Mathematisches Forschungsinstitut Oberwolfach

Report No. 45/2005

Low-Dimensional Manifolds

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September 25th – Oktober 1st, 2005

Mathematics Subject Classification (2000): 57N10,57N16,20F67.

Introduction by the Organisers

The workshop on Low-Dimensional was organized by M. Boileau (Toulouse), K. Johannson (Frankfurt) and P. Scott (Ann Arbor). The aim was to bring together an international audience in order to expose some of the recent results in Low-Dimensional Topology and related areas. There were altogether 22 talks from a broad range of topics including Heegaard surfaces, the curve complex, hyperbolic manifolds, orbifolds and geometric group theory. There was ample opportunity for ample interaction. The organizers would like to thank the Institute for providing a pleasant and stimulating atmosphere.

Workshop: Low-Dimensional Manifolds

Table of Contents

Abstracts

Families of Discrete Characters and Non-Zero Degree Maps

Steve Boyer
(joint work with Michel Boileau and Shicheng Wang)

The existence of a proper map of non-zero degree between two compact, connected, orientable manifolds of the same dimension is a difficult question to handle in topology. M. Gromov pointed out the interest in using the existence of such maps to define a partial ordering on the set of homeomorphism classes of compact, connected, orientable manifolds of the same dimension. This ordering is defined as follows: we say that M *dominates* N, written $M \geq N$, if there is a continuous, proper map from M to N of non-zero degree. Moreover if N is not homeomorphic to M we say that M *strictly dominates* N. Though it is not always true that $M \geq N$ and $N \geq M$ implies that $M \cong N$ or even $M \simeq N$, this is often the case and in low dimensions it is understood when it isn't. For instance for closed, connected, orientable surfaces $\geq$ is a total ordering which agrees with that given by the genus: $M \geq N$ if and only if the genus of M is greater than or equal the genus of N. In this talk we are interested in the 3-dimensional situation where things are much more subtle and difficult to study. In this case it is known that $\geq$ is a partial order when restricted to manifolds which have infinite fundamental groups and are either Haken or admit geometric structures b! ut are neither torus bundles, torus semi-bundles, or Seifert fibred manifolds with zero Euler number.

We assume below that our manifolds are compact, connected, orientable, and 3-dimensional. Further we shall suppose that they are *small*, that is they contain no closed, essential surfaces. This is a simplifying hypothesis and though many of the results discussed below have analogues in the general setting, we will not discuss them.

By a *hyperbolic* manifold we mean a manifold whose interior admits a complete, finite volume hyperbolic structure. Consider a hyperbolic manifold M with a torus boundary. We are interested in understanding families of dominations $f_n : M(\alpha_n) \geq V_n$ where $\{\alpha_n\}$ is sequence of distinct slopes on ∂M and V_n is hyperbolic. There is a close connection between such dominations and non-zero volume representations $\rho : \pi_1(M) \rightarrow PSL_2(\mathbb{C})$ whose images are torsion-free, co-compact, and Kleinian. For instance, given such a domination we have a representation ρ_n defined by the composition $\pi_1(M) \rightarrow \pi_1(\alpha_n) \rightarrow \pi_1(V_n) \subset PSL_2(\mathbb{C})$. Note

$$vol(\rho_n) = \text{degree}(f_n) vol(V_n) \neq 0.$$

Conversely, given a non-zero volume representation $\rho : \pi_1(M) \rightarrow PSL_2(\mathbb{C})$ whose image, Γ say, is torsion-free, co-compact, and Kleinian, there is at least one slope α on ∂M such that $\rho(\alpha) = \pm I$. It follows that there is an induced map $f : M(\alpha) \rightarrow V = \mathbb{H}^3/\Gamma$. Further, $0 \neq vol(\rho) = \text{degree}(f) vol(V)$ and therefore degree$(f) \neq 0$.

We say that a representation $\rho : \pi_1(M) \to PSL_2(\mathbb{C})$ is *peripherally nontrivial* if $\rho(\pi_1(\partial M))$ does not equal $\{\pm I\}$. If the image of such a ρ is torsion-free, there is a unique slope α on ∂M such that $\rho(\alpha) = \pm I$. In this case we call α the *slope of* ρ.

Theorem 1. *Suppose that $\{\chi_n\}$ is a sequence of distinct characters of peripherally nontrivial, non-zero volume representations $\rho_n : \pi_1(M) \to PSL_2(\mathbb{C})$ with image Γ_n, the fundamental group of a close hyperbolic 3-manifold. Let α_n be the slope of χ_n. Then up to taking a subsequence, one of the following two possibilities arises.*
(a) The slopes α_n converge projectively to the class of a boundary slope of M; or
(b) $\lim \chi_n = \chi_{\rho_0}$ where $\rho_0|\pi_1(\partial M)$ is 1-1, $\rho_0(\pi_1(M))$ is the fundamental group of a 1-cusped hyperbolic manifold V, and there are slopes β_n on ∂V such that $\pi_1(V(\beta_n)) \cong \Gamma_n$ and χ_n is the character of $\pi_1 M) \to \rho_0(\pi_1(M)) = \pi_1(V) \to \pi_1(V(\beta_n)) \cong \Gamma_n$. □

Here is a version of Theorem 1 stated in terms of non-zero degree maps.

Theorem 2. *Suppose that there is a slope α_0 such that $M(\alpha_0)$ does not strictly dominate a hyperbolic manifold. Let α_n be a sequence of distinct slopes on ∂M which do not subconverge to a boundary slope of M and assume that there are strict dominations $f_n : M(\alpha_n) \geq V_n$ where V_n is hyperbolic. Then there are a strict domination $f : M \geq V$, where V is hyperbolic, a subsequence $\{j\}$ of $\{n\}$, slopes β_j on ∂V such that $V(\beta_j) \cong V_j$, and homotopy commutative diagrams*

$$\begin{array}{ccc} M & \xrightarrow{f} & V \\ \downarrow & & \downarrow \\ M(\alpha_j) & \xrightarrow{f_j} & V_j \cong V(\beta_j) \end{array}$$

□

Corollary 1. *Under the hypotheses of Theorem 2 we have*

$$|\{n : f_n \text{ is not induced by a strict domination } M \geq V\}| < \infty.$$

□

Remark 1. In certain circumstances we can remove the condition on the projective convergence of the slopes α_n in Theorems 1 and 2. For instance, this is the case when the χ_n lie on the canonical component X_0 of the $PSL_2(\mathbb{C})$-character variety of $\pi_1(M)$ (the one defined by the hyperbolic structure on M). It follows that except for a finite number of exceptions, the discrete, non-zero volume characters on X_0 are those determined by the hyperbolic structure on M and its Dehn fillings. More generally, suppose that the χ_n lie on a curve component X_1 of $X_{PSL_2}(M)$ such that for each essential surface S_0 associated to an ideal point of X_1 either

(1) $|\partial S_0| \leq 2$, or

(2) there is a character $\chi_\rho \in X_1$ which restricts to an irreducible character on $\pi_1(S_0)$.

then (b) of Theorem 1 holds and we can remove the convergence hypothesis from Theorem 2.

Let $\mathcal{M}$ be a family of 3-manifolds. A manifold $V \in \mathcal{M}$ is called $\mathcal{M}$-*minimal* if the only manifold in $\mathcal{M}$ it dominates is itself. We focus on the families $\mathcal{H}$ of hyperbolic manifolds and $\mathcal{G}$ of manifolds which are either Haken, geometric, or a connected sum of such manifolds.

Example 1.
(1) The total space of a (hyperbolic) punctured torus bundle is $\mathcal{G}$-minimal if and only if its monodromy is not a proper power. This can be verified using elementary 3-manifold topology.
(2) Hyperbolic twist knots are $\mathcal{G}$-minimal as are the exteriors of the $\frac{p}{q}$ rational knots when p is prime. These claims, and those in the next two examples (3) and (4), can be proven using $PSL_2(\mathbb{C})$-character variety arguments.
(3) The exterior of the $(-2, 3, n)$ pretzel knot is $\mathcal{G}$-minimal if and only if $n \not\equiv 0 \pmod 3$. If the knot is hyperbolic, it is $\mathcal{H}$-minimal.
(3) (Reid-Wang) $\frac{1}{2}$ surgery on the figure eight knot is hyperbolically minimal.

We pointed out above a connection between non-zero degree maps and non-zero volume representations. In the case of a 1-cusped manifold, a similar connection shows that the decomposition of its $PSL_2(\mathbb{C})$-character into algebraic components is intimately related to the set of manifolds it dominates. On the other hand, in general it is extremely difficult to compute the $PSL_2(\mathbb{C})$-character varieties, especially those of closed manifolds, so that besides those listed above, there are very few other examples known of minimal manifolds. Theorem 2 shows that in the closed case, they are quite plentiful.

Corollary 2. (of Theorem 2) *If M is hyperbolically minimal and there is a slope α_0 such that $M(\alpha_0)$ does not strictly dominate a hyperbolic manifold, then $M(\alpha)$ is hyperbolically minimal for infinitely many slopes α on ∂M.* □

For the 1-cusped manifolds described in Example 1 we can use Remark 1 to prove

Corollary 3. *If M is any of the hyperbolic 1-cusped manifolds described in Example 1, we have $|\{\alpha : M(\alpha) \text{ is not } \mathcal{H}\text{-minimal }\}| < \infty$.*

In order to construct families of $\mathcal{G}$-minimal manifolds it is necessary to prove a version of Theorem 1 for representations to $PSL_2(\mathbb{C})$ whose images are Fuchsian. Though the presence of torsion complicates matters somewhat, the arguments go through much as before. We content ourselves with noting that these theorems imply, for instance, that when M is a hyperbolic twist knot exterior, then

$$|\{\alpha : M(\alpha) \text{ is not } \mathcal{G}\text{-minimal }\}| < \infty.$$

Limit groups and their subdirect products

Martin Bridson

What natural class of finitely generated groups should best be described as the “approximately free groups”? A remarkable fact that emerged in recent years is that when one defines “approximately” in various different ways, the class of groups one obtains is the same — *Limit groups.* One may, for example, take Gromov-Hausdorff limits of free groups, or ask which groups have the same residual properties as free groups (on arbitrary finite subsets), or take groups that are indistinguishable from free groups in first order logic (existential theory to obtain all limit groups, the full first order theory to obtain the subclass of *elementarily free groups*).

The first purpose of this lecture is to provide a quick tour through a circle of ideas indicating how these different notions of “approximately free” are connected. The second purpose is to a sketch of a body of work by myself and others that pursues the following line of thought: given an interesting property of free groups one should explore the extent to which it holds for all limit groups (or the elementarily free groups) and look for geometric consequences when such properties do hold. The third part of the lecture describes recent progress on the understanding of subdirect products of free and surface groups, and the extent to which these results can be extended to other limit groups.

1. Different Approaches to Limit Groups

The first non-trivial examples of elementarily free groups are the fundamental groups of surfaces of Euler characteristic less than -1. I begin this lecture with an explanation of why, as $n \to \infty$, the Cayley graphs of the following sequence of rank-3 free groups $F^{(n)}$ converge in the pointed Gromov-Hausdorff topology[1] to the Cayley graph of $\Sigma_2 = \langle a, b, c, d \mid [a, b] = [c, d]\rangle$

$$F^{(n)} = \langle a, b, \alpha_n, \beta_n \mid \alpha_n = [a, b]^n a[a, b]^{-n},\ \beta_n = [a, b]^n b[a, b]^{-n}\rangle.$$

A discusion of this example leads to an examination of the solutions of the *equation* $[x, y] = [a, b]$ in free groups, and the observation that solutions correspond to homomorphisms $\langle a, b, x, y \mid [x, y] = [a, b]\rangle \to F(a', b')$ sending a to a' and b to b'. This leads us naturally to examine sets $\mathrm{Hom}(G, F)$ where G is an arbitrary finitely generated group and F is a free group. A homomorphism $G \to F$ induces an action of G on the Cayley graph of F with respect to a fixed basis, and hence one can regard $\mathrm{Hom}(G, F)$ as a subset of the space of isometric actions of G on $\mathbb{R}$-trees. Such spaces have played an important role in 3-dimensional topology and geometric group theory in the last twenty years and the machinery developed in this context provides the basic tools that Zlil Sela and others used to prove powerful structure theorems for limit groups.

From the description of limit groups as GH-limits of free groups, it is not difficult to pass to the equivalent definition of them as those finitely generated groups Γ

[1] and indeed in a slightly stronger sense sometimes called the Grigorchuk topology

that are fully residually free: for every finite subset $S \subset \Gamma$ there is a homomorphism $\phi : \Gamma \to F$ where F is a free group and $\phi|_S$ is injective. One property of such groups is that they are *commutative-transitive*, meaning that for all $x, y, z \in \Gamma$, if $[x, y] = [y, z] = 1$ and $y \neq 1$, then $[x, z] = 1$. Indeed if $[x, z] \neq 1$, one could find a *free* quotient in which the images of y and $[x, z]$ were non-trivial, which is nonsense since non-trivial elements of a free group commute only if they lie in the same maximal cyclic subgroup. Being commutative-transitive is described by the first-order predicate

$$\forall x, y, z \ : \ (y = 1) \vee ([x, y] \neq 1) \vee ([y, z] \neq 1) \vee ([x, z] = 1).$$

Thus we have a hint that if a group Γ shares the first order logic of a free group, then one can deduce non-trivial group-theoretic properties.

Solving an old problem of Tarski, Zlil Sela characterized the groups with the same first order logic as free groups — elementarily free groups — as a certain geometrically-defined subclass of the limit groups. He and, independently, Kharlampovich-Myasnikov also give a topological characterization of all limit groups, which Remenslenikov showed are the groups in which the set of true first order sentences using only one quantifier $\exists$ is the same as for a free group.

