Mathematisches Forschungsinstitut Oberwolfach

Report No. 52/2007

Professional Development of Mathematics Teachers - Research and Practice from an International Perspective

Organised by
Kristina Reiss, München
Alan Schoenfeld, Berkeley
Günter Törner, Duisburg

November 11th – November 17th, 2007

Abstract. The Oberwolfach Workshop on "Teachers Professional Development" gathered a wide spectrum of topics of international research and practice in the field of mathematics education. Well-known researchers from all over the world explored the theme from various angles and with different emphasis ranging from theoretical and conceptual perspectives to empirical interventions.

Mathematics Subject Classification (2000): 97C70, 97C50.

Introduction by the Organisers

The workshop *Professional Development of Mathematics Teachers - Research and Practice from an International Perspective*, organized by Kristina Reiss (Ludwig-Maximilians-Universität München), Alan Schoenfeld (University of California, Berkeley) and Günter Törner (Universität Duisburg-Essen) was held November 11th–November 17th, 2007. The meeting was attended by more than 40 participants with broad geographic representation from all continents. The researchers were mostly mathematics educators, some research mathematicians as well as colleagues working in the field of psychology and education. Due to the different backgrounds, the talks presented a variety of views on professional development and provided broad conceptual and theoretical information. The presentations were scheduled for half an hour and led to intensive discussion afterwards. Further, the workshop was organized around the following subtopics: aspects of professional development in general, examples for "best practice", perspectives on teaching and learning, various themes in the field of professional development, teaching and teacher education. It consisted of thirty-seven talks encompassing the aforementioned areas.

Our meeting started with talks regarding aspects of professional development in general. One of the organizers gave an overview on theoretical and practical issues of teachers' professional development (Alan Schoenfeld). The following talks were concerned with conceptual aspects (Martin Simon) and issues of effectiveness (John Mason). The next session was dedicated to examples of "best practice". First, some basic mathematical assumptions in teacher education were outlined (Rina Zazkis), second "best examples" from various countries were presented. That is, two professional development initiatives in Germany were pointed out (Peter Baptist, Bettina Rösken), one from Canada (Sharon Friesen), followed by information about the tradition of mathematics teaching in Hungary (Ödön Vansco).

The next subtopic was perspectives on teaching and learning. Since the focus of the first day was on qualitative studies, these talks were primarily concerned with quantitative approaches. The presentation of a quasi-experimental design regarding the effect of an in-service teacher training on students' achievement (Kristina Reiss) was followed by information about the COACTIV project, a study elaborating on the professional knowledge of German mathematics teachers (Werner Blum). In the afternoon, the use of different media as a tool to investigate in the field of teacher education was introduced (Miraim Sherin, Aiso Heinze, Jürgen Richter-Gebert and Hans Georg Weigand) as well as some general thoughts about teaching mathematics in the classroom (Abrahm Arcavi and Klaus Hoechsmann). The talks on Wednesday discussed several approaches to initiate professional development of teachers. First, a model to describe professional growth was presented and applied to the use of video within teacher development programs (David Clarke). The next contributions described how institutional changes inspired and catalyzed practical changes and theoretical progress in the French system of teacher education (Michele Artigue) and how teachers and didacticians can work together as practitioners and researchers in a co-learning inquiry model. One of two parallel talks called attention to awareness of teachers for their decisions in complex situations and the (possibly less aware) basic assumptions that guide them (Chris Breen). The other one introduced a case study describing the journey of a Taiwanese teacher towards the teaching of mathematical modeling (Kai-Lin Yang).

On the fourth day of the workshop the first talk gave an overview of mathematics teacher education as a field with two aspects, practice and research, that gained contact in the recent years, (Konrad Krainer), followed by a report on a large teacher education project that incorporated and combined both aspects consistently (Malcolm Swan). The next talk on professional knowledge of mathematics teachers and the related TEDS-M study presented current quantitative research in the field (Gabriele Kaiser). The end of the morning formed a case study with two elementary teachers and their approaches to mathematical modeling and problem solving (Lieven Verschaffel). In the afternoon, a three-hour workshop challenged the participants to deal with professional knowledge, particularly with those aspects of mathematics teachers' knowledge that is distinctive for their profession

(Dina Tirosh, Tommy Dreyfuss). In a parallel session, a special aspect of professionality, the identity as a mathematics teacher, was focused, presenting the results from a study with ten experienced teachers (Fulvia Furinghetti). A second parallel talk introduced the approach of "lesson study" as a means to assure and improve quality in Japanese mathematics classrooms (Makoto Yoshida). The last contribution of the day dealt with the mathematical part of teacher knowledge, described by the model of "mathematical knowledge for teaching" (Hyman Bass).

A final discussion formed the official end of the workshop. The last day, Friday, was reserved - and widely used - for informal discussions based on the large number of ideas and concepts presented during the week.

Additionally, three working groups were offered in the evenings: Alan Bishop invited the participants to work on values in education and a group around Barbara Jaworski focused on the roles of teachers and teacher educators, both as professionals and learners within the context of teacher education. The third group, organized by Aiso Heinze on Wednesday and Thursday, dealt with the notion of pedagogical concept knowledge of teachers and discussed possible item formats to measure this construct. During the week, there were also several informal sessions and lively group discussion. On Monday evening, we celebrated a birthday reception in honor of the 60th birthdays of two of the organizers. Wednesday afternoon, the excursion gave us an opportunity to experience the first snow of the year during a walk to Oberwolfach-Walke. Of course we did not miss the opportunity to taste some of the famous local cake.

The workshop provided a good overview on professional development and substantial information where the field is currently located. The different strands that were presented gave an impression of the diversity of the field. This research area is characterized by multiple perspectives leading to partial conceptual diffuseness, and sometimes seems to lack an overarching research-based theory. Due to cultural diversity, it is usually almost impossible to agree on the different notions and, what became obvious, we need cultural awareness to deal with this problem. One prerequisite for this awareness is the possibility to discuss concepts and notions in a stimulating and constructive atmosphere. The meeting was one attempt to provide an appropriate opportunity. It was fruitful and informative, brought people together although we could have ended in more concrete collaborations for ongoing research. Unfortunately, the railway strike had an effect on our meeting. We felt impelled to bring forward the talks announced for Friday in order to give all participants the opportunity to present their results. Though this was possible, it resulted in a tight schedule and there was less time for informal discussions than we could have filled - which is of course a regular experience for workshops like this one.

The contributions to this report are extended abstracts of most talks. In cases were the extended abstracts could not be submitted in time, we included the original (shorter) abstracts provided in advance of the meeting.

We would like to thank the Oberwolfach Institute, its director and the staff for a perfectly organized week that enabled an intensive and inspiring meeting concerned with mathematics teachers' professional development.

Workshop: Professional Development of Mathematics Teachers - Research and Practice from an International Perspective

Table of Contents

Abstracts

Theoretical and practical Issues in the professional development of teachers

Alan H. Schoenfeld

In broad theoretical terms, I argue that the choices that people make while engaged in purposeful activities such as teaching and problem solving are a function of those individuals' knowledge, goals, beliefs/orientations, and a certain kind of decision-making that follows from these. In particular, the decisions that teachers make in the classroom are shaped in fundamental ways by what they know mathematically and pedagogically, what their personal, mathematical, and pedagogical goals are, and the ways that these goals and understandings are shaped by their (often unconscious and culturally influenced) beliefs, values, and orientations concerning the nature of mathematics, the nature of learning, what constitutes effective teaching, external constraints, and the capacities of their students. This theoretical perspective has been used as a basis for creating detailed analytic models of classroom lessons, and promises to be quite general. If it is indeed correct, then it raises many questions for professional development (PD) experiences, from in-service through pre-service. The main question is, how can one arrange PD, both externally and as part of teachers' professional lives, so that they develop the kinds of knowledge, goals, and beliefs/orientations that will enable them to teach in productive ways that are valued? (“By whom?” is a question we shall explore.)

Pedagogical concepts as goals for teacher education: Towards an agenda for research in teacher development

Martin Simon

The arguments advanced in this theoretical article are based on two assumptions: 1. Aspects of the knowledge base in mathematics education are critical content (goals) for mathematics teacher education. 2. Identification of goals for mathematics teacher education is critical to both effective teacher education and productive research on teacher education. In this article, I identify key pedagogical concepts that derived from our work and the work of others in order to discuss goals for teacher education and agendas for research on teacher development. Following are brief discussions of four pedagogical concepts that are important to consider because of their impact on mathematics teaching and because we have found them to be generally lacking among mathematics teachers in US classrooms. Most of these concepts are overlapping and interrelated.

(1) Negotiation of classroom norms. The notion that classroom norms are negotiated, not imposed McNeal et al. [2], Yackel [5] allows teachers to be conscious of their contribution to the constitution of classroom norms.

(2) Assimilation. An understanding of assimilation is essential for teachers to understand what determines what students perceive and understand, and the resources they bring to learning situations. [1] described a representational view of mind in arguing this point. Our study of teachers involved in the reform Simon et al. [4] highlighted the distinction between teachers with perception-based perspectives (lacking a concept of assimilation) and those with conception-based perspectives.
(3) Reflective abstraction versus empirical learning. Learning of mathematical concepts is a process of reflective abstraction, not an empirical learning process Simon [3]. The former develops anticipation of the logical necessity of a mathematical relationship. The latter only leads to a prediction without an understanding of why it must be that way.
(4) A new mathematical operation. Teachers struggle with what it means for students to develop a new operation, for example multiplication. Teachers tend to teach about multiplication to students who have no concept of multiplication. Important is the idea that the term "multiplication" must label for the student a commonality that they perceive in their actions in particular situations. It is only when a student observes that what I did in this problem about the cost of 5 candy bars is "the same" as what I did in this problem about 7 boxes of pencils, that she has something to label as multiplication - that commonality. This perception of commonality builds on the learner's anticipation of the activity needed and the effect of that activity. This pedagogical concept is based on the concept of assimilation and the concept of learning through activity discussed next. It is part of a larger issue involving students' development of new mathematical objects.

