to polynomial matrix completion problems. This involves the characterization of all autonomous subbehaviors of a given behavior. See Fuhrmann [2005a] and Fuhrmann and Trumpf [2005] for the details.

References

[1969] G. Basile and G. Marro, "Controlled and conditioned invariant subspaces in linear system theory", J. Opt. Th. and Appl., 3, 306-315.

[1980] E. Emre and M.L.J. Hautus, "A polynomial characterization of (A, B)-invariant and reachability subspaces", *SIAM J. Contr. & Optim.* 18, 420-436.

[1976] P.A. Fuhrmann, "Algebraic system theory: An analyst's point of view", *J. Franklin Inst.* 301, 521-540.

[1977] P.A. Fuhrmann, "On strict system equivalence and similarity", *Int. J. Contr.*, 25, 5-10.

[1981] P.A. Fuhrmann, "Duality in polynomial models with some applications to geometric control theory", *IEEE Trans. Aut. Control*, AC-26, 284-295.
[2002] P.A. Fuhrmann, "A study of behaviors", *Lin. Alg. Appl.*, vols. 351-352, 303-380.
[2005a] P.A. Fuhrmann, "Autonomous subbehaviors and output nulling subspaces", submitted to *Int. J. Contr.*.
[2005b] P.A. Fuhrmann, "On duality in some problems of geometric control", submitted, *Acta Applicandae Mathematicae*.
[2005] P.A. Fuhrmann and J. Trumpf, "On observability subspaces", submitted, *Int. J. Contr.*.
[1997] J.W. Polderman and J.C. Willems, *Introduction to Mathematical System Theory*, Springer, New York.

Convolutional Codes as Discrete-Time Systems and their Weight Distribution

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(joint work with Gert Schneider)

1. Introduction

Convolutional codes and block codes are the two most important classes of codes used for securing the reliability of data transmission. Mathematically, convolutional codes can be considered as direct summands of $\mathbb{F}[z]^n$ where $\mathbb{F}$ is a finite field, the alphabet for the symbols. More precisely, a k-dimensional *convolutional code* $\mathcal{C} \subseteq \mathbb{F}[z]^n$ can be represented as $\mathcal{C} = \operatorname{im} G := \{uG \mid u \in \mathbb{F}[z]^k\}$ where the *generator matrix* $G \in \mathbb{F}[z]^{k\times n}$ is a right invertible polynomial matrix. In this notation the vector $u \in \mathbb{F}[z]^k$ plays the role of the message while $v := uG \in \mathbb{F}[z]^n$ is the associated codeword. The parameter δ defined as the maximum degree of the k-minors of G is known as the *degree* of the code. Block codes can be regarded as convolutional codes with degree zero. They have a constant generator matrix $G \in \mathbb{F}^{k\times n}$ and thus form subspaces of $\mathbb{F}^n$. Despite recent progress for convolutional codes mathematical block code theory is still by far more developed than convolutional coding theory. Throughout this note we will only consider codes with positive degree.

The encoding process $u \longmapsto uG =: v$ can be regarded as an input/output system mapping finite input sequences $u_0, \dots, u_N \in \mathbb{F}^k$, represented by the the polynomial $u = \sum_{i=0}^{N} u_i z^i$, into finite output sequences $v_0, \dots, v_M \in \mathbb{F}^n$ where $v = uG = \sum_{i=0}^{M} v_i z^i$. Since polynomial multiplication needs memory, the input/output system is actually a state space system $x_{t+1} = x_t A + u_t B,\ v_t = x_t C + u_t D$ with initial condition $x_0 = 0$. It turns out that the minimum state space dimension is the degree of the code and that the system is a realization of the proper rational transfer function $G(z^{-1})$. For details see [3].

The most important parameter of a code is its *distance* defined as

$$\operatorname{dist}(\mathcal{C}) := \min\{\operatorname{wt}(v) \mid v \in \mathcal{C},\, v \neq 0\}$$

where the *weight* $\mathrm{wt}(v)$ of the word $v = \left(\sum_{j=0}^{M} v_{1j} z^j, \ldots, \sum_{j=0}^{M} v_{nj} z^j\right)$ is given by $\mathrm{wt}(v) := \#\{(i,j) \mid v_{ij} \neq 0\}$. Hence the weight is the number of nonzero terms in the polynomial vector v. The distance is a measure for the error-correcting capability of the given code, see, e. g., [2, 5].

2. Weight Distribution and Weight Adjacency Matrix

Even more information about the quality of a code is given by its weight distribution. It basically counts the number of all codewords of a given weight. In order to make this a meaningful notion, one has to restrict to *atomic* codewords, that is, to codewords $v \in \mathcal{C}$ which start at time zero (i. e. $v_0 \neq 0$) and which cannot be written as the sum of two nonoverlapping codewords (that is, v is not of the form $v = \hat{v} + z^N \tilde{v}$ where $\hat{v}$, $z^N \tilde{v} \in \mathcal{C} \backslash \{0\}$ and $\deg \hat{v} < N$). In state space description these codewords correspond to state sequences which start and end at the zero state and do not pass through zero in between. Keeping also track of the lengths of the corresponding state sequences all this amounts to the *weight distribution* of the code $\mathcal{C}$ being defined as $\Omega_{\mathcal{C}}(W,L) := 1 + \sum_{l=1}^{\infty} \sum_{\alpha=1}^{\infty} \omega_{l,\alpha} W^\alpha L^l \in \mathbb{Q}[\![W,L]\!]$ where $\omega_{l,\alpha} := \#\{v \in \mathcal{C} \mid v \text{ atomic}, \mathrm{wt}(v) = \alpha, \deg(v) = l-1\}$ (notice that codewords of degree $l-1$ correspond to state sequences of length l).

Computing the distance or even the weight distribution for a given code is in practice, of course, quite a costly task. However, by introducing the *weight adjacency matrix* it can be shown that it is at least a finite problem. Recall that the state space of a code of degree δ over the field $\mathbb{F}_q$ with q elements has dimension δ and thus q^δ elements. Thus the finitely many state transitions $y = xA + uB$ from state x to state y driven by the input $u \in \mathbb{F}^k$ can be displayed in a (finite) directed graph. The weight adjacency matrix is a refined version of the classical adjacency matrix of the graph where also the weights of the associated output $v = xC + uD$ is taken into account. Precisely, we define $\Lambda := \left(\sum_{\alpha=0}^{n} \lambda_{x,y}^{(\alpha)} W^\alpha\right)_{x,y \in \mathbb{F}_q^\delta} \in \mathbb{Q}[W]^{q^\delta \times q^\delta}$ where $\lambda_{x,y}^{(\alpha)} = \#\{u \in \mathbb{F}^k \mid y = xA + uB, \mathrm{wt}(xC + uD) = \alpha\}$. Hence $\lambda_{x,y}^{(\alpha)}$ is the number of edges from x to y where the weight of the corresponding output is α. One should note that the weight adjacency matrix depends on the choice of the generator matrix G of the code as well as on the chosen realization (A,B,C,D) of $G(z^{-1})$. However, it can be shown [3] that two such adjacency matrices Λ and Λ' for a given code differ only by similarity via a permutation matrix. More precisely, there exists some $U \in Gl_\delta(\mathbb{F}_q)$ such that $\Lambda'_{x,y} = \Lambda_{xU,yU}$ for all $x, y \in \mathbb{F}_q^\delta$.

The adjacency matrix can be used to compute the weight distribution of the code. Indeed, it has been shown in [6] that

$$\Omega_{\mathcal{C}}(W,L) = 2 - \left(\left[(I - L\hat{\Lambda})^{-1}\right]_{0,0}\right)^{-1}$$

where $\hat{\Lambda} \in \mathbb{Q}[W]^{q^\delta \times q^\delta}$ is such that $\hat{\Lambda}_{0,0} := \Lambda_{0,0} - 1$ while $\hat{\Lambda}_{x,y} = \Lambda_{x,y}$ for all other pairs of states (x,y) (which simply means that we have to exclude the trivial state transition from zero to zero with zero input from the weight adjacency matrix).

3. Does there exist a MacWilliams Duality for Convolutional Codes?

For a code $\mathcal{C} \subseteq \mathbb{F}[z]^n$ the dual code is defined as its "orthogonal complement" with respect to the standard bilinear form on $\mathbb{F}[z]^n$. Precisely, $\mathcal{C}^\perp := \{w \in \mathbb{F}[z]^n \mid wv^\mathsf{T} = 0 \text{ for all } v \in \mathcal{C}\}$. It is easily seen by examples that the distance of a code does not determine the distance of its dual. However, a famous result of block code theory states that the weight distribution of a block code fully determines the weight distribution of its dual code. Precisely, for a k-dimensional code $\mathcal{C} \subseteq \mathbb{F}_q^n$ one has

$$\Omega_{\mathcal{C}^\perp}(W) = q^{-k}\big(1 + (q-1)W\big)^n \Omega_{\mathcal{C}}\Big(\frac{1-W}{1+(q-1)W}\Big),$$

see [4] (here we left out the indeterminate L since all atomic codewords have length 1). Such a formula helps to determine the weight distribution of certain codes without searching through all codewords if the dual code is well-known (which, for instance, is the case for 1-dimensional codes).

This classical result motivates the question whether such a duality holds true for convolutional codes as well. But this has been answered to the negative already in [7] for the weight distribution $\Omega_{\mathcal{C}}$ of a convolutional code. However, the weight adjacency matrix also forms a generalization of the weight distribution from block to convolutional codes and one might study duality in this context. So far a first case could be answered to the positive. Indeed, based on the paper [1] we could recently establish for k-dimensional codes $\mathcal{C} \subseteq \mathbb{F}_q[z]^n$ of degree 1 the duality

$$\Lambda_{\mathcal{C}^\perp} = q^{-k}\big(1 + (q-1)W\big)^n \Big(H^{-1}\Lambda_{\mathcal{C}}^\mathsf{T} H\Big)\Big|_{\frac{1-W}{1+(q-1)W}}.$$

Here the transformation matrix H is defined as $H_{x,y} = \big(\chi(xy)\big)_{x,y\in\mathbb{F}_q}$ where χ is a non-trivial character on $\mathbb{F}_q$, i. e., a non-trivial group homomorphism from $(\mathbb{F}_q, +)$ into $(\mathbb{C}^*, \cdot\,)$ We strongly believe that such a duality theorem holds true for much more general classes of convolutional codes but that has to remain open for future research.

References

[1] K. A. S. Abdel-Ghaffar, *On unit constrained-length convolutional codes*, IEEE Trans. Inform. Theory **38** (1992), 200–206.

[2] G. D. Forney Jr., *Convolutional codes I: Algebraic structure*, IEEE Trans. Inform. Theory **16** (1970), 720–738.

[3] H. Gluesing-Luerssen, *On the weight distribution of convolutional codes*, Preprint 2004. E-Print math.IT/0501016 (http://front.math.ucdavis.edu).

[4] F. J. MacWilliams and N. J. A. Sloane, *The Theory of Error-Correcting Codes* (1977), North-Holland.

[5] R. J. McEliece, *The algebraic theory of convolutional codes*, In V. Pless and W. Huffman, editors, *Handbook of Coding Theory, Vol. 1* (1998), Elsevier, Amsterdam, 1065–1138.

[6] R. J. McEliece, *How to compute weight enumerators for convolutional codes*, In M. Darnell and B. Honory, editors, *Communications and Coding (P. G. Farrell 60th birthday celebration)* (1998), Wiley, New York, 121–141.

[7] J. B. Shearer and R. J. McEliece, *There is no MacWilliams identity for convolutional codes*, IEEE Trans. Inform. Theory **23** (1977), 775–776.

Redesign techniques for nonlinear sampled–data control

Lars Grüne
(joint work with Dragan Nešić, Jürgen Pannek)

Nowadays, modern controllers are typically implemented digitally and this fact strongly motivates investigation of sampled–data systems that consist of a continuous time plant controlled by a discrete time (digital) controller. While tools for analysis and design of linear sampled–data systems are well developed (see, e.g., [1], [2]), similar results for nonlinear systems still need development.

In this talk we consider the problem of static state feedback stabilization of the origin of a finite dimensional control system

$$\dot{x} = f(x, u)$$

with $x \in \mathbb{R}^n$ and $u \in U \subseteq \mathbb{R}^m$, i.e., we are looking for a map $u : \mathbb{R}^n \to U$ such that for the closed loop system

$$\dot{x}(t) = f(x(t), u(x(t))) \tag{1}$$

the origin $x^* = 0$ is a globally asymptotically stable equilibrium. In order to model a sampled–data implementation of this problem with a zero order hold device we consider the corresponding sampled–data system with constant sampling rate $T > 0$ given by

$$\dot{x}(t) = f(x(t), u_T(x(iT))), \quad t \in [iT, (i+1)T),\ i = 0, 1, \ldots \tag{2}$$

and construct a controller u_T for this model. Assuming that a suitable controller u for the continuous time system (1) has been designed, a possible approach for sampled-data controller design is to first design a continuous-time controller for the continuous-time plant ignoring sampling and then discretize the obtained controller for digital implementation, i.e., set $u_T = u$, an approach which is often termed *emulation design*. This approach was shown in [5] to recover the performance of the continuous-time system in a semi-global practical sense. However, due to hardware limitations on the minimum achievable T there may exists critical regions, where this approach yields bad performance as in Figure 1, below, where sampling introduces overshoot, or even instability as in Figure 2, below.

Our goal is hence to design a discrete time controller which improves upon the performance of the emulated continuous time controller $u_T = u$, using, however, the available continuous time controller u, i.e., we want to redesign u.

In our first approach, the *Lyapunov redesign technique* developed in [6], we consider control affine single input systems, i.e., $f(x, u) = f_0(x) + g(x)u$ and assume that there exists a Lyapunov function V corresponding to the continuous time system (1) for which the $\mathcal{KL}$ function β obtained from integrating the Lyapunov inequality $L_f(x, u(x))V(x) \leq -\alpha(V(x))$ yields a good reference estimate

$\|x(t,x_0)\| \leq \beta(\|x_0\|,t)$ for the trajectories $x(t,x_0)$ of (1). On the discrete time level this estimate is induced by the Lyapunov difference

$$\Delta V(x) := V(x(T,x)) - V(x).$$

Denoting the trajectories of the sampled data system (2) by $x_T(t,x_0,u_T)$ we can define the sampled–data Lyapunov difference by

$$\Delta V_T(x,u_T) := V(x_T(T,x,u_T)) - V(x).$$

and design u_T in such a way that this difference ΔV_T assumes "good" values. In this context, a "good value" can have several meanings which depend on the redesign objective. For example, one can perform a model reference type redesing by matching the continuous time behavior as close as possible by minimizing $\|\Delta V - \Delta V_T\|$. As an alternative, one can increase the convergence speed by minimizing ΔV_T under suitable gain constraints on the controller u_T.

In order to design u_T in practice we need a computationally feasible approximation of the sampled–data Lyapunov difference ΔV_T. Using the Fliess expansion and neglecting the higher order terms yields such an approximation.

As an example, consider the Moore–Greitzer jet engine model given by

$$\begin{aligned} \dot{x_1} &= -x_2 - \frac{3}{2}x_1^2 - \frac{1}{2}x_1^3 - 3x_3x_1 - 3x_3 \\ \dot{x_2} &= -u \\ \dot{x_3} &= -\sigma x_3(x_3 + 2x_1 + x_1^2) \end{aligned}$$

We have applied the Lyapunov redesign technique to the simplified 2d version obtained by setting $x_3 = 0$ using the stabilizing backstepping controller

$$u(x) = -7x_1 + 5x_2,$$

see [4] for details on the model and the controller design.

Using the Lyapunov function $V(x) = x^2/2$ (which is a Lyapunov function outside a neighborhood of the origin, cf. [6]), we obtain the results shown in Fig. 1.

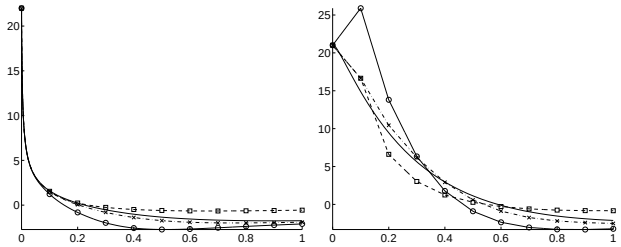

FIGURE 1. Lyapunov based redesign

In this figure, the unmarked curves show the continuous time system, the curves marked with circles show the emulated continuous time controller $u_T = u$. The

Lyapunov difference minimizing redesign is marked with squares while the model reference type redesign is marked with crosses.

In our second approach, the *model predictive redesign* presented in [3, 7], we solve an optimal control problem in order to minimize the distance between x and x_T. While the natural optimal control approach to this problem would be an infinite horizon optimization criterion, this approach is computationally not feasible. Instead we chose a model predictive (or receding horizon) approach by on line solving the finite horizon problem for piecewise constant open loop control $\tilde{u}_T$

$$\min_{\tilde{u}_T} \int_0^{NT} l(x_T(t, x_T^i, \tilde{u}_T) - x(t, x^i), \tilde{u}_T(t))dt + F(x_T(NT, x_T^i, \tilde{u}_T), x(NT, x^i, u))$$

at each sampling instance iT with $x_T^i = x_T(iT, x_0, u_T)$, $x^i = x(iT, x_0)$ and using the sampled–data feedback $u_T(x_T^i) := \tilde{u}_T(0)$. We obtain stability of the closed loop system under mild conditions on l and F and infinite horizon inverse optimality under a local Lyapunov function like condition on the terminal cost F.

We illustrate this method by the 3d Moore–Greitzer model with backstepping stabilizing controller

$$\begin{aligned} u &= -(c_1 - 3x_1)\left(-x_2 - \frac{3}{2}x_1^2 - \frac{1}{2}x_1^3 - 3x_1x_3 - 3x_3\right) \\ &\quad + c_2\left(x_2 - c_1x_1 + \frac{3}{2}x_1^2 + 3x_3\right) - x_1 - 3\sigma x_3\left(x_3 + 2x_1 + x_1^2\right) \end{aligned}$$

using the parameters $\sigma = 2$, $c_1 = 1$ and $c_2 = 50$. The result is shown in Figure 2.

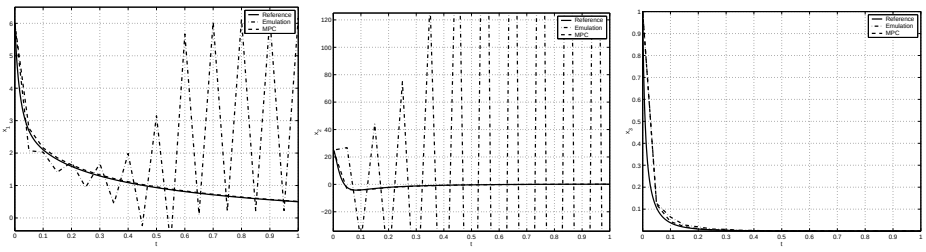

FIGURE 2. Model predictive (MPC) redesign

REFERENCES

[1] B.D.O.Anderson, *Controller design: moving from theory to practice*, IEEE Control Systems Magazine **13** (April 1993), 16–25.

[2] T.Chen and B.A.Francis, *Optimal sampled-data control systems*. Springer, 1995.

[3] L.Grüne, D.Nešić and J.Pannek, *Model predictive sampled–data redesign for nonlinear systems*, submitted to CDC–ECC05

[4] M.Krstić, I.Kanellakopoulos, P.V.Kokotović, *Nonlinear and Adaptive Control Design*, John Wiley & Sons, 1995.

[5] D.S.Laila, D.Nešić and A.R.Teel, *Open and closed loop dissipation inequalities under sampling and controller emulation*, Europ. J. Contr. **8** (2002), 109–125.
[6] D.Nešić and L.Grüne, *Lyapunov based continuous–time nonlinear controller redesign for sampled–data implementation*, Automatica (2005), to appear
[7] D.Nešić and L.Grüne, *A receding horizon control approach to sampled–data implementation of continuous–time controllers*, Syst. Contr. Lett., Special Issue on New Trends in Applied Nonlinear Control (2005), to appear.

How to prove the stability of a nonlinear cascaded loop - An open problem

Peter Hippe

The problem considered arises in the design of reference shaping filters for systems with input saturation. In order to get a fast small amplitude behavior in spite of input saturation, several nonlinear loops are cascaded such, that for each overlying loop with input saturation, the linear part of the underlying system meets the circle criterion. This, however, only assures stability of the filter if saturation and desaturation occur in the correct order, namely saturation staring in the outer cascade and progressing inwards, while desaturation starts in the inner cascade and progresses outwards. If in such a multi loop cascade only the inner saturation was active, the describing function method would predict the possibility of limit cycles. It turns out, that an instability of this filter does not occur in simulations when modifying the reference signal injections such, that for stationary input signals the steady state control signals coincide in all cascades. Unfortunately there exists no possibility to prove the fact, that by this appropriate reference signal injection the unstable filter behavior is not excited.

The second problem considered is related to an elastic system of order seven, where an inner loop moves the six complex conjugate poles to the real axis without modifying the natural frequency, whereas the outer loop shifts the real poles to the natural frequency of the zeros (giving an "asymptotic compensation" of the zeros) and the integrating pole far into the left half s-plane. Though the linear part of the inner loop violates the circle criterion so badly, that the describing function method predicts the possibility of limit cycles, also this filter cannot be destabilized by any reference input sequence. If the inner saturation, however, never became active, the stability of this filter could be proven, since the outer loop with input saturation meets the circle criterion. By reducing the saturation limit in the outer loop appropriately, an activation of the inner saturation can be prevented for arbitrary reference input signals. This becomes possible by use of the L_1-norm. Unfortunately, the resulting transient behavior is much slower than the one of the filter without proven stability. However, by a modified design of the inner loop, an improved transient behavior with assured stability becomes feasible. Though this solution gives a transient behavior quite similar to the one without proven stability, it only solves the special problem considered for elastic systems. It does not constitute a solution to the general problem addressed here. Consequently

the problem of proving the stability for such nonlinear multi-cascaded nonlinear systems remains an open problem.

Interpolation on Manifolds by Rolling and Wrapping

Knut Hüper
(joint work with Fatima Silva Leite)

Many engineering applications call for efficient methods to generate smooth interpolating curves on non-Euclidean spaces. This is the case for instance in path planning for mechanical systems whose configuration spaces have components which are Lie groups or symmetric spaces. Interpolation over a spherical surface, also has immediate applications in manufacturing industry. Several methods to generate interpolating curves on Riemannian manifolds are available in the literature. They correspond to appropriate generalizations of classical methods which have been around for many years. We mention the variational approach to splines on manifolds [1, 3, 7], which may also be reformulated via a Hamiltonian formalism; the geometric approach that corresponds to the De Casteljau algorithm [2, 8]; and the analytic approach undertaken in [4]. These methods posed interesting new mathematical problems and challenges regarding implementation. Even for the simplest cases, say SO_3 or S^2, explicit solutions are hard to obtain. Following [5], we present a method to generate interpolating curves on smooth manifolds, which is based on a rolling and unwrapping technique. Examples considered in this talk are S^n and SO_n. The solution of the interpolating problem is given explicitly in terms of the coordinates of the embedding space. Moreover, because our solution curves are given in closed form they are easily implemented, see [10, 11].

Let M be a smooth n-dimensional manifold embedded into $\mathbb{R}^N$ so that, for all $p \in M$, the tangent space T_pM can be considered as an affine subspace of $\mathbb{R}^N$.

Problem 1: *Find a C^2-smooth curve $\gamma : [0, \tau] \to M$ satisfying $\gamma(t_i) = p_i$ and $1 \leq i \leq k-1$, for a given set of distinct points $p_i \in M$ and fixed times t_i, where $0 = t_0 < t_1 < \cdots < t_{k-1} < t_k = \tau$, and in addition,*

$$\gamma(0) = p_0,\ \gamma(\tau) = p_k, \qquad \dot\gamma(0) = \xi_0 \in T_{p_0}M,\ \dot\gamma(\tau) = \xi_k \in T_{p_k}M,$$

where ξ_0 and ξ_k are given tangent vectors to M at p_0 and p_k, respectively.