One can distinguish the rank of a free abelian group in first order logic; for example the following sentence is true[2] in $\mathbb{Z}^2$ but not in $\mathbb{Z}^3$.

$$\exists s,\, t,\, u \ : \ \forall x \ \exists y \ (y^2 = x) \vee (y^2 = xs) \vee (y^2 = xt) \vee (y^2 = xu).$$

In contrast, answering a question of Tarski, Sela shows that the first order theories of all finitely generated non-abelian free groups are identical.

1.1. **Towers.** All of the results mention in Section 2 are proved using the topological characterisations of limit groups and elementarily free groups alluded to above. Limit groups are the finitely generated subgroups of π_1 of compact aspherical spaces assembled in a hierarchy from graphs,n-tori and hyperbolic surfaces that admit pseudo-Anosov diffeomorphisms. Elementarily free groups are obtained by excluding tori and not passing from the construction and not allowing passage from π_1 to a subgroup.

2. Free-Group-Like Theorems for Limit Groups

Here are some interesting properties of the class of finitely generated free groups: They are the fundamental groups of compact, 1-dimensional negatively curved spaces (graphs); they have (cohomological and topological) dimension 1; the class is closed under passage to finitely generated subgroups; they are LERF (all finitely generated subgroups are closed in the profinite topology); any finitely generated normal subgroup must be of finite index; every non-trivial element $t \in F$ is primitive in a subgroup of finite index; a subgroup is finitely generated if and only its first homology with rational coefficients is finite dimensional. At a deeper level, one has the remarkable properties of subdirect products of free groups.

[2]in $\mathbb{Z}^2$, take s, t, u to be $(0, 1)$, $(1, 0)$, $(1, 1)$ and think of odd/even

2.1. **Analogous Theorems for Limit Groups.** Since an obvious and natural generalisation of a free group is the fundamental group of a compact, locally CAT(-1) space of dimension ≤ 2, one would be worried indeed if elementarily free groups did not have such classifying spaces, and the tower construction of (1.1) shows that they do. Alibegovic and Bestvina proved that general limit groups are also the fundamental groups of compact non-positively curved spaces. It is unknown if all hyperbolic limit groups have compact negatively curved classifying spaces.

It is obvious that, as the fully residually free groups, the class of limit groups is closed under passage to finitely generated subgroups. However, the subclass formed by elementarily free groups is not closed under this operation (the larger subclass of hyperblic limit groups is).

My student Henry Wilton recently proved that elementarily free group are LERF, and has made progress on the case of arbitrary limit groups.

Jim Howie and I proved that in any limit group Γ, if $S \subset \Gamma$ has $H_1(S, \mathbb{Q})$ finitely generated, then S is finitely generated and has finite index in its normaliser N unless N abelian. And we prove an analogue of the virtual primitivity of non-trivial elements.

2.2. **Measure Equivalence and Limit Groups.** Countable groups Γ_1, Γ_2 are defined to be *measure equivalent* if there exist commuting, measure-preserving, essentially-free actions of these groups on a measure space (Ω, μ) with fundamental domains of finite measure. The prototypical examples are pairs of lattices (uniform or not) in a Lie group. For example, surface groups of negative Euler characteristic and finitely generated free groups are measure equivalent as they're lattices in $\mathrm{PSL}(2, \mathbb{R})$.

Largely to the work of Damien Gaboriau, much progress has been made in recent years in understanding measure equivalence for groups. However, it is unknown which groups are measure-equivalent to free groups. Building on work of Gaboriau, Tweedale, Wilton and I recently proved that all elementarily free groups are. The situation for general limit groups is unclear.

3. Subdirect Products

Subdirect products of free group can be remarkably wild, for example there exist finitely generated subgroups of $F_2 \times F_2$ that are not finitely presented and finitely presented subgroups $S \subset F_2 \times F_2 \times F_2$ with $H_3(S, \mathbb{Z})$ not finitely generated. And there is no algorithm to decide isomorphism among the finitely generated subgroups of $F_2 \times F_2$. Moreover, if one restricts the homological finiteness of subdirect products, then they are remarkably rare: Howie, Miller, Short and I proved that if a subdirect product S of n free groups intersects each factor non-trivially, then either it has finite index in the whole group, or else it has a subgroup of finite index $S_0 \subset S$ such that $H_k(S_0)$ is not finitely generated for some $k \leq n$. We proved the same result for subdirect products of surface groups, although the algebraic nature of the proof rendered the true nature of this extension rather mysterious.

More recently, Howie and I found a more geometric proof that extends to all elementarily free groups. A different technique allowed us to extend the result to certain subdirect products of arbitray limit groups (including the case of two factors) but the general case remains open.

The case of most geometric interest is that of subdirect products of surface groups. Indeed the remarkable work of Delzant and Gromov shows that understanding such subgroups is of central importance in the quest to understand which groups arise as fundamental groups of compact Kähler manifolds. In contrast to the above theorems, it does not seem natural here to impose homological finiteness conditions: one wants to undestand all finitely presented subgroups. Miller and I have recently made substantial progress in this direction proving, for example, that if S is a finitely presented subgroup of a direct product of at most three surface groups, then either S is virtually a product of (at most 3) free and surface groups, or else G is virtually the kernel of a map from a product of surface groups to an abelian group (in which case one can analyze S using BNS theory). A weaker result (involving nilpotent quotients) is proved for subgroups of arbitrarily many surface groups. Using these structural results, we prove that (in contrast to the situation for finitely generated subgroups) the conjugacy problem and membership problem are solvable for every finitely presented subgroups of direct products of surface groups.

Heegaard splittings, pants decompositions, and volumes of hyperbolic 3-manifolds

Jeff Brock
(joint work with Juan Souto)

Let M be a closed 3-manifold admitting a hyperbolic structure. Then M has a *Heegaard splitting* if it decomposes as a union of two handlebodies $M = H_1 \cup H_2$ along their common boundary. It is a standard result that each 3-manifold admits a Heegaard splitting, so one searches for efficient Heegaard splittings by some measure.

A *meridian* of a handlebody H is a simple closed curve on ∂H that bounds a disk in H, but not on ∂H. If M is not the 3-sphere, a Heegaard splitting of M is *strongly irreducible* if meridian for H_1 intersects each meridian for H_2. (The standard splitting of the 3-sphere into two tori is not considered strongly irreducible).

A *pants decomposition* of a closed surface S of negative Euler characteristic is a maximal collection of isotopy classes of pairwise disjoint simple closed curves on the surface, each of which is homotopically non-trivial in S (each component of the complement of these curves is a three-holed sphere). Then we say two pants decompositions P and P' differ by an *elementary move* of P' is obtained from P by removing one isotopy class α from P and replacing it by another β so that the minimal intersection number of representatives of α and β is minimal.

Let $P(S)$ be the graph with one vertex for each pants decomposition of S and an edge joining pairs of vertices whose corresponding pants decompositions differ by an elementary move. This graph, called the *pants graph* was proven to be connected by Hatcher and Thurston. It gives a notion of distance $d_P(.,.)$ between pants decompositions by assigning each each to have length one and taking $d_P(P,P')$ to be the distance in the graph $P(S)$ from P to P'.

Then pants distance gives some measure of the complexity of a Heegaard splitting in the following sense. Given a handlebody H, let $\mathcal{P}(H)$ denote the set of pants decompositions P of ∂H so that P contains a set of meridians whose corresponding disks decompose the interior $\mathrm{int}(H)$ of H into solid tori. Then $\mathcal{P}(H_1)$ and $\mathcal{P}(H_2)$ sit as subsets of the vertices of $P(S)$ where $S = \partial H_1 = \partial H_2$. Then we let

$$\delta(H_1 \cup H_2) = d_P(\mathcal{P}(H_1), \mathcal{P}(H_2)).$$

Then we prove the following theorem:

Theorem 1. *Let $M = H_1 \cup H_2$ be a strongly irreducible Heegaard splitting of M. There is a constant K, depending only on the topological type of S so that*

$$\frac{1}{K}\delta(H_1 \cup H_2) < \mathrm{vol}(M) < K\delta(H_1 \cup H_2)$$

where $\mathrm{vol}(M)$ is the hyperbolic volume of M.

We derive a similar result for handlebodies. A *hyperbolic handlebody* M is a complete hyperbolic structure on the interior of a compact handlebody H of genus g. We say M is *geometrically finite* if the convex core $CC(M)$, namely, the minimal convex subset carrying the homotopy of M, has finite volume.

Thurston observed that in the geometrically finite case the boundary $\partial CC(M)$ has the structure of a finite area hyperbolic surface X homeomorphic to ∂H when it is equipped with its path metric. A theorem of Bers guarantees that there is an L_g, depending only on g, so that X admits a pants decomposition P_X all of whose curves have length at most L.

Then we have the following theorem.

Theorem 2. *Let M be a geometrically finite hyperbolic structure on a handlebody H of genus g. Let X denote the convex core boundary of M and let P_X be a pants decomposition all of whose curves have length at most L_g on X. Then there is a K depending only on g so that we have*

$$\frac{1}{K}d_P(P_X, \mathcal{P}(H)) < \mathrm{vol}(CC(M)) < K d_P(P_X, \mathcal{P}(H))$$

where $\mathrm{vol}(CC(M))$ is the hyperbolic volume of the convex core of M and $d_P(P_X, \mathcal{P}(H))$ denoters the minimal distance from P_X to the set $\mathcal{P}(H)$.

The density conjecture for Kleinian groups with bounded geometry

Kenneth Bromberg
(joint work with Juan Souto)

We will discuss the proof of the following theorem:

Theorem 1. *Let Γ be a singly degenerate Kleinian group without parabolics. Then Γ is a an algebraic limit of quasifuchsian groups.*

Let $M = \mathbb{H}^3/\Gamma$ be the quotient hyperbolic 3-manifold. The manifold M has *ϵ-bounded geometry* if every closed geodesic in M has length $\geq \epsilon$. If M doesn't have ϵ-bounded geometry for any $\epsilon > 0$ then M has *unbounded geometry.* The proof of Theorem 1 when M has bounded geometry was first given in [11]. The unbounded geometry case was first proven in [4]. Here we will discuss a unified approach along the lines of the proof in [4]. For more general versions of Theorem 1 see [2] and [3].

Our proof has three ingredients:

(1) grafting in degenerate ends of hyperbolic 3-manifolds
(2) the deformation theory of hyperbolic cone-manifolds
(3) geometric limits of manifolds with bounded geometry

The proof of Theorem 1 when M has unbounded geometry only requires (1) and (2) and was given in [4]. The new ingredient is (3) and that is where we will concentrate our discussion.

We begin with (1). We will not define grafting. Instead will we describe the output of the construction. By work of Bonahon ([1]) M is homeomorphic to $S \times \mathbb{R}$ where S is a closed surface. The following theorem was proven in [4]:

Theorem 2. *Let γ be a simple closed curve on S and assume that the product structure on M can be chosen such that $\gamma \times \{0\}$ is a geodesic. Then there exists a quasifuchsian hyperbolic cone-manifold M_γ with cone-angle 4π such that the metrics on M and M_γ agree on $S \times (-\infty, 0)$.*

The manifold M_γ is again homeomorphic to $S \times \mathbb{R}$. It has a smooth hyperbolic metric except at $\gamma \times \{0\}$ where there is a one-dimensional cone singularity with cone-angle 4π.

Now we are ready to apply (2). Hodgson and Kerckhoff developed a deformation theory of 3-dimensional hyperbolic cone-manifolds in [8, 9, 10]. This was extended to the geometrically finite setting in [5, 6]. If the geodesic is sufficiently short this work allows us to deform the cone-manifold to a smooth cone-manifold and to bound the change in the geometry throughout the deformation.

We are now in position to sketch the proof in the case when M has unbounded geometry. Unbounded geometry implies that there exists a sequence of geodesics γ_i whose lengths limit to zero. Then a theorem of Otal ([12, 13]) implies that (after possibly passing to a subsequence) the product structure on M can be chosen such that γ_i is a simple closed curve on $S \times \{i\}$. For each γ_i we then produce a hyperbolic cone-manifold M_i as given by Theorem 2. Next we apply the deformation theory of cone-manifolds to deform the M_i to smooth quasifuchsian manifolds M_i'. These M_i' will be the approximating manifolds, proving Theorem 1 for manifolds with unbounded geometry.

Now assume M has bounded geometry. We no longer have a family of geodesics with lengths limiting to infinity. However, we can find a family of geodesics γ_i whose lengths are uniformly bounded. There are now two problems with our previous strategy. First we do not know that the γ_i are isotopic to simple curves on $S \times \{0\}$. Second we cannot apply the deformation theory of cone-manifolds because the geodesics are not short.

We can solve both problems simultaneously by lifting to a finite cover. In particular given a closed geodesic γ we can find a finite cover $\hat{M} \longrightarrow M$ such that γ has a homeomorphic lift $\hat{\gamma}$ in $\hat{M}$ that is isotopic to a product surface. Furthermore for any $R > 0$ we can choose this cover such that $\hat{\gamma}$ has an embedded tubular neighborhood of radius R. The first condition always us to apply Theorem 2 while the second condition allows us to deform the resultant cone-manifold.

We need to find this cover not for a single closed geodesic but simultaneously for the entire family of geodesics γ_i. This is where we need to discuss geometric limits of manifolds of bounded geometry.