Although we have worked with and studied a number of fine, reform-oriented teachers, they have generally not demonstrated an understanding of the concepts identified above. Thus, understanding teacher learning is essential. Each of these pedagogical concepts can serve as an agenda item for research on mathematics teacher development. For each concept we can ask the questions:

(1) To what extent can teachers at different stages come to understand this concept?
(2) What is the process of development and how can development be fostered?
(3) How are concepts related in terms of prerequisite concepts and co-developing concepts?

The identification of pedagogical concepts that can serve as goals of teacher education is the first step in establishing and enacting a research agenda on mathematics teacher development.

References

[1] P. Cobb, E. Yackel, & T. Wood, A constructivist alternative to the representational view of mind in mathematics education, Journal for Research in Mathematics Education, 23, (1992), 2-33.

[2] B. McNeal, & M. Simon, Mathematics culture clash: Negotiating new classroom norms with prospective teachers, Journal of Mathematical Behavior, 18, (2000), 475-509.
[3] M. Simon, Key developmental understandings in mathematics: A direction for investigating and establishing learning goals, Mathematical Thinking and Learning, 8, 4, (2006), 359-371.
[4] M. Simon, R. Tzur, K. Heinz, M. Kinzel, & M. Smith, Characterizing a perspective underlying the practice of mathematics teachers in transition, Journal for Research in Mathematics Education, 31, (2000), 579-601.
[5] E. Yackel, & P. Cobb, Sociomathematical norms, argumentation, and autonomy in mathematics, Journal for Research in Mathematics Education, 27 (4), (1996), 458-477.

PD as Educating Awareness so as to Inform Behaviour

John Mason

A longer version of these notes is available from the author.

My approach is experientially phenomenological, and so I began my presentation with a mathematical task, through which it was possible to experience some aspects of what teachers (need to) do:

- Creating, and sustaining a mathematically appropriate milieu;
- Selecting, augmenting and creating not just tasks, but task-domains (suites of related tasks including variations and extensions);
- Interacting with learners and supporting learner-learner interaction, choosing both pedagogic strategies and didactic tactics;
- Mediating between conventional and idiosyncratic, local notation and meanings;
- Validating learners' own criteria (evaluation).
- Reflecting on intended and actualised actions, and 'proflecting' by imagining acting in desired ways in similar situations in the future.
- In order to initiate and engage in these actions it is necessary that teachers have access to a structured view of what constitutes a topic.

I distinguish between knowledge as a possession probed through essays, tests and interviews which at best reveal knowing-about (made up of knowing-that, knowing-why, and knowing-how in a theoretical manner). What matters professionally is knowing-to act, in the moment, participating in a moment of choice as some possibility comes to mind. The contrast is evidenced by the acknowledged difference between espoused and enacted beliefs, principles and theories.

I also distinguish between reactions (based on metonymic triggers usually below the surface of consciousness and making use of reinforced behaviours), and responses based on metaphoric (structural) resonance and making use of possibilities that are brought to mind with some element of participation in choosing whether or not to act.

Metaphors

An endemic but inappropriate 'frozen' metaphor of social forces acting on a situation as if they were physical forces for which the resultant force causes the resultant effect employs the inappropriate mechanism of cause-and-effect despite wilful and both reactive and responsive agents acting in non-causal ways. An alternative might draw on the chemistry of (re)agents, whose mutual presence transforms the very nature and influence of the components. An even more suitable metaphor might be organic growth, development and evolution arising from individual and group responses to stimuli from the environment.

Effective PD

I suggested that PD can only be considered to be effective under two conditions, one immediate, one longer term: participants are able to imagine themselves acting differently in the future, and learners benefit in some way from participating in those actions. Effective PD offers

- Fresh insights and language or labels for referring to those insights;
- Resonance with past and-or experience, to make fresh sense of the past or to make it possible to imagine oneself acting freshly in the future;
- Crystallising nascent awareness previously below the surface of consciousness;
- Reconsideration of habits and assumptions;
- Alerts to other ways of perceiving, acting and articulating.

There is an essential circularity in accepting the existence of beliefs, attitudes and values as components of the psyche: what you think you detect when collecting data arising from probes is the reaction-response which people have internalised (through enculturation) as appropriate when being probed for the causes or sources of their behaviour.

Obstacles to Effective PD

Pitfalls in providing PD include

- Displaying desire that teachers change;
- Using a discourse based on deficiency rather than one based on ongoing development from a basis of current strengths and expertise;
- Someone no longer in the classroom telling teachers 'what to do' or 'how to think';
- Trying to 'give' teachers 'what they need', or focusing solely on 'what they say they need';
- Failing to be consistent between what is practiced and what is espoused;
- Stressing changes in behaviour without opportunity for teachers to educate their awareness, since it reinforces a procedural focus for their teaching;
- Failing to engage teachers in mathematics as the basis for immediate experience on which to reflect, so that discerned elements can be described and negotiated so as to serve as a reminder of learner experience.

Revisiting basic mathematical assumptions in teacher education

Rina Zazkis

There are many goals we are trying to achieve in education of teachers. With a focus on mathematics, the following are my main goals:

(a) to improve/enhance teachers' personal understanding of mathematics
(b) to examine/introduce the variety of students' possible understandings or misunderstandings of mathematics

I suggest that examination of 'basic assumptions' provides a possible means towards achieving these goals.

'Basic assumptions' are pieces of information that are essential in mathematical problem solving, but are not mentioned explicitly. These can be conventions, such as positive direction of a rotation, or shared understandings with respect to the context of any given task. One of the goals in teacher education is to raise awareness of these implicit assumptions, and in such increase teachers' mathematical understanding and pedagogical sensitivity. Further, when saying 'basic assumptions' I refer to assumptions related to mathematical content, rather than those related to the nature of learners or learning processes. I distinguish between different kinds of assumptions: assumptions that are mathematical conventions, assumptions that are shared understandings, and assumptions that present unintended constraints to problem solving.

Among several tasks that exemplify a mathematical convention I ask teachers to consider the following claims:

(a) The sum of interior angles in a triangle ABC is 280 degrees
(b) The graph of a function y=x is a parabola.
(c) A number is divisible by 5 if and only if the sum of its digits is divisible by 5.

There usually is a tendency to claim that the statements are wrong or to consider the elements that did not fit as misprints or typographical errors and therefore to make some corrections, like taking x to the second power in (b), changing 5's to 3's in (c), and changing 280 to 180 in (a). However these statements can be true if the triangle ABC is on a sphere, if the function is represented with focus-directrix coordinates and the divisibility statement is made about numbers represented in base 6.

Among several problems that exemplify a shared understanding I ask teachers to consider the following:

- *Grandma baked 12 cookies for three of her grandchildren. How many cookies will each child have?*
- *280 students of ABC elementary school will go to a field trip by buses. There are 40 seats on a bus, how many buses are needed?*

Immediate answers to these and similar textbook problems do not account for the complexity of the "real world". For example, the normative solution assumes that cookies are shared equally and that teachers or parent are not accompanying students on their trip.

An example of a task that that presents unintended constraint is a request to cut a paper square into 10 squares, using all the material. Many people experience difficulty with this task, assuming that the squares must be the same size. The problem becomes easy when this unintended constraint is removed.

Revisiting basic mathematical assumptions tasks are used to

(a) increase teachers' awareness of underlying mathematical ideas, an
(b) increase teachers' awareness of students' possible mathematical ideas.

According to Skemp, "to understand something means to assimilate it into an appropriate schema" (p. 46) . A question that pertains to mathematics teacher education is how can one understand better what has been already understood, that is, assimilated. I extend Skemp's claim by suggesting that to understand something better means to assimilate it in a richer or more abstract schema. A central tenet of Piaget's theory is that an individual, dis-equilibrated by a perceived problem situation in a particular context, will attempt to re-equilibrate by assimilating the situation to existing schemas or, if necessary, reconstruct particular schemas enabling the individual to accommodate the situation. I argue that tasks that require revisiting basic mathematical assumptions dis-equilibrate teachers and therefore create a need for reconstruction of their schemas.

References

[1] Skemp, R. R. (1971). The Psychology of Learning Mathematics. Penguin Books.

Improving the learning of mathematics: Addressing the professional development challenge

Sharon Friesen

Mathematicians and mathematics educators have been working together to provide professional development opportunities to teachers. We have created ways to engage teachers in learning mathematics, increasing their pedagogical effectiveness, engaging with and developing more robust mathematical problems, and connecting with colleagues. We have started to actively research the impact of some of the initiatives. In this session, I hope to provide and overview of the work we are doing and to solicit input and feedback on ways to strengthen our efforts.

"Mathematics Done Differently": a German Initiative for Teachers' Professional Development

Bettina Rösken

Much research has been conducted in the area of teachers' professional development focusing on different issues and analyzing the topic from various perspectives. Certainly the most important person is the teacher him or herself. Every passing year of the career, he or she becomes increasingly experienced and in so doing, aspects of teaching are explicably connected with learning (Krainer & Llinares, 2006). Furthermore, this learning takes place every day and professional development is at first just another name for the everyday life of the job (Tenorth, 2007). Even though, from this viewpoint, professional development occurs even without any intervention from outside there are educational reforms and policies that constitute demands teachers are supposed to meet (Day & Sachs, 2004). In this regard, the challenge is balancing both positions. Top-down implementation is easy to get started (Kohonen, 2007) but the question emerging than is, *Who owns the project?* Innovative approaches, in contrast, are sensitive to teachers' needs and their conception of professional development. Consequently, initiatives should not be designed in terms of "either/or" but "both/and". The initiative "Mathematics Done Differently", founded by Deutsche Telekom Stiftung, is concerned with spreading and broadening existing local or regional in-service programs in different thematic fields and to design new ones accordingly to teachers' needs. The main characteristics of the initiative are: showing appreciation for the teachers, connecting research and practice (trainers operate as *tandems* of researchers and teachers), involving expertise of other teacher educators (courses*a la carte*), considering explicitly teachers' needs by designing courses *on demand*, addressing groups of teachers from one school or neighboring ones, offering elaborated course material on the homepage and evaluating all courses at three measuring points. Courses *on demand* initiate a broad procedure of mentoring teachers which includes providing help to concretize their needs, classifying the request with regard to research and finally, identifying experts that may serve as "trainers". The long term goal is to establish a net of experts for the different thematic fields suggested by teachers. First evaluation results indicate that a successful approach has been established by the initiative. Teachers' expectations regarding courses and trainers were confronted with their assessment after the in-service training. The findings showed that in most cases the assessment exceeded most expectations.