We are interested in rolling maps that describe how M rolls without slipping or twisting on its affine tangent space V at a point $p_0 \in M$. ($M, V \subset \mathbb{R}^n$ submanifolds). Being this a rigid body motion, it can be described by an action of SE_n on $\mathbb{R}^n$. The general definition of a rolling map [9] can be adapted to the present situation.

Definition 1: *A map*

$$\begin{aligned} h : [0,\tau] &\to \mathrm{SE}_n = \mathrm{SO}_n \ltimes \mathbb{R}^n, \\ t &\mapsto h(t) = (R(t), s(t)) \quad \textit{with} \quad R : [0,\tau] \to \mathrm{SO}_n,\ s : [0,\tau] \mapsto \mathbb{R}^n \end{aligned}$$

satisfying the following properties for each $t \in [0, \tau]$ is called a rolling of M on V without slipping or twisting*:*

(i) *(Rolling) There is a smooth curve* $\alpha : [0,\tau] \to M$*, s.t. for all* $t \in [0,\tau]$ *it is satisfied:* $h(t) \circ \alpha(t) \in V$ *and* $T_{h(t)\circ\alpha(t)}(h(t) \circ M) = T_{h(t)\circ\alpha(t)}V$.
The curve $\alpha_{\mathrm{dev}} : [0,\tau] \to V$ *defined by* $\alpha_{\mathrm{dev}}(t) = h(t) \circ \alpha(t)$ *is called the* development of α on V.

(ii) *(No-slip)* $\dot{h}(t) \circ h(t)^{-1} \circ \alpha_{\mathrm{dev}}(t) = 0$*, for all* $t \in [0,\tau]$.

(iii) *(No-twist) For all* $t \in [0,\tau]$ *it holds a tangential and a normal condition:*
(Tangential) $\dot{h}(t) \circ h(t)^{-1} \circ T_{\alpha_{\mathrm{dev}}(t)}V = \dot{R}(t)R^{\top}(t) \circ T_{\alpha_{\mathrm{dev}}(t)}V \subset (T_{\alpha_{\mathrm{dev}}(t)}V)^{\perp}$,
(Normal) $\dot{h}(t) \circ h(t)^{-1} \circ (T_{\alpha_{\mathrm{dev}}(t)}V)^{\perp} = \dot{R}(t)R^{\top}(t) \circ (T_{\alpha_{\mathrm{dev}}(t)}V)^{\perp} \subset T_{\alpha_{\mathrm{dev}}(t)}V$.

Assume w.l.o.g. that S^n is rolling (without slipping or twisting) over the affine tangent space V at the *south pole* p_0, with rolling curve $t \mapsto \alpha(t)$, satisfying $\alpha(0) = p_0$. The sphere is rotating in $\mathbb{R}^{n+1}$ s.t. its instantaneous axis of rotation is parallel to V and perpendicular to $\dot{\alpha}(t)$. Simultaneously, the center of S^n is miming on another hyperplane the development of α on V. This explains why the kinematic equations for such a motion are, see e.g. [6]:

$$\dot{s}(t) = u(t), \quad \dot{R}(t) = R(t) \textstyle\sum_{i=1}^{n} u_i(t) A_{n+1,i}, \tag{1}$$

with control $u = (u_1, \cdots, u_n)$, $A_{ij} = E_{ij} - E_{ji}$ (E_{ij} having all entries equal to zero except the $(ij)-$th, being equal to 1), $R(t) \in \mathrm{SO}_{n+1}$, and $s = (s_1, \cdots s_n, 0)^T$ are the coordinates of the development of the center of S^n. Let $A(t) := \sum_{i=1}^{n} u_i(t) A_{n+1,i}$. Choosing a control function corresponds to fixing a rolling curve on S^n. For instance, if the control function is constant, this implies that $R(t)$ is a one-parameter subgroup of SO_{n+1} and consequently the rolling curve is a geodesic on S^n. We now show how to construct a rolling map from the kinematic equations (1).

Theorem 1: *If* R *and* s *are the solution of (1), corresponding to a particular choice of the control function and satisfying* $R(0) = I$*,* $s(0) = 0$*, then* $t \mapsto h(t) = (R^{\top}(t), s(t)) \in \mathrm{SE}_{n+1}$ *is a rolling map, in the sense of Definition 1.*

For Problem 1 we propose the following algorithm, which is based on rolling and unwrapping techniques. This approach works for any manifold M embedded in some Euclidean space $\mathbb{R}^N$, if both M and $V \cong T_{p_0}M$ can be considered as submanifolds of $\mathbb{R}^N$. The resulting curve will be given explicitly in terms of the coordinates of the embedding space.

Algorithm 1:

(i) *Compute a smooth curve* $\alpha : [0,\tau] \to M$*, connecting* p_0 *with* p_K*, s.t.* $\alpha(0) = p_0$ *and* $\alpha(\tau) = p_k$.

(ii) *Roll* M *on* V*, with rolling curve* $\alpha([0,\tau])$*. This produces a smooth curve* $\alpha_{\mathrm{dev}} : [0,\tau] \to V$*, which joins the unrolled initial and final point. The rolling conditions ensure that the boundary conditions are mapped to* V*:*

$$\begin{aligned} \alpha(0) = p_0 \mapsto \alpha_{\mathrm{dev}}(0) = p_0 =: q_0, \qquad & \alpha(\tau) = p_k \mapsto \alpha_{\mathrm{dev}}(\tau) =: q_k, \\ \xi_0 \mapsto h(0)\xi_0 = \xi_0 =: \eta_0, \qquad & \xi_k \mapsto h(\tau)\xi_k =: \eta_k. \end{aligned}$$

(iii) *Choose a local diffeomorphism* $\phi : M \supset U \to V$*,* $p_0 \in U$ *open, s.t.* $\phi(p_0) = p_0$ *and* $\mathrm{D}\,\phi(p_0) = \mathrm{id}$*, to unwrap* $\{p_1, \ldots, p_{k-1}\}$ *onto* V*. Each* p_i *is mapped to a* q_i*, by first rotating* p_i *to get the point* $R(t_i)p_i \in M$*, then applying* ϕ *followed by the translation* $s(t_i)$*. I.e.,* $p_i \mapsto \phi(R(t_i)p_i) + s(t_i)$,

or equivalently,

$$p_i \mapsto \phi\big(h(t_i)p_i - \alpha_{\mathrm{dev}}(t_i) + p_0\big) + \alpha_{\mathrm{dev}}(t_i) - p_0 =: q_i,$$

(iv) *On V solve Problem 1 using instead of the data $\{p_0, \ldots, p_k; \xi_0, \xi_k\}$ the mapped data $\{q_0, \ldots, q_k; \eta_0, \eta_k\}$. This will generate $\beta : [0, \tau] \to V$ with*

$$\beta(0) = p_0 = q_0,\ \beta(t_i) = q_i,\ \beta(\tau) = q_k, \quad \dot\beta(0) = \xi_0 = \eta_0,\ \dot\beta(\tau) = \eta_k.$$

(v) *Wrap $\beta([0, \tau])$ back onto the manifold giving the solution γ of Problem 1 by means of the following formula:*

$$\gamma(t) := h(t)^{-1}\big(\phi^{-1}\big(\beta(t) - \alpha_{\mathrm{dev}}(t) + p_0\big) + \alpha_{\mathrm{dev}}(t) - p_0\big) \tag{2}$$

Theorem 2: *The curve defined by* (2) *solves Problem 1,* [5].

The ideas presented can be applied to SO_n as well. By embedding SO_n into $\mathbb{R}^{n\times n}$ and considering SO_n as a rigid body one has to specify how the corresponding rigid body transformations look like. We present the kinematic equations for rolling SO_n without twist or slip along the affine tangent space attached at $P_0 \in \mathrm{SO}_n$:

$$\begin{aligned} \dot X(t) &= \Omega(t)P_0, & X(0) &= 0,\\ \dot U(t) &= \tfrac{1}{2}U(t)\Omega(t), & U(0) &= I,\\ \dot V(t) &= -\tfrac{1}{2}V(t){P_0}^{\top}\Omega(t)P_0, & V(0) &= I. \end{aligned}$$

The function $t \mapsto \Omega(t) = -\Omega^{\top}(t)$ plays the role of the control function, since the motion is entirely determined by the choice of Ω. If, for instance, $\Omega(t) = \Omega$ is constant, then the solution of the kinematic equations is

$$X(t) = t\Omega P_0,\ \ U(t) = \mathrm{e}^{t\frac{\Omega}{2}},\ \ V(t) = {P_0}^{\top}\,\mathrm{e}^{-t\frac{\Omega}{2}}\,P_0.$$

In this case,

$$t \mapsto \alpha(t) = \mathrm{e}^{t\frac{\Omega}{2}}\,P_0{P_0}^{\top}\,\mathrm{e}^{t\frac{\Omega}{2}}\,P_0 = \mathrm{e}^{t\Omega}\,P_0,$$

is a geodesic on SO_n, passing through P_0 at $t = 0$. Consequently, $\alpha_{\mathrm{dev}}(t) = P_0 + X(t) = P_0 + t\Omega P_0$ is also a geodesic in $T^{\mathrm{aff}}_{P_0}\mathrm{SO}_n$, passing through P_0 at $t = 0$.

References

[1] M. Camarinha. *The geometry of cubic polynomials on Riemannian manifolds.* PhD thesis, University of Coimbra, Portugal, 1996.

[2] P. Crouch, G. Kun, and F. Silva Leite. The De Casteljau algorithm on Lie groups and spheres. *J. of Dynamical and Control Systems*, 5(3):397–429, July 1999.

[3] P. Crouch and F. Silva Leite. Geometry and the dynamic interpolation problem. In *Proc. American Control Conference*, pages 1131–1136, Boston, July 1991.

[4] R Giambó, F. Giannoni, and P Piccione. An analytical theory for Riemannian cubic polynomials. *IMA Journal of Mathematical Control and Information*, 19:445–460, 2002.

[5] K. Hüper and F. Silva Leite. Smooth interpolating curves with applications to path planning. In *10-th IEEE Mediterranean Conference on Control and Automation*, Instituto Superior Técnico, Lisboa, Portugal, July, 9-12 2002.

[6] V. Jurdjevic. *Geometric Control Theory.* Cambridge Univ. Press, Cambridge, 1997.

[7] L. Noakes, G. Heinzinger, and B. Paden. Cubic splines on curved spaces. *IMA J. of Math. Control & Inf.*, 6:465–473, 1989.

[8] F.C. Park and B. Ravani. Bézier curves on Riemannian manifolds and Lie groups with kinematics applications. *ASME J. Mechan. Design*, 117:36–40, March 1995.
[9] R.W. Sharpe. *Differential Geometry*. Springer, New York, 1996.
[10] Y. Shen and K. Hüper. Optimal joint trajectory planning for manipulator robot performing constrained motion tasks. In *Australasian Conference on Robotics & Automation*, Canberra, December 2004. Australian Robotics & Automation Association.
[11] Y. Shen and K. Hüper. Optimal joint trajectory planning of manipulators subject to motion constraints. In *International Conference on Advanced Robotics (ICAR 2005)*, Seattle, July 2005. accepted.

Robustness of adaptive controllers

Achim Ilchmann
(joint work with Eugene P Ryan, Philip Townsend)

Despite the title, I have presented a recent result on tracking of linear minimum phase systems with higher relative degree [1] and its nonlinear extension, which is work in progress.
Consider the *system class* of multi-input, multi-output systems

$$\dot{x}(t) = A\,x(t) + B\,u(t), \qquad y(t) = C\,x(t)$$

where the entries of $A \in \mathbb{R}^{n\times n}$, $B, C^T \in \mathbb{R}^{n\times m}$, $x^0 \in \mathbb{R}^n$, are unknown but satisfy the following structural assumptions:

(A1) *strict relative degree and positive high-frequency gain*:
For some known $\rho \in \mathbb{N}$, $CA^iB = 0$ for $i = 1, ..., \rho - 2$ and $CA^{\rho-1}B > 0$.

(A2) *minimum-phase*:

$$\det \begin{bmatrix} sI - A & B \\ C & 0 \end{bmatrix} \neq 0 \qquad \text{for all } s \in \mathbb{C} \text{ with } \operatorname{Re} s \geq 0.$$

The first *control objective* is approximate tracking, by the output y, of reference signals $r \in W^{1,\infty}(\mathbb{R}_{\geq 0}, \mathbb{R}^m)$, i.e. the space of locally absolutely continuous bounded functions with bounded derivative, endowed with norm $\|r\|_{1,\infty} := \|r\|_\infty + \|\dot{r}\|_\infty$. In particular, for arbitrary $\lambda > 0$, we seek an output feedback strategy which ensures that, for every $r \in W^{1,\infty}(\mathbb{R}_{\geq 0}, \mathbb{R}^m)$, the closed-loop system has bounded solution and the tracking error $e(t) = y(t) - r(t)$ is ultimately bounded by λ (that is, $\|e(t)\| \leq \lambda$ for all t sufficiently large). The second *control objective* is prescribed transient behaviour of the tracking error signal. We capture both objectives in the concept of a performance funnel

$$\mathcal{F}_\varphi := \left\{ (t, e) \in \mathbb{R}_{\geq 0} \times \mathbb{R}^m \,\middle|\, \varphi(t)\,\|e\| < 1 \right\}$$

associated with a function φ (the reciprocal of which determines the funnel boundary) belonging to

$$\mathcal{B} := \left\{ \varphi \in W^{1,\infty}(\mathbb{R}_{\geq 0}, \mathbb{R}) \,\middle|\, \varphi(0) = 0,\ \varphi(s) > 0\ \forall\ s > 0,\ \liminf_{s\to\infty} \varphi(s) > 0 \right\}.$$

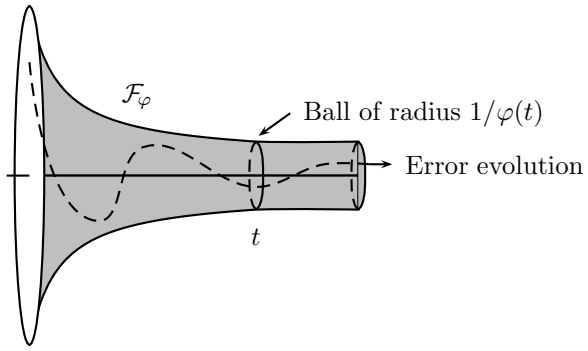

Fig. 1. Prescribed performance funnel $\mathcal{F}_\varphi$.

The aim is an output feedback strategy ensuring that, for every reference signal $r \in W^{1,\infty}(\mathbb{R}_{\geq 0}, \mathbb{R})$, the tracking error $e = y - r$ evolves within the funnel $\mathcal{F}_\varphi$. For example, if $\liminf_{t\to\infty} \varphi(t) > 1/\lambda$, then evolution within the funnel ensures that the first control objective is achieved. If φ is chosen as the function $t \mapsto \min\{t/T, 1\}/\lambda$, then evolution within the funnel ensures that the prescribed tracking accuracy $\lambda > 0$ is achieved within the prescribed time $T > 0$.

The control objective is met by introducing an "observer" of the input

$$\dot{\xi}(t) = F\xi(t) + Gu(t)\,, \text{ where } F \text{ and } G \text{ do not depend on } (A, B, C)$$

in conjunction with a time-varying gain

$$k(t) = \frac{1}{1 - (\varphi(t)\|Cx(t) - r(t)\|)^2}$$

and an input

$$u(t) = -\gamma_\rho\big(k(t), Cx(t) - r(t), \xi(t)\big)\,,$$

where γ_ρ depends polynomially on its arguments.

It is shown that all signals are bounded and the norm of the error e stays uniformly bounded away from the prespecified funnel boundary.

Note that the gain k is not a monotone function as it is in the area of high-gain adaptive control where similar system classes are considered.

References

[1] A. Ilchmann, E.P. Ryan, and P. Townsend. Tracking control with prescribed transient behaviour for systems of known relative degree. *Preprint M02/05 TU Ilmenau,* `http://wcms3.rz.tu-ilmenau.de/ math/preprints/Preprints_en.html`.

Nonlinear observers as internal models

Alberto Isidori
(joint work with C. I. Byrnes)

In a problem of output regulation, we consider a nonlinear system

$$\begin{aligned} \dot{x} &= f(w,x,u) \\ e &= h(w,x) \end{aligned} \tag{1}$$

with state $x \in I\!R^n$, control input $u \in I\!R$ and regulated output $e \in I\!R$. The exogenous (disturbance and/or command) input $w \in I\!R^r$ is generated by an exosystem

$$\dot{w} = s(w)\,. \tag{2}$$

It is assumed that $f(w,x,u)$, $h(w,x)$, and $s(w)$ are C^k functions (for some large k). The set X of admissible initial conditions for x is a compact subset of $I\!R^n$. The set W of admissible initial conditions for w is a compact subset of $I\!R^r$, invariant under the flow of (2).

System (1) is controlled by a system

$$\begin{aligned} \dot{\eta} &= \varphi(\eta,e) \\ u &= \gamma(\eta,e) \end{aligned} \tag{3}$$

with state $\eta \in I\!R^\nu$, in which $\varphi(\eta,e)$ and $\gamma(\eta,e)$ are C^k functions. The set Ξ of admissible initial conditions for η is a compact subset of $I\!R^\nu$.

The forced closed-loop system, i.e. the interconnection of (1), (2) and (3), is the autonomous system

$$\begin{aligned} \dot{w} &= s(w) \\ \dot{x} &= f(w,x,\gamma(\eta,h(w,x))) \\ \dot{\eta} &= \varphi(\eta,h(w,x)) \end{aligned} \tag{4}$$

with output $e = h(w,x)$. The controller (3) solves the problem of output regulation if, in the forced closed-loop system, the positive orbit of $W \times X \times \Xi$ is bounded and $\lim_{t\to\infty} e(t) = 0$, uniformly in the initial condition.

In what follows we describe how the problem of output regulation can be solved for a specific class of nonlinear systems. For the sake of simplicity, we explicitly address the case in which system (1) can be put in the form

$$\begin{aligned} \dot{z} &= f_0(w,z) + f_1(w,z,e)e \\ \dot{e} &= q_0(w,z) + q_1(w,z,e)e + u \end{aligned} \tag{5}$$

with $z \in I\!R^{n-1}$. Accordingly, we let Z and E denote the (compact) sets of admissible initial conditions for z and e.

Assumption 1. The positive orbit of $W \times Z$ under the flow of

$$\begin{aligned} \dot{w} &= s(w) \\ \dot{z} &= f_0(w,z) \end{aligned} \tag{6}$$

is bounded and the set $\omega(W \times Z))$ – the ω-limit set of $W \times Z$ under the flow of (6) – is locally exponentially stable for (6). $\triangleleft$

Consider now a candidate controller of the form

$$\begin{aligned} \dot{\eta} &= \varphi(\eta) + Gv \\ u &= \gamma(\eta) + v \end{aligned} \tag{7}$$

in which v is to be determined later. Controlling the plant by means of (7) yields a system with input v and output e having relative degree one. Its zero dynamics are those of

$$\begin{aligned} \dot{w} &= s(w) \\ \dot{z} &= f_0(w,z) \\ \dot{\eta} &= \varphi(\eta) + G[-\gamma(\eta) - q_0(w,z)] \end{aligned} \tag{8}$$

If the latter have convenient asymptotic properties (namely, if the positive orbit of the set $W \times Z \times \Xi$ is bounded) it is likely that the choice $v = -\kappa e$ would solve the problem.

Now, system (8) can be viewed as the cascade connection of two subsystems: a system with state (w,z) which drives, through the control $u_{\rm ss} := -q_0(w,z))$, a system whit state η. The trajectories of the former are bounded and attracted by the compact invariant set $\omega(W \times Z)$. Thus, a natural hint is to try to design $\varphi(\eta), G, \gamma(\eta)$ so that also in the full system the trajectories are bounded and attracted by a compact invariant set.

Assumption 2. There exists an integer ν and a locally Lipschitz function $f : \mathbb{R}^\nu \to \mathbb{R}$ such that, for any $(w_0, z_0) \in \omega(W \times Z)$, the solution $(w(t), z(t))$ of (6) passing through (w_0, z_0) at $t = 0$ is such that the function $u_{\rm ss}(t)$ satisfies

$$u_{\rm ss}^{(\nu)} + f(u_{\rm ss}, u_{\rm ss}^{(1)}, \ldots, u_{\rm ss}^{(\nu-1)}) = 0 \; . \; \triangleleft \tag{9}$$

Under this assumption it is not difficult to construct (as indicated, for instance, in [2]) a triplet $\{\varphi(\eta), G, \gamma(\eta)\}$ and a C^1 map $\tau : \omega(W \times Z) \to \mathbb{R}^\nu$ whose graph

$$\mathrm{gr}(\tau) := \{(w,z,\eta) : (w,z) \in \omega(W \times Z), \eta = \tau(w,z)\}$$

attracts all trajectories of (8).

Controlling the plant by means of the proposed controller yields a closed-loop system which, in normal form, reads as

$$\begin{aligned} \dot{w} &= s(w) \\ \dot{z} &= f_0(w,z) + f_1(w,z,e)e \\ \dot{\eta} &= \varphi(\eta) + G[-\gamma(\eta) - q_0(w,z)] + \delta(w,z,\eta,e)e \\ \dot{e} &= [\gamma(\eta) + q_0(w,z)] + \theta(w,z,e)e + v \end{aligned} \tag{10}$$

where $\delta(w,z,\eta,e)$ and $\theta(w,z,e)$ are appropriate continuous functions. This can be interpreted as a feedback interconnection of two subsystems: a subsystem with state (w,z,η) driven by the input e, and a subsystem with state e driven by the input (w,z,η). By construction, the former is input-to-state stable, with restrictions, to the compact invariant set $\mathrm{gr}(\tau)$. Moreover, again by construction, the term $[\gamma(\eta)+q_0(w,z)]$ vanishes on the set $\mathrm{gr}(\tau)$. Thus, an (appropriate extended version of) the small-gain theorem can be used to show that the additional choice $v = -\kappa e$, if κ is large enough, makes all trajectories of the closed-loop system

bounded and steers $e(t)$ to zero (details on how this can be proven can be found, for instance, in [3]).

The construction described in [2] exploits the properties of the so-called "high-gain observer" of [5] in the design of $\varphi(\eta), G, \gamma(\eta)$. As a matter of fact $\varphi(\eta)$ and G are chosen as

$$\phi(\eta) = \begin{pmatrix} \eta_2 \\ \eta_3 \\ \cdots \\ \eta_\nu \\ -f(\eta_1, \eta_2, \ldots, \eta_{\nu-1}) \end{pmatrix} \qquad G = \begin{pmatrix} c_{\nu-1} g \\ c_{\nu-2} g^2 \\ \cdots \\ c_1 g^{\nu-1} \\ c_0 g^\nu \end{pmatrix}$$

in which $g > 0$ is a large and $\gamma(\eta) = \eta_1$. In this case, the bottom equation of (8) can be interpreted as the equation of an observer which asymptotically tracks the image $\tau(w, z)$ of the state (w, z) of (6), that is of the upper subsystem of (8).

We see in this way that the design of an internal model for a problem of nonlinear output regulation is intimately related to the design of nonlinear observers. As an alternative to the "high-gain observer" of [5] one could appeal, for the design of $\{\varphi(\eta), G, \gamma(\eta)\}$, to the method presented in [6] and refined in [7, 8, 9].

References

[1] C.I. Byrnes, A. Isidori, *Limit sets, zero dynamics and internal models in the problem of nonlinear output regulation*, IEEE Trans. on Automatic Control **48** (2003), 1712-1723.

[2] C.I. Byrnes, A. Isidori, *Nonlinear internal models for output regulation*, IEEE Trans. on Automatic Control **49** (2004), 2244-2247.