Theorem 3. *Given ϵ, L, R there exist n and K such that for every ϵ-bounded geometry manifold M and a every closed geodesic γ in M of length $\leq L$ there exists a cover $\pi : \hat{M} \longrightarrow M$ such that the following holds.*

(1) *The geodesic γ lifts homeomophically to a simple closed geodesic $\hat{\gamma}$ that is isotopic to a simple closed curve on a product surface.*
(2) *The tube radius of $\hat{\gamma}$ is $\geq R$.*
(3) *The degree of the cover is $\leq n$.*
(4) *Let $\hat{Z}$ be the conformal boundary obtained from grafting along $\hat{\gamma}$. There exists an X in the Teichmüller space of S such that the Teichmüller distance between $\hat{Z}$ and $\pi^*(X)$ is $\leq K$.*

Sketch of Proof. Assume not. For any manifold there is some finite degree cover where (1) and (2) hold. Assume that we have a sequence of examples of manifolds M_i and curves γ_i where the degree of this cover must limit to infinity. Extract a geometric limit of the M_i with the basepoints on γ_i. This limit will again be homeomorphic to $S \times \mathbb{R}$ and therefore in the limit we only need to take a finite degree cover. By pulling this cover back to the approximates we obtain a contradiction.

Repeat this process for (4). □

We can now complete the sketch of the proof of Theorem 1.

Sketch of Proof of Theorem 1. There exists a sequences of closed geodesics γ_i of length $\leq L$ exiting the degenerate end of M. By Theorem 3 there exists an n such that for each i there is cover M_i of degree $\leq n$ such that γ_i lifts homeomorphically to an unknotted geodesic $\hat{\gamma}_i$ with tube radius $\geq R$.

There are only finitely many covers of degree $\leq n$ so we can pass to a subsequence such that each M_i is a fixed cover $\hat{M}$. Graft $\hat{M}$ along $\hat{\gamma}_i$ as in Theorem 2 and then smooth using the Hodgson-Kerckhoff cone deformation theory to obtain quasifuchsian manifolds $Q(\hat{Z}_i, \hat{Y})$ converging to $\hat{M}$. Here we are using an improved version of the deformation theory that can be found in [7].

By (4) of Theorem 3 there exists a K such that for each i there is an $X_i \in \mathcal{T}(S)$ with $d_{\mathcal{T}}(\hat{Z}_i, \pi^*(X_i)) \leq K$. (Here $\mathcal{T}(S)$ is the Teichmüller space of S.) The compactness of K-quasiconformal maps and Sullivan rigidity imply that the sequences $Q(\hat{Z}_i, \hat{Y})$ and $Q(\pi^*(X_i), \hat{Y})$ have the same limits.

Each $Q(\pi^*(X_i), \hat{Y})$ will isometrically cover $Q(X_i, Y)$ and therefore $Q(X_i, Y)$ will converge to M completing the proof of Theorem 1. □

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Quasiconformal Homogeneity of Hyperbolic Manifolds

Richard D. Canary

(joint work with Petra Bonfert-Taylor, Gaven Martin, and Edward Taylor)

A hyperbolic manifold N is said to be K-quasiconformally homogeneous if given any two points $x, y \in N$ there exists a K-quasiconformal homeomorphism $f : N \to N$ such that $f(x) = y$. If N is K-quasiconformally homogenous for some K, we say that N is uniformly quasiconformally homogeneous.

One observes immediately that a closed hyperbolic manifold is uniformly quasiconformally homogeneous (as any diffeomorphism is quasiconformal) and one further sees, with a little more effort, that any regular cover of a closed hyperbolic manifold is uniformly quasiconformally homogeneous.

It then follows quickly from the fact that quasiconformal maps are quasi-isometries that any uniformly quasiconformally homogeneous hyperbolic manifold has bounded geometry. Let $\ell(N)$ be the infimum of the lengths of closed geodesics in N and let $d(N)$ be the supremum of the diameters of embedded hyperbolic balls in N. We further show that

Theorem 1: *For each dimension $n \geq 2$ and each $K \geq 1$, there is a positive constant $m(n, K)$ with the following property. Let $N = \mathbf{H}^n/\Gamma$ be a K–quasiconformally homogeneous hyperbolic n-manifold, which is not $\mathbf{H}^n$. Then*

(1) $d(N) \leq K\ell(N) + 2K \log 4$.
(2) $\ell(N) \geq m(n, K)$, *i.e. there is a lower bound on the injectivity radius of N that only depends on n and K.*
(3) *Every nontrivial element of Γ is hyperbolic and the limit set $\Lambda(\Gamma)$ of Γ is $\partial\mathbf{H}^n$.*

We then use McMullen's version [3] of Sullivan's rigidity theorem [4] to show that, in fact, a hyperbolic manifold of dimension at least three is uniformly quasiconformally homogeneous if and only if it is a regular cover of a closed hyperbolic orbifold.

Theorem 2: *Suppose that $n \geq 3$. A hyperbolic n-manifold is uniformly quasiconformally homogeneous if and only if it is a regular cover of a closed hyperbolic orbifold.*

The same argument produces a uniform lower bound on the quasiconformal homogeneity constant of a uniformly quasiconformally homogeneous hyperbolic n-manifold for all $n \geq 3$ (other than $\mathbf{H}^n$.)

Theorem 3: *If $n \geq 3$, there is a constant $K_n > 1$ such that if N is a K–quasiconformally homogeneous hyperbolic n–manifold which is not $\mathbf{H}^n$, then $K \geq K_n$.*

We note that Theorem 2 does not hold in dimension 2 and that it is unknown whether Theorem 3 holds in dimension 2.

Finally, we observe that in dimension 3, the only non-compact uniformly quasiconformally homogeneous hyperbolic 3-manifolds with finitely generated fundamental group arise as covers associated to the fibers of closed hyperbolic 3-manifolds which fiber over the circle.

Theorem 4: *Let N be a noncompact uniformly quasiconformally homogeneous hyperbolic 3-manifold with finitely generated fundamental group. Then there exists a closed hyperbolic 3-manifold M which fibers over the circle such that N is the cover associated to the fiber.*

As an epilogue, we sketched the proof of a recent result [2] with Chris Leininger, whose proof makes use of techniques related to those used in the proof of Theorem 4. (For our purposes the real length spectrum of a Kleinian group is simply the set of translation lengths of all hyperbolic elements.)

Theorem 5: *Suppose that Γ is a finitely generated, torsion-free Kleinian group. Then the real length spectrum $\mathcal{L}(\Gamma)$ is discrete if and only if either*

(1) *Γ is geometrically finite,*
(2) *there exists a hyperbolic 3-manifold $M = \mathbf{H}^3/G$ which fibers over the circle and Γ is a fiber subgroup of G, or*
(3) *there exists a hyperbolic 3-manifold $M = \mathbf{H}^3/G$ which fibers over $S^1/\langle z \mapsto \overline{z}\rangle$ and Γ is a singular fiber subgroup of G.*

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Real projective structures on 3-Manifolds

Dary Cooper

(joint work with D. Long, M. Thistlethwaite, S. Kerckhoff, and J. Porti (partly))

We discuss various aspects of projective geometry in relation to 3-dimensional topology. A *geometry* in this talk means a pair (X, G) where X is a smooth manifold and G is a group of diffeomorphisms of X. Some examples (1) X is the universal cover of a smooth manifold and G is the group of covering transformations. (2) X is a homogeneous space and G is the group of isomteries of X (3) X is real projective space and G is the real projective general linear group. A *representation* of a geometry (X, G) into a geometry (Y, H) is a local diffeomorphism $dev : X \to Y$ and a homomorphism $hol : G \to H$ such that for all $x \in X$ and all $g \in G$ we have $dev(g.x) = hol(g).dev(x)$. An (X, G)-geometric structure on a manifold M is a representation of $(\tilde{M}, Aut(\tilde{M})$ into (X, G). A *projective representation* of a geometry (X, G) is a representation of (X, G) into $(RP^n, Aut(RP^n))$

A theorem due to Thurston, Molnar and Thiel is that each of the eight Thurston geometries has a projective representation, except that for the product geometries one must restrict to the index-2 subgroup which preserves the fiber orientation.

The only known example of a connected 3-manifold which does not admit a projective structure is the connect sum of two copies of real projective space. We discuss examples of closed hyperbolic 3-manifolds that deform projectively. We discuss the hyperbolic/Euclidean/spherical transition in the proof of the orbifold theorem from the projective viewpoint. Finally we mentioned some work with Porti on the new class of pseudo-conformal geometries.

Non-zero degree maps between three-manifolds

Pierre Derbez

1. introduction

According to [1], we say that a closed orientable 3-manifold M dominates an other one N if there exists a degree one map $f\colon M \to N$. This relation of domination, denoted by $\geq$, gives an interesting way to measure the topological complexity when we consider the set $\mathcal{G}$ of closed orientable 3-manifolds satisfying the Poincaré-Thurston Geometrization Conjecture. This means that $\mathcal{G}$ consists of geometric 3-manifolds, Haken manifolds and connected sums of such manifolds. Then the relation $\geq$ define a partial order on the set $\mathcal{G}$, up to homotopy equivalence. In this talk we study the following question which was raised in the 1980's and formally appeared in [3][3.100].

Question 1. *Given a closed orientable 3-manifold M, are there at most finitely many 3-manifolds N in $\mathcal{G}$, up to homeomorphism, dominated by M?*

Note that in this question we consider always degree one maps to avoid some easy counter examples. For instance for any spherical Lens space $L(p, q)$ there always exists a nonzero degree map (actually a finite covering) from the 3-sphere $\mathbf{S}^3$ to $L(p, q)$. There are many partial results concerning Question 1. More precisely combining the results of [2], [4], [5] and [6] we know that the answer to Question 1 is positive when the targets are geometric 3-manifolds. Hence the next step is to study Question 1 when the targets are Haken manifolds. More precisely, our main result states as follows:

Theorem 1. *Any closed orientable 3-manifold M dominates at most finitely many closed Haken manifolds N with the same Gromov simplicial volume as M.*

This result allows the solve Question 1 for graph manifolds.

Corollary 1. *Any closed orientable graph manifold 1-dominates at most finitely many closed orientable 3-manifolds satisfying the Poincaré-Thurston Geometrization Conjecture.*

Denote by $\hat{\mathcal{G}}_0$ the set of graph manifolds up to homotopy equivalence. Then Corollary 1 implies that the poset $\left(\hat{\mathcal{G}}_0, \geq\right)$ is locally finite.

2. On the Proof of Theorem 1

Given a closed Haken manifold N we denote by $\mathcal{T}_N$ the Jaco-Shalen-Johannson family of canonical tori of N and by N^* the space $N \setminus \mathcal{T}_N$. Denote by $s\colon \underline{\mathrm{N}}^* \to \underline{\mathrm{N}}^*$ the sewing involution of N. Consider now two Haken manifolds N_1 and N_2 with sewing involutions s_1 and s_2. We say that the two ordered pairs $(N_1^*, s_1), (N_2^*, s_2)$ are *equivalent* if there is a homeomorphism $\eta\colon\ N_1^* \to N_2^*$ such that $\eta \circ s_1$ and $s_2 \circ \eta$ are isotopic. Using this notation two Haken manifolds N_1 and N_2 are homeomorphic if and only if the two ordered pairs (N_1^*, s_1) and (N_2^*, s_2) are equivalent. On the other hand we will say, for convenience, that two Haken manifolds N_1 and N_2 are *weakly equivalent* if there is a homeomorphism $\eta\colon\ N_1^* \to N_2^*$.

Let M be a closed orientable 3-manifold. The proof of Theorem 1 contains two steps. In the first one, we show the finiteness of the geometric decomposition of the target manifolds and in the second one we prove the finiteness of the isotopy classes of the sewing involutions of the target.

2.1. **Finiteness of the geometric decomposition of the targets.** Let $\{N_i\}_{i\in\mathbf{N}}$ denote a sequence of closed Haken manifolds dominated by M. The purpose of this step is to prove that the sequence $\{N_i^*\}_{i\in\mathbf{N}}$ is finite up to homeomorphism. This depends on the following key result which says that a nonzero degree map f into a Haken manifold N has a kind of *canonical standard form* with respect to the geometric decompostion of N.

Proposition 1 (Standard Form)**.** *Any closed orientable 3-manifold M admits a finite set $\mathcal{H} = \{M_1, ..., M_k\}$ of closed Haken manifolds satisfying the following property. For any nonzero degree map $g\colon\ M \to N$ into a closed non-geometric Haken manifold N containing no embedded Klein bottles and satisfying* $\mathrm{Vol}(M) = \deg(g)\mathrm{Vol}(N)$ *there exists at least one element M_i in $\mathcal{H}$ and a map $f\colon\ M_i \to N$ with the same degree as g such that:*

(i) $\mathrm{Vol}(M_i) = \deg(f)\mathrm{Vol}(N)$*, and*

(ii) f induces a finite covering between $\mathcal{H}(M_i)$ and $\mathcal{H}(N)$, and

(iii) for any geometric component Q in N^* the preimage $f^{-1}(Q)$ is a canonical submanifold of M.

Using the additivity of the Gromov simplicial volume with respect to connected sums, we may assume in the proof of Proposition 1 that M is Haken. The proof of Proposition 1 is based on the observation that when $\mathrm{Vol}(M) = \deg(f)\mathrm{Vol}(N)$ then we can "control" the "essential part" of $f^{-1}(\mathcal{T}_N)$. Actually one can show, up to homotopy, that this essential part is a subfamily of $\mathcal{T}_M$ which is crucial in our proof since this ensures that the genus of the essential components of $f^{-1}(\mathcal{T}_N)$ is bounded independently of N. This control can not be accomplished when $\mathrm{Vol}(M) \gg \deg(f)\mathrm{Vol}(N)$. The family $\mathcal{H}$ of Haken manifolds in Proposition 1 comes from a finite family of canonical submanifolds $\mathcal{A}$ of M after some Dehn fillings. The main point is the control of the finiteness of the slopes of the Dehn fillings performed along the components of $\mathcal{A}$ to obta! in $\mathcal{H}$.