References

[1] Day, C. & Sachs, J. (2004). Professionalism, perfomativity and empowerment: discourses in the politics, policies and purposes of continuing professional development. In C. Day & J. Sachs, *International Handbook on the Continuing Professional Development of Teachers.* Berkshire: McGraw-Hill.

[2] Kohonen, V. (2007). Towards transformative teacher professionalism. Developing preservice and inservice teacher education in Finland. http://www.glanz-tagung.de/programm.html [11.12.2007]

[3] Krainer, K. & Llinares, S. (2006). Professional Aspects of Teaching Mathematics. In A. Gutierrez & P. Boero, *Handbook of Research on the Psychology of Mathematics Education. Past, Present and Future*, 429-459.

[4] Tenorth, E. (2007). Lehrer wird man im Job und ein guter Lehrer durch Fortbildung im Beruf. http://www.mathematik-anders-machen.de/index2.html [21.02.2007].

SINUS - A trademark for improving mathematics education in Germany

Peter Baptist

"Mathematics Revolution" - that was the headline in the German weekly newspaper DIE ZEIT in fall 2006. Very enthusiastically a journalist reported on math lessons he had monitored. More than 1800 schools had participated in a very successful project with the bulky title "Increasing Efficiency in Mathematics and Science Education". In this project my chair at Bayreuth University was the responsible adviser for mathematics. Therefore I intend to explain our special ''philosophy''. We had succeeded in integrating the teachers in the development of ideas and materials and therefore they have accepted changes in teaching. Mathematics turns out to be an experimental science - at first an unfamiliar point of view for most of our teachers... More information see: `http://sinus-transfer.de`

Can the best tradition of mathematics teaching in Hungary be preserved at the Bologna process?

Ödön Vancsó

In this presentation the Hungarian teacher training tradition will shortly be summarised by some concrete materials and experiences. Afterwards the new system (according to the so called Bologna process) made by 'Hungarian art' will be introduced (during the last two years I have been taking part in the work of the committee of the Eötvös Lóránd University in Budapest). Some 'by-paths' also will be shown which make as far as possible to preserve some advantages of the old system (and parallelly to avoid some obvious disadvantages of the new). Some interesting special course used in teacher training will be mentioned and in my opinion a secret comes to light, why the Hungarian system was (is?) so good for talented students. At the end of the presentation will be reported on two ongoing EU projects involving our department on this field.

Mistake-Handling Activities in the Mathematics Classroom: Effects of an in-service Teacher Training on Student's Performance in Geometry

Kristina Reiss

In a quasi-experimental study with 619 students from 29 classrooms (grades 7/8) we investigated the effects of a teacher training on teachers' mistake-handling activities and students' learning of reasoning and proof in geometry. Teachers of the experi-mental group classrooms received a combined training in mistake-handling and teaching reasoning and proof, whereas the teachers of the control group classrooms only took part in a training on teaching reasoning and proof. Their students participated in a pre- and post-test. Moreover, they were asked to evaluate how the teachers handled their mistakes. Our findings show that the teacher training was successful: the teachers of the experimental group classrooms changed their mistake-handling behavior and, compared to the control group classrooms the students in the experimental group performed significantly better in the post-test.Our study particularly takes into account the work of Oser and colleagues on the role of mistakes for learning processes (Oser & Spychiger, 2005). Accordingly, we postulate that mistakes are necessary to elaborate the individual idea about what is false and what is correct. According to the theory of negative expertise individuals accumulate two complementary types of knowledge: positive knowledge on correct facts and processes, and negative knowledge on incorrect facts and processes (Minsky, 1983). Learning by mistakes is regarded as the acquisition of negative knowl-edge. Detecting one's own errors helps to revise faulty knowledge structures. Storing past errors and the cues that predict failure in memory may prevent individuals from repeating mistakes (Hesketh, 1997). Mistakes are a necessary part of the learning process, and, moreover, error management skills are of particular importance when applying heuristic strategies in the mathematical problem solving process. As empirical findings for Germany indicate teachers and students hardly use mistakes as learning opportunities. Error management trainings in other domains show positive effects for the learning process. In the present study we are interested whether a special in-service teacher training on mistake-handling in mathematics classroom has positive effects on students' performance. In our study we address the question to what extent a teacher training about the role of mistakes for the learning process in mathematics has an effect on

(1) students' perception regarding their teachers' mistake-handling activities in mathe-matics lessons and
(2) students' performance in geometric reasoning and proof.

We trained in-service teachers regarding the role of mistakes in the teaching and learning process. Our findings indicate that this teacher training was successful from two points of view: On the one hand, the teacher of the experimental group changed their mistake-handling behavior in such a manner that it was recognized by the students. The effect sizes indicated moderate effects. On the other hand, the performance of the students in the experimental group improved significantly better in comparison to that of the control group. This improvement is mainly based on a better performance in solving geometrical proof items, i.e. items on a high competency level. Analyzing the data in detail showed an improvement of the affective and cognitive teacher behavior in the perspective from the students. These two factors base partly on common items, however, if we consider only the specific affective or cognitive related items there is a similar tendency. Hence, in-service teacher training might have a positive effect on a teacher's behavior as noticed by the students.In spite of this change in the behavior there is no clear improvement in the self reported students' behavior concerning their own mistakes. Probably the effect of the teacher training is restricted to a modification of the teacher reaction in mistake situations. With respect to achievement in geometric reasoning and proof, students of the experimental group outperformed the control group. This is particularly due to a better solution of complex proof items in the pre- and post-tests; however, there is a small effect size (d = 0.22). We hypothesize that this effect is particularly influenced by the improved mistake-handling activities of the teachers. Since the teachers in the experimental group were more open-minded about students' mistakes in mathematics classroom, a better improvement of students' achievement for complex mathematics tasks is in line with the theoretical assumptions. The results of our intervention study give evidence that an in-service teacher training on mistake-handling activities has positive effects on the mathematics classroom. Nevertheless, further research studies are necessary to optimize the outcome of such training sessions. In particular, one has to think about methods how to guide students to use their individual mistakes for improving their learning in mathematics. The creation and evaluation of specific learning material for this purpose may be one possible way.

References

[1] Hesketh, B. (1997). Dilemmas in training for transfer and retention. *Applied Psychology: An International Review, 46*, 317-339.
[2] Minsky, M. (1983). Jokes and the logic of the cognitive unconscious. In R. Groner, M. Groner & W. F. Bischof (Eds.), *Methods of Heuristics* (pp. 171-193). Hillsdale, NJ: Erlbaum.
[3] Oser, F., & Spychiger, M. (2005). *Lernen ist schmerzhaft. Zur Theorie des Negativen Wissens und zur Praxis der Fehlerkultur.* Weinheim (Germany): Beltz.

The professional knowledge of German mathematics teachers: Investigations in the context of the COACTIV project

Werner Blum

The COACTIV Project (Baumert/Blum/Neubrand, 2002-present) is investigating the professional knowledge of mathematics teachers as well as connections between this knowledge and teaching behaviours and student learning. In my presentation, I will show how we have conceptualised the content knowledge (CK) and the pedagogical content knowledge (PCK) of mathematics teachers and how we have operationalised it by means of tests. I will present some results of these tests carried out with the teachers of the German PISA classes 2004, in particular how CK and PCK influence the progress in mathematics achievement of the PISA classes between 2003 and 2004. The presentation will conclude with some direct implications for teacher education.

Interdisciplinarity in mathematics teacher education ($\varepsilon - C$)

Bharath Sriraman

Introduction

Interdisciplinarity has become relevant for emergent sciences of the 21st century. Yet elementary mathematics is still rooted in rudimentary arithmetic on traditional problems. Nearly all curricular documents in the U.S (NCTM, NRC, NCMST) and elsewhere (OECD) make a call for connecting mathematics (internally) and externally to relevant problems. The goals of this project were to emphasize Connections, Coherence, Civic Consciousness and Cross-curricular competencies with pre-service and in-service teachers. K-8 teachers in the U.S are pre-dominantly generalists (with exceptions in some states where specialists are found in grades 6-8). Teacher education programs typically do not prepare prospective teachers to incorporate interdisciplinary activities that cohere with reading, science, mathematics and societal issues. By Civic Consciousness, I mean shared global problems in our world today e.g., fair trade (setting market prices that are equitable for persons within an economic chain), environmental indexes, ethical issues, information reporting (ability to fact check, source check, detect biases).

Design and Motivation for the Study

A survey conducted on practices in elementary school mathematics with 560 participants (from Montana, Wyoming, Eastern Washington, Southern Alberta, Saskatchwan) indicated that among these practicing (K-8) elementary school teachers (2-27 years of experience) 95% taught mathematics as an isolated subject, 75% taught mathematics at the end of the day in order to keep up with the prescribed curricula, 10% connected reading and science activities, and only 12 out of 560 teachers considered interdisciplinary activities involving mathematics. These findings resulted in the initiation of this project. The goal of this professional development project [ε- Changes, hence forth $\varepsilon - C$] was (1) to create "small" changes in elementary math content courses without compromising content, (2) Follow through with prospective teachers interested in interdisciplinarity through student teaching (clinical experiences) and as practitioners, (3) Promote civic consciousness in communities, and (4) Study the implementation and effectiveness of interdisciplinary activities as self-reported by teachers and through classroom observations.The data gathered so far include written solutions and projects from120 prospective elementary school teachers, 18 of whom are now practitioners. Eight problem scenarios were investigated by these teachers. Three of these problem scenarios were in the process of being implemented in elementary classrooms. Interview and classroom observation data has also been collected.

The Theoretical Framework

The problem scenarios and pedagogical/curricular goals are situated within the framework of prior work done in the areas of (1) Realistic Mathematics Education (Freudenthal,...), (2) Critical Mathematics Education (D'Ambrosio, Skovsmose,...), (3) Wor(l)d Problems (Greer, Verschaeffel) , and (4) Critical pedagogy (Freire, Gutstein, Vithal,...).