[3] C.I. Byrnes, A. Isidori, L. Praly, *On the asymptotic properties of a system arising in non-equilibrium theory of output regulation*, Preprints of the Mittag-Leffler Institute **18**(2003).

[4] C.I. Byrnes, D.S. Gilliam, A. Isidori, J. Ramsey, *On the steady-state behavior of forced nonlinear systems*, in *New Trends in Nonlinear Dynamics and Control, and their Applications*, W.Kang, M. Xiao and C. Borges Eds., Springer Verlag (2003), 119–144.

[5] J.P. Gauthier, I. Kupka, *Deterministic Observation Theory and Applications*, Cambridge University Press, 2001.

[6] N. Kazantzis, C. Kravaris, *Nonlinear observer design using Lyapunov's auxiliary theorem*, Systems and Control Letters **34** (1998), 241-247.

[7] G. Kreisselmeier, R. Engel, *Nonlinear observers for autonomous Lipschitz continuous systems*. IEEE Trans. on Automatic Control **48** (2003).

[8] A. Krener, M. Xiao, *Observers for linearly unobservable nonlinear systems*. Systems and Control Letters, **46** (2002), 281-288.

[9] V. Andrieu, L. Praly, *On the existence of a Kazantzis-Kravaris observer*, 43rd IEEE Conference on Decision and Control, December 14-17, Bahamas.

Minimum-Phase Infinite-Dimensional Second-Order Systems

Birgit Jacob

(joint work with Kirsten Morris)

A finite-dimensional system is *minimum phase* (or *outer*) if and only if it is stable and its transfer function has no right-hand-plane zeros. Right-hand-plane zeros restrict the achievable sensitivity. Also, it can be difficult to robustly stabilize

non-minimum phase systems and most adaptive controllers require the system to be minimum-phase.

It is therefore be advantageous to establish conditions under which infinite-dimensional systems are minimum-phase. There can be difficulties associated with computing the zeros of an infinite-dimensional system [1, 4]. Furthermore, there are aspects of the dynamics that can lead to non-minimum phase behaviour besides right-hand-plane zeros. For example, the transfer function $\exp(-s)$ has no zeros, but is clearly not minimum-phase. Thus, determining minimum-phase behaviour is less straightforward than for finite-dimensional systems.

We study second-order systems of the following form

$$\begin{aligned} \ddot{z}(t) + A_o z(t) + D\dot{z}(t) &= B_o u(t), \\ y(t) &= B_o^* z(t). \end{aligned} \tag{1}$$

We will show that with this choice of output, and certain assumptions on the damping operator, the system is well-posed and has an outer transfer function.

In common with [5, 6] we make the following assumptions

(A1) The stiffness operator $A_o : D(A_o) \subset H \to H$ is a self-adjoint, positive-definite, boundedly invertible linear operator on a Hilbert space H. Under these assumptions A_o^α is well-defined for $\alpha > 0$ and a scale of Hilbert spaces H_α is defined as follows: For positive α, we define $H_\alpha = [D(A_o^\alpha)]$, and $H_{-\alpha} = H_\alpha^*$. Here the duality is taken with respect to the pivot space H, that is, equivalently $H_{-\alpha}$ is the completion of H with respect to the norm $\|x\|_{H_{-\alpha}} = \|A_o^{-\alpha} x\|_H$. Thus A_o extends (restricts) to $A_o : H_\alpha \to H_{\alpha-1}$. We use the same notation A_o to denote this extension. We denote the inner product on H by $\langle \cdot, \cdot \rangle_H$ or $\langle \cdot, \cdot \rangle$, and the duality pairing on $H_{-\alpha} \times H_\alpha$ by $\langle \cdot, \cdot \rangle_{H_{-\alpha} \times H_\alpha}$.

(A2) The control operator B_o is a linear and bounded operator from $\mathbb{C}$ to $H_{-\frac{1}{2}}$.

(A3) The damping operator $D : H_{\frac{1}{2}} \to H_{-\frac{1}{2}}$ is a self-adjoint linear operator satisfying

$$\langle Dx, x \rangle_{H_{-\frac{1}{2}} \times H_{\frac{1}{2}}} \geq \beta \|x\|_H^2, \qquad x \in H_{\frac{1}{2}}.$$

Under these assumptions the system is well-posed and the transfer function, given by

$$G(s) = B_o^*(s^2 I + Ds + A_o)^{-1} B_o,$$

is holomorphic and bounded on the open right half plane.

$G(s)$ is an *outer function* if and only if

$$G(s) := \exp\left[\frac{1}{\pi} \int_{-\infty}^{\infty} \log|G(it)| \frac{ts + i}{t + is} \frac{dt}{1 + t^2}\right]. \tag{2}$$

Note, that in particular an outer function has no zeros in the right half plane and does not contain a factor of the form $e^{-\omega s}$.

Our main result is as follows:

Assume that the second-order system (1) satisfies assumptions (A1)-(A3) with $B_o \neq 0$. Then $G(s)$ is an outer functions times a constant of modulus one.

We note that the assumptions on the damping operator D can be relaxed and the same result holds for the velocity measurement $y(t) = B_o^* \dot{z}(t)$.

As an example we consider a Euler-Bernoulli beam of unit length clamped at each end. Let $w(r,t)$ denote the deflection of the beam from its rigid body motion at time t and position r. Use of the Kelvin-Voigt damping model leads to the following description of the beam vibrations:

$$\frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial r^2}\left[E\frac{\partial^2 w}{\partial r^2} + C_d\frac{\partial^3 w}{\partial r^2 \partial t}\right] = 0, \qquad 0 < r < 1.$$

Here E and C_d are positive constants. The beam is clamped at each end, that is,

$$\begin{aligned} w(0,t) &= 0, \ \left[E\tfrac{\partial^2 w}{\partial r^2} + C_d\tfrac{\partial^3 w}{\partial r^2 \partial t}\right]_{r=0} = 0, \\ w(1,t) &= 0, \ \left[E\tfrac{\partial^2 w}{\partial r^2} + C_d\tfrac{\partial^3 w}{\partial r^2 \partial t}\right]_{r=1} = 0. \end{aligned}$$

A force u is applied at some point ξ, $0 < \xi < 1$, with position measurement at the same point:

$$\begin{aligned} \left[E\frac{\partial^3 w}{\partial r^3} + C_d\frac{\partial^4 w}{\partial r^3 \partial t}\right]_{r=\xi} &= u(t), \\ w(\xi,t) &= y(t). \end{aligned}$$

For this simple example, the conclusion of no right-hand-plane zeros could be seen by analysis of the transfer function, although this is not straightforward. Determining that there is no part $e^{-\omega s}$ is more difficult. The main advantage of the results in this paper is that they can be applied to vibrations on general domains. They also apply to wave problems such as [5, sect. 7], [6, sect. 5.2].

References

[1] A. Cheng and K.A. Morris, *Accurate Zeros Approximation for Infinite-Dimensional Systems*, *42nd IEEE Conference on Decision and Control,* Honolulu, Hawaii, 2003.

[2] J.R. Grad and K.A. Morris, *Calculation of achievable broadband noise reduction using approximations*, Engineering applications and computational algorithms (Guelph, ON, 2003), Dyn. Contin. Discrete Impuls. Syst. Ser. B Appl. Algorithms, suppl., 438–443.

[3] J-L Lin and J-N Juang, *Sufficient Conditions for Minimum-Phase Second-Order Systems*, Journal of Vibration and Control **1** (1995) 183–199.

[4] D.K. Lindner, K.M. Reichard, and L.M. Tarkenton, *Zeros of Modal Models of Flexible Structures*, IEEE Trans. on Automatic Control **38** (1993) 1384–1388.

[5] G. Weiss and M. Tucsnak, *How to get a conservative well-posed system out of thin air, Part I*, ESAIM Control Optim. Calc. Var. **9** (2003), 247–274.

[6] G. Weiss and M. Tucsnak, *How to get a conservative well-posed system out of thin air, Part II*, SIAM J. Control Optim. **42** (2003), 907–935.

Control Synthesis for Uncertain Systems: A New Approach

Hans W. Knobloch

We consider finite dimensional control systems which are given in the form

$$\begin{aligned} \dot{x}_1 &= p_1(x) + B_1(x)u + G_1(x)w \\ \dot{x}_2 &= p_2(x) + B_2(x)u. \end{aligned} \tag{1}$$

The state is $x = (x_1, x_2)^T$, the measured output (= the available information) is x_2. The input consists of the control u the "uncertainty" w (= an unknown function of t and/or x). Available information concerning w: The degree of smoothness (e.g. measurability with respect to t) and a bound for the sup-norm:

$$\|w\| \leq \omega. \tag{2}$$

We deal with the problem of control design: How to specialize u as a function of t and of the information which is available up to time t such that a certain control objective is met (example: Stabilization of the state, i.e. driving it into a target set and keeping it there for all times). We propose a non-conservative strategy based on the notion of "discretized" state feedback (DSF). The definition runs as follows. Given a smooth function $u(\mathcal{P}, \tau)$ depending upon a real variable τ and a set of parameters $\mathcal{P}$. Given also a subdivision of the time axis in sufficiently small subintervals $[t_i, t_{t+1} = t_i + \delta]$, $\delta << 1$. We speak of a DSF if there is specified for each i an open-loop control law

$$u = u(\mathcal{P}_i; \frac{1}{\delta}(t - t_i)),\ t_i \leq t < t_i + \delta,\ i = 0, 1, ... \tag{3}$$

$\mathcal{P}_i$ typically depends upon $x_2(t_i)$. The main reason why DSF's should be studied is that they provide a natural starting point for the problem of disturbance identification. The essential step is the selection of DSF's which give rise to a "Dissipation Equality" (DE). This is for every subinterval $[t_i, t_{i+1}]$ a quadratic relation between the small-time averages

$$\frac{1}{\delta^2} \int_{t_i}^{t_i+\delta} \int_{t_i}^{t} w(\tau) d\tau dt \tag{4}$$

and

$$\frac{1}{\delta^4} \int_{t_i}^{t_i+\delta} (t - t_i) \int_{t_i}^{t} \int_{t_i}^{\tau} w(s) ds d\tau dt. \tag{5}$$

The coefficients depend upon the information which is available in $[t_i, t_i + \delta]$ and upon $x_1(t_i)$. In case of a Lipschitz-continuous $w(t)$ the averages (4), (5) are proportional to $w(t_i)$ plus an error term $\mathcal{O}(\delta)$. Under certain conditions one can find sufficiently many DE's which constitute a system of equations which allows successive determination of $w(t_i)$ using the Implicit Function Theorem. If the $x_1(t_i)$ cannot be measured, one can combine determination of $w(t_i)$ and $x_1(t_i)$ in such a way that both quantities $w(t)$ and the full state $x(t)$ are identified on an interval $[0, T]$ up to an error $\mathcal{O}(\delta)$, provided $w(0)$, $x(0)$ are known. T depends upon the data (1) and (2) only. Thereby the problem of identification has been solved for systems under the influence of an exogenous disturbance without resorting to (deterministic or stochastic) disturbance models. Instead we have to assume merely Lipschitz continuity and boundedness. This is a considerable step beyond the existing techniques in the area of adaptive control. To illustrate the scope of our method, the results are applied to the problem of active damping of vibrations [2].

References

[1] H.W. Knobloch. Disturbance Attenuation in Control Systems, Part I and II. To appear.
[2] M. Corless and G. Leitmann. Deterministic Control of Uncertain Systems: A Lyapunov Theory Approach. In Deterministic Control of Uncertain Systems, A. S. I. Zinober, editors, pages 220–251, Peregrinus, London, 1990.

Nonlinear Model Reduction

Arthur J. Krener

There are several well-known techniques for model reduction of a linear control system of the form

$$\begin{aligned} \dot{x} &= Fx + Gu \\ y &= Hx \end{aligned}$$

$x \in I\!R^n, \quad u \in I\!R^m, \quad y \in I\!R^p$. If we initialize the system at $x(0) = 0$ we get a linear mapping from inputs $u(\cdot)$ to outputs $y(\cdot)$,

$$\mathbf{L}(u)(t) \quad = \quad y(t) = \int_0^t He^{F(t-s)}Gu(s)\ ds$$

called the Hankel map of the system. What is the smallest state dimension necessary to realize $\mathbf{L}$? A realization is of minimal state dimension iff it is controllable and observable. Any realization can be reduced to a minimal one by restricting the state to the controllable directions and projecting out the unobservable directions. Any two minimal realizations differ by a linear change of state coordinates.

The goal of B.C. Moore [1] was to find a reduced order system which captures most of the input/output behaviour of the control system. He assumed that F is Hurwitz, $\sigma(F) < 0$, F, G is a controllable pair and H, F is an observable pair. If the system is not controllable and/or not observable we can make it so by passing to a minimal realization so this is no restriction. The stability of F is needed to insure that if $u(-\infty : 0)$ is bounded then $x(0)$ exists and $y(0 : \infty)$ is bounded.

Moore's insight was that we should restrict to the directions that are easy to excite and also ignore directions where changes don't affect the output very much. To quantify this idea, he introduced the controllablity function of the system.

$$\pi_c(x^0) \quad = \quad \inf \frac{1}{2} \int_{-\infty}^0 |u(t)|^2\ dt$$

subject to the system dynamics and $x(-\infty) = 0,\ x(0) = x^0$ and the observability function

$$\pi_o(x^0) \quad = \quad \frac{1}{2} \int_0^\infty |y(t)|^2\ dt$$

subject to the system dynamics and $x(0) = x^0,\ u(t) = 0$.

Because the system is controllable, $\pi_c(x^0)$ is bounded. Because F is Hurwitz, $\pi_c(x^0)$ is positive definite. Because F is Hurwitz, $\pi_o(x^0)$ is bounded. Because the

system is observable, $\pi_o(x^0)$ is positive definite. Because the system is linear and the criteria are quadratic, $\pi_c(x^0)$ and $\pi_o(x^0)$ are quadratic functions,

$$\pi_c(x) = \frac{1}{2}x' P_c^{-1} x, \qquad \pi_o(x) = \frac{1}{2}x' P_o x$$

where the controllablity and observability gramians P_c, P_o are the unique positive definite solutions of

$$\begin{aligned} 0 &= FP_c + P_c F' + GG' \\ 0 &= F'P_o + P_o F + H'H. \end{aligned}$$

If $\pi_c(x^0)$ is large then it takes a lot of input energy to excite the system in the direction x^0 and so this direction might be ignored in a reduced order model. If $\pi_o(x^0)$ is small then changes in this direction lead to small changes in the output energy and so this direction might be ignored in a reduced order model. There is a linear change of state coordinates so that in these new coordinates

$$P_c = P_o = \begin{bmatrix} \sigma_1 & & 0 \\ & \ddots & \\ 0 & & \sigma_n \end{bmatrix}$$

$$\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n > 0$$

The σ_i are called the Hankel singular values, they are the nonzero singular values of the Hankel map. The reduced model is obtained by Galerkin projection onto the states corresponding to large σ_i.

Suppose $\sigma_k >> \sigma_{k+1}$, let x_1 denote the first k components of x and x_2 denote the last $n-k$ components.

$$P_c = P_o = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$$

Then the full order model is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} u$$

$$y = \begin{bmatrix} H_1 & H_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

and the reduced order model is

$$\begin{aligned} \dot{x}_1 &= F_{11}x_1 + G_1 u \\ y &= H_1 x_1 \end{aligned}$$

Scherpen [2] generalized Moore to nonlinear systems

$$\begin{aligned} \dot{x} &= f(x) + g(x)u = Fx + Gu + \ldots \\ y &= h(x) = Hx + \ldots \end{aligned}$$

She defined the controllability and observability functions as before subject to the nonlinear system. It is not hard to see that π_c satisfies an HJB partial differential

equation and π_o satisfies a Lyapunov partial differential equation. If the linear part of the system is asymptotically stable, controllable and observable then there exists smooth, positive definite local solutions to these PDEs around $x = 0$ whose quadratic parts are $P_c^{-1},\ P_o$

Scherpen made a series of coordinate changes so that

$$\begin{aligned}\pi_c(0,\ldots,x_i,\ldots,0) &= \frac{1}{2}\sum \sigma_i(x_i)^{-1}x_i^2\\ \pi_o(0,\ldots,x_i,\ldots,0) &= \frac{1}{2}\sum \sigma_i(x_i)x_i^2\end{aligned}$$

Hence in these coordinates the controllability and observability functions are balanced along the coordinate curves.

She obtained a reduced order model by neglecting states with small $\tau_i(x)$, It is not hard to see that the controllability function of the reduced order model is the resriction of the controllability function of the full order model but this is not true for the observability functions

There is an important difference beween the linear and nonlinear reductions. The former is intrinsic once k is chosen but the latter is not. Two open questions are as follows. Is there an intrinsic scheme for nonlinear model reduction? Is there a scheme such that both the controllability and observability functions of the reduced order model are the restrictions of the controllability and observability functions of the full order model.

Intuitively to find a reduced order model of state dimension k, we seek a k surface that minimizes $\pi_c(x)$ while holding $\pi_o(x)$ constant. If $k > 1$ then there is no such surface. We offer two altenative definitions that approximate this goal. Unfortunately neither is intrinsic.

Lall, Marsden and Glavski [3] have developed a nonlinear model reduction scheme based on proper orthogonal decomposition. Their scheme is applicable to a locally exponentially stable system and reduces to Moore when the system is linear. We offer extensions of their scheme to unstable systems.

References

[1] B. C. Moore, *Principle Component Analysis in Linear Systems: Controllability, Observability and Model Reduction*, IEEE Trans. Auto. Con., **A**C-26, (1981), 17–32.

[2] J. M. A. Scherpen, *Balancing for Nonlinear Systems*. Systems and Control Letters, **2**1 , (1993), 143–153.

[3] S. Lall, J. E. Marsden and S. Glavaski, *A Subspace Approach to Balanced Truncation for Model Reduction of Nonlinear Control Systems*. Int. J. of Robust and Nonlinear Control, **1**2 , (2002), 519–535.

Reconfigurable control: the concepts of the virtual sensor and the virtual actuator

Jan Lunze
(joint work with Thomas Steffen)

Fault-tolerant control aims at retaining a system in operation after some fault has occurred. Different methods have been elaborated in the recent past to detect and identify faults in a dynamical system and to adjust the controller to the changes of the plant dynamics that are brought about by these faults [1].

This paper concerns the reconfiguration task in case of sensor and actuator failures that make the operation of the nominal control loop impossible. For these failures the adjustment of the controller to the faulty plant includes the selection of alternative sensors or actuators in order to close the control loop which has been brought out of operation by the failures.

This paper develops the idea of a virtual sensor and a virtual actuator, which adapts both the control configuration and the controller parameters to the faulty plant. These virtual blocks are dynamical system, which are put between the faulty plant and the nominal controller. They transform the output vector $\mathbf{y}_f$ of the faulty plant into the output vector $\mathbf{y}_c$ used by the controller and the controller output vector $\mathbf{y}_c$ into the plant input vector $\mathbf{y}_f$. They should "hide" the effects of the sensor or actuator failures from the controller. That is, the reconfigured plant, which consists of the faulty plant and the virtual sensor or virtual actuator, should behave like the nominal plant (*fault-hiding goal*). If this goal is satisfied, the nominal controller can be used to control the faulty plant without changing its structure or parameters.

The idea of using a virtual sensor or a virtual actuator has been first proposed in [1] and [2] and further developed in [4].

The *virtual sensor* uses the idea of the state observer. It reconstructs the state vector $\mathbf{x}$ of the faulty plant by using all sensors that are not affected by the fault. This state is used to get an approximate $\hat{\mathbf{y}}$ of the missing sensor output $\mathbf{y}$. General properties of this scheme can be proved by using the well-known results of state observation.

The original idea of this paper is the development of the *virtual actuator*, which can be thought of as the dual concept of the state observer. This system includes a model of the faulty plant. Intuitively speaking, it distributes the effect of the faulty actuator among the remaining or redundant actuators.

Three important properties have been shown in the given references. First, the virtual actuators ensure that the effects of faulty actuators can actually be "hidden" from the controller so that the nominal controller can be used for the faulty plant. Second, a separation theorem, which is similar to the separation theorem of state observers, holds. Consequently, the set of eigenvalues of the overall system consists of the spectrum of the nominal closed-loop system and the spectrum of the virtual actuator. Third, the design of the virtual actuator can be done completely

automatically without the intervention of a human operator and, thus, is possible under the real-time constraints of practical control reconfiguration.

References

[1] M. Blanke; M. Kinnaert; J. Lunze; M. Staroswiecki, *Diagnosis and Fault-Tolerant Control*, Springer, Heidelberg 2003.

[2] J. Lunze, T,. Steffen, Rekonfiguration linearer Systeme bei Sensors- und Aktorausfall, *Automatisierungstechnik* **51** (2003), 60-68.

[3] J. Lunze, J. Schröder, Sensor and actuator fault diagnosis of systems with discrete inputs and outputs, *IEEE Trans.* **SMC-34** (2004), pp. 1096-1107.

[4] T. Steffen, *Control Reconfiguration of Dynamical System: Linear Approaches and Structural Tests*, Springer-Verlag, Heidelberg 2005.

Phenomena in inverse Stackelberg problems

Geert Jan Olsder

This contribution deals with various types of Stackelberg problems, with a clear emphasis on inverse Stackelberg problems, to be defined later on. Such problems are usually treated within the context of game theory. In its simplest form, there are two players, called leader and follower respectively, each having its own cost function,

$$J_{\mathrm{L}}(u_{\mathrm{L}}, u_{\mathrm{F}}),\ J_{\mathrm{F}}(u_{\mathrm{L}}, u_{\mathrm{F}}),$$

where $u_{\mathrm{L}}, u_{\mathrm{L}} \in \mathrm{R}$. Each player wants to choose its own decision variable in such a way as to minimize its own cost function. Without giving an equilibrium concept, the problem as stated so far is not well defined. Such an equilibrium concept could for instance be the one named after Nash, Stackelberg or after Pareto [1]. In the *inverse Stackelberg equilibrium* concept one player, the leader, announces its strategy $\gamma_{\mathrm{L}}(\cdot)$, which maps u_{F} into u_{L}. This is subsequently made known to the other player, the follower. With this knowledge at hand, the follower chooses its u_{F}. Given the function $\gamma_{\mathrm{L}}(\cdot)$, the follower will make its choice u_{F} according to

$$u_{\mathrm{F}}^* = \arg\min_{u_{\mathrm{F}}} J_{\mathrm{F}}(\gamma_L(u_{\mathrm{F}}), u_{\mathrm{F}}).$$

Optimizing quantities will be provided with an asterisk. Subsequently, u_{L}^* is determined by $u_{\mathrm{L}}^* = \gamma_{\mathrm{L}}(u_{\mathrm{F}}^*)$.

The inverse Stackelberg equilibrium concept was introduced in [5]. Examples of games with such informatiom structures are:

- The leader is the bank and the follower the investor. The investor can buy stocks, with the bank as intermediator, with the money he has in his savings account. Suppose he buys stocks worth u_{F} Euro's. Then the bank will charge him $\gamma_{\mathrm{L}}(u_{\mathrm{F}})$ as transaction costs. The function $\gamma_{\mathrm{L}}(\cdot)$ has been made known by the bank before the actual transaction takes place [2], [7].
- The leader is a producer of electricity in a liberalizing market and the follower is the market (a group of clients) itself. The price of electricity is set to $\gamma_{\mathrm{L}}(u_{\mathrm{F}})$, where u_{F} is the amount of electricity traded [6].