2.2. **Finiteness of the sewing involutions of the targets.** The purpose of this step is to prove the following

Proposition 2. *Let M be a closed orientable 3-manifold. Let N_i be a sequence of weakly equivalent non-geometric closed Haken manifolds such that there exists a degree-one map $g_i \colon M \to N_i$ satisfying* $\mathrm{Vol}(M) = \mathrm{Vol}(N_i)$*. For each $i \in \mathbf{N}$, we denote by $s_i \colon \underline{N}_i^* \to \underline{N}_i^*$ the sewing involution corresponding to N_i. Then the sequence $\{(N_i^*, s_i), i \in \mathbf{N}\}$ is finite, up to equivalence of pairs.*

Throughout the proof of Proposition 2 we will use the collection of closed Haken manifolds $\mathcal{H}$ given by Proposition 1. Points (i), (ii) and (iii) say that the elements of $\mathcal{H}$ dominate the manifolds N_i's in a convenient way. Roughly speaking, the core of the proof of Proposition 2 is to show that the sewing involution associated to each Haken manifold of $\mathcal{H}$ does fix the sewing involutions s_i which produces N_i from N_i^*. More precisely, the sewing involutions $s_i, i \in \mathbf{N}$ are fixed by some equations parametrized by a family of cross sections $\{\mathcal{S}_i\}_{i\in\mathbf{N}}$ of the Seifert pieces of the targets. Then the main point in the proof of Proposition 2 is to show that the sewing involutions of the domain allows to control the finiteness of the family $\{\mathcal{S}_i\}_{i\in\mathbf{N}}$. Note that in this step the condition on the Gromov simplicial volume is still crucial in our proof.

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Deformations of Hyperbolic 3-Manifolds with Boundary

Steve Kerckhoff

(joint work with Craig Hodgson)

A central goal in the geometric theory of 3-manifolds is to understand the connection between the topological, combinatorial, and geometric properties of 3-manifolds. There are well-known methods for describing all 3-manifolds: via Heegaard splittings and via surgery on knots and links. However, any given 3-manifold has many such descriptions. On the other hand, the most prevalent geometric structure in dimension 3, a hyperbolic structure, is unique when it exists. This means that geometric structures can provide valuable topological invariants; it also means that they can be very hard to find.

One method of constructing hyperbolic structures is to cut the manifold into pieces, put geometric structures on the pieces, and then try to match up the structures on the pieces so they agree on the overlap. This has been a successful approach because of what is often called "geometric inflexibility". One can vary the boundary pieces a lot, making it possible to match things up, without causing much change in the structure away from the boundary.

A manifold with boundary has a non-trivial deformation space of hyperbolic structures, and one expects the deformations to be controlled and parametrized by boundary data. Typically, the deformation space of structures is a half-dimensional subset of the deformations of the boundary. A prototype is the theory developed by Alhfors and Bers which shows that geometrically finite hyperbolic structures are parametrized by the conformal structures at infinity of boundary surfaces of genus at least 2. These can be viewed as sitting in the deformation space of representations into $PSL(2, \mathbb{C})$ of the fundamental group of the boundary surfaces, which has twice the dimension.

Requiring that it extend over the entire 3-manifold imposes constraints on a deformation of a boundary surface. A natural problem is to understand these constraints. This type of analysis was the key to Thurston's geometrization of Haken manifolds ([8]). He proved a fixed point theorem for the "skinning map" that allowed him to glue together structures on the pieces that arose from the decomposition of a Haken manifold from its Waldhausen hierarchy. The main drawback is that there is little information about the resulting geometric structure because the proof is not explicitly effective.

Thus, a natural problem is to find a proof of Thurston's theorem that provides more information about the resulting hyperbolic structure.

Problem. *Find an effective proof of Thurston's fixed point theorem for the skinning map.*

There is another important topological process, called Dehn filling, where one attaches a solid torus to a manifold with torus boundary. The resulting topological manifolds are parametrized by the isotopy class of the simple closed curve on the torus boundary which bounds a disk in the solid torus; these are given by pairs of relatively prime integers (p, q). One can attempt to put a hyperbolic structure on the closed manifold by the varying hyperbolic structure on the 3-manifold with torus boundary until it matches one on the solid torus. Thurston showed that this was always possible for all but a finite number of (p, q)-fillings. However, there was little information about the excluded set or the geometry of the resulting hyperbolic structure.

Over the last several years an effective theory of Dehn surgery has been developed by Hodgson and the author ([1] and [2]). Using this, one can find a uniform bound on the number of non-hyperbolic fillings, identify the possible exceptional fillings, and estimate quite sharply the variation in geometry between the non-compact, finite volume complete structure and the filled manifold. It is based on an effective version of local rigidity for hyperbolic cone-manifolds and for manifolds with toral boundary. This results in a type of geometric inflexibility called "effective rigidity".

If a closed 3-manifold M is described by gluing together two handlebodies (a Heegaard splitting), it is natural to ask whether, if M is hyperbolic, the hyperbolic geometry reflects this decomposition. There are many topological Heegaard splittings of a given manifold so ones that reflect the underlying geometry would likely be of a special form. Conversely, information about the underlying geometry can lead to conclusions about the possible types of Heegaard splittings in the topological manifold. Because of the recent work of Minsky ([5]), which constructs quasi-isometric models for hyperbolic structures on a surface crossed with an interval, there has been a lot of recent activity on this problem. An approach is to construct a "hyperbolic Heegaard splitting" in two steps. First, one constructs a hyperbolic structure on a surface crossed with an interval using Minsky's models. Then one uses deformation theory to deform the structure near the boundary components so that it extends over the rest of the manifold. Thus, the Heegaard splitting is viewed geometrically as attaching handlebodies to the two ends of the product. The rationale for this approach is that it should be possible to choose the structure on the boundary of the product so that it will have to be deformed by a uniformly bounded amount. When the boundary components are sufficiently far apart, it should be possible, by geometric inflexibility, to do this deformation without changing the interior of the manifold much.

Namazi ([6]) has partially carried out this program, obtaining a metric with pinched negative curvature, when the attaching map is a high power of a pseudo-Anosov diffeomorphism. Further analysis of these examples by Namazi and Souto provides more information, like the Heegaard genus and the rank of the fundamental group for these manifolds.

A convex, co-compact geometrically finite Kleinian group Γ has a compact core which is the smallest convex subset of $M = \mathbb{H}^3/\Gamma$ carrying the fundamental group of M. Its boundary is a piecewise geodesic hyperbolic surface bent along a measured geodesic lamination, called its bending lamination. There are many submanifolds of M with smooth convex boundary; they all contain the convex core. One can study the deformations of hyperbolic structures on M by studying the deformations of these manifolds with convex boundary.

A convex surface in a hyperbolic 3-manifold has an induced metric with negative curvature at least -1 at each point. It also has a shape operator (second fundamental form) that describes its embedding, and these must satisfy the Gauss and Codazzi equations. The infinitesimal deformations of convex surfaces can be viewed as infinitesimal variations of the induced metric and the shape operator on the surface satisfying, to the first order, the Gauss and Codazzi equations. For such a deformation to extend to an infinitesimal deformation of the hyperbolic structure on a 3-manifold with this surface as boundary puts further restrictions on the deformation. One would like to understand those restrictions and to derive local rigidity results from them. Such local rigidity results should lead to effective rigidity results which will aid in understanding the deformed hyperbolic structures and allow one to piece together new hyperbolic structures with geometric control.

We restrict to constant curvature κ metrics, where $\kappa \in (0, -1)$ is fixed. Then the space of such metrics is parametrized by the Teichmuller space of genus g which has dimension $6g - 6$; the space of compatible shape operators for a given metric also has dimension $6g-6$ so the total dimension of the space of such convex surfaces is $12g - 12$. Labourie [4] has shown that there is always exactly one such surface in the end of a geometrically finite 3-manifold. The space of geometrically finite structures has dimension $6g-6$, so the condition that a convex surface actually be the convex boundary of a hyperbolic structure on M picks out a half-dimensional subset. Thus, the condition on an infinitesimal deformation of the surface extend over the 3-manifold should be a half-dimensional one. This is consistent with the corresponding boundary value problem being elliptic.

A major step in developing a useful deformation theory of this type would be to find model deformations for these convex boundary surfaces, one type that varies the metric in all possible ways, and another that varies the second fundamental form. In the very special case when the surface is totally umbilic, there are very nice model deformations, coming from holomorphic quadratic differentials. In the general case, one expects the correct models to come from some generalized type of quadratic differential that are in the kernel of the $\bar{\partial}$ operator twisted by the second (or third) fundamental form. Operators and differentials of this type play an important role in the work of Labourie and Schlenker ([3], [4], [7]).

Problem. *Find model harmonic deformations for the boundary of a convex hyperbolic 3-manifold and prove local rigidity results using them.*

Recently, Schlenker [7] has shown that, in the case of smooth, convex boundary, a hyperbolic metric on a 3-manifold is uniquely determined by that of its boundary. It is not possible to deform the hyperbolic structure on M in such a way that only the shape operator, and not the metric, on the boundary is changed. It is similarly conjectured that the metric on the (non-smooth) boundary of the convex core uniquely determines the metric on the 3-manifold. It is also conjectured that the 3-manifold is rigid relative to the bending lamination.

Schlenker's proof is quite surprising, proving this hyperbolic result by using classical results on convex surfaces in Euclidean 3-space. Unfortunately, the proof is non-effective, not relating the variation of the metric on M with that of the boundary. It also depends very strongly on smoothness so it is not clear how to apply it to the convex core problem.

The following problems are still unsolved:

Problem. *Give an effective proof that hyperbolic 3-manifolds with smooth convex boundary are locally rigid relative to the metric on their boundaries.*

Problem. *Prove that geometrically finite hyperbolic 3-manifolds are locally rigid relative to the metric on their convex hull boundaries.*

Problem. *Prove that geometrically finite hyperbolic* 3*-manifolds are locally rigid relative to the bending lamination on their convex hull boundaries.*

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Generalizations of Agol's inequality and nonexistence of essential laminations

Thilo Kuessner

Theorem (**Agol-Storm-Thurston**, [2], Cor.2.2.): *If M is a hyperbolic 3-manifold containing an incompressible surface F, then*

$$Vol\,(M) \geq Vol\left(Guts\left(\overline{M-F}\right)\right)$$
$$\geq -V_{oct}\chi\left(Guts\left(\overline{M-F}\right)\right) = V_{oct}\frac{1}{4} \parallel \partial Guts\left(\overline{M-F}\right) \parallel$$

with $V_{oct} = 3.66..$ the volume of a regular ideal octahedron in $\mathbb{H}^3$.

Notation:

- $N = \overline{M-F}$ compact, atoroidal 3-manifold with boundary, JSJ-decomposition (cut along annuli): $N = Char\,(N) \cup Guts\,(N)$, $Char\,(N)$ consists of I-bundles and solid tori, $Guts\,(N)$ is acylindrical, hence admits a hyperbolic metric with geodesic boundary and cusps
- simplicial volume: X compact, orientable manifold, $[X, \partial X] \in H_n\,(X, \partial X; \mathbb{R})$ fundamental class $\parallel X \parallel := inf\,\{\sum \mid a_i \mid : \sum a_i\sigma_i \text{ represents } [X, \partial X]\}$ Theorem (Gromov): X^n hyperbolic of finite volume: $\parallel X \parallel = \frac{1}{V_n} Vol\,(X)$, $V_3 = 1.01..$ volume of regular ideal tetrahedron in $\mathbb{H}^3$

Its proof in [2] uses analytical methods (Perelman's entropy estimate for the Ricci flow, work of Bray and Miao on the Penrose conjecture) and does not seem to generalize to laminations. Using the original topological approach of [1], we want to give a generalization to laminations and to higher-dimensional non-hyperbolic manifolds.

Let $\mathcal{F}$ be a lamination on a manifold M. A singular simplex $\sigma : \Delta^n \rightarrow M$ is **transverse** to $\mathcal{F}$ if the pull-back of $\mathcal{F}$ to Δ^n is an affine lamination, it is **normal** to $\mathcal{F}$ if the pull-back of each single leaf to Δ^n is an affine lamination.

It was first observed in [3] that a refinement of the simplicial volume gives a meaningful invariant for foliations and laminations.

Definition 1. *: Let M be a compact, oriented, connected manifold, possibly with boundary, and $\mathcal{F}$ a foliation or lamination on M. Then define*

$$\parallel M \parallel_{\mathcal{F}} :=$$

$$inf \left\{ \sum_{i=1}^{r} \mid a_i \mid : \psi \left(\sum_{i=1}^{r} a_i \sigma_i \right) \ represents \ [M, \partial M], \sigma_i : \Delta^n \rightarrow M \ transverse \ to \ \mathcal{F} \right\}$$

$$\parallel M \parallel_{\mathcal{F}}^{normal} :=$$

$$inf \left\{ \sum_{i=1}^{r} \mid a_i \mid : \psi \left(\sum_{i=1}^{r} a_i \sigma_i \right) \ represents \ [M, \partial M], \sigma_i : \Delta^n \rightarrow M \ normal \ to \ \mathcal{F} \right\}.$$

There is an obvious inequality

$$\parallel M \parallel \leq \parallel M \parallel_{\mathcal{F}}^{normal} \leq \parallel M \parallel_{\mathcal{F}} .$$

In the case of foliations, equality $\parallel M \parallel_{\mathcal{F}}^{normal} = \parallel M \parallel_{\mathcal{F}}$ holds.