The Scenarios

The goal of $\varepsilon - C$ was to connect the standard curriculum to socially relevant problems, i.e., combine uses of mathematics and critical thinking skills. Scenarios were created that typically involved making Fermi estimates (1) Reasoning in ratios, (2) Estimation and (3) Problem Solving. Critical thinking and interdisciplinary goals included (1) examining assumptions, (2) Sources of data, (3) Fact/Source checking, (4) Explaining discrepancies in sources, (5) Validity. In other words, the purpose of this approach was to create a paradigmatic and humanistic shift about

the purpose/relevance of mathematics in today's global economies and societies. Scenarios emphasized the three key, identified skill sets: (1) Reasoning in ratios, (2) Estimation and (3) Problem Solving. Scenarios in the context of the culture and communities that surround the students *Scenario 1: Consumerism and the Environment* Total wastes in the United States, excluding wastewater amounts to approximately 50 trillion pounds a year.
Scenario 2: Equitable distribution of Global Resources "Simplified" debt calculations between (ex-colonial) countries abundant in natural resources (the debtor) and those that do not have the same magnitude of resources (the collector).
Scenario 3: "Fair" Trade and the Kyoto Protocol Comparative data on GNP, per-capita income and resource consumption between OECD and non-OECD countries (Brazil, China, India) and the reasons for huge discrepancies and their consequences.
Discussion of Scenario 1: Prototypical student solutions, reflections and discussion of scenario 1 are found in an article currently in press in Interchange: A Quarterly Review of Education. The preprint of this article is available at `http://www.umt.edu/math/reports/sriraman/abstract_16.html`.
Discussion of Scenario 2 and 3: A more detailed historical description and relevance of Scenarios 2 and 3 are found at:
`http://www.math.umt.edu/TMME/Monograph1/`.

There are various possibilities and implications for the use of such scenarios in professional development programs. For instance:

(1) Implementation of interdisciplinary activities around Earth week which involves science, math, field trips to local water treatment and recycling plants, as well as local landfills.
(2) Presentation of findings to the community by pre-service students and elementary school children
(3) Elementary school children involved in generating meaningful tasks

Examples of such implementable projects with measures of effectiveness are found in
B.Sriraman, C.Michelsen, A. Beckmann & V. Freiman (Eds). (2008). Proceedings of the Second International Symposium on Mathematics and its Connections to the Arts and Sciences (MACAS2). University of Southern Denmark Press.
Visit `http://www.macas2.sdu.dk/`.

Teacher education that contributes towards professional development: An overview of 25 years of research

João Pedro da Ponte

The last 25 years witnessed the emergence of research on teachers, teaching and teacher education as a specific field of study within mathematics education. The launching and subsequent success of the Journal of Mathematics Teacher Education is an eloquent of this fact. During this period, the knowledge about teachers' professional knowledge and practices developed considerably, taking into account contributions from Piagetian perspectives, theories about professional competence, theories about beliefs and conceptions, theories of cognitive psychology, and sociocultural theories of learning. At the same time the challenges of educating and supporting teachers increased with the dissemination of innovative views regarding the mathematics curriculum that in many countries coexist with an increasing social criticism about the results of students learning in mathematics. New technological possibilities may be put to service to the education of teachers, at the same time that many argue that the Gordian not that ties the teacher activity is the professional culture and the professional status of teachers. This paper reviews progresses made in this period by research on teachers and teacher education, indicating what we may take as main achievements and what current open issues may be tackled by the field.

Teacher's Growth through Participating the Gau-Jan Program in Taiwan

Fou-Lai Lin

The National Science Council of Taiwan has instituted, in 2005, the Gau-Jan program which is a senior high school curriculum development funding program focusing especially on

(1) integrating emerging science and technology into school curriculum,
(2) exploring inquiry teaching approach,
(3) collaborating between schools and universities.

After one year of preparation, 16 senior high schools' projects are reviewed and funded for 2006-2010, and 12 schools' projects are funded for 2007-2011. All Gau-Jan schools are arranged for annual site visiting by a team of inspectors organized by the NSC.

As a critical friend, I have visited all the 16 Gau-Jan schools, funded in the first round during the spring of 2007. To each school, the site visiting program includes project progress report, interviews with participating students, teachers, school principles, university professors and other project consultants. Based on the site visiting data, a story about the professional development of those participating senior high school teachers is described.

One of the teachers summarized her first year of Gau-Jan life as

"Panic about the unknown leads me to intensive study;
intensive study enhances my progress;
successive progress gives me a provision of harvest."

Based on participated teachers' words, their awareness is classified into five components.

(1) ***Awareness of the problematique of designing teaching module***, e.g., I always feel dissatisfied with my self-designed teaching materials.
(2) ***Awareness of the problematique of inquiry teaching approach***, e.g., conventional teaching materials have standardized answers to questions. Now with inquiry-oriented teaching materials of which the questions

are open and answers uncertain, I'm concerned as to how to respond to students' alternative thinking.

(3) ***Awareness of the contemporary goal of self learning***, e.g., after years of teaching I feel that the knowledge I possess is like a pool of dead water; the words given by university professors are indeed enlightening.
(4) ***Awareness of learners' learning***, e.g., we switched on the students' will to learn with hands-on materials. Students were poor in learning by doing with hands-on materials, before they joined Gau-Jan; after they tried to make things themselves for one semester, they could even begin with buying the materials themselves to make things.
(5) ***Awareness of a new teaching perspective of teaching becomes learning***, e.g., Gau-Jan is a big challenge to the students and the teachers; Both the students as well as the teachers are learning.

The use of computers in interdisciplinary teaching szenarios

Jürgen Richter-Gebert

Computers offer a variety of versatile tools for teaching mathematics. In particular programs for function plotting, dynamic geometry and computer algebra systems play a crucial role in mathematics education.

The talk demonstrates how computers can be used in interactive teaching scenarios for interdisciplinary topics. Here in particular connections of mathematics to computer science, physics, biology and music are explored. Simple examples are presented that can be used to show the connection of mathematics to each of these disciplines. By exploring these examples with a computer very often the student experiences mathematics from a new perspective that emphasizes the usefullness of mathematical modeling. The examples are presented with the interactive mathematics software Cinderella whose new version emphasizessuch interdisciplinary scenarios. Besides a kernel that supports dynamic geometry the program offers a multi purpose scripting language, an environment for particle simulations and an interface to a music synthesizer.

Using video to investigate mathematics teachers' professional vision

Miriam Sherin

The introduction of portable video equipment in the 1960s offered a great deal to social science researchers who were interested in studying human behavior. In a sense, video allows researchers to "freeze time" and move slowly through human behavior on the order of seconds or minutes. This is precisely the time-frame in which events take place in a classroom. Therefore, it seems reasonable to think that video would be a useful tool for analyzing classrooms. Furthermore, video might be a useful tool not only for researchers but also for teachers who want to look closely at their own practice. In this report I briefly consider three questions surrounding the use of video with teachers: (a) What is the space of possible uses of video with teachers? (b) What kinds of video are useful for teachers? and (c) What might video help teachers learn?

First, one way to distinguish among the range of video-based programs that currently exist is through three key dimensions. One dimension refers to the media tool that is used, in other words, the kind of access to video that the program provides - videotape, DVD, multimedia format, etc. Another dimension concerns the nature of the video that is viewed. Do teachers view video from their own classrooms or from other teachers? Is the video raw footage or edited? Finally, we can consider the task and social setting of the program. This includes, for example, whether teachers have volunteered to participate, what kind of facilitation is provided, and what program goals have been established.

Second, within all of these formats, a critical decision is the choice of video that will be viewed. I argue that some video excerpts are likely to be more productive for teacher learning that others. Specifically, in recent work we identified several types of video clips that consistently promoted substantive teacher discussions of students' mathematical thinking (Linsenmeier & Sherin, 2007). Of particular importance is the extent to which a video provides a window into student thinking, that is, whether there is evidence of student reasoning in the video clip. Similarly, it is important to consider the depth of student thinking revealed in the video, whether students are exploring substantive mathematical ideas or are engaged in more superficial problem-solving activities. In addition, we found it valuable to consider the clarity of the student thinking in the video, whether or not a students' idea was easily understood on an initial viewing of the videos.

Finally, in consdering what video might help teachers learn, I believe is it useful to explore a construct introduced by Goodwin (1994) called "professional vision." The idea is that as we become part of a professional discipline we are trained to see events in a particular way. Thus teachers' professional vision involves the ability to notice and interpret significant events in a classroom. Furthermore, it is this component of teaching expertise that I believe video is particular well-suited to help develop. In my research on video clubs in which groups of teachers watch and discuss video excerpts from their classrooms, I found that teachers extended their professional vision over the course of a year-long video club. Specifically, teachers came to pay increased attention to student mathematical thinking and developed a range of strategies for interpreting the ideas that students raised (Sherin, 2007; Sherin & Han, 2004; van Es & Sherin, in press).

References

[1] Linsenmeier, K., & Sherin, M. G. (2007). What?, Wow!, and Hmm...: Video clips that promote discussion of student math thinking. Journal of Mathematics Education Leadership, 10(1), 32-41.

[2] Goodwin, C. (1994). Professional vision. American Anthropologist, 96, 606-633.

[3] Sherin, M. G. (2007). The development of teachers' professional vision in video clubs. In R. Goldman, R. Pea, B. Barron, & S. Derry (Eds.) R. Goldman, R. Pea, B. Barron, & S. Derry, Video research in the learning sciences (pp. 383-395). Hillsdale, NJ: Erlbaum.

[4] Sherin, M. G., & Han, S. (2004). Teacher learning in the context of a video club. Teaching and Teacher Education, 20,163-183.

[5] van Es, E. A. & Sherin, M. G. (in press) Mathematics teachers "learning to notice" in the context of a video club. Teaching and Teacher Education.

Professional development through historical research in mathematics

Jan van Maanen

The ICMI-study on *History in Mathematics Education* [1] presents a variety of fruitful connections between history and pedagogy of mathematics. History shows the context in which mathematical theories were created, and this is useful knowledge for the teacher who wants to teach mathematics in a problem-oriented manner. Another argument for integrating history in teaching is that mathematics would not exist without mathematicians. Some students prefer learning mathematics in a very detached way, but others tend to ask questions about human aspects. These students are more convinced that descriptive statistics is worth learning when they know, for example, about the work of Florence Nightingale.