The leader, before announcing its $\gamma_{\mathrm{L}}(\cdot)$, will of course realize how the follower will play and he should exploit this knowledge in order to choose the best possible γ-function, such that ultimately its own cost function J_{L} becomes as small as possible. Symbolically we could write

$$\gamma_{\mathrm{L}}^*(\cdot) = \arg\min_{\gamma_{\mathrm{L}}(\cdot)} J_{\mathrm{L}}(\gamma_L(u_{\mathrm{F}}^*(\gamma_{\mathrm{L}}(\cdot))), u_{\mathrm{F}}^*(\gamma_{\mathrm{L}}(\cdot))).$$

In this way one enters the field of composed functions [4], which is known to be notoriously complex. From here onward it turns out to be difficult to proceed in an analytic way. However, there is a trick that often works and which can be elucidated in a graphical way. Suppose that the curve $\gamma_L(\cdot)$ can be chosen in such a way as to

- pass through the absolute minimum of J_L, to be indicated by $(u_{\mathrm{L}}^*, u_{\mathrm{F}}^*)$,
- have, apart from this absolute minimum, no points in common with the set
  $\{(u_{\mathrm{L}}, u_{\mathrm{F}}) | J_{\mathrm{F}}(u_{\mathrm{L}}, u_{\mathrm{F}}) \leq J_{\mathrm{F}}(u_{\mathrm{L}}^*, u_{\mathrm{F}}^*)\}$.

Then the follower, by minimizing its own cost function subject to the constraint $u_{\mathrm{L}} = \gamma_{\mathrm{L}}(u_{\mathrm{F}})$ announced by the leader, helps the leader in obtaining its absolute minimum. Such a strategy for the leader is not always possible, however. See [1] as to what the leader can still achieve in those cases.

Examples are given which are motivated by the liberalization of the energy market in Western Europe, see for example [3]. The players are the producers of electricity (the leaders) and the consumers of electricity (the followers). If there is more than one leader, it is assumed that they play Nash among themselves. The same assumption is made if there is more than one consumer. These examples are rather academic. As a side issue, expressions like "two captains on a ship", "the laughing third party" and "divide and conquer" are given a mathematical foundation.

In the second part of this contribution dynamic generalizations of the inverse Stackelberg problem are considered. The main problem now is

$$\dot{x} = f(x,u),\ x(0) = x_0, \tag{1}$$

$$\min_{u_{\mathrm{F}}} J_{\mathrm{F}} = \min_{u_{\mathrm{F}}}(q(x(T)) + \int_0^T g(x, u_{\mathrm{F}})\mathrm{d}t + \int_0^T \gamma_{\mathrm{L}}(u_{\mathrm{F}}(t))\mathrm{d}t), \tag{2}$$

$$\max_{\gamma_{\mathrm{L}}(\cdot)} J_{\mathrm{L}} = \max_{\gamma_{\mathrm{L}}(\cdot)} \int_0^T \gamma_{\mathrm{L}}(u_{\mathrm{F}}(t))\mathrm{d}t. \tag{3}$$

The function γ_{L} is up to the choice of the leader subject to the restriction

$$\gamma_{\mathrm{L}}(0) = 0,\ \ \gamma_{\mathrm{L}}(\cdot) \geq 0.$$

Occasionally we will also require that γ is nondecreasing with respect to $||u||$ and that $\gamma_{\mathrm{L}}(u) = \gamma_{\mathrm{L}}(-u)$. The inverse Stackelberg equilibrium should be recognized easily by means of the notation: the leader announces the function γ_{L} which thus becomes known to the follower who subsequently chooses u_{F}. Note that $\gamma_{\mathrm{L}}(\cdot)$ does not explicitly depend on time t.

A possible interpretation of this model is that the follower is an investor who wants to maximize his wealth

$$-q(x(T)) - \int_0^T g(x, u_{\mathrm{F}})\mathrm{d}t,$$

equivalently wants to minimize $\overline{J}_{\mathrm{F}} \stackrel{\mathrm{def}}{=} q(x(T)) + \int_0^T g(x, u_{\mathrm{F}})\mathrm{d}t$. Please note the difference in the notations J_{F} and $\overline{J}_{\mathrm{F}}$. The term $-q(x(T))$ in the criterion represents the wealth of the investor at the final time T and the term $-\int_0^T g(x, u_{\mathrm{F}})\mathrm{d}t$ represents the consumption during the time interval $[0, T]$. The decision variable $u_{\mathrm{F}}(t)$ denotes the transaction density with the bank at time t (e.g. selling or buying stocks), i.e. during the time interval $[t, t + \mathrm{d}t]$ the number of transactions equals $u(t)\mathrm{d}t$. For $u_{\mathrm{F}} = 0$ no transactions take place ($\gamma_{\mathrm{L}}(0) = 0$). Transactions cost money and we assume that the bank wants to maximize these transaction costs as indicated by (3). These costs are added to the costs of the follower as indicated in (2).

Routine ways of analysis, e.g. by means of Pontryagin's maximum principle, are not very fruitful due to the phenomenon of composed functions. Other solution methods are investigated and some partial results are given.

References

[1] T. Başar and G.J. Olsder. *Dynamic Noncooperative Game Theory.* SIAM, Philadelphia, 1999.

[2] P. Bernhard. *A robust control approach to option pricing including transaction costs.* In A. Nowak (ed.), *Annals of the ISDG 7*, Birkhauser, 2002.

[3] M. Bossy, N. Maizi, G.J. Olsder, O. Pourtallier and E. Tanré. Electricity prices in a game theory context. To appear in A. Haurie, G. Zaccour (eds), Dynamic Games: theory and applications, 2005.

[4] M. Kuczma. *Functional Equations in a Single Variable.* Polish Scientific Publishers, 1968.

[5] Y.-C. Ho, P.B. Luh and G.J. Olsder. A control-theoretic view on incentives. *Automatica*, Vol. 18, 1982, pp. 167-179.

[6] R. Madlener and M. Kaufmann. *Power exchange spot market trading in Europe: theoretical considerations and emperical evidence* Contract no. ENSK-CT-2000-00094 of the 5th Framwork Programme of the European Community, 2002.

[7] G.J. Olsder. Differential game-theoretic thoughts on option pricing and transaction costs. *International Game Theory Review*, Vol. 2, 2000, pp. 209-228.

Bifurcations and Phase Portraits of nonlinear control systems on the plane

Witold Respondek

(joint work with Bronislaw Jakubczyk)

In this talk we define and study bifurcations of 1-parameter families of smooth, control-affine systems

$$\Sigma \ : \ \dot{\xi} = f(\xi, \epsilon) + g(\xi, \epsilon)u,$$

where ξ lies in an open subset $X \subset \mathbb{R}^2$ or in a differential manifold $X = M^2$ and $u \in \mathbb{R}$. We classify the generic bifurcations at control-regular points (i.e., with $g(\xi, \epsilon) \neq 0$). The reader is sent to [7] for proofs and more details.

A local bifurcation of a parameter dependent dynamical system $\dot{\xi} = f(\xi, \epsilon)$ occurs at an equilibrium if there is a change, when the parameter ϵ varies, of topological character of the solution curves nearby the equilibrium (see, e.g., [8], [11]). Understanding bifurcations of such equations is important from several points of view and the already known classification is rather rich (see, e.g., [3]).

Analogous definition of bifurcation applied to a control system is not suitable since the set of trajectories of Σ is rich (local invariants of the feedback group include functional invariants, already for generic systems on $\mathbb{R}^2$, see [5], [6], [13]).

Therefore we consider only the most characteristic trajectories: constant trajectories, time-critical trajectories and, so called, fast (quasi) trajectories. Thus, we attach to our system three basic invariants (equivariants) of feedback transformations. Namely, the *equilibria set* and the *critical set* are defined, respectively, by

$$E_\epsilon = \{p \in X \mid f(p, \epsilon) \text{ and } g(p, \epsilon) \text{ are linearly dependent}\},$$

$$C_\epsilon = \{p \in X \mid [g, f](p, \epsilon) \text{ and } g(p, \epsilon) \text{ are linearly dependent}\},$$

where $[g, f] = Df\, g - Dg\, f$ is the Lie bracket of g and f. The *canonical foliation* $\mathcal{G}_\epsilon$ (foliation of *fast trajectories*) consists of orbits (non-parameterized integral curves) of the control vector field $g(\cdot, \epsilon)$. Note that the fast trajectories are not true trajectories of Σ but only "asymptotic trajectories", corresponding to "arbitrarily large" control.

The choice of these invariants is justified by the fact that for a generic system $\Sigma : \dot{\xi} = f(\xi) + ug(\xi)$, $\xi \in X$, $u \in \mathbb{R}$, first, E consists of all points, where we can produce an equilibrium (by an appropriate choice of control), second, the pair of invariants $(E, \mathcal{G})$ determines the set of unparameterized trajectories of Σ in the region X where $g(\xi) \neq 0$, and, third, locally time-minimal and time-maximal trajectories are contained in C.

We will study bifurcations of these invariants. A bifurcation occurs if the triplet of basic invariants $(E_\epsilon, C_\epsilon, \mathcal{G}_\epsilon)$ at ϵ_0 is not topologically conjugated to the triplets at nearby values of ϵ. The same definition can be used for any subset of the triplet $(E_\epsilon, C_\epsilon, \mathcal{G}_\epsilon)$. In particular, we define bifurcations of the equilibrium set E_ϵ (E-bifurcations), of the critical set C_ϵ (C-bifurcations), as well as bifurcations of the pairs (E_ϵ, C_ϵ), $(E_\epsilon, \mathcal{G}_\epsilon)$, and $(C_\epsilon, \mathcal{G}_\epsilon)$.

Our first main result says that, generically, there are only six nonequivalent bifurcations of planar systems Σ at control-regular points. Throughout the talk by a generic system Σ we mean a 1-parameter family of pairs (f, g) of vector fields that belongs to a dense set, which is a countable intersection of open and dense subsets in the C^∞ Whitney topology of the space of all pairs (f, g) defined on $X \times I$ (see [4] for properties of the Whitney topology).

Theorem *Let Σ be a smooth, generic, 1-parameter family of control-affine systems. If $g(p, \epsilon_0) \neq 0$ and Σ bifurcates locally at (p, ϵ_0), then the bifurcation is equivalent to one of the following:*

(i) *an E-bifurcation which can be of two types: birth of equilibria or cross of equilibria;*
(ii) *a C-bifurcation which can be of two types: birth of critical curve or cross of critical curve;*
(iii) *a $C\mathcal{G}$-bifurcation;*
(iv) *an $E\mathcal{G}$-bifurcation (or an EC-bifurcation).*

Here equivalence of bifurcations is understood as equivalence of the triples of invariants $(E_\epsilon, C_\epsilon, \mathcal{G}_\epsilon)$ under smooth, invertible local transformations of the form

$$\tilde{\xi} = \phi(\xi, \epsilon), \qquad \tilde{\epsilon} = \eta(\epsilon).$$

The study of bifurcations of control systems was initiated by Abed and Fu [1], [2], in a different setting, for systems of the form $\dot{\xi} = f(\xi, u, \epsilon)$. They assumed that the uncontrolled system, defined by taking $u = 0$, undergoes a bifurcation at $\epsilon = \epsilon_0$ and they studied stabilizability of the system by quadratic and cubic feedbacks. A control system does not need a parameter to bifurcate, the control can play the same role. This point of view is presented by Krener et al [10]. They consider systems $\dot{\xi} = f(\xi, u)$, for which the set of equilibria is conveniently parameterized by the control u. According to their definition, a control bifurcation takes place at an equilibrium if the linear approximation of the system looses stabilizability.

Our approach is close in spirit to that of Kang [9] who studied bifurcations of the set of equilibria and of the linear approximation of the system at an equilibrium.

The above theorem is a consequence of our second main result, which locally classifies generic families Σ under smooth orbital feedback equivalence (see [7] for details, in particular, the list of normal forms). In order to establish that classification we use Mather theorem on universal unfoldings (see, e.g., [12]) as well as equivalence of ratios of smooth functions.

References

[1] E. H. Abed and J.-H. Fu, Local feedback stabilization and bifurcation control, Part I, Hopf bifurcations, *Syst. Contr. Lett.* **7** (1986), 11-117.

[2] E. H. Abed and J.-H. Fu, Local feedback stabilization and bifurcation control, Part I, Stationary bifurcations, *Syst. Contr. Lett.* **8** (1987), 463-473.

[3] V.I. Arnold, V.S. Afraimovitch, Yu.Ilyashenko, L.P. Shilnikov, *Theory of Bifurcations*, Encyclopaedia of Mathematics, Vol. 5, Springer.

[4] M. Hirsch, *Differential Topology*, Springer, New York, 1976.

[5] B. Jakubczyk and W. Respondek, Feedback equivalence of planar systems and stabilizability, in *Robust Control of Linear Systems and Nonlinear Control*, M.A. Kaashoek, J.H. van Schuppen, A.C.M. Ran (eds), Birkhäuser, Boston, 1990, pp. 447-456.

[6] B. Jakubczyk and W. Respondek, Feedback classification of analytic control systems in the plane, in: *Analysis of Controlled Dynamical Systems*, B. Bonnard et al. (eds.), Birkhäuser, Boston, (1991), pp. 262–273.

[7] B. Jakubczyk and W. Respondek, Bifurcations of 1-parameter families of control-affine systems in the plane, to appear in *SIAM J. Control and Optim.*

[8] G. Iooss and D. Joseph. *Elementary Stability and Bifurcation Theory*, Springer-Verlag, New York-Heidelberg-Berlin, (1980).
[9] W. Kang, Bifurcation and normal form of nonlinear control systems - part I and part II, *SIAM J. Control and Optim.* **36** (1998), pp. 193-212 and 213-232.
[10] A. J. Krener, W. Kang, and D. E. Chang, Control bifurcations, *IEEE Trans. Autom. Control* **49** (2004), pp. 1231-1246.
[11] Y.A. Kuznetsov, *Elements of Applied Bifurcation Theory*, Springer-Verlag, New York, 1998.
[12] J. Martinet, *Singularities of Smooth Functions and Maps*, London Math. Soc. Lecture Notes Series 58, Cambridge University Press, Cambridge 1982.
[13] M. Zhitomirskii, Finitely determined 1-forms ω, $\omega|_0 \neq 0$ are reduced to the models of Darboux and Martinet, *Funct. Anal. and its Appl.* **19**, 1985, pp. 71-72.

Matrix sum-of-squares relaxations in robust control

Carsten W. Scherer
(joint work with Camile Hol)

It is well-established that a whole variety of analysis and synthesis problems in control can be reduced to scalar polynomially constrained polynomial programs. Only rather recently it has been suggested how to construct semi-definite programming (SDP) relaxations of such non-convex optimization problems based on the sum-of-squares (SOS) decomposition of multivariable polynomials [10, 3, 9, 5]. In particular in control engineering many problems actually involve semi-definite constraints on symmetric-valued polynomial matrices, such as the spectral factorization of multidimensional transfer functions to asses dissipativity of linear shift-invariant distributed systems [12] or the synthesis of H_∞-optimal output feedback controllers with a constraint on the controller structure, such as an a priori bound on its McMillan degree [6].

Control systems are typically affected by uncertainty which captures the mismatch between the employed model and the real plant under consideration for analysis or synthesis. For different important classes, such as parametric or dynamic, time-invariant or time-varying deterministic uncertainties it is well-understood how to reduce robust stability and performance analysis or robust state-feedback and estimator synthesis problems to so-called robust semi-definite programs [4, 1]. Although dynamic model-mismatch in feedback interconnections leads to complex uncertainties which enter in a rational fashion, it is not difficult to reduce to real uncertainty and polynomial dependence [7].

This leads us to the core subject of this presentation, the following robust polynomial semi-definite program:

$$(1) \qquad \begin{array}{ll} \text{infimize} & c^T y \\ \text{subject to} & F(x,y) \succ 0 \ \text{ for all } \ x \in \mathbb{R}^m \ \text{ with } \ G(x) \preceq 0. \end{array}$$

Here $F : \mathbb{R}^m \times \mathbb{R}^n \mapsto \mathcal{S}^p$ and $G : \mathbb{R}^m \mapsto \mathcal{S}^q$ are symmetric-valued functions which depend polynomially on the uncertainty parameter $x \in \mathbb{R}^m$ while F depends affinely on the design parameter $y \in \mathbb{R}^n$. Therefore $F(x,y) \succ 0$ is a standard linear

matrix inequality (LMI) in y for fixed x, while the robust counterpart requires to satisfy the LMI for all x in the uncertainty set

$$\mathcal{G} = \{x \in \mathbb{R}^m : \ G(x) \preceq 0\} \tag{2}$$

which itself admits a very general description in terms a polynomial semi-definite constraint. We stress that in many interesting practical cases $\mathcal{G}$ turns out to admit an LMI representation (G is affine) or is even just a compact polytope (G is diagonal and affine). Moreover polynomial semi-definite programs $\sup\{f(x) : \ x \in \mathbb{R}^m, \ G(x) \preceq 0\}$ as considered in [8, 6] are recovered from (1) with $F(x, y) = y - f(x)$ and $c = 1$. If in addition $G(x) = \mathrm{diag}(-g_1(x), -g_2(x), \ldots, -g_q(x))$ is scalar-diagonal we arrive at the problem class considered in [9, 11, 14]. Note that multiple polynomial SDP-constraints can be easily collected into one inequality by diagonal augmentation.

If F depends also affinely on the uncertainty x and $\mathcal{G}$ is the convex hull of a moderate number of explicitly given generators, it is clear that (1) amounts to solving a standard LMI problem. If the number of extreme points to describe $\mathcal{G}$ is large, it is often possible to construct efficiently computable relaxation with beautiful a priori guarantees on the relaxation error [2]. The situation drastically differs if the uncertainties enter nonlinearly, since then such a priori guarantees are out of reach. Still, however, various relaxation schemes in robust control (such as multiplier relaxation in structured singular value theory) have been applied to construct efficiently computable relaxations.

In general it cannot be expected that these relaxations are exact, and the only known techniques to systematically reduce the relaxation gap with guaranteed convergence is restricted to finitely generated polytopes with known generators [13]. As the main goal of this talk we show how such asymptotically exact relaxation families can be constructed on the basis of matrix SOS decompositions for the much larger class of uncertainty sets $\mathcal{G}$, based directly on the implicit polynomial matrix inequality description (2) if $G(x)$ satisfies a suitable constraint qualification. In contrast to approaches based on scalarization and a subsequent application of existing relaxation techniques [9, 11, 14], we will be able to show that the sizes of the constructed LMI relaxations grow at most quadratically in the dimension p of $F(x, y)$ and q of $G(x)$ respectively. Moreover we will reveal how the techniques in [13] can be applied in order to verify whether a given finite relaxation does not involve any conservatism.

References

[1] A. Ben-Tal and A. Nemirovski. *Lectures on Modern Convex Optimization: Analysis, Algorithms, and Engineering Applications*. SIAM-MPS Series in Optimizaton. SIAM Publications, Philadelphia, 2001.

[2] A. Ben-Tal and A. Nemirovski. On tractable approximations of uncertain Linear Matrix Inequalities affected by interval uncertainties. *SIAM J. Optim.*, 12(3):811–833, 2002.

[3] G. Chesi, A. Garulli, A. Tesi, and A. Vicino. An LMI-based approach for characterizing the solution set of polynomial systems. In *Proc. 39th IEEE Conf. Decision and Control*, Sydney, Australia, 2000.

[4] L. El Ghaoui and S.I. Niculescu, editors. *Advances in Linear Matrix Inequality Methods in Control*. SIAM, Philadelphia, 2000.
[5] D. Henrion and A. Garulli, editors. *Positive polynomials in control*. Lecture Notes in Control and Information Sciences. Springer-Verlag, Berlin, Heidelberg, New York.
[6] C.W.J. Hol and C.W. Scherer. Fixed order H_∞-synthesis: Computing optimal values by robust performance analysis. In S.M. Phillips, editor, *Proceedings of the 2004 American Control Conference*, Boston, Mass, 2004.
[7] D. Jibetean and E. De Klerk. Global optimization of rational functions: a semidefinite programming approach. Technical report, CWI (Centrum voor Wiskunde en Informatica), 2003.
[8] M. Kojima. Sums of squares relaxations of polynomial semidefinite programs. Technical report, Tokyo Institute of Technology, 2003.
[9] J. B. Lasserre. Global optimization with polynomials and the problem of moments. *SIAM J. Optim.*, 11(3):796–817, 2001.
[10] P. Parrilo. *Structure semidefinite programs and semialgebraic geometry methods in robustness and optimization*. PhD thesis, California Institute of Technology, 2000.
[11] P. Parrilo and B. Sturmfels. Minimizing polynomial functions. *DIMACS Series in Discrete Mathematics and Theoretical Computer Science*, 2001.
[12] H. Pillai and J.C. Willems. Lossless and dissipative distributed systems. *SIAM J. Control Optim.*, 40(5):1406–1430, 2002.
[13] C.W. Scherer. Relaxations for robust linear matrix inequality problems with verifications for exactness. *to appear in SIAM Journal on Matrix Analysis and Applications*, 2005.
[14] M. Schweighofer. Optimization of polynomials on compact semialgebraic sets. *to appear in SIAM Journal on Optimization* 2005.

A power-based description of mechanical systems

Jacquelien M.A. Scherpen
(joint work with Dimitri Jeltsema)

It is well-known that a large class of physical systems (e.g., mechanical, electrical, electro-mechanical, thermodynamical, etc.) admits, at least partially, a representation by the Euler-Lagrange or Hamiltonian equations of motion, see e.g. [1, 5, 6, 7] and the references therein. A key aspect of both sets of equations is that the energy storage in the system plays a central role.

For standard conservative mechanical systems with n degrees of freedom, and locally represented by n generalized displacement coordinates $q = \mathrm{col}(q_1, \ldots, q_n)$, the Euler-Lagrange equations of motion are given by[1]

$$\frac{d}{dt}\mathcal{L}_{\dot{q}}(q,\dot{q}) - \mathcal{L}_q(q,\dot{q}) = \tau, \tag{1}$$

where $\dot{q} = \mathrm{col}(\dot{q}_1, \ldots, \dot{q}_n)$ denote the generalized velocities, and $\mathcal{L}(q,\dot{q})$ represents the Lagrangian function, which is defined by the difference between the kinetic co-energy $\mathcal{T}^*(q,\dot{q}) = \frac{1}{2}\dot{q}^T M(q)\dot{q}$ and a potential energy function $\mathcal{V}(q)$, i.e., $\mathcal{L}(q,\dot{q}) = \mathcal{T}^*(q,\dot{q}) - \mathcal{V}(q)$. The positive definite symmetric $n \times n$ matrix $M(q)$ is a called the inertia (or generalized mass) matrix, Usually the forces τ are decomposed

[1]We use the notation that for any scalar function $\mathcal{J} : \mathbb{R}^n \to \mathbb{R}$, the gradient of $\mathcal{J}(x)$ is denoted by $\mathcal{J}_x(x) = \partial\mathcal{J}(x)/\partial x$, and the Hessian by $\mathcal{J}_{xx}(x) = \partial^2\mathcal{J}(x)/\partial x^2$.

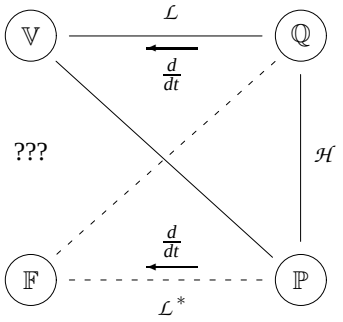

FIGURE 1. Mechanical configuration-space quadrangle: $\mathbb{Q}$, $\mathbb{P}$, $\mathbb{V}$ and $\mathbb{F}$ denote the spaces of the generalized displacements, momenta, velocities and forces, respectively; The solid and dashed diagonal lines represent Legendre transformations.

into dissipative forces and generalized external forces. Equations of the form (1) represent a force-balance.