Theorem 1. *(K., [5], Thm.1): Let M be a compact, orientable n-manifold and $\mathcal{F}$ a lamination (of codimension one) of M. Assume that there exists a compact, aspherical, n-dimensional submanifold $Q \subset \overline{M - \mathcal{F}}$ such that, if we let $N = \overline{M - \mathcal{F}}, \partial_0 N = \partial N \cap \partial M, \partial_1 N = \overline{\partial N - \partial_0 N}, \partial_{00} Q = \overline{\partial Q - (\partial N \cap \partial Q)}, \partial_{01} Q = \partial Q \cap \partial_0 N, \partial_0 Q = \partial_{00} Q \cup \partial_{01} Q, \partial_1 Q = \partial_1 N \cap \partial Q$, then*
i) each component of $\partial_{00} Q := \overline{\partial Q - (\partial M \cap \partial Q)}$ has amenable fundamental group,
ii) $(Q, \partial_1 Q)$ is pared acylindrical,
iii) the decomposition $N = Q \cup \overline{(N - Q)}$ is essential (i.e. the inclusions $Q \rightarrow N, \overline{N - Q} \rightarrow N, \partial_0 Q \rightarrow Q, \partial_0 Q \rightarrow \overline{N - Q}$ are π_1-injective) . Then

$$\parallel M \parallel_{\mathcal{F}}^{normal} \geq \frac{1}{n+1} \parallel \partial Q \parallel, \parallel M \parallel_{\mathcal{F}} \geq \frac{1}{\left[\frac{n}{2}\right] + 1} \parallel \partial Q \parallel .$$

Proof: To give a flavor of the argument, we describe it in the simplest case: M is a hyperbolic 3-manifold, $\mathcal{F} = F$ a geodesic surface (i.e. $\partial Guts \left(\overline{M - \mathcal{F}} \right) = 2F$).

Let $\sum a_i \sigma_i$ be a normal cycle representing $[M]$. Then $\sum a_i \left(\sigma_i \cap F \right)$ represents $[F]$ and to get the wanted inequality $\sum \mid a_i \mid \geq \frac{1}{4} \parallel 2F \parallel$ it would suffice to have that each σ_i intersects F in at most 4 simplices.

Of course, there is a priori no reason to have an upper bound on the number of intersections that a normal simplex may have with the geodesic surface.

However, one can easily see that, whenever a 3-simplex intersects the surface more than 4 times, than two of the intersection triangles must have a parallel edge, i.e. cut out a square on one boundary face of the standard 3-simplex.

If σ_i mapped this square to a cylinder (i.e. mapped the opposite edges to the same edge), then one could use the acylindricity of $\overline{M-F}$ to argue that the cylinder degenerates after homotopy, hence can be removed without changing the homology class, and thus the number of intersections can be reduced.

Then the proof consists of defining a straightening which produces the maximally possible number of cylinders. (Some care is needed because the subdivided 1-skeleton can, of course, not be straightened arbitrarily. Even though each 1-simplex can be moved freely, the 2-skeleton imposes homotopy relations between concatenations of 1-simplices, which have to be respected by the straightening.)

Roughly the same argument works whenever $Q = N$. To handle the general case $Q \neq N$, one would like to define a retraction $r : C_*(N) \to C_*(Q)$. It seems not possible to define r directly, but, using Gromov's work on multicomplexes, one can at least define it up to some 'amenable ambiguity' and use this to prove the general case. □

Corollary 1. *: Let M be a compact Riemannian n-manifold of negative sectional curvature and finite volume. Let $F \subset M$ be a geodesic $n-1$-dimensional hypersurface of finite volume. Then $\| F \| \leq \frac{n+1}{2} \| M \|$.*

Proof: : Consider the lamination given by F. Its complement $N = \overline{M-F}$ is aspherical and pared acylindrical. If N is compact we can choose $Q = N$, in which case the other assumptions of Theorem 1 are trivially satisfied. If N is not compact, we cut off the cuspical ends along submanifolds with amenable fundamental groups to get Q. From Theorem 1 we conclude $\| M \|_F^{normal} \geq \frac{1}{n+1} \| \partial N \|$. The boundary of N consists of two copies of F, hence $\| \partial N \| = 2 \| F \|$. Moreover $\| M \|_F^{normal} = \| M \|$, because any cycle can be homotoped to be normal to F. The claim follows. □

Corollary 2. *: Let M be a compact hyperbolic 3-manifold and $\mathcal{F}$ an essential lamination of M. Then*

$$\| M \|_{\mathcal{F}}^{normal} \geq \frac{1}{2} \| \partial Guts\left(\overline{M-\mathcal{F}}\right) \| .$$

Proof: The inequality $\| M \|_{\mathcal{F}}^{normal} \geq \frac{1}{4} \| \partial Guts\left(\overline{M-\mathcal{F}}\right) \|$ follows from Theorem 1. The improvement by a factor 2 can be obtained as in [1] by considering more general polyhedral (rather than simplicial) norms. □

If $\mathcal{F}$ consists of one compact, incompressible surface, then $\| M \|_{\mathcal{F}}^{normal} = \| M \|$, and the above inequality is exactly the weaker form of Agol's inequality, from [1]. We discuss the application to non-existence results for laminations on 3-manifolds.

For a lamination $\mathcal{F}$ on a 3-manifold M, its leaf space is defined as $T = \widetilde{M}/\sim$, where two points of the universal covering $\widetilde{M}$ are identified if either they belong to the same leaf of the pull-back lamination $\widetilde{\mathcal{F}}$, or they belong to the same connected component of the complement $\overline{\widetilde{M} - \widetilde{\mathcal{F}}}$. (One assumes that $\mathcal{F}$ has no isolated leaf, which can be assured by replacing each isolated leaf with a one-parameter family of leaves.)

Corollary 3. *: If M is a hyperbolic 3-manifold and $\mathcal{F}$ a tight lamination (i.e. the leaf space T is Hausdorff), then $Vol(M) \geq V_3 \frac{1}{2} \parallel \partial Guts\left(\overline{M - \mathcal{F}}\right) \parallel$.*

Proof: If $\mathcal{F}$ is tight, then each cycle can be homotoped to be normal to $\mathcal{F}$, thus $\parallel M \parallel_{\mathcal{F}}^{normal} = \parallel M \parallel$. The claim follows then from corollary 2. □

Tao Li has proved that the existence of a transv. orientable essential lamination on a given hyperbolic 3-manifold M implies that the same M must also carry a tight lamination.

If M is hyperbolic and carries a tight lamination with empty guts, then Calegari and Dunfield have shown ([4], Theorem 3.2.) that there is an injective homomorphism $\pi_1 M \rightarrow Homeo\left(S^1\right)$. Calegari and Dunfield also observed that a generalization of Agol's inequality to tight laminations would give obstructions to existence of laminations with nonempty guts, and, e.g., exclude existence of tight laminations on the Weeks manifold.

The following corollary applies, for example, to all hyperbolic manifolds M obtained by Dehn-filling the complement of the figure-eight knot in S^3. (It follows from work of Hatcher that each of these M contains tight laminations. By the following corollary, all these tight laminations have empty guts.)

Corollary 4. *: If M is a closed hyperbolic 3-manifold with $Vol(M) < 2V_3 = 2.02...$, then each tight lamination has empty guts.*

Proof: If $Guts\left(\overline{M - \mathcal{F}}\right)$ is not empty, then $\parallel \partial Guts\left(\overline{M - \mathcal{F}}\right) \parallel \geq 4$. □

Corollary 5. *: The Weeks manifold W admits no transv. orientable essential lamination.*

Proof: Calegari-Dunfield have shown that $\pi_1 W$ is not a subgroup of $Homeo^+ \mathbb{S}^1$. Moreover, $Vol(W) = 0.94.. < 2.02...$ Apply corollary 4. □

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The Waldhausen Conjecture

Tao Li

A *Heegaard* splitting of a closed and orientable 3–manifold M is a decomposition $M = H_1 \cup_S H_2$, where $S = \partial H_1 = \partial H_2 = H_1 \cap H_2$ is a closed embedded separating surface and each H_i $(i = 1, 2)$ is a handlebody. The surface S is called a *Heegaard surface*, and the genus of S is the genus of this Heegaard splitting. Every closed and orientable 3–manifold has a Heegaard splitting.

A Heegaard splitting is *reducible* if there is an essential curve in the Heegaard surface that bounds compressing disks in both handlebodies. A Heegaard splitting $M = H_1 \cup_S H_2$ is *weakly reducible* [1] if there exist a pair of compressing disks $D_1 \subset H_1$ and $D_2 \subset H_2$ such that $\partial D_1 \cap \partial D_2 = \emptyset$. If a Heegaard splitting is not reducible (resp. weakly reducible), then it is *irreducible* (resp. *strongly irreducible*). A lemma of Haken [4] says that if M is reducible, then every Heegaard splitting is reducible. Casson and Gordon [1] showed that if a Heegaard splitting of a non-Haken 3–manifold is irreducible, then it is strongly irreducible.

A conjecture of Waldhausen asserts that a closed orientable 3–manifold has only a finite number of Heegaard splittings of any given genus, up to homeomorphism. Johannson [6, 7] proved this conjecture for Haken manifolds. If M contains an incompressible torus, one may construct an infinite family of homeomorphic but non-isotopic Heegaard splittings using Dehn twists along the torus. The so-called generalized Waldhausen conjecture says that a closed, orientable and atoroidal 3–manifold has only finitely many Heegaard splittings of any genus, up to isotopy. This is also proved to be true for Haken manifolds by Johannson [6, 7]. In this report, we outline a proof of the (generalized) Waldhausen conjecture, see [8] for details. A much stronger theorem for non-Haken 3–manifolds is proved in [9].

Theorem 1. *A closed, orientable, irreducible and atoroidal 3–manifold has only finitely many Heegaard splittings in each genus, up to isotopy.*

The first ingredient of the proof is normal surface theory. A normal disk in a tetrahedron is a triangle or a quadrilateral that meets each edge in at most one point. A surface is called a normal surface, if the intersection of the surface with each tetrahedron consists of normal disks. Any incompressible surface is isotopic to a normal surface [3].

An almost normal piece in a tetrahedron is either an octagon, or an annulus obtained by connecting two normal disks using an unknotted tube, see [14] for a picture. An embedded surface S is *almost normal* if S is normal except in one tetrahedron T, where $T \cap S$ consists of normal disks and at most one almost normal piece. A theorem of Rubinstein and Stocking [12, 14] says that every strongly irreducible Heegaard surface is isotopic to an almost normal surface.

The second ingredient of the proof is the theory of branched surfaces. A branched surface is a 2-dimensional generalization of a train track, see [2, 11] for a picture and basic properties. Given an almost normal surface, by identifying the parallel normal disks, we get a branched surface, and we say that this branched surface *fully carries* the almost normal surface. Since there are only a finitely number of normal disk types, there are only finitely many different such branched surfaces. This is basically a construction in [2], where the authors showed that every incompressible surface is fully carried by one of a finite collection of branched surfaces. It follows trivially that, after isotopy, every strongly irreducible Heegaard surface is fully carried by one of a finite collection of branched surfaces.

For any branched surface B, there is a one-to-one correspondence between compact surfaces carried by B and nonnegative integer solutions to the system of branch equations, see [11] for basic definitions. Given any two compact surfaces F_1 and F_2 carried by the B, the sum $F_1 + F_2$ is obtained by a canonical cutting-and-pasting along the double curves. The sum $F_1 + F_2$ is also a surface carried by B and its corresponding integer solution is the vector sum of the two integer solutions corresponding to F_1 and F_2.

A crucial part of the proof is to analyze normal surfaces with nonnegative Euler characteristic. Since the 3–manifold is orientable, if a branched surfaces does not carry any normal 2–sphere or torus, then it dose not carry any closed normal surface with nonnegative Euler characteristic.

Theorem 2. *Let M be a closed orientable, irreducible and atoroidal 3–manifold, and suppose M is not a small Seifert fiber space. Then, M has a finite collection of branched surfaces, such that*

(1) *each branched surface in this collection is obtained by gluing together normal disks and at most one almost normal piece, similar to* [2]*,*
(2) *up to isotopy, each strongly irreducible Heegaard surface is fully carried by a branched surface in this collection,*
(3) *no branched surface in this collection carries any normal 2–sphere or normal torus.*

Proof of Theorem 1 using Theorem 2. Suppose a branched surface B in Theorem 2 fully carries an infinite family of almost normal surfaces of genus g. Then one can find two surfaces S_1 and S_2 in this family such that $S_2 = S_1 + T$, where T is a closed surface carried by B. Since S_1 and S_2 have the same genus, $\chi(S_1) = \chi(S_2)$ and hence $\chi(T) = 0$. This implies that a component of T has nonnegative Euler characteristic. Moreover, T must be a normal surface, since S_1 and S_2 have the same almost normal piece. This contradicts Theorem 2. □

The first two conditions in Theorem 2 follow trivially from the construction of Floyd and Oertel mentioned above. To prove Theorem 2, we show that, given a branched surface B, one can perform some splittings and obtain a finite collection of branched surfaces such that any Heegaard surface fully carried by B is carried by a branched surface in this collection and no branched surface in this collection carries any normal 2–sphere of torus.

To eliminate 2–spheres, we use a 0–efficient triangulation. A 0–efficient triangulation is a triangulation with only one vertex and the only normal S^2 is the vertex-linking one. It is shown in [5] that, except for S^3 and certain Lens spaces, every closed and orientable 3–manifold admits a 0–efficient triangulation. Since the branched surfaces in our construction fully carry some Heegaard surfaces, we may assume our branched surfaces do not carry the vertex-linking normal S^2.

To eliminate normal tori, we need to consider measured laminations. In general, a measured lamination (carried by a branched surface) corresponds to an irrational point in the solution space of the branch equations. Given any measured lamination μ carried by B, the Euler characteristic of μ can be defined using a linear equation of the weights of μ at the branch sectors of B [10].