One fundamental problem is that generally the historical knowledge of mathematics teachers is restricted, and also the pedagogical possibilities are not always within reach. As a consequence the efforts that some textbooks make to present historical elements (contexts, biographical notes, isolated exercises) are annihilated by the decisions of teachers to 'skip all this dust'.

In the years since 2000 history of mathematics appeared to be an interesting and productive area of research for mathematics teachers (in this discussion I shall concentrate on teachers who worked supervised by me). At the same time the historical research functioned as a domain for professional development. The historical content knowledge, the research skills and for some researchers also the educational outcomes were valuable.

In The Netherlands it is possible to do a PhD-project on a part-time basis. Barbara van Amerom and Iris van Gulik worked in this manner. Van Amerom studied the question wherher historical problems and texts help pupils to learn algebraic problem solving skills [2]. A survey appeared in [3]. In the same vein, but concentrating on geometry, was the study by Iris van Gulik–Gulikers. The PhD [5], about the 'reinvention of geometry', was in Dutch, but the first chapter also appeared as a journal article in English [4].

Other historical projects by teachers were done within the framework the Dutch Science Foundation ('NWO'). The branch of 'Exact Sciences' has a programme called *Leraar in Onderzoek* (The teacher researcher), which allows teachers together witn an academic supervisor to apply for a grant. The grant enables the school to appoint a substitute teacher for one day a week during two years, in which time the teacher do research. The programme, started 1999, has a double goal: it wants to create research output, but it also aims at professional development. Several teacher researchers did and do historical research witin this programme. One study recently led to a complete PhD [7], another is currently producing very interesting teaching materials which are available through the internet [6].

Does this type of small scale research tuition have an effect on the professional development. There are several reasons to answer this question positively. The effect on the academic level of the teacher community as a whole may be ignored, but the teacher–researchers served as an important source of inspiration. They gave many presentations for their fellow teachers and reported in the Dutch professional journals. The lesson sequences about similar triangles by Van Gulik were tested in 46 school classes throughout The Netherlands. They had impact on teachers and their pupils. A maybe unwanted effect is that the teacher–researchers directly develop a higher professional profile, and employers clearly notice this. One of the NWO-projects (not one mentioned above) had to be stopped soon after the start, because the teacher went to a job in higher education. On the other hand, this observation may stimulate more teachers to start with such a project, since it clearly improves their career perspective.

References

[1] John Fauvel, Jan van Maanen (eds.), *History in Mathematics Education. The ICMI-Study*, Dordrecht: Kluwer Academic Publishers 2000 (ISBN 0-7923-6399-X, 456 pp.)

[2] B.A. van Amerom, *Reinvention of early algebra. Developmental research on the transition from arithmetic to algebra*, Utrecht: Freudenthal Instituut 2002 (ISBN 90-73346-48-7, 340 pp.)

[3] B.A. van Amerom, *Focusing on informal strategies when linking arithmetic to early algebra*, Educational Studies in Mathematics, (54)(1) (2003), 63–75

[4] Iris Gulikers, Klaske Blom, *'A historical angle', a survey of recent literature on the use and value of the history in geometrical education*, Educational Studies in Mathematics, (47)(2) (2001), 223–258

[5] Iris van Gulik–Gulikers, *Meetkunde opnieuw uitgevonden. Een studie naar de waarde en de toepassing van de geschiedenis van de meetkunde in het meetkundeonderwijs*, Utrecht: PhD-thesis 2005

[6] Henk Hietbrink, website with teaching materials based on the book with *Geometrical Exercises* (1659) by Frans van Schooten: `http://home.planet.nl/~hietb062/home_wis.htm` (inspected 11 December 2007)

[7] M.H. Sitters, *Sybrandt Hansz. Cardinael 1578-1647, rekenmeester en wiskundige: zijn leven en zijn werk*, Groningen: PhD-thesis 2007

Internet-supported Teaching and Learning in Mathematics Teacher Education

Hans-Georg Weigand

There is an ongoing debate how to integrate the Internet into university teacher education. Despite the huge number of Internet pages in the area of mathematics, very little is known whether and how these pages support the*l*earning of mathematics. There are only a few empirical studies about this crucial didactical question.

MaDiN-System

In a joint project of five German universities the Internet-based system MaDiN (Mathematics Education on the Web – www.madin.net) was developed. It is an Internet-based teaching and learning system for the teaching of students and preparing them to become mathematics teachers. MaDiN will be integrated into a traditional lecture, helps lecturers to prepare and to give their lectures and supports students to prepare and repeat the contents of the lectures. It is used in addition to the lecture and not instead of the lecture. The system stimulates the student to learn on his own, provides possibilities of controlling his knowledge and fosters the communication both between students and the lecturer and among the students.

Additional Online-Courses
More over, there were three Online Learning Courses developed: "Basics of Arithmetics", "Basics of Geometry", and "Mathematics and Computers". These courses are provided by the Virtual University of Bavaria and we use these courses in our lectures as a repetition of school mathematics, as a preparation for the didactics courses and to make students acquainted with new technologies, Computer Algebra Systems and Dynamic Geometry Software.

Compared with a book ...
... we expect a number of advantages of the Internet-based knowledge-base. In particular we have the possibility of:

- A permanent update of the pages with new and current contents; especially a permanent update with authentic materials, which gives students a feeling of working within the frame of "up-to-date" circumstances;
- Permanent accessibility and especially availability during the lectures (if there is an Internet access);
- Video-based explanations of hand-oriented activities;
- Explaining problems and their solutions on different levels;
- Providing interactive tools and experiments with real-world models.

Due to the use of new media (e. g. videos, downloads) and the appropriateness of hypertext (HTML), it is necessary to provide the students with clear structures, so they can easily organize their studies on their own without getting lost.

Evaluation
We integrated the system into some semester-long regular course, and we have done a summative evaluation. In short form a few results:

- The animations and the interactive elements played a crucial role for many students.
- The students sometimes missed the relationship between the topics in Internet courses and those dealt with in the lecture.
- Nearly all students didn't use further written materials like books or articles.
- There were two categories of students. The first group appreciated the electronic course, for the others it was only an obligation.
- Different types of activities are necessary: questions for students' individual work, open problems and problems which foster collaborative working.
- The way students understand the contents is slightly changed. The understanding is more based on a dynamical view.
- With some students we noticed an influence on the didactical attitude or the students' view of the contents, their mathematical belief.

Although we used the systems in different lectures, we are not able to answer the questions: Have students learned more? Did they gain better results? It is very difficult to isolate the variable "electronic system" from the many other variables (different lecturers, teaching styles, contents) in an empirical investigation.

References

[1] Weigand, H.-G., (2002), New Ways of Communicating in Mathematics Teacher Education: Linking to the Internet

[2] Weigand, H.-G. et al. (ed.), Selected Papers from the Annual Conference on Didactics of Mathematics, Potsdam, Verlag Franzbecker, Hildesheim and Berlin

[3] See also: `http://webdoc.sub.gwdg.de/ebook/e/gdm/2000/index.html`

Paper and pencil test or video based instruments: How to measure teacher competence?

Aiso Heinze

(joint work with Anke Lindmeier)

Introduction

In the last years several studies were conducted to measure teacher competence. Many of these studies used an approach where test persons had to describe their actions in hypothetical teaching situations. This situational testing framework is well known and used, for example, in assessment centers and for research in situational interviews, as well as in paper-and-pencil tests. The basic idea is to value a reaction in the test situation as an indicator for a future real life situation.

Still the validity problem is not to be neglected as it is dependent on at least three major factors: the understanding of the construct teacher competence, the criteria for the quality of this construct, and last but not least the modality of measuring these aspects.

Teacher competence

Analyzing current studies on mathematics teacher competence (cf. reports of H. Bass, W. Blum, or G. Kaiser in this volume) one can state that research is focused on at least three major components of teacher competence: basic knowledge, reflective competence, and situative competence. Under basic knowledge we understand in the sense of Shulman ([2]) content knowledge, pedagogical content knowledge, and pedagogical knowledge (each related to mathematics instruction). This includes particularly knowledge of typical errors, different teaching approaches for certain topics, etc. Reflective competence encompasses the didactical organization of lessons and composing of task sequences. Further is included the ability to analyze student learning processes, and to choose between possible actions or reactions suitable for the student's position. This category aims at a competence showing up in a reflective process before or after the active teaching situation. The third major component, situational competence, could be identified as the active component. It shows up in form of spontaneous reactions in critical or challenging teaching and learning situations. Here teachers typically have just a few seconds to find a decision.

Whatever aspect one takes into account, an open question is how to evaluate responses or reactions of teachers on given test items. On the one hand this question leads back to cultural-sensitive values, beliefs and norms of good mathematics teaching. On the other hand, the problem of lacking empirical evidence for clear criteria on the effectivity of different teaching methods shows up. It is not clear how to rate reactions and responses of teachers obtained in these testing situations.

Different scoring methods are established, e.g. expert scoring, consensus or majority scoring, and target scoring. For example, these methods are discussed in the field of emotional intelligence ([1]). Presently, only expert scoring seems to be a passable way, i.e. expert reactions are taken as a norm for an adequate teacher reaction. However, what happens if the experts do not agree at all? The consistency of the expert reactions must be considered as one criterion for the quality of a test item.

Video-based diagnostics and a new tool

The main advantage of video-based research in teaching and learning is the possibility to expose teachers to a situation as realistic as possible in a constructed teaching situation. Having a look at the existing video-based studies, teachers were asked to answer in either multiple choice or Likert scale formats, other closed answer format (keyboard, paper-and-pencil), or open answer format (keyboard, paper-and-pencil). In most cases no immediate reaction of the teachers was expected, so they have time to reflect on the situation before typing/writing an answer.

For some testing situations this is a suitable - or at least unproblematic way of evaluation, e.g. evaluating a student's written solution for an exercise, planning a lesson, or choosing tasks for a lesson. For other situations, e.g. if immediate reaction of the teacher is required in a critical classroom situation, or if students ask questions awaiting a direct answer, this method of testing is obviously problematic. Presently, we are working on a computer environment that, besides the above mentioned answering possibilities, allows to record immediate teacher reactions. Video clips are shown to the teachers, who then must react by giving a verbal answer, either in form of a direct response to the student in the video clip, or as an explanation how he/she would react.