The relation between the Euler-Lagrange equations and the Hamiltonian equations is classically established as follows. If we define the generalized momenta $p = \mathcal{L}_{\dot{q}}(q, \dot{q})$, with $p = \mathrm{col}(p_1, \ldots, p_n)$, then the equations of motion, as originally described by the set of second-order equations (1), can be described by a set of $2n$ first-order equations:

$$\begin{aligned} \dot{q} &= \mathcal{H}_p(q, p) \\ \dot{p} &= -\mathcal{H}_q(q, p) + \tau. \end{aligned} \tag{2}$$

Here $\mathcal{H}(q, p)$ is the Hamiltonian function, obtained by taking the Legendre transformation [1] of $\mathcal{L}(q, \dot{q})$, i.e., $\mathcal{H}(q, p) = \mathcal{T}(q, p) + \mathcal{V}(q)$, where $\mathcal{T}(q, p) = \frac{1}{2} p^T M^{-1}(q) p$ represents the kinetic energy. (Notice that $M^{-1}(q)$ is well defined since $M(q) > 0$ for all q by definition.) Equations of the form (2) are referred to as the Hamiltonian system equations of motion.

The relationship between (1) and (2) is graphically represented in the diagram shown in Figure 1 (solid lines). Denote the generalized displacement, velocity, momentum and force spaces by $\mathbb{Q}$, $\mathbb{V}$, $\mathbb{P}$ and $\mathbb{F}$, respectively. Figure 1 also suggests that there exists a dual form of the equations (1) in the sense that a mechanical system is expressed in terms of a set of generalized momenta and its time-derivatives, which represent a set of generalized forces. In [5], a description of the dynamics in $\mathbb{P}$ and $\mathbb{F}$ is called a co-Lagrangian system, where the Lagrangian function $\mathcal{L}(q, \dot{q})$ in (1) is replaced by its dual form $\mathcal{L}^*(p, \dot{p})$, which is defined as $\mathcal{L}^*(p, \dot{p}) = \mathcal{V}^*(\dot{p}) - \mathcal{T}(p, \dot{p})$, and the external forces are replaced by external velocities τ^*. Hence, in contrast to (1), the co-Lagrangian second-order equation set represents a velocity-balance equation. This co-Lagrangian formulation is less known but is sometimes used for solving special modeling problems, see e.g., [5].

So far we have considered three representations of the dynamics of a mechanical system. The underlying relationship between the three different sets of equations

is the existence of a (well-defined) Legendre transformation between $\mathbb{Q}$, $\mathbb{V}$, $\mathbb{P}$ and $\mathbb{F}$. Furthermore, the quadrangle depicted in Figure 1 also suggest a fourth equation set, represented by the question marks. Intuitively, at this point, one could be tempted to call a dynamic description on the spaces $\mathbb{V}$ and $\mathbb{F}$ the co-Hamiltonian equations of motion in terms of a co-Hamiltonian given by $\mathcal{H}^*(\dot{q},\dot{p}) = \mathcal{T}^*(\dot{q},\dot{p}) + \mathcal{V}^*(\dot{p})$. However, with $\mathcal{H}^*$ as "Hamiltonian" the Hamiltonian equations of motion do not correctly describe the dynamics.

It is our purpose to consider the left-hand side (represented by the question marks) of the quadrangle depicted in Figure 1. This fourth equation set can be given by the mechanical analogue to what is known in electrical circuit theory as the Brayton-Moser equations. These equations describe a very general class of nonlinear electrical circuits using a single scalar function called the mixed-potential function that has the units of power, the time derivative of the energy. These equations were first proposed in the early sixties in [2]. Although it is well-known that there is a standard analogy between simple mechanical and electrical systems, like e.g. the mass-inductor or the spring-capacitor analogy, with the corresponding velocity-current (resp. flux-momenta) or force-voltage (resp. displacement-charge) analogies, the existence of a well-defined analogy for more general mechanical systems is not straightforward. One of the main reasons for making such analogy difficult is the presence of the so-called Coriolis and centrifugal forces in the mechanical domain, which do not appear as such in the electrical domain. For that reason we can, in general, not equate the dynamics of a mechanical systems mutatis-mutandis along the lines of [2]. Another difficulty is that, in contrast to electrical circuits, mechanical systems are in general not nodical. Hence, a mechanical system can not always be considered as an interconnected graph.

Although there have been earlier attempts towards the formalization of a mechanical analogue of [2], e.g., [3], [4], here we present an analogue that is natural and general in the sense that if the Legendre transformation are well-defined, then we have a Brayton-Moser description for a mechanical system in coordinates of the spaces $\mathbb{V}$ and $\mathbb{F}$. If these Legendre transformations are not well-defined, we can still obtain a Brayton-Moser description where the mixed-potential function has the units of power, and the form of the equations is equal to the Brayton-Moser form, but the coordinates are still in the spaces $\mathbb{Q}$ and $\mathbb{P}$. Besides the motivation to have a description for the missing part of the quadrangle in case the Legendre transformations are well defined, the new power-based description may be used for controller design and stability analysis in a different manner than is usual. The basic results on which the description relies is given as follows:

Theorem Consider a standard conservative mechanical system described by the Hamiltonian equations (2). The dynamics of (2) can be expressed with $x = (q^T, p^T)^T$ as

$$Q^m(x)\dot{x} = \mathcal{P}_x^m(x), \quad \text{with}$$

$$Q^m(q,p) = \begin{bmatrix} \mathcal{V}_{qq}(q) + \frac{1}{2}\Big(p^T M^{-1}(q)p\Big)_{qq} & -\Big(p^T M^{-1}(q)\Big)_q \\ \Big(M^{-1}(q)p\Big)_q & -M^{-1}(q) \end{bmatrix},$$

and $\mathcal{P}^m(x)$ represents the mixed-potential (with units of power) of mechanical type:

$$\mathcal{P}^m(q,p) = \Big\langle \mathcal{V}_q(q), M^{-1}(q)p \Big\rangle + \frac{1}{2}\Big\langle \Big(p^T M^{-1}(q)p\Big)_q, M^{-1}(q)p \Big\rangle.$$

The proof uses a similar construction as in [2], and dissipation can be included easily. Now, power-based control techniques can be applied. Also extensions to more general (electro-)mechanical systems can be considered.

References

[1] R. Abraham and J. E. Marsden, *Foundations of Mechanics*, Benjamin / Cummings, 2nd edition, 1978.

[2] R.K. Brayton and J.K. Moser, *A theory of nonlinear networks I*, Quart. Appl. Math., **12(1)** (April and July 1964), 1–33.

[3] D. Jeltsema and J. M. A. Scherpen, *On mechanical mixed potential, content and co-content*, Proc. of the European Control Conference (ECC), Cambridge, England, (Sept. 2003), 73–78.

[4] A. Kugi, *Nonlinear Control Based on Physical Models*. Springer-Verlag, 2001.

[5] A.G.J. MacFarlane, *Dynamical System Models*, George G. Harrap & Co. Ltd., 1970.

[6] R. Ortega, A. Loría, P.J. Nicklasson, and H. Sira-Ramírez, *Passivity-Based Control of Euler-Lagrange Systems; Mechanical, Electrical and Electromechanical Applications*, Springer-Verlag, 1998.

[7] A.J. Van der Schaft, $\mathcal{L}_2$*-Gain and Passivity Techniques in Nonlinear Control*, Springer-Verlag, 2000.

Two Examples and some Remarks on Flatness Based Control

Kurt Schlacher

(joint work with Stefan Fuchshumer, Johann Holl)

Control theory and automatic control are important scientific fields, where engineering and mathematics meet each other. In this talk two industrial examples are presented, where a special mathematical property of the plant, the flatness of its mathematical model, is exploited for the system analysis, as well as for the tracking and disturbance rejection problem. The first example is a problem from nonlinear vehicle dynamics, where the planar bicycle model of a car is used for the investigations. The second describes the so-called chatter phenomenon in steel rolling mills, which can be explained by a Hopf bifurcation.

In the first example, the vehicle dynamics, an approach is discussed, which is essentially based on the observation that the dynamics of the planar holonomic bicycle model are differentially flat, with the flat output being associated with a clear physical meaning [4]. The contact between the tires and the road is modeled in terms of contact forces, which implies that the tires are enabled to slip and slide

on the road. The steering angle and the longitudinal tire forces are regarded as control inputs. The bicycle model emerges from the four-wheel vehicle by gluing together the front and the rear wheels to a single (mass-less) front and rear wheel, respectively, located at the longitudinal axis of the car. This planar model, known as a well-established basis for the design of vehicle dynamics control systems, see, e.g., [1], [2], is capable of rendering the longitudinal, lateral and yaw dynamics of the vehicle. The pitch and roll dynamics of a vehicle are clearly not involved in the scope of this model.

The components of a suitable flat output of the bicycle model are identified as the longitudinal and lateral component of the vehicle's velocity (v_x, v_y) of a distinguished point located on the longitudinal axis. The location of this point is determined in terms of the mass, the moment of inertia and the distance between the front wheel and the center of gravity, which can be regarded as well-known parameters in practical applications. Additionally, the representative of the flat output does not depend on the particular actuation of the vehicle, i.e., it holds for the rear-, front- and all-wheel driven bicycle equivalently. As a matter of particular interest, the system analysis does not refer to particular representatives for the functions describing the lateral tire forces, up to a certain family of functions which have to be excluded. However, these exclusions are shown not to imply any practically relevant restrictions.

The task of (real-time) trajectory planning amounts to mapping the inputs supplied by the driver, i.e., the current position of the throttle/brake pedal and the angle of the steering wheel, to suitable trajectories for the flat output. As a (long-term) future issue, informations gathered by scanning the environment, e.g., regarding the conditions of the road, or the position of a detected obstacle, might also be incorporated for the real-time shaping of the trajectories. Clearly, the trajectory shaping task inherently involves subjective sensations of the driver. A possible approach to the trajectory shaping task is given as follows: The position of the throttle/brake pedal is thought of as to reflect the driver's demands on the longitudinal dynamics, thus, this input (supplied at the current time) is used to define the desired trajectory for v_x for a future time horizon. Accordingly, the angle of the steering wheel may be regarded as the driver's demand on the lateral dynamics, so, this input might be used to adjust a desired trajectory for v_y.

Clearly, the performance of the proposed vehicle dynamics control depends on the accuracy of the tire model. Robustness issues and the identification of the tire parameters are addressed by current research.

The second example discusses the phenomenon of chatter, a vibration problem in steel cold rolling mills. In a multi-stand configuration of a rolling mill plant the strip deformation takes place in several consecutive mill stands under a considerable strip tension. Therefore, these multi-stand rolling mills represent a coupled system of individual mill stands where the dominant coupling is given by the rolling product. Moreover, mathematical models with different levels of complexity are arranged for simulation and controller design purposes, respectively. For simulation purposes a mill stand model based on a finite element analysis is used.

Whereas for the analysis of the mathematical multi-stand model with regard to the so-called third octave chatter phenomenon, as well as for a controller design, a reduced mill stand model is considered. As a result of the analysis it can be shown that the interaction of the roll force, the strip tensions, and the strip thicknesses allows an explanation of third octave chatter by a Hopf bifurcation and thus by an unstable equilibrium point of the mechanical system of an interconnected rolling mill stand. Without any change of the mill plant operating point, e.g. by a reduction of the processing speed, these vibrations would result in rejected products, damages of the mill plant, and consequently lead to lasting production delays.

In order to overcome this instability a so-called looper system is used to adjust the strip tension between two adjacent mill stands with high dynamics. For an active rejection approach of the third octave chatter phenomenon a flatness-based strip tension control of the multi-stand mill plant is considered where the looper system as well as the main mill drives are used as actuators for control. It is the crucial observation that this arrangement meets the property of differential flatness and moreover, the flat output is determined by the strip tension, the angular velocity of the main drives, and the strip elongation due to the main drives. The concept of differential flatness allows a systematic approach for the problem of trajectory planning, as well as of trajectory tracking, such that the bifurcation is avoided.

The obtained nonlinear control laws are commonly implemented on a digital processor in a quasi-continuous-time manner under the assumption that the sample time is sufficiently small. By considering two different sample times it is possible to derive a control sequence such that by means of the slow sample time the trajectories of the plant coincide with the trajectories of a desired closed loop system. Thus, one succeeds to implement these nonlinear control laws for the additional cost of solving nonlinear algebraic equations only. This approach is of advantage especially if the sampling time is low. Therefore, for active chatter rejection it does not mean an improvement since the frequency of chatter vibrations is typically greater than 100Hz.

Summarizing we may state that good tracking and disturbance rejection could be achieved in both examples by flatness based control. For the chatter rejection problem also passivity-based approaches give good results. In general a mathematical property like flatness alone is not sufficient for a successful solution of an engineering problem. The flat output of the presented examples has a clear physical meaning, furthermore it coincides with the output to be controlled. This fact allows us to plan trajectories in a simple and straightforward manner. The successful application of model based control to engineering problems has two needs. Good mathematical design methods, which are competitive with the industrial praxis, and precise mathematical models of the plants to be controlled such that the advantages of the mathematical approach are not neutralized by the lack of knowledge. If these prerequisites are met then a successfully theory based control system design is possible.

References

[1] J. Ackermann, T. Bünte, and D. Odenthal, *Advantages of Active Steering for Vehicle Dynamics Control*, in Proc. 32nd International Symposium on Automotive Technology and Automation (1999), Vienna, Austria, 263–270.

[2] T. Bünte, *Beiträge zur robusten Lenkregelung von Personenkraftwagen*, PhD thesis (1998), Technische Hochschule Aachen.

[3] M. Fliess, J. Lévine, P. Martin, and P. Rouchon, *Flatness and Defect of Non-linear Systems: Introductory Theory and Examples*, International Journal of Control **61(6)** (1995), 1327–1361.

[4] S. Fuchshumer, K. Schlacher, and T. Rittenschober, *Ein Beitrag zur nichtlinearen Fahrdynamikregelung: Die differentielle Flachheit des Einspurmodells*, accepted for publication in e&i Elektrotechnik und Informationstechnik (2005).

[5] J. Holl and K. Schlacher, *Analysis and Nonlinear Control of Implicit Discrete-time Dynamic Systems*, accepted for publication on 16th IFAC World Congress, Prague, Czech Republic (2005).

[6] J. Rudolph, *Flatness Based Control of Distributed Parameter Systems* (2003), Shaker, Aachen.

[7] K. Schlacher, S. Fuchshumer, G. Grabmair, J. Holl, and G. Keintzel, *Active Vibration Rejection in Steel Rolling Mills*, accepted for publication on 16th IFAC World Congress, Prague, Czech Republic (2005).

Control and Realization of Piecewise-Affine Hybrid Systems

Jan H. van Schuppen

Control problems for hybrid systems are motivated by the use of computers for control of engineering systems and by the appropriateness of discontinuous systems as models in engineering. Examples are automotive control, electric power networks, and chemical plants.

The choice of a subclass of hybrid systems for control is difficult. Most control problems for arbitrary nonlinear hybrid systems are undecidable. Therefore attention is limited to piecewise-affine hybrid systems. Even for this class the problems of characterizing controllability and observability are undecidable. Control theory therefore has to focus on sufficient conditions for existence of control laws for particular control problems.

The subclass of continuous piecewise-affine hybrid systems selected, consists at the discrete level of a finite-state automaton and at the continuous level, at each state of the automaton, of an affine system on a polytope. This subclass of hybrid systems is useful for many engineering control problems.

A major problem of control theory is to establish that the system is reachable and to derive a control law to go from an initial state to a final state. A sufficient condition for the existence and the computation of a control law will be stated. This leads to the problems: (1) control to an admissable exit facet, for an affine system on a polytope (2) control to a fixed point of an affine system on a polytope, and (3) to a reachability problem for a nondeterministic automaton. The approach will be illustrated for an academic example.

Due to the rapid automation of cars there is currently a considerable research effort for control problems for cars. One such problem is the control of a car engine when it is idle. Due to disturbances caused by the driver (switching on the radio or the air-conditioning, etc) the engine could stall. The control objective is thus to keep the speed in the car in a very narrow range even in case of disturbances. Control synthesis for idle speed control is based on the above formulated control law for reachability of piecewise-affine hybrid systems.

The realization problem is to obtain necessary and sufficient conditions for the existence, for the characterization of minimality, and a procedure to construct a minimal realization of piecewise-affine hybrid systems based on the relation between input-output functions. An overview of the results has been presented on the realization problem for switched linear systems.

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Guide to the references. Reachability and control synthesis of piecewise-affine hybrid systems [9, 10, 11, 12, 13, 18]. Observability of PAHS [4, 3, 8]. Realization of switched systems [14, 17, 15, 16]. Computability of hybrid systems [2, 6, 5, 7]. Idle speed control [1].

References

[1] Andrea Balluchi, Federico Di Natale, Alberto Sangiovanni-Vincenteli, and Jan H. van Schuppen. Synthesis for idle speed control of an automotive engine. In Rajeev Alur and George J. Pappas, editors, *Hybrid Systems: Computation and Control*, number 2993 in Lecture Notes in Computer Science, pages 80–94, Berlin, 2004. Springer.

[2] Pieter Collins. Continuity and computability of reachable sets. In *Proceedings of the International Symposium MTNS*, pages CD–ROM, Leuven, Belgium, 2004. Katholieke Universiteit Leuven.

[3] Pieter Collins and Jan H. van Schuppen. Observability of hybrid systems and turing machines. In *Proceeedings of the 43th IEEE Conference on Decision and Control*, New York, 2004. IEEE Press.

[4] Pieter Collins and Jan H. van Schuppen. Observability of piecewise-affine hybrid systems. In Rajeev Alur and George J. Pappas, editors, *Hybrid Systems: Computation and Control*, number 2993 in Lecture Notes in Computer Science, pages 265–279, Berlin, 2004. Springer.

[5] P.J. Collins. Computability of reachable sets. Preprint, CWI, Amsterdam, 2004.

[6] P.J. Collins. Continuity and computability of reachable sets. Report MAS-R0401, CWI, Amsterdam, 2004.

[7] P.J. Collins. Hybrid trajectory spaces. preprint, CWI, Amsterdam, 2004.

[8] P.J. Collins and J.H. van Schuppen. Observability of hybrid systems and Turing machines. Report MAS-E0413, CWI, Amsterdam, 2004.

[9] L.C.G.J.M. Habets and J.H. van Schuppen. Control of piecewise-linear hybrid system on simplices and rectangles. In M.D. Di Benedetto and A. Sangiovanni-Vincentelli, editors, *Hybrid Systems; Computation and Control, 4th Int. Workshop, Rome, Italy, March 2001*, pages 261–274, Berlin, 2001. Springer.

[10] L.C.G.J.M. Habets and J.H. van Schuppen. A controllability result for piecewise-linear hybrid systems. In *Proceedings of European Control Conference (ECC2001)*, pages 3870–3873, Porto, 2001.
[11] L.C.G.J.M. Habets and J.H. van Schuppen. A control problem for affine dynamical systems on a full-dimensional polytope. *Automatica*, 40:21–35, 2004.
[12] L.C.G.J.M. Habets and J.H. van Schuppen. Reachability of affine systems on polytopes in the plane. In *Proceedings International Symposium MTNS.2004 (CD-ROM)*, Leuven, 2004.
[13] L.C.G.J.M. Habets and J.H. van Schuppen. Reachability of affine systems on polytopes in the plane. Report MAS-E0423, CWI, Amsterdam, 2004.
[14] Luc C.G.J.M. Habets and Jan H. van Schuppen. Reduction of affine systems on polytopes. In *Proceedings of the International Symposium MTNS 2002*, 2002.
[15] M. Petreczky. Realization theory for linear switched systems. Report MAS-R0402, CWI, Amsterdam, 2004.
[16] M. Petreczky. Realization theory for linear switched systems: Formal power series approach. Report MAS-R0403, CWI, Amsterdam, 2004.
[17] Mihály Petreczky. Realization theory for linear switched systems. In *Proc. International Symposium MTNS.2004, Leuven, Belgium, July 2004*, 2004.
[18] J.H. van Schuppen. A sufficient condition for controllability of a class of hybrid systems. In T.A. Henzinger and S. Sastry, editors, *Hybrid systems: Computation and control*, number 1386 in Lecture Notes in Computer Science, pages 374–383, Berlin, 1998. Springer.

A very non-smooth maximum principle with state-space constraints

Héctor J. Sussmann

We consider a Bolza optimal control problem with state space constraints and a fixed time-interval, in which the objective is to minimize a cost functional $\varphi(x) + \int_a^b f_0(\xi(t), \eta(t), t)\, dt$ among all $(\xi, \eta) \in \mathcal{A}$, where $\mathcal{A}$ is the set of all pairs (ξ, η) such that (i) η is a map from $[a, b]$ to U, (ii) $\eta \in \mathcal{U}$, (iii) ξ is an absolutely continuous map from $[a, b]$ to $\mathbb{R}^n$ such that $\dot{\xi}(t) = f(\xi(t), \eta(t), t)$ for a.e. $t \in [a, b]$, (iv) if we let $\psi(t) = f_0(\xi(t), \eta(t), t)$, then ψ is measurable and the function $[a, b] \ni t \mapsto \max(0, -\psi(t)) \in \mathbb{R}$ is integrable, and (v) the curve ξ satisfies the endpoint constraints $\xi(a) = \bar{x}$, $\xi(b) \in S$, and $h_j(\xi(b)) = 0$ for $j \in \{1, \dots, \tilde{m}\}$, as well as the state space constraints $g_i(\xi(t), t) \leq 0$ for all $t \in [a, b]$, and all $i \in \{1, \dots, m\}$.

Here the prescribed data 14-tuple $\mathcal{D} = (n, m, \tilde{m}, U, a, b, \varphi, f_0, f, \bar{x}, g, h, S, \mathcal{U})$ is such that (using "ppd" for "possibly partially defined," and writing "$\theta : A \hookrightarrow B$" for "$\theta$ is a ppd map from A to B") (1) n, m and $\tilde{m}$ are nonnegative integers, (2) U is a set, (3) $a, b \in \mathbb{R}$ and $a < b$, (4) $f = (f_1, \dots, f_n)^\dagger : \mathbb{R}^n \times U \times \mathbb{R} \hookrightarrow \mathbb{R}^n$, so f is a column of n ppd functions $f_i : \mathbb{R}^n \times U \times \mathbb{R} \hookrightarrow \mathbb{R}$, (5) $f_0 : \mathbb{R}^n \times U \times \mathbb{R} \hookrightarrow \mathbb{R}$, (6) $\bar{x} \in \mathbb{R}^n$, (7) $g = (g_1, \dots, g_m)$ is an m-tuple of ppd functions $g_i : \mathbb{R}^n \times \mathbb{R} \hookrightarrow \mathbb{R}$, (8) $h = (h_1, \dots, h_{\tilde{m}})$ is an $\tilde{m}$-tuple of ppd functions $h_j : \mathbb{R}^n \hookrightarrow \mathbb{R}$, (9) φ is a ppd function from $\mathbb{R}^n$ to $\mathbb{R}$, (10) $S \subseteq \mathbb{R}^n$, and (11) $\mathcal{U}$ is a set of maps from $[a, b]$ to U.