Let $\mathcal{PL}(B)$ be the projective solution space of the branch equations of B ($\mathcal{PL}(B)$ is also called the projective measured lamination space). Let $\mathcal{T}(B) \subset \mathcal{PL}(B)$ be the projective space of measured laminations with Euler characteristic 0. For any Heegaard surface S fully carried by B and $\mu \in \mathcal{T}(B)$, we may assume that S and μ lie in $N(B)$, where $N(B)$ is a regular neighborhood of B and $N(B)$ can be regarded as an I–bundle over B. We call an isotopy in $N(B)$ a B–isotopy if the isotopy is invariant on each I–fiber of $N(B)$. We call an arc $\alpha \subset S$ a *splitting arc* relative to μ if $\alpha \cap \mu \neq \emptyset$ under any B–isotopy of μ in $N(B)$. Note that if we delete a small neighborhood of α from $N(B)$, we get a regular neighborhood of a new branched surface B' that still carries S but does not carry μ. We say that B' is obtained by splitting !B along α. The following lemma is an important step in the proof of Theorem 2. In the lemma, we use a combinatorial length, and one can consider the length of an arc to be the number of intersection points of the arc with the 2–skeleton.

Lemma 3. *Let $\mu \in \mathcal{T}(B)$. Then, for any strongly irreducible Heegaard surface S fully carried by B, there is a surface S' carried by B and isotopic to S in M, such that either (1) $S' \cap \mu = \emptyset$ or (2) there is a splitting arc $\alpha \subset S'$ relative to μ with length$(\alpha) < K(B,\mu)$, where $K(B,\mu)$ is a number depending on B and μ.*

A theorem in [10] says that any measured lamination is a disjoint union of a finite number of sublaminations of the following types: (1) a lamination consisting of compact leaves and (2) a minimal exceptional set (every leaf is dense). The proof of Lemma 3 is a discussion of two cases: (1) μ is a normal torus and (2) μ is an exceptional minimal set. In the first case, we use the fact that a normal torus in a 0–efficient triangulation always bounds a solid torus. In the second case, we first show that μ is contained in a nice solid torus. Then we apply a theorem in [13], which says that the intersection of a strongly irreducible Heegaard surface with a certain solid torus is very simple.

Let α be the splitting arc in Lemma 3. If we split B along α, we get a branched surface B' that carries a surface isotopic to S but does not carry μ. In fact, for each splitting arc α, there is an open neighborhood N of μ in $\mathcal{PL}(B)$ such that none of the measured laminations in N is carried by B'. Since we use a combinatorial length, up to isotopy fixing the 2–skeleton, there are only finitely many different splitting arcs with length less than $K(B,\mu)$. Thus, for each $\mu \in \mathcal{T}(B)$, we can define N_μ to be the intersection of all the neighborhoods of μ that correspond to these splitting arcs. So, N_μ is an open neighborhood of μ in $\mathcal{PL}(B)$. By compactness, there are a finite number of measured laminations $\mu_1, \dots, \mu_n$ in $\mathcal{T}(B)$ such that $\mathcal{T}(B) \subset \bigcup_{i=1}^{n} N_{\mu_i}$.

For each strongly irreducible Heegaard surface S, we have a finite set of splitting arcs $\alpha_1, \dots, \alpha_n$ relative to $\mu_1, \dots, \mu_n$ respectively. After splitting along these arcs we obtain a branched surface B_s that carries a surface isotopic to S but does not carry any measured lamination in $\bigcup_{i=1}^{n} N_{\mu_i}$. Since $\mathcal{T}(B) \subset \bigcup_{i=1}^{n} N_{\mu_i}$, B_s does not carry any normal torus. As the length of each α_i is bounded, there are only finitely many possibilities for the set $\{\alpha_i\}$. Hence, if we apply such splittings to every strongly irreducible Heegaard surface, we end up with only finitely many different branched surfaces, none of which carries any normal torus. This proves Theorem 2.

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Canonical splittings of open 3-manifolds

Sylvain Maillot

We are interested in the following question: does the theory of canonical splittings of compact 3-manifolds extend to noncompact manifolds? Here we will investigate this question for the Kneser-Milnor Prime Decomposition and for the Jaco-Shalen/Johannson Decomposition.

1. Spheres

Let M be an open, orientable 3-manifold. Obvious examples, such as connected sums of infinitely many copies of compact irreducible 3-manifolds, show that it is not reasonable to expect to split M along a *finite* system of spheres into irreducible manifolds. Let us agree to call *spherical decomposition* of M a *locally finite* collection $\mathcal{S}$ of pairwise disjoint embedded 2-spheres in M such that when one splits M along $\mathcal{S}$ and caps off a 3-ball to each boundary component, one gets irreducible manifolds.

Maybe surprisingly, it is not true that every open 3-manifold has a spherical decomposition. The first example was constructed by Peter Scott. Here is a simpler example: let F be an orientable surface with one end, one boundary component, and infinite genus. Let M be obtained by Dehn filling on $S^1 \times F$ so that the S^1 factor bounds a meridian disk. Then M does not have a spherical decomposition.

However, we have the following positive result:

Theorem 1. *Let (M, g) be a complete Riemannian manifold of bounded geometry. Suppose that there is a constant C such that $\pi_2(M)$ is generated, as a $\pi_1(M)$-module, by 2-spheres of area at most C. Then M admits a spherical decomposition.*

For those (if any) who do not like Riemannian geometry, there is a combinatorial version of this theorem, where the Riemannian metric g is replaced by a triangulation, and 'area' is to be interpreted as 'weight'. In fact, the Riemannian version is proven by first reducing it to the combinatorial version, which in turn is proven using (a variant of) the Jaco-Rubinstein theory of PL minimal surfaces.

2. Tori

From now on, we assume that M is irreducible. It may at first seem natural to look for a locally finite collection of tori that split M into atoroidal manifolds, but simple examples show that this would be wrong. For instance, take $F \times \mathbf{R}$ (where F is as before) and attach to it along its boundary A an atoroidal manifold, not $\mathbf{R}^2 \times [0, +\infty)$. The resulting manifold contains many incompressible tori, but the only reasonable splitting for it is the single annulus A, with two pieces: one atoroidal piece, and a Seifert piece.

Hence we look for a locally finite collection $\mathcal{T}$ of pairwise disjoint, properly embedded tori and annuli that split M in pieces that are Seifert or atoroidal. To really deserve to be called a 'canonical splitting', $\mathcal{T}$ should have some additional properties. For instance, let us call 'canonical torus' an incompressible torus T with the property that any other incompressible torus T' can be isotoped away from T. Then any canonical torus T should be isotopic to a member of $\mathcal{T}$.

There are (quite complicated) examples that do not have such splittings. However, we again have a positive result. Its statement is more involved than Theorem 1. Roughly, if M has a triangulation with respect to which least PL area canonical tori have small area, and there are sufficiently many least PL area noncanonical tori of small width, then M has canonical splitting along tori and open annuli.

The geometry of the pants complex

Howard Masur
(joint work with Jeffrey Brock)

This represents joint work with Jeffrey Brock. Let $S = S_{g,n}$ a surface of genus g with n punctures. We assume that $3g-3+n > 0$. Associated to the surface is the pants graph $C_P(S)$. The vertices are a collection of $3g-3+n$ disjoint isotopically nontrivial, non isotopic simple closed curves. These are called pants decompositions. Two vertices P_1, P_2 are connected by an edge if they differ by an elementary move. This means P_1, P_2 share $3g-3+n-1$ curves. The complement of the shared curves is either a punctured torus or a four times punctured sphere. The curve in P_2/P_1 is required to intersect the curve in P_1/P_2 once in the first case and twice in the second. The pants graph is given a metric by requiring each edge to have length 1 and the distance between vertices is the minimum number of edges joining them.

If $g \geq 2$ then the surface has dividing curves γ separating the surface into components S_1 and S_2. In this case the set of pants P_γ that contain γ is a product $C_P(S_1) \times C_P(S_2)$. This means that $C_P(S)$ is not a Gromov hyperbolic space.

We restrict now to the case of $g=2$ and $n=0$. Let $\hat{C}_P(S)$ be the electric space which is found by replacing P_γ by a single point x_γ for each separating curve γ, and assigning distance 1 from any pair of pants P to x_γ if the distance in $C_P(S)$ of P to P_γ is 1.

Theorem 1. *If $g=2$ and $n=0$ then $\hat{C}_P(S)$ is a Gromov hyperbolic space*

We say a space has geometric rank n if there is a quasi-isometric embedding of $\mathbf{R^n}$ into the space but no embedding of $\mathbf{R^{n+1}}$.

Theorem 2. *If $g=2$ and $n=0$ the space $C_P(S)$ has geometric rank 2.*

Twisted torus knots and distance one Heegaard splittings

Yoav Moriah
(joint work with Eric Sedgwick)

One of the more tantalizing problems in the Heegaard theory of 3-manifolds is that there are no known examples of non-minimal genus Heegaard splittings which are weakly reducible and non-stabilized. Or, in alternate terminology; Non-minimal genus Heegaard splittings which are distance one . The problem is that there are no known techniques to show that a non-minimal genus Heegaard splitting is non-stabilized.

In this paper which is joint work with Eric Sedgwick we wish to describe an infinite family of candidates for such Heegaard splittings which are indeed weakly reducible and about which we have a great deal of additional information. We discuss the surprising difficulties which arise it the attempt to prove that they are indeed non-stabilized.

It is a generally accepted rule that Heegaard splittings of small genus are easier to handle than those of large genus. Furthermore since we are dealing with questions of *reducibility* there is an advantage to dealing with Heegaard splittings of manifolds with boundary as having a boundary reduces the possibilities for the disks inside the compression bodies.

Since we are trying to prove a negative i.e., that a Heegaard splitting is not stabilized, we are forced into a proof by contradiction. Hence the argument should follow more or less the following theme: Let M be a 3-manifold of genus g. Assume that M has a weakly reducible Heegaard splitting which is stabilized of genus $g+1$. Destabilize it to obtain a Heegaard splitting of minimal genus g and somehow obtain a contradiction. If we can find such manifold which has a unique minimal Heegaard splitting we would have the additional option of getting a contradiction by showing that the surface we obtain after the destabilization cannot possibly be isotopic to the unique minimal genus Heegaard surface.

To sum up we are looking preferably for a tunnel number one knot $K \subset S^3$ so that $E(K)$ has a genus three weakly reducible Heegaard splitting and a unique genus two Heegaard splitting.

An obvious place to look for weakly reducible Heegaard splittings is Heegaard splittings which are *amalgamated* or in this case a ∂-stabilization. Heegaard splittings which are ∂-stabilizations can be stabilizations if they are (μ or) γ-primitive.

Definition: The knot $K \subset S^3$ obtained by taking the (p,q)-torus knot $K(p,q) \subset S^3$ (embedded on a standard torus $V \subset S^3$) removing a neighborhood of a small unknotted S^1 around two adjacent strands, which we denote by C, and doing a $\frac{1}{r}$ - Dehn filling along C will be called a *r-twisted torus knot* and denoted by $T(p,q,2,r)$. Since we always will take two strands we abbreviate to $T(p,q,r)$.

For these knots we have the following facts:

Theorem 1 (Morimoto, Sakuma, Yokota). *All the knots in S^3 of the form $K_m = T(7,17,10m-4, m \in \mathbb{Z})$ are tunnel number one knots which are not μ-primitive.*

Theorem 2. *The knots $K = T(7, 17, 10m - 4)$ are not γ-primitive for all curves $\gamma \subset \partial S^3 - N(K)$.*

Theorem 3. *The knots $K = T(7, 17, 10m - 4)$ are hyperbolic knots for all $m \in \mathbb{Z}$.*

Theorem 4. *Let $K_m = T(p, q, r)$ be a twisted torus knot with $(p, q) = (7, 17)$ and $r = 10m - 4, m \in \mathbb{Z}$. The knot complement $S^3 - N(K)$ has a unique minimal genus two Heegaard splitting for sufficiently large $|m|$.*

Question 1. *Is the weakly reducible Heegaard splitting obtained by ∂-stabilization from the unique genus two Heegaard splitting stabilized ?*

It is a well known theorem of Casson-Gordon that if a *closed* irreducible 3-manifold has a weakly reducible Heegaard splitting then it is Haken. It is a natural question wether this theorem can be extended to manifolds with boundary. In a previous paper the authors found examples of manifolds with two or more boundary components which have weakly reducible and non-stabilized minimal genus Heegaard splittings so that when the Heegaard surface is weakly reduced the surface obtained is non-essential. It is still an open question if such an example exists for manifolds with a single boundary component.

If we can show that the Heegaard splittings described above for the complements of the knots $K_m \subset S^3$ are indeed not stabilized then we obtain an example for a manifold with a single boundary component with a weakly reducible Heegaard splitting which when weakly reduced produces a 2-torus which must boundary parallel and hence non-essential. Such an example will resolve the above question.

Cut points of CAT(0) groups

Panagiotis Papasoglu
(joint work with Eric Swenson)

It is known by work of Bowditch that splittings over 2-ended groups of a hyperbolic group are reflected on its boundary. We show that this holds for CAT(0) groups too under the assumption of no infinite torsion subgroups.

Theorem 1. *Let G be a finitely generated group acting discretely and co-compactly on a $CAT(0)$ space X. Suppose that G has no infinite torsion subgroup. If ∂X is connected, and $\partial X - \{a, b\}$ is not connected then either G is commensurable to a planar group or G splits over a two-ended group.*

The proof has two parts. In the first part we show that we can associate to a continuum X, with no cut points, an $\mathbb{R}$-tree T encoding the set of cut pairs of X. We call T the JSJ-tree of the continuum. This tree is canonical i.e. any homeomorphism ϕ of X induces a homeomorphism $\bar{\phi} : T \to T$.