Additionally it is planned to give teachers the possibility to write down some notes (geometric figures, formulas, etc.), simultaneously recorded by a small camera. In case teachers are reacting slowly, the pressure can be increased after a given time, e.g. by an additional question or reaction of the student presented in form of another short video clip. The big advantage of the new testing environment is a less biased reaction of the teacher. Evaluative studies will show whether this new video-based instrument really allows getting a more detailed and more differentiated picture of mathematics teacher competence.

References

[1] J. D. Mayer, P. Salovey, D. R. Caruso, & G. Sitarenios, *Emotional intelligence as a standard intelligence.* Emotion **1** (2001), 232–242.
[2] L. S. Shulman, Paradigms and research programs in the study of teaching: A contemporary perspective. In M. C. Wittrock, (Ed.), *Handbook of research on teaching.* New York: Macmillan (1986), 3–36.

Mathemapedia: a wikipedia for mathematics education

John Mason

In response to a request from the National Centre for Excellence in Teaching Mathematics which was set up by the UK government to coordinate PD in mathematics education for England and Wales, I have initiated a wikipedia for mathematics education.

Called the MathemaPedia it is accessible from the portal site

`http://www.ncetm.org.uk`

or directly by appending /mathemapedia. The portal as a whole provides a range of support and stimulus for professional development, from communities (blogs and forums both open and closed) and news about events, to self-evaluation and links to UK government standards, and to resources and research summaries. Increasingly these are interlinked with each other and with the mathemapedia.

At the core of the site is the mathemapedia which, unlike the original factually based wikipedia, is focused particularly on stimulating PD. Thus each record in the mathemapedia provides probes (questions to ask yourself which might prompt you to make observations, try something out, and link to the literature) and also further action, often in the form of suggestions for working with colleagues on related issues. It is intended that a collection of case studies (brief accounts of classroom incidents) will begin to emerge, with commentary relating the case study to constructs in the mathemapedia. Currently records are assigned to one or more categories:

Mathematical Concepts, Pedagogical Constructs (which have proved informative), didactical devices and tactics (pertinent to specific mathematical topics or manipulatives including ICT), Issues & Concerns, Pedagogic Strategies, Professional Development, and Mathematical Themes, Powers and Heuristics.

The mathemapedia is open to anyone to use, and anyone who registers can offer new records or can suggest edits to existing records.

In response to the question, why not simply augment the wikipedia, the idea is to have a discipline self-contained site which can be used easily by PD providers (for example as follow-up to or inspiration for PD provision) and by teachers working by themselves or in groups on some aspect of teaching and learning mathematics.

The mathemapedia will only be of use of colleagues contribute. For example, in the UK, pre-service teachers and masters students have to provide evidence of making use of ICT to develop their professionalism. Contributing to a forum, modifying or initiating a mathemapedia record would be a useful way to provide such evidence.

Taking theoretical ideas into the practice of professional development

Abraham Arcavi
(joint work with Alan Schoenfeld)

On the basis of a theoretical model of the teaching process (Schoenfeld, 1998; 1989), we developed a workshop designed to provide teachers with analytical tools with which to reflect upon a variety of teacher practices. Two rounds of this workshop were implemented in Israel with teacher leaders and mathematics educators. During the workshop participants collectively watched videotapes that included examples of a range of teacher decisions and actions. The tapes, four of which are the TIMSS tapes of Japanese and American classrooms, served as stimuli for conversation. Spontaneous comments made by the participants resulted in discussions among workshop members. The workshop leader (the first author) attempted to guide the discussion with a set of prepared questions concerning issues of teacher knowledge, goals and beliefs. Initially, two main kinds of comments predominated: "evaluative" and "research oriented." As the workshop progressed, largely as a result of explicit re-direction, a third and potentially more productive mode of analysis of teacher practices evolved. We briefly described the design of the workshop, the settings in which they were implemented and the first two main "watching modes" and comments. We then describe the third mode, with both its initial difficulties and its productive aspects. It appears that teachers and researchers can be induced to think about why other teachers (and perhaps themselves) make particular instructional moves, and that they can engage substantially with the roles of knowledge, goals and beliefs in shaping teachers' instructional choices. The full experience is described in Arcavi & Schoenfeld (2006).

References

[1] A. Arcavi & A.H. Schoenfeld, 2006, "Using the unfamiliar to problematize the familiar" (In Portuguese) Chapter 4 in M. Borba (Ed.) Tendencias Internacionais em Formaçâo de Professores de Matemática (International Perspectives in Mathematics Teacher Education), pp. 87-111 (English version available).

[2] Schoenfeld, Alan H. (1998). Toward a theory of teaching-in-context. Issues in Education, Volume 4, Number 1, pp. 1-94.

[3] Schoenfeld, Alan H. (1999) (Special Issue Editor). Examining the Complexity of Teaching. Special issue of the Journal of Mathematical Behavior , 18 (3).

Narrowing Klein's Gap in Western Canada

Klaus Hoechsmann

This presentation, planned as a sequel to that of Sharon Friesen, my colleague from Calgary, did comment on Klein's *Merano Reform* as well as on the *New Math* and the NCTM *constructivist* initiative. Each of these was to inspire a particular kind of *thinking*, (functional, conceptual, creative), but failed to show teachers the sort of mathematics apt to nurture their enthusiasm in the face of drab curricula with nothing but seemingly unrelated university courses under the belt.

School teachers, too, need to see part of the mental landscape mathematicians call beautiful, through windows uncluttered with daunting formalisms. The modest proposal of this talk is to begin filling the trench between school and university with a coherent body of mathematics of the kind now relegated to disconnected "recreational" vignettes. It lacks neither logic nor clarity, but is *informal* in the sense of Schoenfeld [2]. The present brief report unfortunately allows no more than a few hints at what is possible.

Dandelin's insight (cf. `www.clowder.net/hop/dandelin/dandelin.html`), for instance, ought to make conic sections widely appreciated and understood, as it displays planes, cones, and spheres in an amazingly transparent set of relations — which easily *could* (but need not) be used to derive the formulas usually crowding the foreground and hiding the core.

Adding the *volumes* of cone and sphere can be done soon after the Pythagorean Theorem: replacing its three squares by, say, quarter circles, cutting different sets of these (with constant hypotenuse) out of heavy paper, and then glueing the cut-outs into three cardboard corners at appropriate levels, one obtains a tangible and yet theoretically convincing model of the "cone + sphere = cylinder" result of Archimedes. For his famous 1:2:3, it remains to note the relation between cone and cylinder, echoed as 1:3 by pyramid and prism.

When numerical measures get involved, they call for new operations, e.g., the square root, found by iteratively "squaring up" rectangles of a fixed area $A = wh$, i.e., computing $w := (w + h)/2$ and then $h := A/w$ as the new width and height each time around. Together with Pythagoras, this opens up formerly inaccessible distances, whence areas like that of a disc. Obtained by iterating $u := \sqrt{(u+1)/2}$ and then $A := A/u$ (these having been duly derived!) with $u = 0$, $A = 2$ at the start, the latter lands squarely in Viète's π-recipe.

Such computational merry-go-rounds are quite suitable for work in small groups, where each participant receives input from one neighbour and hands output to the next. They are not as rote as might be feared, since a host of observations occurs along the way, and the individual steps, beginning as utterly simple, can be gradually concatenated to formulas typically found in algebra texts. Calculators should be allowed, with any new button becoming "legal" only when its action can be fluently simulated by more primitive ones.

If, in the preceding loop, $A := A/u$ is replaced by $v := v/(u + u)$, with $v = 1$ initially, it yields Cartesian coordinates (u, v) for binary fractions of a right angle, i.e., a rudimentary trig-table, useful for improved precision in scale drawings based on angles. As in Feynman et al. [1], repeated square roots similarly yield binary fractional powers of 10, i.e., a simple log-table, which helps with problems about growth and decay. Time allowing, Feynman's naively ingenious treatise can be followed all the way to Euler's $e^{i\theta} = \cos\theta + i\sin\theta$.

The loop initialising $m = 800$, $n = 801$, $y = 100$ (say) and repeating $y := y * m/n$, $m := m - 1$, $n := n + 1$ computes a scale model of the 1600th line in Pascal's triangle. After time spent on the arithmetic and combinatorial properties of the latter, the histogram (bars 1 mm thick) of the y-values comes as a surprise: a bell-curve accurate enough to reproduce the kind of table found in elementary probability texts. Take logs, and you are back to the parabola! This unexplained (experimental) result can serve as the starting point for further exploration.

References

[1] R.P. Feynman, R. B. Leighton, M. Sands, The Feynman Lectures on Physics (1965), Vol. 1, Chapter 22, Table 22-1. Reading, MA: Addison-Wesley

[2] Schoenfeld, A. H., On mathematics as sense-making: An informal attack on the unfortunate divorce of formal and informal mathematics. (1990). In J. Voss, D. Perkins, & J. Segal (Eds.), Informal reasoning and Education, pp. 311-343. Hillsdale, NJ: Erlbaum.

Educating teachers about values in mathematics education

Alan Bishop

In my research I have defined values in mathematics education as the deep affective qualities which education aims to foster through the teaching of mathematics. Values are not the same as beliefs, although the two constructs are related, and also I argue that although there is much research on beliefs in mathematics education there is little on values. One task for researchers is to thoroughly review the research on beliefs to see if it is more appropriately considered as research on values. I relate the two constructs by seeing values as 'beliefs in action', that is, one may hold several beliefs, but when one is faced with action choices it is one's values which determine which choice one decides to adopt. Beliefs seem rather to be the support or justification for one's choices.

From a research perspective, there is only a limited understanding at present of what values are being transmitted, and of how effectively they are being transmitted (see Bishop [2]). Perhaps this is because most values appear to be taught and learnt implicitly rather than explicitly in mathematics classrooms.

My initial analytic work on values appeared in the book Mathematical Enculturation (Bishop [1]), and was linked with my general analysis of mathematics as a form of cultural knowledge. In 1999 the first empirical project, the Values and Mathematics Project (VAMP), started and the findings and papers from that project can be found at:

`http://www.education.monash.edu.au/research/groups/smte/index.html/`

and follow the links.