We assume that, in addition to $\mathcal{D}$, we are given a reference TCP ("trajectory-control pair") $(\xi_*, \eta_*) \in \mathcal{A}$. Furthermore, we make some technical assumptions. In order to state these assumptions, we let $\hat{\mathcal{U}}$ be the set of all maps from $[a, b]$ to U and, for $\eta \in \hat{\mathcal{U}}$, $\delta > 0$, we write $f^{\eta,\delta}$, $f_0^{\eta,\delta}$, $\mathbf{f}^{\eta,\delta}$, to denote, respectively, the maps $\mathcal{T}^n(\xi_*, \delta) \ni (x, t) \mapsto f(x, \eta(t), t) \in \mathbb{R}^n$, $\mathcal{T}^n(\xi_*, \delta) \ni (x, t) \mapsto f_0(x, \eta(t), t) \in \mathbb{R}$, and

$\mathcal{T}^n(\xi_*,\delta) \ni (x,t) \mapsto (f_0^\eta(x,t), f^\eta(x,t)) \in \mathbb{R}\times\mathbb{R}^n$, where $\mathcal{T}^n(\xi_*,\delta)$ is the tube given by $\mathcal{T}^n(\xi_*,\delta) = \{(x,t)\in\mathbb{R}^n\times[a,b] : \|x-\xi_*(t)\|\le\delta\}$. We use $\mathcal{U}_c$ to denote the set of all $\eta\in\hat{\mathcal{U}}$ that are constant, and write $\mathcal{U}_{c,*} = \mathcal{U}_c\cup\{\eta_*\}$. If $a\le\alpha\le\beta\le b$ and $k\in L^1([a,b],[0,+\infty])$, we let $\mathrm{ARC}_k([\alpha,\beta],\mathbb{R}^n)$ be the space of all absolutely continuous maps $\zeta:[\alpha,\beta]\mapsto\mathbb{R}^n$ such that $\|\dot\zeta(t)\|\le k(t)$ for almost all $t\in[\alpha,\beta]$, and we define $\mathrm{ARC}_k([[a,b]],\mathbb{R}^n) = \bigcup\Big\{\mathrm{ARC}_k([\alpha,\beta],\mathbb{R}^n) : a\le\alpha\le\beta\le b\Big\}$. We then define a metric $d_{a,b,n;k}$ on $\mathrm{ARC}_k([[a,b]],\mathbb{R}^n)$ by letting the distance $d_{a,b,n;k}(\zeta_1,\zeta_2)$ be equal to $|\alpha_1-\alpha_2|+|\beta_1-\beta_2|+\max\{\|\zeta_1^{ext}(t)-\zeta_2^{ext}(t)\| : t\in\mathbb{R}\}$ if $\zeta_i\in\mathrm{ARC}_k([\alpha_i,\beta_i],\mathbb{R}^n)$ for $i=1,2$, where ζ_i^{ext} is the continuous extension of ζ_i to $\mathbb{R}$ which is constant on $]-\infty,\alpha_i]$ and on $[\beta_i,+\infty[$. We let $ARC_k(\mathcal{T}^n(\xi_*,\delta))$ be the set of $\zeta\in\mathrm{ARC}_k([[a,b]],\mathbb{R}^n)$ such that $(\zeta(t),t)\in\mathcal{T}^n(\xi_*,\delta)$ for all $t\in\mathrm{Do}(\zeta)$.

Our first three technical hypotheses are then as follows.

(H1) For every $\eta\in\mathcal{U}_{c,*}$ there exist $\delta_\eta\in]0,+\infty[$, $k_\eta\in L^1([a,b],[0,+\infty])$ such that, if we let $f^\eta = f^{\eta,\delta_\eta}$, $f_0^\eta = f_0^{\eta,\delta_\eta}$, $\mathbf{f}^\eta = \mathbf{f}^{\eta,\delta_\eta}$, then
  (H1.a) for every $(x,t)\in\mathcal{T}^n(\xi_*,\delta_\eta)$, $\mathbf{f}^\eta(x,t)$ is defined and satisfies the bounds $\|f^\eta(x,t)\|\le k_\eta(t)$ and $f_0^\eta(x,t)\ge -k_\eta(t)$,
  (H1.b) the maps f^η and f_0^η are Borel⊗Lebesgue measurable,
  (H1.c) the maps f^η and f_0^η are, respectively, integrally continuous and integrally lower semicontinuous on $ARC_{k_\eta}(\mathcal{T}^n(\xi_*,\delta_\eta))$. (This means that, if $\{\xi_j\}_{j\in\mathbb{N}}$ is a sequence in $ARC_{k_\eta}(\mathcal{T}^n(\xi_*,\delta_\eta))$ that converges in $ARC_{k_\eta}(\mathcal{T}^n(\xi_*,\delta_\eta))$ to a limit ξ_∞, and $\xi_j\in ARC_{k_\eta}([\alpha_j,\beta_j],\mathbb{R}^n)$ for $j\in\mathbb{N}\cup\{\infty\}$, then $\lim_{j\to\infty}\int_{\alpha_j}^{\beta_j} f^\eta(\xi_j(t),t)\,dt = \int_{\alpha_\infty}^{\beta_\infty} f^\eta(\xi_\infty(t),t)\,dt$ and $\liminf_{j\to\infty}\int_{\alpha_j}^{\beta_j} f_0^\eta(\xi_j(t),t)\,dt \ge \int_{\alpha_\infty}^{\beta_\infty} f_0^\eta(\xi_\infty(t),t)\,dt$.)

(H2) There exist a positive number δ_g such that, on the tube $\mathcal{T}^n(\xi_*,\delta_g)$, for each $i\in\{1,\dots,m\}$, (a) $g_i(x,t)$ is defined for all (x,t), (b) the function $t\mapsto g_i(x,t)$ is upper semicontinuous for every $x\in\mathbb{R}^n$, and (c) the function $x\mapsto g_i(x,t)$ is continuous for each $t\in[a,b]$, uniformly with respect to t.

(H3) The class $\mathcal{U}$ is a "variational neighborhood" of η_*, in the following sense: if N is an arbitrary positive integer, and $\mathbf{u}=(u_1,\dots,u_N)$ is an arbitrary N-tuple of members of U, then there exists a positive number $\varepsilon=\varepsilon(N,\mathbf{u})$ such that whenever η is a function obtained from η_* by selecting an N-tuple $(I_1,\dots,I_N)$ of pairwise disjoint subintervals of $[a,b]$ such that the sum of the measures of the I_j is $\le\varepsilon$, and substituting the constant value u_j for the value $\eta_*(t)$ for every $t\in I_j$, $j=1,\dots,N$, it follows that $\eta\in\mathcal{U}$.

Finally, we make technical assumptions on five new objects called Λ^F, Λ^G, Λ^h, Λ^φ, and C. The conditions on Λ^h and Λ^φ involve the concept of "generalized differential quotient," defined, for example, in [1], [2], and [4], and say that

(H4) Λ^h is a generalized differential quotient of h at $(\xi_*(b),h(\xi_*(b)))$ in the direction of $\mathbb{R}^n$, and Λ^φ is a generalized differential quotient of $\check\varphi$ at $(\xi_*(b),\varphi(\xi_*(b)))$ in the direction of $\mathbb{R}^n$;

(Here $\check{\varphi}$ is the epimap of φ, i.e., the set-valued map from $\mathbb{R}^n$ to $\mathbb{R}$ given by $\check{\varphi}(x) = [\varphi(x), +\infty[$ if $\varphi(x)$ is defined, and $\check{\varphi}(x) = \emptyset$ if $\varphi(x)$ is not defined.) Regarding C, we assume that

(H5) C is a Boltyanskii approximating cone of S at $\xi_*(b)$.

(This means that there exist a natural number ν, a closed convex cone D in $\mathbb{R}^\nu$, a neighborhood N of 0 in $\mathbb{R}^\nu$, a continuous map $\Phi : N \cap D \mapsto S$, and a linear map $L : \mathbb{R}^\nu \mapsto \mathbb{R}^n$, such that $L(D) = C$ and $\Phi(v) = \xi_*(b) + L \cdot v + o(\|v\|)$ as $v \to_D 0$.)

The conditions for Λ^F and Λ^G are more complicated. They involve the set-valued maps $F : \mathbb{R}^n \times [a,b] \mapsto\!\!\!\!\to \mathbb{R} \times \mathbb{R}^n$, $G : \mathbb{R}^n \times [a,b] \mapsto\!\!\!\!\to \mathbb{R}^m$, given by the formulas $F(x,t) = \check{f}_0^{\eta_*}(x,t) \times \{f^{\eta_*}(x,t)\}$, $G(x,t) = G_1(x,t) \times \cdots \times G_m(x,t)$, where $G_i(x,t) = [0, +\infty[$ if $g_i(x,t) > 0$, and $G_i(x,t) = \mathbb{R}$ if $g_i(x,t) \leq 0$. Furthermore, the condition for Λ^G also involves the compact subsets K_i of $[a,b]$ defined by

$$K_i = \{t \in [a,b] : (\forall \delta > 0)(\exists (x,s) \in \mathbb{R}^n \times [a,b])(\|x - \xi_*(t)\| + |t - s| \leq \delta \wedge g_i(x,s) > 0\}\,.$$

The conditions say, essentially, that Λ^F and Λ^G are set-valued maps whose values, for each t, are convex generalized differential quotients of $F(\cdot,t)$ and $G(\cdot,t)$ at $\xi_*(t)$. However, we need to require a certain amount of uniformity, and then the precise statements of the conditions turn out to be as follows.

(H6) Λ^F is a measurable set-valued map on $[a,b]$ with values nonempty compact convex subsets of $\mathbb{R}^n \times \mathbb{R}^{n \times n}$, such that there exists an integrable function $\kappa^{\Lambda^F} : [a,b] \mapsto [0,\infty]$, a positive number δ^{Λ^F}, and a family $\{\kappa_\delta\}_{0 < \delta \leq \delta^{\Lambda^F}}$ of nonnegative integrable functions on $[a,b]$, having the property that $\lim_{\delta \downarrow 0} \int_a^b \kappa_\delta(t)\, dt = 0$, and such that for every $\delta \in]0, \delta^{\Lambda^F}]$ and every $(h,t) \in \mathbb{R}^n \times [a,b]$ for which $\|h\| \leq \delta$ there exist a $y \in F(\xi_*(t) + h, t)$ and an $(L_0, L) \in \Lambda^F(t)$ such that $\|y - \sigma^F(t) - (\langle L_0, h\rangle, L \cdot h)\| \leq \delta \kappa_\delta(t)$. (Here $\sigma^F(t) = (f_0^{\eta_*}(\xi_*(t), t), f^{\eta_*}(\xi_*(t), t))$.)

(H7) Λ^G is an m-tuple $(\Lambda^{G_1}, \ldots, \Lambda^{G_m})$ such that, for each i, Λ^{G_i} is an upper semicontinuous set-valued map from K_i to $\mathbb{R}^n$ with nonempty compact convex values, such that whenever $\{(h_j, t_j)\}_{j \in \mathbb{N}}$ is a sequence of points of $(\mathbb{R}^n \backslash \{0\}) \times [a,b]$ such that $\lim_{j \to \infty} t_j = t$ for some $t \in K_i$ and $\lim_{j \to \infty} h_j = 0$, it follows that there exist $r_j \in G_i(\xi_*(t_j) + h_j, t_j)$ and $\omega_j \in \Lambda^{G_j}(t)$ such that $\lim_{j \to \infty} (\|h_j\|^{-1} |r_j - \langle \omega_j, h_j \rangle|) = 0$.

Theorem. Assume that $\mathcal{D}$, ξ_*, η_*, Λ^F, Λ^G, Λ^h, Λ^φ, C satisfy Hypotheses (H1) to (H7). Let $I = \{i \in \{1, \ldots, m\} : K_i \neq \emptyset\}$. Then there exist (1) a vector $\bar{\pi}$ belonging to the negative of the polar cone of C (i.e., such that $\langle \bar{\pi}, c \rangle \geq 0$ for all $c \in C$), (2) a nonnegative real number π_0, (3) an $\tilde{m}$-tuple $(\lambda_1, \ldots, \lambda_{\tilde{m}})$ of real numbers, (4) measurable selections $[a,b] \ni t \mapsto (L_0(t), L(t))$ of Λ^F, and $K_i \ni t \mapsto \gamma^i(t)$ of Λ^{G_i} for $i \in I$, (5) a member $L^h = (L_1^h, \ldots, L_{\tilde{m}}^h) \in (\mathbb{R}^n)^{\tilde{m}}$ of $\Lambda^{\mathbf{h}}$, (6) a member L^φ of Λ^φ, and (7) finite nonnegative Borel measures μ_i on $[a,b]$, such that $\mathrm{support}(\mu_i) \subseteq K_i$, having the property that, if we let $\pi : [a,b] \mapsto \mathbb{R}^n$ be

the unique solution of the adjoint equation

$$\pi(t) = \bar{\pi} - \sum_{j=1}^{\tilde{m}} \lambda_j^2 L_j^h + \int_t^b \pi(s) \cdot L(s) ds - \pi_0 \int_t^b L_0(s) ds - \pi_0 L^\varphi - \sum_{i \in I} \int_{[t,b]} \gamma^i(s) d\mu_i(s),$$

then (I) $H_{\pi_0}(\xi_*(\bar{t}), \eta_*(\bar{t}), \pi(\bar{t})) \geq H_{\pi_0}(\xi_*(\bar{t}), u, \pi(\bar{t}))$ whenever $u \in U$, $\bar{t} \in [a,b]$ are such that $(\xi_*(\bar{t}), \bar{t})$ is a point of approximate continuity of both augmented vector fields (f_0^u, f^u) and $(f_0^{\eta_*}, f^{\eta_*})$, and $\bar{t}$ is not an atom of any of the μ_i, and (II) $\|\bar{\pi}\| + \pi_0 + \sum_{j=1}^{\tilde{m}} |\lambda_j| + \sum_{i \in I} \|\mu_i\| > 0$. (The notion of "point of approximate continuity" is defined in [3]. The *Hamiltonian* $H_\alpha : \mathbb{R}^n \times U \times \mathbb{R}^n \times \mathbb{R} \hookrightarrow \mathbb{R}$ is defined by $H_\alpha(x,u,p,t) = \langle p, f(x,u,t) \rangle - \alpha f_0(x,u,t)$.)

References

[1] H. J. Sussmann, *Résultats récents sur les courbes optimales.* In "15 ^{e} Journée Annuelle de la Société Mathémathique de France (SMF)," Publications de la SMF, Paris, 2000, 1-52.

[2] H. J. Sussmann, *New theories of set-valued differentials and new versions of the maximum principle of optimal control theory.* In "Nonlinear Control in the year 2000," A. Isidori, F. Lamnabhi-Lagarrigue and W. Respondek Eds., Springer-Verlag, London, 2000, 487-526.

[3] H. J. Sussmann, *Needle variations and almost lower semicontinuous differential inclusions.* Set-valued analysis **10** (2002), 233-285.

[4] H. J. Sussmann, *Set-valued differentials and the hybrid maximum principle.* In "Proc. 39th IEEE Conf. Decision and Control, Sydney, Australia, December 12-15, 2000," IEEE publications, New York, 2000.

Observability of Nonlinear Systems

Bernd Tibken

In this contribution the observability of systems given by the state space representation

$$\dot{x}(t) = f(x(t)) \ , \ x(0) = x_0 \ , \tag{1}$$

$$y(t) = h(x(t)) \tag{2}$$

where $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$ are state vector and output vector, respectively, is investigated. The functions f and h are assumed to be real analytic and it follows immediately that $y(t)$ is an analytic function of t with Taylor series given by the Lie series

$$y(t) = \sum_{k \geq 0} \left(L_f^k \, h \right)(x_0) \tag{3}$$

where the Lie derivative $L_f h$ defined by

$$L_f \, h(x) = \sum_{i=1}^{n} \frac{\partial h}{\partial x_i} f_i(x) \ , \ L_f^0 h = h \ , \ L_f^{k+1} = L_f \left(L_f^k \, h \right) \tag{4}$$

has been used. Two points x_0^1, x_0^2 in state space can be distiguished by the output if the corresponding outputs are distinct. If all pairs can be distinguished the

system is called observable. In order to formalize this we define the observability mapping [1, 2]

$$F(x) = \begin{pmatrix} h(x) \\ L_f\, h(x) \\ \vdots \\ L_f^k\, h(x) \\ \vdots \end{pmatrix} \tag{5}$$

and two points can be distinguished if they satisfy $F(x_0^1) \neq F(x_0^2)$. This condition can be translated into a geometric condition if we introduce the following sets in $\mathbb{R}^{2n}$

$$G = \left\{ \begin{pmatrix} x \\ z \end{pmatrix} \in \mathbb{R}^{2n} \mid F(x) = F(z) \right\}, \tag{6}$$

$$D = \left\{ \begin{pmatrix} x \\ z \end{pmatrix} \in \mathbb{R}^{2n} \mid x = z \right\} \tag{7}$$

from which it is obvious that the system is observable in $G = D$ holds. Unfortunately this geometric condition can not be tested in general. In the following we will specialize f and h in order to derive conditions which can be tested via computer algebra systems.
As a first simplification we assume that f and h are polynomial functions of the state vector. It immediately follows that the components of F are polynomial functions also, thus, we will employ the machinery of algebraic geometry [3] in order to find conditions for $G = D$. In order to use the full power of algebraic geometry it is necessary to allow all variables to be complex. Accordingly we define

$$G_{\mathbb{C}} = \left\{ \begin{pmatrix} x \\ z \end{pmatrix} \in \mathbb{C}^{2n} \mid F(x) = F(z) \right\}, \tag{8}$$

$$D_{\mathbb{C}} = \left\{ \begin{pmatrix} x \\ z \end{pmatrix} \in \mathbb{C}^{2n} \mid x = z \right\} \tag{9}$$

and it is clear that we have $G = G_{\mathbb{C}} \cap \mathbb{R}^{2n}$ and $D = D_{\mathbb{C}} \cap \mathbb{R}^{2n}$. Now we represent these sets via the algebraic varieties associated to the ideals generated by the polynomials defining the sets. In the following the notation $< g_1, g_2, \ldots, g_m >$ denotes the ideal generated by the polynomials $g_i, i = 1, \ldots m$. The ideal most important ideal is given by

$$\mathcal{G} =< F_1(x) - F_1(z), F_2(x) - F_2(z), \ldots > \tag{10}$$

and it is important to note that due to the fact that the ring of polynomials is noetherian this ideal is finitely generated. The second ideal which is important is given by

$$\mathcal{D} =< x_1 - z_1, x_2 - z_2, \ldots, x_n - z_n > \tag{11}$$

and we have the representation

$$G_{\mathbb{C}} = V(\mathcal{G}) , \tag{12}$$

$$D_{\mathbb{C}} = V(\mathcal{D}) \tag{13}$$

and the observability condition is given by

$$\sqrt{\mathcal{G}} = \mathcal{D} \tag{14}$$

or equivalently using the ideal quotient

$$\sqrt{\mathcal{G}} : \mathcal{D} =< 1 > \tag{15}$$

which is a very strong condition which ensures the observability for the complexification. In the real case in which we are primary interested we can give a relaxed condition which is given as

$$V(\sqrt{\mathcal{G}} : \mathcal{D}) \cap \mathbb{R}^{2n} \subseteq D \tag{16}$$

which can be checked in principle because a Gröbner basis for the ideal

$$\mathcal{Q} = \sqrt{\mathcal{G}} : \mathcal{D} \tag{17}$$

can be computed using computer algebra systems like SINGULAR.

References

[1] J. Baillieul, *Controllability and observability of polynomial dynamical systems*, Nonlinear Analysis, Theory, Methods and Applications **5** (1981), 543–552.
[2] X. Xia, and M. Zeitz, *On nonlinear continuous observers*, International Journal of Control **66** (1997), 943–954.
[3] D. Cox, J. Little, and D. O'Shea, *Ideals, Varieties and Algorithms*, Springer (1992), Berlin.

A behavioral approach to the estimation problem and its applications to state-space models

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(joint work with Mauro Bisiacco, Università di Padova, Padova, Italy)

The original theory of state observers was concerned with the problem of reconstructing (equivalently, estimating) the state from the corresponding inputs and outputs. This problem has been later generalized in various ways, and in relatively recent years there has been a great deal of research aiming at designing state observers (also for nonlinear state-space models) in the presence of unknown inputs (disturbances).

Another research issue, which originated in the eighties and flourished in the nineties, but still represents a very lively research topic, is the fault detection and isolation (FDI) problem. The problem of detecting and identifying the faults affecting the system functioning, possibly in the presence of disturbances, may be naturally seen as an estimation problem.

The last few years have witnessed a renewed interest for these issues: estimation problems and observer synthesis (in a deterministic context) have been investigated for wider classes of dynamic systems, described, for instance, in a behavioral setting or by means of polynomial/rational models, thus enlightening interesting connections between the problem solutions obtained via different approaches [1, 3, 4].

The aim of this talk is that of fully addressing the observer design problem for linear time-invariant (discrete-time) dynamic systems, defined on $\mathbb{Z}_+$, that are described in behavioral terms by means of a set of difference equations. The results of this analysis can be then exploited for providing very compact and elegant solutions to all the aforementioned classical problems, posed for state-space models.

The present results extend those recently appeared in [4] for the continuous time case and in [3] for discrete time systems.

Specifically, we consider a dynamic system $\Sigma = (\mathbb{Z}_+, \mathbb{R}^{\mathtt{w}}, \mathfrak{B})$, with $\mathbb{Z}_+$ the time axis, $\mathbb{R}^{\mathtt{w}}$ the signal alphabet and $\mathfrak{B} \subset (\mathbb{R}^{\mathtt{w}})^{\mathbb{Z}_+}$ the system behavior. Independently of the physical meaning of the system variables which are grouped together in the vector $\mathbf{w}$, when dealing with any type of estimation problem a first natural distinction is introduced between measured variables, denoted by $\mathbf{w}_m$, and unmeasured variables. These latter, in turn, may be naturally split into the subvector of all system variables which are (unmeasured and) the object of our estimation problem (the "relevant" variables for the specific estimation problem), $\mathbf{w}_r$, and the subvector of all variables which are both unmeasured (for instance because they represent disturbances or modeling errors) and "irrelevant" for our estimation problem. We refer to such a subvector as $\mathbf{w}_i$. As a consequence, the vector $\mathbf{w}$ is naturally split into three subvectors, i.e., $\mathbf{w}^T = \begin{bmatrix} \mathbf{w}_r^T & \mathbf{w}_m^T & \mathbf{w}_i^T \end{bmatrix}^T$.

Accordingly, the behavior trajectories in $\mathfrak{B}$ satisfy a difference equation of the following type

$$\begin{bmatrix} R_r(\sigma) & -R_m(\sigma) & -R_i(\sigma) \end{bmatrix} \begin{bmatrix} \mathbf{w}_r(t) \\ \mathbf{w}_m(t) \\ \mathbf{w}_i(t) \end{bmatrix} = 0, \quad t \in \mathbb{Z}_+,$$

for suitable polynomial matrices R_r, R_m, R_i in the left shift operator σ. The natural goal is that of designing an estimator of $\mathbf{w}_r$ based on the knowledge of $\mathbf{w}_m$, such that its estimation error goes to zero in a finite number of steps, independently of $\mathbf{w}_i$.

To this end, we first introduce the concepts of reconstructibility and observability of the "relevant" variable $\mathbf{w}_r$ from the "measured" variable $\mathbf{w}_m$, and later the definition of dead-beat observer (or of consistent dead-beat observer) of $\mathbf{w}_r$ from $\mathbf{w}_m$, in the presence of the unmeasured (and irrelevant) variables $\mathbf{w}_i$. Necessary and sufficient conditions for the existence of such (consistent) dead-beat observers, are introduced, and a complete parametrization of all dead-beat observers is given.

Equivalent conditions for the existence of causal dead-beat observers, by this meaning dead-beat observers endowed with a proper rational transfer matrix and hence realizable by means of a causal discrete-time state-space model, are also derived.

Finally, several classical problems addressed for state-space models, like standard state estimation, the design of unknown input observers or the design of fault detectors and identifiers (possibly in the presence of disturbances), are casted in this general framework, by accordingly partitioning all system variables (state, input, output, disturbance and fault) into the three disjoint subvectors $\mathbf{w}_r, \mathbf{w}_m$ and $\mathbf{w}_i$. As a result, the aforementioned equivalent conditions and parametrizations provide specific answers to all these special instances.

References

[1] P.A. Fuhrmann. On observers and behaviors. In *Proceedings Fifteenth International Symposium on Mathematical Theory of Networks and Systems, Ed. D.S. Gilliam, J. Rosenthal*, pages 1–7, Univ. Notre Dame, Notre Dame, IL, USA, 2002.

[2] M.E. Valcher. On some special features which are peculiar of discrete-time behaviors with trajectories on $\mathbf{Z}_+$. *Linear Algebra and its Appl.*, 351-352:719–737, 2002.

[3] M.E. Valcher and J.C. Willems. Dead beat observer synthesis. *Systems & Control Letters*, 37:285–292, 1999.