In the second part we use the action of G on the JSJ-tree of the boundary of X. The situation is more complicated than in the hyperbolic case as this action is not a convergence action but we show it satisfies a weaker condition, namely it is a $\pi/2$-convergence action.

Separating incompressible surfaces in 3-manifolds of Heegaaard genus 2

Hyam Rubinstein
(joint work with Kazuhiro Ichihara and Makoto Ozawa.)

The idea is to construct a large class of closed orientable 3-manifolds having genus 2 Heegaard splittings and contain embedded separating incompressible surfaces.

Until recent work of Agol, all known non Haken 3-manifolds had Heegaard genus either 2 or 3. Moreover most of the low volume examples in the census of Hodgson and Weeks are also of low Heegaard genus. So the question of whether low Heegaard genus means that a manifold is more likely to be non Haken or Haken is of interest.

A 'generic' Heegaard splitting will produce a manifold of finite first homology, so that there are no closed orientable non separating surfaces. Consequently the problem is to decide whether or not there are separating incompressible surfaces. We build a large number of examples where there are such surfaces, which are in nice position relative to the height function associated with the Heegaard splitting.

The basic idea is to first attach a separating 2-handle to both ends of a product of a genus 2 orientable surface and a closed interval, to form a manifold N with 4 boundary tori. It is then shown that in many cases, two disjoint separating surfaces of the same genus g can be constructed in N which each have one maximum, one minimum and $2g$ saddles relative to the obvious height function. Since these surfaces decompose N into 3 regions, it is relatively easy to decide if the surfaces have compressing disks.

Finally, well known results can be used to show that these two surfaces remain incompressible under most Dehn fillings on the boundary tori of N, to produce many examples of Haken closed orientable 3-manifolds of Heegaard genus 2 and finite first homology.

Similar but more complicated methods apply to higher genus examples, where a whole fumily of separating surfaces can be bullt, to again make it easier to decide if there are compressing disks.

We would like to thank Ian Agol for bringing to our attention some examples of small 4 component Montesinos links which were very useful to elucidate exactly what conditions were required on the attachment of the separating 2-handles.

Punctured torus groups and 2-bridge knot groups

Makoto Sakuma
(joint work with Hirotaka Akiyoshi, Masaaki Wada and Yasushi Yamashita)

By the late 70's Troels Jorgensen had made a series of detailed studies on the space $\mathcal{QF}$ of quasifuchsian (once) punctured torus groups from the view point of their Ford fundamental domains. These studies are summarized in his famous unfinished paper [2]. In it, he gave a complete description of the combinatorial structure of the Ford domain of every quasifuchsian punctured torus group, and showed that the space $\mathcal{QF}$ can be described in terms of the combinatorics of the faces of the Ford domain. In the talk, I explained one of the main ideas of our preprint [1] in which we give a full proof to the results announced in [2].

The preprint [1] forms Part I of our joint work which extends Jorgensen's theory to the outside of the quasifuchsian space as follows: Let $\mathcal{P}(\lambda^-, \lambda^+)$ be a pleating variety of $\mathcal{QF}$ (see [3] corresponding to a pair (λ^-, λ^+) of rational projective measured laminations of the once-punctured torus. Then the following hold:

(1) The pleating variety $\mathcal{P}(\lambda^-, \lambda^+)$ of $\mathcal{QF}$ has a natural extension to the outside of $\mathcal{QF}$ in the space of type-preserving representations of the $\pi_1(T)$.
(2) Each point in the extension is the holonomy representation of a certain hyperbolic cone manifold, which is commensurable with a hyperbolic cone manifold, $M(\theta^-, \theta^+)$, whose underlying space is the complement of a 2-bridge knot and whose cone singularity is the union of the upper and lower tunnels, which have the cone angles θ^+ and θ^-, respectively. Moreover the 2-bridge knot is of type (p, q), or of slope q/p, if (λ^-, λ^+) is equivalent to $(1/0, q/p)$ by a modular transformation.
(3) If the distance $d(1/0, q/p)$ in the Farey triangulation is ≥ 2, namely if $q \not\equiv \pm 1 \pmod{p}$, then the hyperbolic cone manifold $M(\theta^-, \theta^+)$ exists for every pair of cone angles in $[0, 2\pi]$. Thus we have a continuous family of hyperbolic cone manifolds connecting $M(0, 0)$, the quotient hyperbolic manifold of a double cusp group, with $M(2\pi, 2\pi)$, the complete hyperbolic structure of the 2-bridge knot complement.
(4) If $d(1/0, q/p) = 1$, namely if $q \equiv \pm 1 \pmod{p}$, then the hyperbolic cone manifold $M(\theta^-, \theta^+)$ exists for every pair of cone angles in $[0, 2\pi]$, except the pair $(2\pi, 2\pi)$. If both cone angles approach 2π, then $M(\theta^-, \theta^+)$ collapses to the base orbifold of the Seifert fibered structure of the knot complement.
(5) The holonomy group of $M(\theta^-, \theta^+)$ is discrete if and only if $\theta^\pm \in \{2\pi/n \,|\, n \in \mathbb{N}\} \cup \{0\}$. In particular, that of $M(2\pi, 2\pi/n)$ is generated by two parabolic transformations and is called a *Heckoid group* by Riley.

Actually, we have constructed these hyperbolic cone manifolds by explicitly constructing "Ford fundamental polyhedra". In other words, we have extended Jorgensen's description of the Ford fundamental polyhedra for quasifuchsian punctured torus groups to those of the hyperbolic cone manifolds arising from the 2-bridge knots. In particular, we have shown that the canonical decompositions of hyperbolic 2-bridge knot complements are isotopic to the topological ideal tetrahedral decompositions constructed in [4], proving the conjecture which motivated our project. We hope to write down a full proof of these results in a forthcoming paper.

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Alternate Heegaard genus bounds distance

Martin Scharlemann

(joint work with Maggy Tomova)

We illustrate that a theorem of Hartshorn's, limiting the distance of a Heegaard surface to twice the genus of any embedded closed surface, naturally extends to embedded essential surfaces in 3-manifolds with boundary. More surprisingly, it extends also to alternate Heegaard splittings and even beyond. To be precise:

Suppose M is a compact orientable irreducible 3-manifold with Heegaard splitting surfaces P and Q. Then either Q is isotopic to a possibly stabilized copy of P or the Hempel distance of the splitting P is no greater than twice the genus of Q.

More generally, if P and Q are bicompressible but weakly incompressible connected closed separating surfaces in M then either

(a) P and Q can be well-separated (roughly, separated by an incompressible surface) or
(b) P and Q are isotopic or
(c) the Hempel distance of P is no greater than twice the genus of Q.

The technique used is to examine the two-parameter family of positionings of P and Q given by natural sweep-outs which they induce on M. The chief challenge is to find a labeling scheme for these positionings that is delicate enough to provide useful information from the graphic associated to the 2-parameter family.

A metric survey of curve complexes

Saul Schleimer

(joint work with Howard Masur)

The *graph of curves* of a surface S has as its vertex set all isotopy classes of simple closed essential, nonperipheral curves in S. Two distinct vertices are connected by an edge if the classes in question have disjoint representatives. This graph $\mathcal{C}(S)$ (or rather, its clique complex) was introduced by Harvey [2] and has been used to study the mapping class group [7], Kleinian groups [9], and Heegaard splittings [4].

There is a veritable zoo of similar objects: $\mathcal{A}(S)$ the graph of arcs [1], Sep(S) the separating curve complex, the Hatcher-Thurston complex, the pants complex [3], and so on. Let $\mathcal{G}(S)$ be any of these. The vertices are all isotopy classes of multicurves in S, and the edges of $\mathcal{G}(S)$ are the relation "small geometric intersection," typically disjointness. All edges are given length one. We wish to study the coarse properties of the resulting metric space. The model theorem in this direction is due to Masur and Minsky [6]:

Theorem 1. *The graph of curves is Gromov hyperbolic.*

Following [7] or [5], if X is an essential subsurface of S then any of the above graphs admits a "cut-and-paste" map to $\mathcal{C}(X)$ as follows: Pick α a vertex of $\mathcal{G}(S)$. Isotope α to intersect X tightly. Pick any component of $\alpha \cap X$. This gives an arc α' in X. Let α'' be any nonperipheral (in X) component of the boundary of a neighborhood of $\alpha' \cup \partial X$. Then α'' is a *subsurface projection* of the vertex α to X and we have a coarse map $\pi_X : \mathcal{G}(S) \to \mathcal{C}(X)$. If every vertex of $\mathcal{G}(S)$ meets X nontrivially then the subsurface projection is everywhere defined. In this case we call X a *hole* for $\mathcal{G}(S)$.

From Lemma 2.3 of [7] it is straight-forward to show:

Lemma 1. *For any $\mathcal{G}(S)$ there is a constant $K > 0$ so that subsurface projection to any hole is K-Lipschitz.*

It is then easy to deduce:

Corollary 1. *Suppose $\mathcal{G}(S)$ admits an action by the mapping class group and X and Y are disjoint holes. Then $\mathcal{G}(S)$ is not Gromov hyperbolic.*

It is a pleasant exercise to classify the holes for any of the standard examples given above. In particular one finds examples where all holes intersect. It is natural to conjecture a converse to Corollary 1:

Conjecture 1. *Suppose that $\mathcal{G}(S)$ admits an action by the mapping class group and any pair of holes X and Y intersect. Then $\mathcal{G}(S)$ is Gromov hyperbolic.*

A crucial step in proving the conjecture for any fixed $\mathcal{G}(S)$ would be to verify the *distance estimate*:

Conjecture 2. *The sum $\sum' d_X(\alpha, \beta)$ is within uniform multiplicative and additive error of the distance between α and β in $\mathcal{G}(S)$.*

Here the summation ranges over all holes X for $\mathcal{G}(S)$. The quantity $d_X(\alpha, \beta)$ equals the distance between $\pi_X(\alpha)$ and $\pi_X(\beta)$ in $\mathcal{C}(X)$. The "prime" on the summation indicates that all summands less than certain size are omitted.

We have verified the distance estimate and hyperbolicity for the arc complex. The techniques required are essentially contained in the two papers [6] and [7].

It is much more difficult to obtain the two conjectures for the *graph of disks*, $\mathcal{D}(V_g)$, defined by McCullough [8]. This graph has as vertex set all proper isotopy classes of essential disks in a genus g handlebody V_g. As usual the edges come from disjointness. As work-in-progress we have classified the holes for $\mathcal{D}(V)$ using the techniques of Masur and Minsky, Jaco-Shalen-Johannson theory, and an analysis of which surfaces admit pseudo-Anosov maps.

Suppose now that ∂V is identified with S. Then there is a relationship between $\mathcal{D}(V)$ and $\mathcal{C}(S)$. The former is included in the latter by the natural boundary map. In fact a pair of handlebodies V and W, both glued to S, specifies a three-manifold with *Heegaard splitting surface* S. Hempel [4] then defines the *distance* of S to be $d_S(V, W)$: the minimal distance in $\mathcal{C}(S)$ between the subgraphs $\mathcal{D}(V)$ and $\mathcal{D}(W)$. As an application of the classification of holes we obtain:

Algorithm 1. *Fix a genus g. There is a constant K and an algorithm which, given a Heegaard diagram $(S, \mathbb{D}, \mathbb{E})$, computes the distance $d_S(V, W)$ up to an additive error of at most K.*

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Homological stability for mapping class groups of non-orientable surfaces

Nathalie Wahl

Let $S = S_{n,r}$ be a non-orientable surface of genus n with r boundary components, i.e. S is the connected sum of n copies of $\mathbb{R}\mathrm{P}^2$ with r discs removed. The mapping class group of S is

$$\mathcal{M}_{n,r} := \pi_0 \mathrm{Diff}(S_{n,r}; \partial),$$

the group of path components of the space of diffeomorphisms of S which fix its boundary pointwise.

When $r \geq 1$, there are stabilization maps $\mathcal{M}_{n,r} \to \mathcal{M}_{n+1,r}$, obtained by gluing a punctured Moebius band (or a twice punctured $\mathbb{R}\mathrm{P}^2$) to the surface and extending the diffeomorphisms by the identity on the added part, and $\mathcal{M}_{n,r} \to \mathcal{M}_{n,r+1}$, obtained similarly by gluing a pair of pants. Gluing a disc on the added pair of pants defines a right inverse to the second map. This means in particular that the map $H_i(\mathcal{M}_{n,r};\mathbb{Z}) \to H_i(\mathcal{M}_{n,r+1};\mathbb{Z})$ is always injective. Our main theorem is the following.

Theorem 1. *The map $H_i(\mathcal{M}_{n,r};\mathbb{Z}) \longrightarrow H_i(\mathcal{M}_{n+1,r};\mathbb{Z})$ is surjective when $n \geq 4i-1$ and injective when $n \geq 4i+2$ for any $r \geq 1$.*

The map $H_i(\mathcal{M}_{n,r};\mathbb{Z}[\frac{1}{2}]) \longrightarrow H_i(\mathcal{M}_{n,r+1};\mathbb{Z}[\frac{1}{2}])$ is an isomorphism when $n \geq 4i+2$ for any $r \geq 1$.

In work in progress, we prove that the second map is an isomorphism integrally when $n \geq 3i+3$ and that the map $H_i(\mathcal{M}_{n,r};\mathbb{Z}) \to H_i(\mathcal{M}_{n+1,r-1};\mathbb{Z})$, induced by gluing a Moebius band, is an isomorphism in a similar range. An analogous theorem was proved by Harer [1] (improved by Ivanov [2]) in the case of orientable surfaces.