The second empirical project, the Mathematics and Science project, investigated primary and secondary teachers' values and practices in mathematics and science teaching. (Bishop, Clarke, Corrigan, D. & Gunstone, R. [4]). It used the six value classification of Rationalism, Objectism (or Empiricism as the science educators preferred it), Control, Progress, Openness, Mystery (Bishop [1]). There were important differences between the two groups of teachers, and between the subjects they taught. Those latter differences show that teachers' values in the classroom are shaped to some extent by the values embedded in each subject, as perceived by them. In terms of values teaching, the subject context of mathematics is different from that of science. This implies that changing teachers' perceptions and understandings of the subject being taught may well change the values they can emphasise in class.

References

[1] Bishop, A. J.: 1988, Mathematical enculturation: A cultural perspective on mathematics Education, New York: Springer.

[2] Bishop, A.J. (1999) "Mathematics teaching and values education - an intersection in need of research". In Zentralblatt fur Didaktik der Mathematik, 99/1, 1-4.

[3] Bishop, A. (2001). Research into practice: what values do you teach when you teach mathematics? Teaching Children Mathematics, 7(6), 346-349.

[4] Bishop, A.J., Clarke, B., Corrigan, D. & Gunstone, R. (2006) Values in mathematics and science education: researchers' and teachers' views on the similarities and differences. For the learning of mathematics, 26(1), 7-11.

Facilitating Reflection and Action: The Possible Contribution of Video to Mathematics Teacher Education

David Clarke

Our capacity to conceive of, develop and implement effective teacher education programs is limited by the simplistic nature of the models on which such programs are explicitly or implicitly based. In the Interconnected Model of Teacher Professional Growth Clarke et al. [1], change in teacher beliefs, knowledge and practice is mediated by either enaction or reflection (see Figure 1). Key aspects of this model are its non-linearity, specification of the mechanisms of change, capacity to accommodate previous models (eg. Guskey [2]), and its use of "Salient Outcomes" rather than simply "Student Outcomes" as the mediating change domain between Professional Experimentation and Changes in Teacher Knowledge and Beliefs.

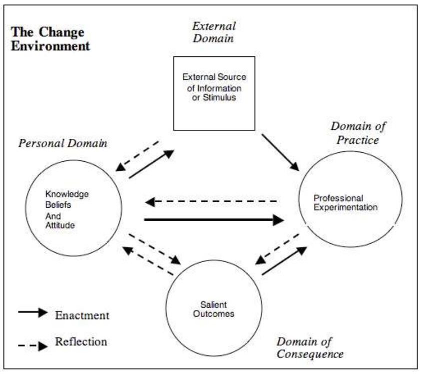

Figure 1 - The Interconnected Model of Teacher Professional Growth

The stimulus for change can be provided by an external source such as a professional development program or it can result from the teacher's inevitable classroom experimentation and her reflection on the consequences of that experimentation. Video can play a key role in catalysing change and facilitating teacher reflection. In particular, video can play a catalytic role with respect to:

(i) International research employing video and the capacity of such research to inform practice in both pre-service and inservice settings Clarke et al. [3]. Analyses of well-taught classrooms overseas can offer teachers insights into the novel, interesting and adaptable practices employed in other school systems and into the strange, invisible, and unquestioned routines and rituals of their own classrooms. International comparative research can provide insurance against the inevitable insularity of our attempts to document, theorise and improve the practices of our classrooms.

(ii) The use of video in professional development programs and the choice between images of exemplary and problematic practice as catalysts for teacher reflection in both pre-service and inservice programs. Case-based professional development programs can catalyse teacher reflection by using video excerpts of classroom situations and posing the question, "What might the teacher do in this situation?"

(iii) Video as one tool by which a standards-based professional culture can be both problematised and realised. The use of video as a tool to illustrate professional standards must recognise that the integration by teachers of any such standards into their professional practice is mediated by the teachers' perception of what they consider to be the salient features of that modelled practice; and

(iv) Teacher agency can be promoted through the use of video in supporting teachers' reflection on their own practice. In particular, video discussion groups provide a forum for teachers to share video excerpts of their own classroom practice (with or without an academic facilitator). In such a culture, video becomes the communicative medium to sustain a professional community of reflective practitioners.

Inevitably, the language by which we describe classroom practice is culturally bounded. This language determines the visibility of key events and constrains our theorising about the classroom. A more sophisticated technical language is needed to describe the mathematics classroom and video can play a key role in the development of such a language.

References

[1] Clarke, D.J. & Hollingsworth, H. (2002). *Elaborating a model of teacher professional growth.* Teaching and Teacher Education, 18(8), 947-967.

[2] Guskey, T. R. (1986). *Staff development and the process of teacher change.* Educational Researcher, 15(5), 5-12.

[3] Clarke, D.J., Emanuelsson, J., Jablonka, E., & Mok, I.A.C. (Eds.). (2006). *Making Connections: Comparing Mathematics Classrooms Around the World.* Rotterdam: Sense Publishers.

From changes in the preparation of mathematics teachers to changes in practice: reflecting on the IUFM experience

Michéle Artigue

I. The creation of the IUFMs and its consequences on teachers' preparation
Before the creation of the IUFMs, the preparation of mathematics secondary teachers was taken in charge by universities. After obtaining a *licence* in mathematics, students prepared a national competition. Those who passed this competition had then one year of practical training, with one class in full responsibility and some professional lectures. If no major problem was detected, at the end of that year, the student became civil servant and got a permanent position in a secondary school.

In the eighties, social changes progressively destabilized this system leading to the creation of the IUFMs (University Institutes for Teacher Preparation) in 1990. As reported in [1], one important aim of this creation was to find a better balance between the academic and professional parts of the formation, and to have teacher preparation benefit more from the results of educational research for facing the new challenges met by the profession. The creation of the IUFMs did not strongly affect the first year of preparation that remained mainly devoted to the consolidation of basic university mathematics knowledge, to the establishment of connections between maths domains, to the clarification of the mathematics underlying the secondary curriculum, the selection and analysis of educational tasks on a mathematical basis.

In the second year, conversely the changes were important. Beyond the practical training, a substantial professional preparation was organized at the IUFM (1 to 2 days per week), structured around three main dimensions: mathematics knowledge for teaching with a specific emphasis on topics generally under-represented in the students' previous preparation, didactical and pedagogical knowledge, and knowledge of the educational system. In addition, students were asked to prepare a professional dissertation, trying to answer a question raised by their practice.

Pre-service teachers had also a supervisor in the IUFM visiting them in their classroom, maintaining contact with the secondary school supervisor, supervising also the preparation of the professional dissertation.

As shown by several studies, the interest of these changes was not immediately perceived by pre-service teachers. For them, the most important part was their professional work in the class they had in full responsibility. What they aimed at was a class functioning normally at a reasonable cost. They showed a spontaneous tendency at interpreting all the difficulties they experienced in pedagogical, institutional or social terms, and looked for solutions at these same levels. Having them understand the role played in the observed problems by their mathematical and didactical choices was much more difficult. They would have liked the IUFM formation to provide them with solutions to the specific difficulties they met, easy to implement and close to their actual practices. Instead of that, the formation proposed was quite generic, trying to install the basis for a long term professional development which could just be initiated in the IUFM years. It wanted to introduce students to some important ideas about mathematics epistemology, learning processes, classroom interactions, tools for the understanding of the students' difficulties, for the design and analysis of classroom situations. Moreover when teaching scenarios were proposed, these were generally very expert and distant from what the students would have designed themselves. Getting the pre-service teachers adhere such a project was not at all obvious.

II. The intertwined evolution of research and practices

These difficulties were the motor of a progressive adaptation. On the research side, they induced an increasing interest in didactic research regarding the teacher and her professional development [2]. The regular recruitment of French didactic researchers in the IUFMs indeed created a new context for French didactic research, which had developed mainly through collaborative work with experienced teachers within the IREM network. This new context questioned its theoretical frames and methodologies, led to an exponential growth of research on teachers' practices and professional development, an international phenomenon but whose dynamics for French research was influenced by the specific didactic culture and context.

This increasing interest on teachers influenced the existing frames and was also the source of new constructions. If one looks at the recent developments of the two main frames: the theory of didactic situations (TDS) initiated by Brousseau and the anthropological theory of didactics (ATD) initiated by Chevallard, this is quite evident. For the TDS, it led Brousseau to refine the notion of *didactic contract* creating a scale for situating contracts along levels of increasing didacticity [3]. It also led to complement the vertical structure of the *milieu* by introducing positive levels and positions corresponding to the different milieux involved in the design work of the teacher [4]. For the ATD, it led to complement the notion of *mathematics praxeology* by the notion of *didactic praxeology*, and to investigate the dialectic relationships between these [5], to the structure of *didactic moments* and, more recently, to the hierarchy of *levels of determination* for praxeologies from the mathematical topic to the civilization level [6].

New constructions were also developed and, among these, the *didactic-ergonomic* approach of teachers' practices inititated by Robert and Rogalski [7] is becoming more and more influential. Within this approach relying on activity theory, the teacher is seen as a professional working in a dynamical and open environment, building her own coherence through the combination of determinations of different nature: cultural, social, institutional and personal. Teachers' practices are seen as a coherent complex system. They are analyzed at different scale levels and this analysis is structured around five components: the cognitive, mediative, social, institutional and personal components. The picture that this analysis provides is used in order to identify and explain regularities and variations, understand the *genre* of the teacher profession [8] as well as personal *styles*, and infer from this understanding potential dynamics for professional development.

It results from such a construction a more realistic look at the teacher's professional work, the teacher being seen as a professional working in complex, changing and at times risky environments to which she has to adapt permanently; a specific attention to the *teacher's workload*, a crucial notion in cognitive ergonomy, and the sensibility to the fact that optimizing a complex system cannot be achieved through the separated optimization of its different components. This induces a different look at the formation, the evolution of teachers' practices being seen as a long term dynamics which has to be sensitive to the genre and styles of the profession. Moreover, all along this dynamics, the *distance* between the *old* and the *new* is seen as a critical variable conditioning workload and thus viability. Many doctoral thesis attest today the productivity of such an evolution (see for instance [9],[10]).