[4] M.E. Valcher and J.C. Willems. Observer synthesis in the behavioral approach. *IEEE Trans. on Aut. Contr.*, 44, no.12:2297–2307, 1999.

ARMAX System Identification: First X, then AR, finally MA

Jan C. Willems

(joint work with Ivan Markovsky, Bart L.M. De Moor)

In this extended abstract, 'process' means: a zero mean, gaussian, stationary, ergodic vector process on $\mathbb{Z}$, $\perp$ means 'independence', and 'white noise' means a process ε for which the $\sigma^t\varepsilon(0)$'s are all $\perp$ for $t \in \mathbb{Z}$, and σ denotes the shift ($\sigma f(t) := f(t+1)$). Consider the difference equation

$$\text{(ARMAX)} \qquad W(\sigma)w = E(\sigma)\varepsilon,$$

with W, E suitably sized polynomial matrices. The *behavior* of (ARMAX) consists of all processes w such that (ARMAX) holds for some white noise process ε. The identification (ID) problem is to obtain estimates of (W, E) from observation of a realization of w:

$$\tilde{w}(1), \tilde{w}(2), \ldots, \tilde{w}(T).$$

In this extended abstract, we will assume for simplicity of exposition that $T = \infty$. In the actual algorithm, we assume T finite, and study the behavior of the estimates as $T \to \infty$.

Every ARMAX system admits a more refined representation

$$\text{(AR-MA-X)} \qquad A(\sigma)R(\sigma)w = M(\sigma)\varepsilon$$

with A square, $\det(A)$ non-zero and without unit circle roots, and R left-prime. Note that $R(\sigma)w = 0$ corresponds to the 'exogenous' part of the AR-MA-X system (obtained by setting $\varepsilon = 0$). We call R the 'X' (exogenous) part, A the 'AR' part, and M the 'MA' part of the AR-MA-X system. We present an algorithm that identifies first R, then A, and finally M.

Many interesting problems emerge: When do two systems (A, R, M) define the same behavior? Obtain canonical forms. If $w = \left[\begin{smallmatrix} u \\ y \end{smallmatrix}\right]$, when is u a 'free input', in the sense that for any process u, there exists a process y such that $w = \left[\begin{smallmatrix} u \\ y \end{smallmatrix}\right]$ belongs to the behavior of (AR-MA-X)? When is this y unique? In [1] these issues are studied in depth.

It is easy to see that for all $n \in \mathbb{R}[\xi]$ in the $\mathbb{R}[\xi]$-module generated by the transposes of the rows of R, $n(\sigma)^\top w \perp \varepsilon$. Assume that $R = [\ P\ Q\ ]$ with P square, and correspondingly $w = \left[\begin{smallmatrix} u \\ y \end{smallmatrix}\right]$, with $u \perp \varepsilon$. Now look for the finite linear combinations of the rows of the observed

$$\tilde{W} = \begin{bmatrix} \tilde{w}(1) & \tilde{w}(2) & \tilde{w}(3) & \cdots & \tilde{w}(t) & \cdots \\ \tilde{w}(2) & \tilde{w}(3) & \tilde{w}(4) & \cdots & \tilde{w}(t+1) & \cdots \\ \tilde{w}(3) & \tilde{w}(4) & \tilde{w}(5) & \cdots & \tilde{w}(t+2) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

that are orthogonal to the rows of the observed

$$\tilde{U} = \begin{bmatrix} \tilde{u}(1) & \tilde{u}(2) & \tilde{u}(3) & \cdots & \tilde{u}(t) & \cdots \\ \tilde{u}(2) & \tilde{u}(3) & \tilde{u}(4) & \cdots & \tilde{u}(t+1) & \cdots \\ \tilde{u}(3) & \tilde{u}(4) & \tilde{u}(5) & \cdots & \tilde{u}(t+2) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}.$$

Call these linear combinations 'orthogonalizers'. Obviously each orthogonalizer is a vector of the form $\pi = \mathrm{col}(\pi_0, \pi_1, \cdots, \pi_{\mathtt{n}}, \cdots)$, with the $\pi_{\mathtt{n}}$'s $\in \mathbb{R}^{\mathtt{w}}$, and all but a finite number of them non-zero. Organize the orthogonalizers as polynomial vectors $\pi(\xi) = \pi_0 + \pi_1\xi + \cdots + \pi_{\mathtt{n}}\xi^{\mathtt{n}} + \cdots \in \mathbb{R}^{\mathtt{w}}[\xi]$.

It can be shown that if $\tilde{u}$ is persistently exciting, then the orthogonalizers form exactly the $\mathbb{R}[\xi]$-module generated by the transposes of the rows of R. This yields an algorithm for identifying R from the observations via the orthogonalizers. As we have described it here, this algorithm requires an infinite number of rows of $\tilde{W}$ and $\tilde{U}$, but if we assume that (upper bounds for) the lag L and the dynamic order n of the AR-MA-X system are known, we can restrict attention to the first L rows of $\tilde{W}$ and the first $L + n$ rows of $\tilde{U}$.

Once R has been estimated, we compute

$$\tilde{a} = \hat{R}(\sigma)\tilde{w},$$

and obtain an estimate $\hat{A}$ of A from $\tilde{a}$, and proceed by computing

$$\tilde{m} = \hat{A}(\sigma)\tilde{a},$$

to obtain an estimate $\hat{M}$ of M, leading to an estimate $(\hat{R}, \hat{A}, \hat{M})$ for (R, A, M).

This extended abstract reports on research in progress. A full paper is in preparation.

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References

[1] E.J. Hannan and M. Deistler, *The Statistical Theory of Linear Systems*, Academic Press, 1979.

Internet congestion control and linear inclusions

Fabian Wirth
(joint work with Douglas Leith, Robert Shorten, Rade Stanojević)

We consider a communication network in which n sources send data through a bottleneck router. The sources are assumed to employ an additive-increase-multiplicative-decrease (AIMD) algorithm, as it is used in current transfer control protocol (TCP) implementations.

Under some mild assumptions, the evolution of the window size of each source, i.e. the number of unacknowledged packets in flight, is approximately of the form shown in Figure 1, if time is measured in round-trip-time (RTT).

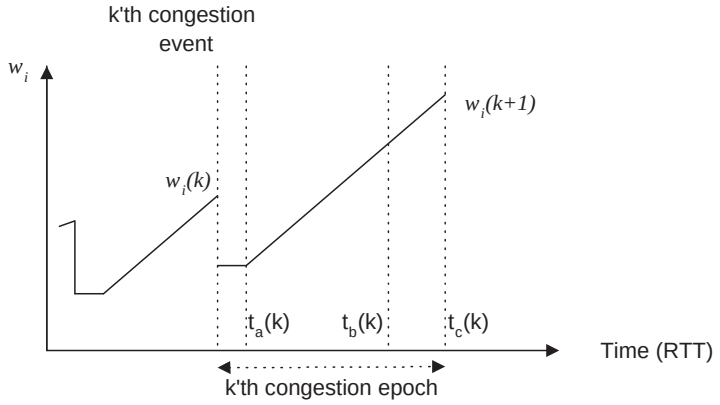

Figure 1. Evolution of window size

The drops occur at congestion events detected by the source through unacknowledged packages. It can be shown that the dynamics of the vector of window sizes

from congestion event k to event $k+1$ is given by a linear inclusion defined by a set of column stochastic matrices.

Assuming that experiencing congestion is an independent and identically distributed random process, the system becomes a Markov e-chain. For this chain we characterize the support of the invariant measure and get explicit formulas for the long term average of the window sizes. These results can be reinterpreted for the design of AIMD protocols in terms of fairness and efficient use of the resources.

Design of Nonlinear Feedforward Control

Michael Zeitz
(joint work with Christian Bermes and Knut Graichen)

The swing–up maneuver of a two–link planar robot with a single actuator at the shoulder is used to present a new approach for nonlinear feedforward control design under input constraints [1] – [4]. The pendubot model and experiment are widely used in nonlinear control education and research due to challenging features like multiple steady states, unstable internal dynamics, and the lack of feedback linearizability [5]. A particularly difficult control task is swinging up the pendubot from its stable downward equilibrium position to the unstable upward equilibrium position. The proposed feedforward control treats the swing–up maneuver as a two–point boundary value problem (BVP) defined on a finite–time interval, which enables to directly incorporate input constraints.

The considered equations of motion are given by the second order ODEs

$$A\,\ddot{y} + B\,\ddot{\eta} + C = u, \quad D\,\ddot{y} + E\,\ddot{\eta} + F = 0 \tag{1}$$

for the inner and outer angles $y(t)$ and $\eta(t)$ of the pendubot in dependence of the constrained input $|u(t)| \leq u_{max}$. The coefficients A to F are nonlinear functions of $y, \eta, \dot{y}$ and $\dot{\eta}$, as given in [3]. The equations (1) can be re–written in the nonlinear input–output normal form [6]

$$\ddot{y} = \frac{E}{G}\Big(\frac{BF}{E} - C + u\Big) = \alpha(y, \dot{y}, \eta, \dot{\eta}, u), \;\; \ddot{\eta} = \frac{D}{G}\Big(C - \frac{AF}{D} - u\Big) = \beta(\eta, \dot{\eta}, y, \dot{y}, u) \tag{2}$$

with $G = AE - BD > 0$ and $E \neq 0$. An intrinsic feature of the pendubot model is that the zero dynamics associated to the η–ODE is stable for the downward equilibrium position $y = \eta = -\pi$ and unstable for the upward equlibrium position $y = \eta = 0$, i.e. both minimum phase and non–minimum phase behavior occur during the swing–up maneuver.

The swing–up within the finite–time interval $t \in [0, T]$ means that the solutions $y(t)$ and $\eta(t)$ of the ODEs (2) have to satisfy the boundary conditions (BCs)

$$y(0) = -\pi, \; y(T) = 0, \; \dot{y}\,|_{0,T} = 0, \; \eta(0) = -\pi, \; \eta(T) = 0, \; \dot{\eta}\,|_{0,T} = 0. \tag{3}$$

The second order ODEs (2) and the BCs (3) form two coupled nonlinear BVPs in dependence of the constrained input $|u(t)| \leq u_{max}$.

The inversion–based feedforward control design uses the inverse input–output dynamics[1]

$$u^* = \alpha^{-1}(y^*, \dot{y}^*, \ddot{y}^*, \eta^*, \dot{\eta}^*), \tag{4}$$

in order to determine the input trajectory $u^*(t)$ in dependence of the trajectories $y^*(t)$ and $\eta^*(t)$ of the output and the internal dynamics, whereby an asterik $(^*)$ is used to characterize the feedforward variables [1], [2], [7]. For the determination of $\eta^*(t)$, the input (4) is inserted into the ODE of the internal dynamics in (2), i.e.

$$\ddot{\eta}^* = \bar{\beta}(\eta^*, \dot{\eta}^*, y^*, \dot{y}^*, \ddot{y}^*). \tag{5}$$

Here, the output $y^*(t)$ and its time derivatives serve as inputs. Note that the second order ODE (5) and the four BCs in (3) for $\eta^*(t)$ define an overdetermined BVP. Its solution technique plays a keyrole in the inversion–based feedforward control design.

Devasia et.al. [7] consider stable system inversion and propose to split the internal dynamics into stable and unstable parts in order to numerically solve the respective ODEs by applying an iterative forward and backward integration scheme. However, the separation in stable and unstable parts is difficult for nonlinear systems, as in case of the pendubot. Furthermore, the integration leads to a pre– and/or a post–actuation time interval. This means that the feedforward control has to start in advance of the transition (pre–actuation), or reaches the final stationary input value only asymptotically (post–actuation), although the output transition is performed in a finite time.

The main idea of the approach presented in [1], [2] is that the solvability of the BVP (3), (5) of the internal dynamics can be ensured by introducing two free parameters $p^* = (p_1^*, p_2^*)$ which are provided in the construction of the output trajectory $y^*(t) = \Upsilon(t, p^*)$.

A further elaboration of this approach enables to directly incorporate the input constraints within the feedforward control design [3, 4]. Therefore, a new function $\hat{\alpha} = \ddot{y}^*$ is introduced to parametrize the highest time derivative of the output in the ODEs (2) and (5):

$$\ddot{y}^* = \hat{\alpha}, \quad \ddot{\eta}^* = (\eta^*, \dot{\eta}^*, y^*, \dot{y}^*, \hat{\alpha}). \tag{6}$$

The solutions $y^*(t)$ and $\eta^*(t)$ of the ODEs (6) with the BCs (3) as well as the feedforward trajectory $u^*(t)$ in (4) mainly depend on the set–up of the function $\hat{\alpha} = \ddot{y}^*$ with respect to the following objectives:

(i) In order to guarantee that the feedforward trajectory $u^*(t)$ in (4) is $\mathcal{C}^0$–continuous at the bounds $t = 0, T$, the function $\hat{\alpha} = \ddot{y}^*(t)$ must meet the BCs $\ddot{y}^*(0) = 0$ and $\ddot{y}^*(T) = 0$.

(ii) The solvability of the BVPs defined by two second order ODEs (6) and eight BCs (3) requires at least four free parameters. The needed parameters $p^* = (p_1^*, \ldots, p_4^*)$ are provided in the set–up $\Phi(t, p^*) = \hat{\alpha}$ of the function $\hat{\alpha} = \ddot{y}^*$. Thereby, $\Phi(t, p^*)$ has to satisfy the BCs assumed in (i),

[1] Due to $E \neq 0$, the relative degree $r = 2$ is well defined and the inverse α^{-1} exists globally.

i.e. $\Phi(0,p^*)=0$ and $\Phi(T,p^*)=0$. Two suitable alternatives to construct this function are

$$\Phi(t,p^*)=\begin{cases} -\sum_{k=1}^{n} p_k^* \frac{t}{T} + \sum_{k=1}^{n} p_k^* \left(\frac{t}{T}\right)^{k+1} & \text{(a)} \\ \sum_{k=1}^{n} p_k^* \sin\frac{k\pi t}{T} & \text{(b)} \end{cases}$$

It is easily seen for $t=0$ and $t=T$ that both the polynomial set–up (a) and the sine series (b) meet the BCs in (i).

(iii) In order to account for the input constraints $|u(t)| \le u_{max}$, the resulting feedforward control

$$u_\Phi^* = \alpha^{-1}\left(y^*, \dot{y}^*, \Phi(t,p^*), \eta^*, \dot{\eta}^*\right)$$

is calculated, which follows from (4) with the set–up $\ddot{y}^* = \Phi(t,p^*)$.

(iv) By means of u_Φ^*, it can be decided whether the bounds $\pm u_{\max}$ are met or violated, such that the right–hand side $\hat{\alpha}$ of the y^*–ODE in (6) must be re–planned:

$$\hat{\alpha} = \begin{cases} \Phi(t,p^*) & \text{if } |u_\Phi^*| \le u_{\max} \\ \alpha(y^*,\dot{y}^*,\eta^*,\dot{\eta}^*,-u_{\max}) & \text{if } u_\Phi^* < -u_{\max} \\ \alpha(y^*,\dot{y}^*,\eta^*,\dot{\eta}^*,u_{\max}) & \text{if } u_\Phi^* > u_{\max} \end{cases}$$

The calculation of the feedforward control $u^*(t)$, $t\in[0,T]$ in (4) requires the solution of the two BVPs (3), (6) with the set–up function $\hat{\alpha}$ constructed via the design steps (i)–(iv) in dependence of the free parameters p^*. In [3], the swing–up time T is chosen as one of the free parameters p^* in the function $\Phi(t,p^*)$. Thereby, a time transformation $t=\epsilon\tau$ with $T=\epsilon T_0$ is required to re–define the BVPs (3) and (6) on a fixed time interval $\tau\in[0,T_0]$. The scaling factor ϵ is used as new free parameter and denotes the variation of the swing–up time T with respect to the fixed transition time T_0 in the new time coordinate τ.

A difficult question concerns the existence and uniqueness of a solution to the two nonlinear BVPs (3), (6). On the other hand, the solvability of the BVPs implies the existence of a feedforward control and therefore can be seen as a constructive controllability proof of the considered system. However, the analytic conditions of controllability are still an open problem for general nonlinear systems.

The solution of BVPs with free parameters p^* as given in (3), (6) is a standard task in numerics. For instance, MATLAB provides the function bvp4c[2], which can be applied in a straight–forward manner to compute both the parameters p^* and the solutions $y^*(t)$ and $\eta^*(t)$ [1] – [4]. The feedforward control $u^*(t)$, $t\in[0,T]$ is finally determined by substituting the trajectories $y^*(t)$, $\eta^*(t)$ and their time derivatives in (4). Due to the algebraic approach of the BVP solver bvp4c, there

[2] ftp://ftp.mathworks.com/pub/doc/papers/bvp

is no distinction whether the internal dynamics is stable or unstable, since its solution is obtained without numerical time integration, in contrast to the stable system inversion [7]. Therefore, the feedforward control design is applicable in the same manner for both minimum–phase and nonminimum–phase systems.

The experimental validation of the feedforward control for the swing-up maneuver requires stabilization by an additional feedback. Due to the accuracy of the nonlinear feedforward control, the feedback part can be designed by linear methods with the pendubot model linearized along the nominal trajectories, such that

$$u = u^* + k^T(x^* - x) \tag{7}$$

is chosen as the stabilizing input to track the desired trajectories for the swing–up [3]. The time–variant row vector $k^T(t)$ with the gains $k_i(t)$, $i = 1, 2, 3, 4$ of the states $x = [y, \dot{y}, \eta, \dot{\eta}]^T$ is calculated point–wise in time by an LQR technique with suitable weighting matrices. Thereby, it must be considered that the controllability of the linearized model is lost in the midth of the swing–up maneuver, which leads to a finite–time escape of the controller gains $k_i(t)$. This problem can be avoided by interpolating the feedback gains through the singularity, which has been experimentally validated by the swing–up and swing–down maneuver of the pendubot. Finally, this feedforward/feedback control scheme has been implemented in order to track the finite–time transitions between the four equilibrium points of the pendubot [8].

References

[1] K. Graichen, V. Hagenmeyer, M. Zeitz, *Van de Vusse CSTR as a benchmark problem for nonlinear feedforward control design techniques*, F.Allgöwer (edit.), Preprints 6th IFAC–Symposium on Nonlinear Control Systems (NOLCOS), Stuttgart/Germany 2004, Vol.3, 1415–1420.

[2] K. Graichen, V. Hagenmeyer, M. Zeitz, *A new approach to inversion–based feedforward control design for nonlinear systems*, Automatica (accepted 2005).

[3] K. Graichen, M. Zeitz, *Nonlinear feedforward and feedback tracking control with input constraints solving the pendubot swing–up problem*, IFAC World Congress, Prague/CZ 2005.

[4] K. Graichen, M. Zeitz, *Feedforward control design for nonlinear SISO systems under input constraints*, (submitted 2005).

[5] M.W. Spong, *The swing up problem for the acrobot*, IEEE Control Systems Magazin **15** (1995), 49–55.

[6] A. Isidori, *Nonlinear Control Systems*, 3rd Ed., Springer, 1995.

[7] S. Devasia, D. Chen, B. Paden, *Nonlinear inversion–based output tracking*, IEEE Transactions on Automatic Control **41** (1996), 930–942.

[8] C. Bermes, *Entwurf einer Vorsteuerung und Folgeregelung für den Ruhelagenwechsel des polaren Doppelpendels*, Student Thesis (in German), Universität Stuttgart 2005.

Algebraic analysis of time-varying linear behaviors

Eva Zerz

Consider a left Noetherian ring $\mathcal{D}$ (with unity) and a left $\mathcal{D}$-module $\mathcal{A}$. In systems theory, $\mathcal{D}$ is typically a ring of (differential) operators, and $\mathcal{A}$ is a set of signals on which the elements of $\mathcal{D}$ operate. An abstract linear system takes the form

$$\mathcal{B} = \{w \in \mathcal{A}^q \mid Rw = 0\},$$

and $R \in \mathcal{D}^{g\times q}$ is called a representation of $\mathcal{B}$. Thus, an abstract linear system is the solution set, in the signal module $\mathcal{A}^q$, of the linear system of (differential) equations $Rw = 0$. Such solution sets are known as "behaviors" in systems theory [9].

Define the left $\mathcal{D}$-module $\mathcal{M} = \mathcal{D}^{1\times q}/\mathcal{D}^{1\times g}R$. There is an isomorphism of (additive) Abelian groups $\mathcal{B} \cong \mathrm{Hom}_{\mathcal{D}}(\mathcal{M}, \mathcal{A})$, which is known as the Malgrange isomorphism [5]. Thus, the contravariant functor $\mathrm{Hom}_{\mathcal{D}}(\cdot, \mathcal{A})$ links the algebraic object $\mathcal{M}$ with the analytic object $\mathcal{B}$. In the $\mathcal{D}$-module theoretic approach [2, 8], a linear system is identified with $\mathcal{M}$, whereas in the behavioral approach, $\mathcal{B}$ is the central object to be investigated. The theories are parallel over signal sets $\mathcal{A}$ that are injective cogenerators [4]. This means that the functor $\mathrm{Hom}_{\mathcal{D}}(\cdot, \mathcal{A})$ is exact and faithful, i.e., it preserves and reflects exactness. Over an injective signal module $\mathcal{A}$, we have the so-called fundamental principle, which says that the solvability of an inhomogeneous equation $Py = v$, where the $\mathcal{D}$-matrix P and the $\mathcal{A}$-vector v are given, is equivalent to a condition on the right hand side v of the form $Qv = 0$, where Q is a $\mathcal{D}$-matrix whose rows generate the left kernel of P. The existence of Q is guaranteed since $\mathcal{D}$ is assumed to be left Noetherian. Moreover, the cogenerator property implies that the inclusion of behaviors can be characterized in terms of a divisibility condition on the representations matrices, namely, $\mathcal{B}_1 \subseteq \mathcal{B}_2$ if and only if $R_2 = XR_1$ for some $\mathcal{D}$-matrix X. Injective cogenerators provide an extremely fruitful framework for systems theory, because they make it possible to translate any statement on $\mathcal{B}$ that can be expressed in terms of kernels and images, into an equivalent statement on $\mathcal{M}$, and vice versa. This is true, for example, for the ring $\mathcal{D}$ of ordinary or partial differential operators with real or complex coefficients, and the signal sets $\mathcal{A}$ of entire or smooth functions, or of distributions [6].

Assume additionally that $\mathcal{D}$ is a domain. Over a signal set $\mathcal{A}$ that enjoys the injective cogenerator property, the systems theoretic property of autonomy (i.e., absence of free variables or inputs) can be characterized by the fact that $\mathcal{M}$ is torsion. Equivalently, all representations of an autonomous system possess full column rank, where the rank is defined over the quotient field of left fractions of the left Noetherian domain $\mathcal{D}$. An abstract linear system is called controllable if it possesses an image representation, that is,

$$\mathcal{B} = \{w \in \mathcal{A}^q \mid \exists \ell \in \mathcal{A}^l : w = L\ell\}$$

for some $L \in \mathcal{D}^{q\times l}$. If $\mathcal{D}$ is also right Noetherian, then controllability of $\mathcal{B}$ is equivalent to torsion-freeness of $\mathcal{M}$. Note that the assumption that $\mathcal{D}$ should be left and right Noetherian can be weakened [7], but this is not necessary for the special case to be considered next.