Using the work of Madsen and Weiss [3], we obtain the following consequence of Theorem 1:

Theorem 2. *$H_i(\mathcal{M}_{n,r},\mathbb{Z}[\frac{1}{2}]) \cong H_i(\Omega^\infty\Sigma^\infty_0(BO(2)_+);\mathbb{Z}[\frac{1}{2}])$ when $n \geq 4i+2$ and $r \geq 1$.*

Here $\Omega^\infty\Sigma^\infty_0(BO(2)_+)$ denotes the 0th component of the infinite loop space of the suspension spectrum of $BO(2)$ with an added basepoint.

Let $\mathcal{M}_\infty = \mathrm{colim}_{n\to\infty}(\mathcal{M}_{n,1} \to \mathcal{M}_{n+1,1} \to \dots)$ be the stable non-orientable mapping class group. An immediate corollary of Theorem 2 is the non-orientable analogue of the Mumford conjecture:

Corollary 1. *$H^*(\mathcal{M}_\infty,\mathbb{Q}) \cong \mathbb{Q}[\lambda_1,\lambda_2,\dots]$ with $|\lambda_i| = 4i$.*

The homological stability theorem (Theorem 1) is proved using complexes of arcs in non-orientable surfaces.

Let S be a surface, orientable or not, and let $\overrightarrow{\Delta}$ be a set of *oriented points* in ∂S, that is each point comes with the choice of an orientation of the component of ∂S it lies in. We say that an arc in $(S,\overrightarrow{\Delta})$ is *1-sided* if its boundary points are in $\overrightarrow{\Delta}$ and its normal bundle identifies the orientations of its endpoints. Note that a 1-sided arcs from a point to itself is a 1-sided curves in the usual meaning of the word. If S is orientable, the choice of an orientation for S decomposes $\overrightarrow{\Delta}$ as $\overrightarrow{\Delta} = \Delta^+ \sqcup \Delta^-$, where Δ^+ is the set of "positive" points and Δ^- the set of negative ones. The 1-sided arcs in this case are exactly the arcs with one boundary point in Δ^+ and the other in Δ^-. This complex, in the oriented case, was studied by Harer in [1]. He shows that it is highly connected. We generalize his result in two ways: first to the complex of arcs between two sets of points Δ_0 and Δ_1 in a non-orientable surface, and then to the complex of 1-sided arcs in $(S,\overrightarrow{\Delta})$ for a set of oriented points $\overrightarrow{\Delta}$. Our proof is different from Harer's and uses techniques from [4].

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Surfaces in finite covers and the group determinant

Genevieve S. Walsh
(joint work with Daryl Cooper)

Let Y be a 3-manifold with one torus boundary component and $\tilde{Y} \to Y$ a finite regular covering of Y induced by a map $\pi_1(Y) \to G$. Let $K(\tilde{Y})$ denote the kernel of the map $i_* : H_1(\partial\tilde{Y};\mathbb{Q}) \to H_1(\tilde{Y};\mathbb{Q})$. For a slope α on ∂Y, denote by $V(\alpha,\tilde{Y})$ the subspace of $H_1(\partial\tilde{Y};\mathbb{Q})$ spanned by pre-images of α. We say that a slope α on ∂Y is a *virtual homology slope* (for $\tilde{Y}$ of rank n) if $dim(K(\tilde{Y}) \cap V(\alpha,\tilde{Y})) = n > 0$. Note that if α is a virtual homology slope, then there is a non-separating surface in $\tilde{Y}$ whose boundary components map down to curves that have slope (a multiple of) α in ∂Y.

We relate the group determinant of G, as studied by Frobenius and Dedekind, to a matrix that encodes $K(\tilde{Y})$. This enables us to prove the following theorems:

Theorem 1. *Let Y be a hyperbolic 3-manifold with one torus boundary component. Then either:*

i) for all $n \in \mathbb{N}$, there is a regular cover $\tilde{Y} \to Y$ and a slope α on ∂Y so that α is a virtual homology slope for $\tilde{Y}$ of rank at least n, or

ii) every slope on ∂Y is a virtual homology slope.

Theorem 2. *Let Y be a hyperbolic 3-manifold with one torus boundary component. Then infinitely many fillings of Y are virtually Haken.*

Theorem 2 is implied for the case that Y is not fibered by [2].

Let G be a finite group, and $R : G \to Aut(\mathbb{C}^{|G|})$ the representation induced by the right regular action of G on itself, $g : h \mapsto hg^{-1}$. Let $\{X_g\}$ be a collection of commuting variables, one for each element of G. For any representation ρ of G, the representation matrix $M(\rho)$ is the matrix $\sum_{g\in G} \rho(g)X_g$. Then the group matrix of G, $M(G)$, is the representation matrix for R. Thus $M(G)$ has $X_{g_i^{-1}g_j}$ as the ij-th entry. The *group determinant* of G is $det(M(G))$. Important to our computations is the fact that if a representation is reducible, i.e., $\rho = \rho_1 \oplus \rho_2$, then the representation determinant $det(\rho) = det(M(\rho))$ is a product $det(\rho_1)det(\rho_2)$. Since we will ultimately be interested in linear factors of the group determinant, we will look for irreducible representations that have linear factors in their determinants.

In our applications we will have a matrix that is a specialization of the group matrix that is symmetric. Thus to simplify matters we work with the symmetrized group matrix, $M^{sym}(G)$ which is obtained from $M(G)$ by setting $X_g = X_{g^{-1}}$. Similarly $det^{sym}(G) = det(M^{sym}(G))$. For example, the group determinant of $\mathbb{Z}_3$ is $(a+b+c)(a^2-ab+b^2-ac-bc+b^2)$, while $det^{sym}(\mathbb{Z}_3) = (a+2b)(a-b)^2$.

Given a regular cover $\tilde{Y} \to Y$ induced by a map $\pi_1(Y) \to G$, we define a matrix $B(\tilde{Y})$ that encodes the vector space $K(\tilde{Y})$. Rational eigenvalues of $B(\tilde{Y})$ are virtual homology slopes, and the dimension of the associated eigenspace equals the rank of the slope. When $\pi_1(\partial Y) \to 1 \in G$, the boundary matrix is a specialization of $M^{sym}(G)$. When $det(M^{sym}(G))$ has a rational linear factor of rank n, any regular covering $\tilde{Y} \to Y$ with covering group G where the boundary torus lifts will have a virtual homology slope of rank at least n.

In general however, the boundary torus will not lift, and in this case we identify the variables X_{g_1} and X_{g_2} whenever g_1 and g_2 are in the same element of $\{HgH \cup Hg^{-1}H\}_{g\in G}$. This yields the group matrix of G with respect to H, $M(G,H)$, and we can determine the rational eigenvalues of $B(\tilde{Y})$ from this matrix.

The covers we use are induced by the surjections to $PSL(2,\mathbb{F}_p)$ given in [5]. This implies that $\pi_1(Y)$ surjects $PSL(2,\mathbb{F}_p)$ for infinitely many p where $\pi_1(\partial M)$ maps onto the cyclic subgroup of order p. By analyzing the permutation representation induced by the action of $PSL(2,\mathbb{F}_p)$ on $\mathbb{P}^1(\mathbb{F}_p)$, we show that there is an invariant slope of rank at least p for any such cover. This is the idea of the proof of Theorem 1. An application of [4] shows that if $p \geq 2$, there is a non-separating surface S in $\tilde{Y}$ that is not the fiber of a fibration and whose boundary curves all map down to curves of the same slope in ∂Y. By results in [6], [2] and [1], a large enough cyclic cover of $\tilde{Y}$ dual to S will contain a closed incompressible surface, a lift of Freedman-Freedman tubing of two copies of S! , and this will survive fillings that are distance greater than 1 from this slope. Since infinitely many fillings of Y lift to this cover, this implies Theorem 2.

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The rank problem of Kleinian groups

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(joint work with Ilya Kapovich)

The rank problem for the group G is the problem to compute the minimal number of elements needed to generate the group G. Like most decision problems, the rank problem is not decidable in the class of finitely presented group. However as shown by Baumslag, Miller and Short [2] the rank problem is not even decidable in the class of hyperbolic groups, a class in the which the three classical decision problems are all decidable.

Under more restrictive hypotheses, namely in the case that all finitely generated subgroups are quasi-convex, it has been shown that the rank problem is decidable [6]. This condition however is not always satisfied for Kleinian groups, in fact the virtual fibre conjecture of Thurston would imply that it is never satisfied for fundamental groups of closed hyperbolic 3-manifolds. Using results of Canary [4] and the recently established tameness conjecture [1],[5] the result of [6] can nevertheless be generalized to also deal with Kleinian groups.

Theorem 1. *There exists an algorithm which, given a finite presentation of torsion-free word-hyperbolic Kleinian group G, determines the rank of G.*

Note that an explicit solution to the rank problem for Seifert manifolds is due to Boileau and Zieschang [3].

The proof uses the folding methods for groups acting of hyperbolic spaces as developed in [6]. The same methods further show that any generating tuple can be replaced by a Nielsen equivalent tuple that only contains short elements, more precisely the following holds:

Theorem 2. *Let G be a torsion-free word-hyperbolic Kleinian group. For any integer k there exist only finitely many Nielsen equivalence classes of k-tuples generating G.*

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Deforming Euclidean cone-3-manifolds

Hartmut Weiss
(joint work with Joan Porti)

A cone-manifold of curvature $\kappa \in \{-1, 0, 1\}$ is a metric space X, which is homeomorphic to a manifold and whose local geometry is modelled on the κ-cone over a cone-manifold of curvature $+1$ and one dimension lower. The set of points whose link (i.e. the cross-section of the model cone) is isometric to the standard round sphere is called the smooth part of the cone-manifold and will be denoted by M, its complement is called the singular locus and will be denoted by Σ. In the following we will be concerned with 2- and 3-dimensional cone-manifolds only.

In two dimensions the singular locus consists of isolated points. The link of each cone point is isometric to the circle of a certain length, which we will refer to as the cone angle associated to that point. In three dimensions the singular locus is an embedded geodesic graph. The cone-angle associated to an edge will be the cone-angle of a transverse disk.

If cone-angles are $\leq 2\pi$, these spaces satisfy a lower curvature bound in the triangle comparison sense. If cone-angles are $\leq \pi$, the geometry is even more restricted, for example the Dirichlet-polyhedron will be convex and the valency of a vertex of the singular locus (in the 3-dimensional case) will be at most 3.

The concept of cone-3-manifold is a natural generalization of the concept of geometric 3-orbifold, where the cone-angles are of the form $2\pi/n$, $n \in \mathbb{Z}, n \geq 2$. Cone-3-manifolds play a significant role in the proof of the Orbifold Theorem, which has recently been accomplished by M. Boileau, B. Leeb and J. Porti, cf. [1]. The Orbifold Theorem states that a similar geometric decomposition as conjectured for 3-manifolds holds true for 3-orbifolds with non-empty singular locus. It was announced by W. Thurston around 1982.

To state the main theorem of [3], let us say that a Euclidean cone-3-manifold is *almost product* if it is the quotient of $E^2 \times S^1$ by a finite group of isometries respecting the product structure, where E^2 is a 2-dimensional Euclidean cone-manifold.

Theorem 1. *Let X be a closed, orientable Euclidean cone-3-manifold with cone angles $\leq \pi$. If X is not almost product, then for all multiangles $\bar{\alpha} \in (0, \pi)^N$ there exists a unique cone-manifold structure of curvature $\kappa \in \{-1, 01\}$ on X with those cone angles.*

Furthermore, if all cone angles of X are π, then every point in $(0, \pi)^N$ is the multiangle of a hyperbolic cone-manifold structure.

If some of the cone angles is $< \pi$, then the subset $E \subseteq (0, \pi)^N$ of multiangles of Euclidean cone-manifold structures is a smooth, properly embedded hypersurface that splits $(0, \pi)^N$ into two connected components S and H, corresponding to multiangles of spherical and hyperbolic cone-manifold structures respectively. Moreover, for each $\bar{\alpha} \in E$ the tangent space of E at $\bar{\alpha}$ is orthogonal to the vector of lengths of singular edges $\bar{l} \in \mathbb{R}_+^N$.

The following corollary of Theorem 1 gives an alternative argument to the last step in the proof of the orbifold theorem in [1], which is more natural from the point of view of cone-manifolds:

Corollary 1.1. *Let $\mathcal{O}$ be a closed, orientable, irreducible 3-orbifold. If there exists a Euclidean cone-manifold structure on $\mathcal{O}$ with cone angles strictly less than the orbifold angles of $\mathcal{O}$, then $\mathcal{O}$ is spherical.*

The proof of Theorem 1 uses ideas from [2] and a theorem about L^2-cohomology in [4], namely that in the Euclidean case the first L^2-cohomology group of the smooth part M with values in the flat tangent bundle TM is represented by parallel forms. This gives us infinitesimal information about the $SU(2)$-character variety, and hence about the deformations of the rotational part of the Euclidean holonomy.

In the hyperbolic and the spherical case, the corresponding theorem in [4] states that the first L^2-cohomology group of the smooth part M with values in the flat bundle of infinitesimal isometries of M vanishes. From this one concludes rather directly, that deformations of the hyperbolic, resp. spherical holonomy through cone-manifold structures are parametrized by the cone-angles.

In the Euclidean case the situation is more subtle. In [2] a criterion is established that describes precisely which deformations of the rotational part of a Eulidean holonomy (inside the larger group of hyperbolic, resp. spherical isometries) actually correspond to regenerations into hyperbolic, resp. spherical structures. This is the key – along with the cohomological information as described above – to analyze the situation and prove Theorem 1.

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