Let $\mathcal{D}$ be the ring of linear ordinary differential operators with rational (meromorphic) coefficients. Then $\mathcal{D}$ is a non-commutative simple principal ideal domain [1]. A signal set with the injective cogenerator property is given by the set of functions that are smooth except for a finite (discrete) number of points [10]. To prove this, it suffices to show [4] that any scalar equation $dy = u$, where $0 \neq d \in \mathcal{D}$ and $u \in \mathcal{A}$ are given, possesses a solution $y \in \mathcal{A}$, and that for any $d \in \mathcal{D}$ that is not a unit, there exists a non-zero solution to $dy = 0$. Having established these properties, one finds that any $\mathcal{B}$ can be represented by a matrix of full row rank, that two representations of full row rank differ only by a unimodular left factor, and that autonomy is equivalent to the existence of a square non-singular representation. Controllability amounts to the existence of a right invertible representation matrix. Analytical characterizations of autonomy and controllability can be derived [3, 10]. Moreover, any $\mathcal{B}$ can be decomposed into a direct sum of its largest controllable subsystem and an autonomous subsystem. Also the existence of row-proper representations, input-output structures with proper transfer matrix, and state space realizations, which are well-known in the time-invariant case, carry over, in a straightforward manner, to the time-varying setting.

The situation is more involved, however, when addressing the interpretation of the state as the system's memory. The solution space of functions that are smooth almost everywhere is not appropriate for tackling this question. One needs to restrict to solutions with some sort of regularity such as locally integrable solutions. Differentiation can be interpreted in the distributional sense, but additionally, one has to restrict to representations with polynomial (analytic) coefficients. The resulting ring of differential operators is much more complex than $\mathcal{D}$. However, using algebraic manipulations over $\mathcal{D}$, we can still make statements about the generic properties of $\mathcal{B}$ by considering local versions on open intervals outside a finite (discrete) set of points.

References

[1] P. M. Cohn, *Free rings and their relations*, Academic Press, 1971.

[2] M. Fliess, J. Lévine, P. Rouchon, Index of an implicit time-varying linear differential equation: a noncommutative linear algebra approach, *Linear Algebra Appl.* **186** (1993), 59–71.

[3] A. Ilchmann, V. Mehrmann, A behavioural approach to time-varying linear systems, Preprint, 2004.

[4] T. Y. Lam, *Lectures on rings and modules*, Springer, 1999.

[5] B. Malgrange, Systèmes différentiels à coefficients constants, *Semin. Bourbaki* **15** (1962/63), No. 246 (1964), 11 p.

[6] U. Oberst, Multidimensional constant linear systems, *Acta Appl. Math.* **20** (1990), 1–175.

[7] J.-F. Pommaret, A. Quadrat, Generalized Bezout identity, *Appl. Algebra Eng. Commun. Comput.* **9** (1998), 91–116.

[8] J. Rudolph, Duality in time-varying linear systems: a module theoretic approach, *Linear Algebra Appl.* **245** (1996), 83–106.

[9] J. C. Willems, Paradigms and puzzles in the theory of dynamical systems, *IEEE Trans. Autom. Control* **36** (1991), 259–294.

[10] E. Zerz, An algebraic analysis approach to linear time-varing systems, accepted for publication in *IMA J. Math. Control Information*, 2005.

Open Problem: Model Reduction by Projection

Athanasios C. Antoulas

In the large-scale setting linear systems are described in terms of state and output equations of the form $\dot{x}(t) = Ax(t)+Bu(t)$, $y(t) = Cx(t)+Du(t)$, respectively, where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, $D \in \mathbb{R}^{n\times m}$, with n the state dimension (which depending on the application may range from several thousand to several million states), m the number of inputs, and p the number of outputs. Given the projection $\Pi = VW^*$, $\Pi^2 = \Pi$, where V, $W \in \mathbb{R}^{n\times k}$, $k \ll n$, and $(\cdot)^*$ denotes transposition, followed by complex conjugation if necessary, the associated system $\hat{\Sigma} = (\hat{A}, \hat{B}, \hat{C}, \hat{D})$, of reduced order k, is obtained as follows

$$\hat{A} = W^*AV,\ \hat{B} = W^*B,\ \hat{C} = CV,\ \hat{D} = D. \tag{1}$$

The Rational Lanczos Procedure. In the sequel for simplicity of exposition, we will assume that the systems $\Sigma = (A, B, C, D)$, are single-input single-output ($m = p = 1$). Let $2k$ pairwise distinct points $\lambda_i \in \mathbb{C}$ be given ($\lambda_i \neq \lambda_j$, $i, j = 1, \cdots, 2k$). Consider the projection $\Pi = VW^*$, where

$$\begin{aligned} \operatorname{im} V &= \operatorname{im} \left[(\lambda_1 I - A)^{-1}B \ \cdots \ (\lambda_k I - A)^{-1}B\right] \\ \operatorname{im} \bar{W}^* &= \operatorname{im} \begin{bmatrix} C(\lambda_{k+1} I - A)^{-1} \\ \vdots \\ C(\lambda_{2k} I - A)^{-1} \end{bmatrix}, \ W^* = (\bar{W}^*V)^{-1}\bar{W}^*, \end{aligned} \tag{2}$$

where $\operatorname{im} Z$ denotes the *image* or the *span of the columns* of the matrix Z. V, W defined above are called *generalized controllability* and *generalized observability* matrices respectively; it can readily be shown that the non-singularity of the $k \times k$ matrix $\bar{W}^*V$ is a consequence of the controllability and observability of the triple (A, B, C).

Interpolation Property. The above construction leads to the following important property. The transfer function of the reduced system defined by (1) where the projection is defined by (2), *interpolates* the transfer function of the original system at the given points λ_i:

$$\begin{aligned} H(\lambda_i) &= D + C(\lambda_i I_n - A)^{-1}B &&= \\ &= \hat{D} + \hat{C}(\lambda_i I_k - \hat{A})^{-1}\hat{B} &&= \hat{H}(\lambda_i), \end{aligned}$$

for $i = 1, \cdots, 2k$.

Furthermore, if $\lambda_i = \lambda_{k+i}$, $i = 1, \cdots, k$, then

$$H(\lambda_i) = \hat{H}(\lambda_i) \text{ and } \left.\frac{dH(s)}{ds}\right|_{s=\lambda_i} = \left.\frac{d\hat{H}(s)}{ds}\right|_{s=\lambda_i},$$

for $i = 1, \cdots, k$, that is, the transfer function of the reduced system interpolates both the values and those of the first derivatives of the transfer function of the original system, at the first k points.

Generality of model reduction by rational Krylov. It turns out that model reduction by rational Krylov is quite general. First, it can be shown that in the single-input single-output case, given the original system and *any* reduced system, the latter can be obtained from the former by rational Krylov, for an appropriate choice of the interpolation points λ_i. In the multi-input multi-output case, this property holds generically.

Thus, given this fact the following question arises:

How to choose the interpolation points, so that the reduced system satisfies desired properties?

Open Problem: Performance of sampled–data systems

Lars Grüne

Consider a single input control affine closed loop system

$$\dot{x}(t) = f(x(t)) + g(x(t))u(x(t))$$

with $x \in \mathbb{R}^n$ and controller u and the corresponding sampled–data system

$$\dot{x}_T(t) = f(x_T(t)) + g(x_T(t))u_T(x_T(iT))), \quad t \in [iT, (i+1)T),\ i = 0, 1, \ldots$$

with controller u_T. We consider the mismatch after one time step given by

$$\Delta_T := \|x(T, x_0, u) - x_T(T, x_0, u_T)\|.$$

It is known that for $u_T = u$ we obtain $\Delta_T = O(T^2)$ while for

$$u_T(x) = u(x) + \frac{T}{2}\frac{\partial u(x)}{\partial x}[f(x) + g(x)u(x)]$$

we obtain $\Delta_T = O(T^3)$ (this follows from [1, Theorem 4.11] setting $V(x) = x_i$ observing that positive definiteness of V is not needed). Remark 4.12 in [1] suggests that higher order cannot be obtained in general.

Problem: Find conditions on f, g, u under which $\Delta_T < O(T^3)$ can be achieved.

References

[1] D.Nešić and L.Grüne, *Lyapunov based continuous–time nonlinear controller redesign for sampled–data implementation*, Automatica (2005), to appear

Open problem: What is the right definition of observability for polynomial systems?

Uwe Helmke

One reasonable requirement for a sensible definition of observability is that the property holds for a generic class of systems. This issue strongly depends on the system class under consideration. Recall, that a nonlinear system

$$\dot{x}(t) = f(x(t)), \quad y(t) = h(x(t)) \tag{1}$$

is called observable, if the initial states $x_1(0), x_2(0)$ of two trajectories are distinguishable by the corresponding output functions $h(x_1(\cdot)), h(x_2(\cdot))$. For linear systems, generic observability is well-known and indeed an easy consequence of the Kalman observability rank condition. For nonlinear systems the situation becomes much more complicated. In the 1980s, D. Aeyels and F. Takens proved that observability is a generic property for smooth continuous–time systems. In fact, Aeyels and Takens [1, 4] proved genericity for a much stronger condition than observability, i.e. that for generic smooth systems (1) the map

$$\Phi^N : \mathbb{R}^n \to \mathbb{R}^{N+1}, x \mapsto (h(x), L_f h(x), \cdots, L_f^N h(x))$$

is an embedding, provided $N \geq 2n$. Here $L_f h(x) = dh(x)f(x)$ denotes the Lie derivative. Similar genericity results for observability of real analytic systems have been obtained by Gauthier and Kupka. It seems strange that such genericity results do not hold for discrete-time systems. In fact, elementary counterexamples, first constructed by Vivaldi, show that there exist open classes of unobservable discrete-time systems. Indeed, consider the one-dimensional system

$$x_{t+1} = x_t^2, \quad y_t = x_t^3 + 2x_t^2 + 3x_t + 4. \tag{2}$$

This system is not observable. Moreover, any small perturbation in the system parameters results in an unobservable system. In fact, the image $F(\mathbb{R})$of the map $F(x) = (x^2, x^3 + 2x^2 + 3x + 4)$ has a simple self-intersection point at $(0, 0)$. Thus, small perturbations of the system parameters will also result in a set $\tilde{F}(\mathbb{R})$ having a simple self-intersection point. Thus, there exists an open set of discrete–time systems which are not observable. The above counterexample shows that, even in the restricted class of polynomial discrete-time systems, genericity of observability cannot be expected. The question remains whether or not observability is a generic property for polynomial continuous–time systems. Pollicott has shown that observability holds generically for continuous-time polynomial systems on compact algebraic manifolds, provided the degrees of the class of polynomial functions are assumed to be bounded.

The compactness assumption excludes the case of unconstrained continuous–time polynomial systems on $\mathbb{R}^n$. We believe that genericity holds here, too. Moreover, we conjecture that even the much more restricted condition of algebraic observability is generic. The concept of algebraic observability was first introduced by Sontag [3]. A polynomial system (2) is *algebraically observable* if there

exists a finite integer $N \geq 1$ and a polynomial $\pi \in \mathbb{R}[y_0, \cdots, y_N]$ such that for all $x \in \mathbb{R}^n$

$$x = \pi\big(h(x), h(f(x)), \ldots, h(f^N(x))\big)$$

holds, that is, the initial state is expressible as a polynomial function of the $N+1$ observations.

Conjecture 1. *Algebraic observability is a generic property for continuous–time polynomial systems (f, h) on $\mathbb{R}^n$ of bounded degrees.*

References

[1] D. Aeyels. Generic observability of differentiable systems. *SIAM J. Control and Opt.* 19(5):595–603, 1981.

[2] J. P. Gauthier and I. A. Kupka. Observability for systems with more outputs than inputs and asymptotic observers. *Math. Z.* 223:47–78, 1996.

[3] E. D. Sontag. On the observability of polynomial systems. *SIAM J. Control and Opt.* 17:139–152, 1979.

[4] F. Takens. Detecting strange attractors in turbulence. *Proc. of the Symposium on Dynamical Systems and Turbulence; Lecture Notes in Mathematics* 898:366–381, Springer–Verlag, New York, 1981.

Open problem: Non-adaptive polynomial stabilization of linear minimum phase systems with known upper bound of the relative degee

Achim Ilchmann

Consider the *system class* of single-input, single-output systems

$$\dot{x}(t) = A\,x(t) + b\,u(t), \qquad y(t) = c\,x(t) \tag{1}$$

where the entries of $A \in \mathbb{R}^{n\times n}$, $b, c^T, x^0 \in \mathbb{R}^n$, are unknown but satisfy the following structural assumptions:

(A1) *known upper bound of the relative degree and positive high-frequency gain*: For some known $r \in \mathbb{N}$, there exists $\rho \in \{1, \ldots, r\}$ such that $cA^ib = 0$ for $i = 1, ..., \rho - 2$ and $cA^{\rho-1}b > 0$.

(A2) *minimum-phase*:

$$\det \begin{bmatrix} sI - A & b \\ c & 0 \end{bmatrix} \neq 0 \qquad \text{for all } s \in \mathbb{C} \text{ with } \operatorname{Re} s \geq 0.$$

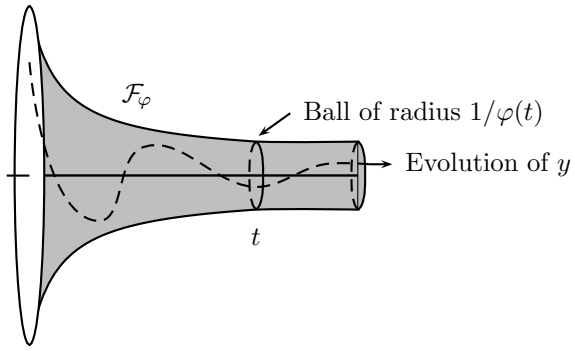

Fig. 1. Prescribed performance funnel $\mathcal{F}_\varphi$.

The *control problem* is as follows: consider an arbitrary performance funnel

$$\mathcal{F}_\varphi := \big\{(t,e) \in \mathbb{R}_{\geq 0} \times \mathbb{R}^m \big| \ \varphi(t)\,\|e\| < 1\big\}$$

associated with a function φ (the reciprocal of which determines the funnel boundary) belonging to

$$\mathcal{B} := \Big\{\varphi \in W^{1,\infty}(\mathbb{R}_{\geq 0}, \mathbb{R}) \Big| \ \varphi(0) = 0,\ \varphi(s) > 0\ \forall\ s > 0,\ \liminf_{s\to\infty} \varphi(s) > 0\Big\}\ ;$$

design a feedback law

$$u(t) = -\gamma_r\big(k(t), y(t), \xi(t)\big)\,, \qquad \gamma_r \text{ polynomial}$$

where

$$k(t) = [1 - (\varphi(t)\|y(t)\|)^2]^{-1}\,, \qquad \dot{\xi}(t) = f(y(t), u(t), \xi(t)) \quad \text{with polynomial } f,$$

such that y evolves within the funnel $\mathcal{F}_\varphi$.

In [1], we have introduced a control strategy for the above class of system in the special case that $r = \rho$. The novelty is that the high-gain parameter k is not a monotone function as it is in the context of adaptive control.
In adaptive control, similar system classes have been considered, however, apart from [2], transient behaviour in terms of a funnel has not been addressed. In [3], the problem is solved for systems where additionally a bound on the dimension of the system is known; however, their control strategy involves a monotone high-gain parameter k and transient behaviour is not addressed either.

References

[1] A. Ilchmann, E.P. Ryan, and P. Townsend. Tracking control with prescribed transient behaviour for systems of known relative degree. *Preprint M02/05 TU Ilmenau,* `http://wcms3.rz.tu-ilmenau.de/ math/preprints/Preprints_en.html`.

[2] Miller, D.E. and E.J. Davison (1991). An adaptive controller which provides an arbitrarily good transient and steady-state response. *IEEE Trans. Aut. Control* **36**(1), 68–81.

[3] Mudgett, D.R. and A.S. Morse (1985). Adaptive stabilization of linear systems with unknown high-frquency gains. *IEEE Trans. Aut. Control* **30**(6), 549–554.

Open Problem: A Stochastic Control Problem for the Capital Adequacy Ratio of a Bank

Jan H. van Schuppen
(joint work with Mark A. Petersen)

Does there exist a control law for a stochastic system which will prevent the capital adequacy ratio of a bank to drop below a threshold? If not, how to determine a control law such that the drop below the threshold is postponed as long as possible?

Motivation. The experience is that banks do default. The Bank of International Settlements in Basel, Switzerland, has issued a set of rules to assist national supervisory bodies with their task, see [1]. One of these rules states that the capital adequacy ratio has to stay above the threshold value 0.08. The capital adequacy ratio is defined as the quotient of the risk-weighted assets over the eligible regulatory capital. If this ratio drops below the threshold then the national supervisory body can take action which include the closure of the bank. Typically the ratio will be far above the threshold, say in the range [0.20,0.25].

Stochastic system. The authors have formulated a model for the behavior of the capital adequacy ratio in the form of a stochastic system driven by Brownian motion. Due to space limitations, the system cannot be included in this short problem formulation.

Conjecture. It is conjectured that there does not exist a control law which will prevent the capital adequacy ratio to drop below 0.08. The consequence of this conjecture is that every bank behaving like this model will almost surely default eventually. Therefore the second part of the problem is of interest. A publication on the problem is in preparation.

References

[1] Basel Committee on Banking Supervision. *International convergence of capital measurement and capital standards - A revised framework.* Bank for International Settlements, Basel, 2004.

Open Problem: When is a Linear System Optimal?

Jan C. Willems

The following problem was presented during the open problem session. Its title is taken from a well-known seminal paper [1] by R.E. Kalman. The questions posed in [1] and here are similar in spirit, but the setting is quite different.

Let $\Phi \in \mathbb{R}^{\mathtt{w}\times\mathtt{w}}(\zeta,\eta)$, $\Phi(\zeta,\eta) = \sum_{\mathtt{k},\ell} \Phi_{\mathtt{k},\ell}\zeta^{\mathtt{k}}\eta^{\ell}$, with $\Phi_{\mathtt{k},\ell} = \Phi_{\ell,\mathtt{k}}^{\top} \in \mathbb{R}^{\mathtt{w}\times\mathtt{w}}$, hence $\Phi = \Phi^{\star}$, where $\Phi^{\star}(\zeta,\eta) := \Phi(\eta,\zeta)^{\top}$. Denote by Q_{Φ} the 'quadratic differential form' [5] which maps as follows

$$w \in \mathcal{C}^{\infty}(\mathbb{R},\mathbb{R}^{\mathtt{w}}) \mapsto \sum_{\mathtt{k},\ell} (\frac{d^{\mathtt{k}}}{dt^{\mathtt{k}}}w)^{\top}\Phi_{\mathtt{k},\ell}(\frac{d^{\ell}}{dt^{\ell}}w) \in \mathcal{C}^{\infty}(\mathbb{R},\mathbb{R}).$$

Consider for $w, \Delta \in \mathcal{C}^{\infty}(\mathbb{R},\mathbb{R}^{\mathtt{w}})$, with Δ of compact support, the integral

$$\int_{-\infty}^{\infty} \big(Q_{\Phi}(w+\Delta) - Q_{\Phi}(w)\big)\, dt. \tag{1}$$

Expand (1) in a term which is bilinear in w, Δ, and one which is quadratic in Δ. We obtain

$$(1) \;= \int_{-\infty}^{\infty} \Delta^{\top}\big(\Phi(-\frac{d}{dt},\frac{d}{dt})w\big)\, dt + \int_{-\infty}^{\infty} Q_{\Phi}(\Delta)\, dt.$$

The trajectory $w \in \mathcal{C}^{\infty}(\mathbb{R},\mathbb{R}^{\mathtt{w}})$ is said to be *stationary* with respect to Φ, relative to variations Δ, if the linear term in Δ in (1) vanishes, i.e. if

$$\int_{-\infty}^{\infty} \Delta^{\top}\Phi(-\frac{d}{dt},\frac{d}{dt})w\, dt = 0$$

for all $\Delta \in \mathcal{C}^{\infty}(\mathbb{R},\mathbb{R}^{\mathtt{w}})$ of compact support. It is said to be *optimal* with respect to Φ, relative to variations Δ, if (1) is non-negative for all $\Delta \in \mathcal{C}^{\infty}(\mathbb{R},\mathbb{R}^{\mathtt{w}})$ of compact support. Hence $w \in \mathcal{C}^{\infty}(\mathbb{R},\mathbb{R}^{\mathtt{w}})$ is stationary if and only if

$$\Phi(-\frac{d}{dt},\frac{d}{dt})w = 0, \tag{2}$$

and optimal if and only if in addition

$$\int_{-\infty}^{\infty} Q_{\Phi}(\Delta)\, dt \geq 0 \tag{3}$$

for all $\Delta \in \mathcal{C}^{\infty}(\mathbb{R},\mathbb{R}^{\mathtt{w}})$ of compact support. It is easy to prove (see [5]) that (3) holds if and only if the following frequency domain condition is satisfied:

$$\Phi(-i\omega, i\omega) \geq 0 \text{ for all } \omega \in \mathbb{R}. \tag{4}$$

Denote by $\mathcal{L}^{\mathtt{w}}$ the set of linear time-invariant differential systems in $\mathtt{w}$ variables, i.e. $\mathcal{B} \in \mathcal{L}^{\mathtt{w}}$ means that $\mathcal{B} \subseteq \mathcal{C}^{\infty}(\mathbb{R},\mathbb{R}^{\mathtt{w}})$ and that there exists a polynomial matrix $R \in \mathbb{R}^{\bullet\times\mathtt{w}}[\xi]$ such that

$$R(\frac{d}{dt})w = 0 \tag{5}$$

has $\mathcal{B}$ as its $\mathcal{C}^{\infty}$ solutions. Note that, while R specifies $\mathcal{B}$, the converse is not true (see [3]). The open problem is to

Characterize the behaviors $\mathcal{B} \in \mathcal{L}^{\mathtt{w}}$ that are stationary or optimal.

In other words, under what conditions on $\mathcal{B} \in \mathcal{L}^{\mathtt{w}}$ does there exist $\Phi = \Phi^{\star} \in \mathbb{R}^{\mathtt{w}\times\mathtt{w}}[\zeta,\eta]$ such that $\mathcal{B} = \ker\big(\Phi(-\frac{d}{dt},\frac{d}{dt})\big)$? We are looking for conditions on $\mathcal{B}$ directly, more than on representations of $\mathcal{B}$. This open problem is, of course, an

high-order analogue of a well-studied problem in the calculus of variations and classical mechanics. The ideas and results from [4] are very relevant and partly solve the open problem stated above.

It is straightforward to settle the case $\mathtt{w} = 1$. In this case $\mathcal{B}$ is stationary if and only if either $\mathcal{B} = \mathcal{C}^\infty(\mathbb{R}, \mathbb{R})$, or $\mathcal{B}$ is a finite dimensional subset of $\mathcal{C}^\infty(\mathbb{R}, \mathbb{R})$ (i) of even dimension and (ii) time-reversible (in the sense that $t \in \mathbb{R} \mapsto w(t) \in \mathbb{R}$ belongs to $\mathcal{B}$ if and only if $t \in \mathbb{R} \mapsto w(-t) \in \mathbb{R}$ belongs to $\mathcal{B}$). It is optimal if and only if in the finite dimensional case the following additional condition on the oscillatory solutions holds: (iii) whenever $t \in \mathbb{R} \mapsto t^{\mathtt{k}} \sin \omega t$ belongs to $\mathcal{B}$ for some even integer $\mathtt{k}$ and some $\omega \in \mathbb{R}$, then also $t \in \mathbb{R} \mapsto t^{\mathtt{k}+1} \sin \omega t$ belongs to $\mathcal{B}$.

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References

[1] R.E. Kalman, *When is a linear control system optimal?,* Transactions of the ASME, Journal of Basic Engineering, Series D, **86** (1964), 81–90.

[2] J.C. Willems, *Paradigms and puzzles in the theory of dynamical systems,* IEEE Transactions on Automatic Control, **36** (1991), 259–294.

[3] J.W. Polderman and J.C. Willems, *Introduction to Mathematical Systems Theory: A Behavioral Approach*, Springer-Verlag, New York, 1998.

[4] P. Rapisarda and H.L. Trentelman, *Linear Hamiltonian behaviors,* SIAM Journal on Control and Optimization, **43** (2004), 769–791.

[5] J.C. Willems and H.L. Trentelman, *On quadratic differential forms,* SIAM Journal on Control and Optimization, **36** (1998), 1703–1749.

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