Along the years and intertwining with research evolution, training practices in the IUFMs have also evolved. Retrospectively the two evolutions look quite coherent. An increasing emphasis is put on the reflection on practices, work with concrete cases and questions proposed by the students themselves through videos, narratives or cross-observations; the formation is much more integrated and supported by the use of technology, ICT contributing to the development of communities of practice; the operational dimension of didactic knowledge is much more focused on; a better coherence is achieved between the formation at the IUFM and the terrain formation in secondary schools thanks to the development of specific programs for the terrain supervisors; moreover the formation has been extended to the two first years of professional activity. Thanks to these changes, the situation has progressively improved. The system is nevertheless entering now a new period of turbulence due to several factors: the decision taken by the government of integrating the IUFMs into universities, the recent definition of standards for teachers' competences and the limited place these give to the mathematics and didactic competences, the way European harmonisation is going to affect the qualification and recruitment procedures, the reduction in the number of students entering mathematics programs in universities and its logical consequence on the number of students preparing the national competition. We can only hope that the knowledge built in the last decade will be helpful for facing these new changes and the associated challenges.

References

[1] Henri, M., Cornu, B. (2001). Mathematics Teachers' Education in France: From academic Training to Professionalization. In, D. Holton (Ed.), *The Teaching and Learning of Mathematics at University Level: An ICMI Study*, pp. 481-500. Dordrecht: Kluwer Academic Publishers.

[2] Margolinas C. & Perrin-Glorian M.J. (Eds). (1997). Recherches sur la modélisation de l'enseignant. *Recherches en Didactique des Mathématiques*, vol. 17/3.

[3] Brousseau G. (1997). *Theory of Didactical Situations in Mathematics*. Dordrecht: Kluwer Academic Publishers.

[4] Margolinas C. (1998). Le milieu et le contrat, concepts pour la construction et l'analyse de situations d'enseignement. In R. Noirfalise (Ed.), *Actes de l'Université d'Eté " Analyse des pratiques enseignantes et didactique des mathématiques "*, 3-16. IREM of Clermont-Ferrand.

[5] Chevallard Y. (1999). L'analyse des pratiques enseignantes en anthropologie du didactique. *Recherches en Didactique des Mathématiques*, vol. 19/2, 221-265.

[6] Chevallard Y. (2002). Organiser l'étude. In J.L. Dorier y al. (Eds), *Actes de la Xème Ecole d'Eté de Didactique des Mathématiques,* pp. 3-22 & 41-56. Grenoble : La Pensée Sauvage.

[7] Robert A., Rogalski J. (2002). Le système complexe et cohérent des pratiques des enseignants de mathématiques : une double approche. *La revue canadienne des sciences, des mathématiques et des technologies*, vol 2 .4, 505-528.

[8] Clot, Y. (1999). *La fonction psychologique du travail*. Paris: PUF.

[9] Lenfant A. (2002). *De la position d'étudiant à la position d'enseignant : l'évolution du rapport à l'algèbre de professeurs stagiaires.* Doctoral thesis. University Paris 7.

[10] Cirade, G. (2006). *Devenir professeur de mathématiques : entre problèmes de la profession et formation en IUFM. Les mathématiques comme problème professionnel.* Doctoral thesis. University Aix Marseille.

Developing mathematics teaching: Teachers and didacticians as practitioners and researchers in a co-learning inquiry model

Barbara Jaworski

The research team of which I am a part at AUC seeks to know more about how opportunities for learners of mathematics in school classrooms can be enhanced through development of understandings and practices in mathematics teaching. A model of co-learning inquiry assumes that teachers and didacticians together participate in social practice in research and development to promote better learning of mathematics for students. Our projects LCM and TBM (Learning Communities in Mathematics and Teaching Better Mathematics) are rooted in theoretical perspectives of communities of practice/inquiry (Wenger, 1998; Jaworski 2006).

According to Wenger, belonging to (or having identity in) a community of practice involves engagement, imagination and alignment. We engage with ideas through communicative practice, develop those ideas through exercising imagination and align ourselves "with respect to a broad and rich picture of the world" (p. 218). Align, literally 'to line up with', indicates that we are positioned according to, or in line with the practices and activities in the communities in which we participate. The totality of such communities, according to Wenger, offers a "broad and rich picture of the world". The terms participation, belonging, engagement and alignment all point towards the situatedness of doing and being and the growth of knowledge in practice. In LCM, teachers and didacticians engage with mathematics in practices in workshops and school settings and align themselves with existing or emerging practices related to the particular setting. Imagination contributes to the emergence of new practices.

We provide for interactivity of teachers and didacticians in workshop and school situations in which we explore (inquire into) possibilities for enabling pupils to inquire into and hence learn mathematics. Fundamental to this inquiry process is that "belonging" to the project community transforms Wenger's "alignment" to "critical alignment" through which we question overtly both established practices and the new approaches we design within the project. We start by using "inquiry" as a tool for learning and work towards inquiry as a way-of-being-in-practice.

Workshops introduce mathematical tasks and provide opportunity for all participants to work together on these tasks and reflect on their activity and participation. In schools, teacher teams decide how they want to approach development in their classrooms, building on workshop practices, modifying workshop practices for classrooms, or taking workshop ideas into their own design for classrooms according to goals within their school situation. Didacticians visit schools and offer support or involvement according to teachers' expressed needs. Didacticians collect data from all situations (largely qualitative). A parallel longitudinal study collects data (quantitative and qualitative) on pupils' mathematical knowledge and achievement and pupils' and teachers' attitudes towards mathematics learning and teaching.

A tension that is perhaps not surprising, is that of teachers' expressed perceptions of the role of didacticians in the project in contrast with didacticians' interpretation of their role. Teachers expect didacticians to show them new ways of teaching and how to enable high mathematical achievement by pupils, whereas didacticians wish to engage teachers in exploration of approaches to teaching and learning with observation and analysis. Over the time of the project these differing perceptions have been reconciled to some extent and teachers and didacticians are learning side by side about what can be achieved in classrooms and how, and what are the tensions and issues we must face. The processes of critical inquiry in which we engage offer the roots of this learning.

References

[1] Jaworski B. (2006). Theory and Practice in Mathematics Teaching Development: critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education.* Special Issue: Relations between theory and practice in mathematics teacher Education. **9** (2), 187-211.

[2] Wenger, E.: 1998, *Communities of practice*. Cambridge: Cambridge University Press.

Observing Mathematics Education through the Eyes of Complexity and Diversity

Chris Breen

Introductory Context

My teaching and research in the past five years has been strongly influenced by ideas coming from Complexity Science and Ecology [1] and in particular from the theory of enactivism [2]. This approach has been enriched by an attempt to focus on my own and my students' awareness of what is happening and the choices that each of us are making at hinge-moments [3]. The Discipline of Noticing [4] has proved to provide a useful method for collecting rich situations to discuss and explore.

In an interesting parallel development, in recent years I have also been asked to teach courses on the topic of Complexity and Diversity at the Graduate School of Business at the University of Cape Town. These courses take place against a specific contextual background where the government has introduced policies of Employment Equity and Affirmative Action in an attempt to redress some of the effects of the apartheid regime which prevented many women and Black South Africans from having sufficient opportunities to occupy senior positions of responsibilities in the corporate world.

All these courses on Complexity and Diversity run for at least one full day – on longer certificated courses there is another full day's follow-up on subsequent modules. My aim in the course is to introduce participants to Complexity Thinking as a foundation for an increasingly espoused alternative management and leadership paradigm. The course emphasizes the importance of multiple perspectives, conversation, mindfulness and the creation of an environment that encourages diverse ideas and voices.

Some Experiences.

Research in mathematics education is far more likely to focus on mistakes or misconceptions than on the problems that occur from too much success. The majority of those business school students who display a comfortable regard for their own abilities and a reluctance to turn to others for advice claim to have been successful students at mathematics at school. They report that they were rewarded for displaying their abilities in class. Again, it is these same people who, after missing the gorilla, continue trying to make a case for why they were correct in what they did. They will often be obsessed with finding out what the correct answer is, or for claiming that the person who saw the gorilla could not have been concentrating on what she as supposed to have been doing.

Working in an enactive paradigm requires a leader to consider seriously the input from all members of the team. In both these activities the response shows that those most comfortable with their own abilities are most unlikely to consider it possible that they might be mistaken and to listen to (or even ask for) different perceptions. In the South African situation, those showing the most self-reliance and suspicion of those who think differently from them are often white males.

Concluding Thoughts.
In 1986 during the height of the uprising in schools against the apartheid regime in South Africa, there was a call for teachers to think about their particular subject in terms of 'alternative education'. I led a group of students who explored what this might mean for mathematics teachers [5].

In the current milieu, mathematics teacher education research has focused almost exclusively on finding ways of identifying the best way to teach mathematics at different levels. Yet there are strong social and environmental movements which make strong statements about education, such as the **Earth Charter (**Integrate into formal education and life-long learning the knowledge, values, and skills needed for a sustainable way of life) and the **Declaration of the Rights of the Child** (He shall be given an education which will promote his general culture and enable him, on a basis of equal opportunity, to develop his abilities, his individual judgment, and his sense of moral and social responsibility, and to become a useful member of society).

What are the responsibilities of the mathematics teacher educator in trying to address broader societal considerations as to the effects of current teaching methods on the production of citizens who are suitably prepared for society in the 21st century?

References

[1] C. Breen (2002). *Researching Teaching: Moving from gut feeling to disciplined conversation.* South African Journal of Higher Education, **16**, 2, 25-31.

[2] H. Maturana & F. Varela (1986). *The Tree of Knowledge.* New York: Shambhala.

[3] F. Varela (1999). *Ethical Know-How.* Stanford: Stanford University Press.

[4] J. Mason (2002). *Researching Your Own Practice: The Discipline of Noticing.* London: RoutledgeFalmer.

[5] C. Breen (1986). Alternative Mathematics Programmes. *Proceedings of the 8th National MASA Conference, Stellenbosch*, 1-4 July.

An experienced teacher's journey to teach mathematical modeling

Kai-Lin Yang

(joint work with Fou-Lai Lin)

It's challenging for our teachers to implement an innovative and time-consuming learning activities in mathematics classrooms. There are three sources of reasons: textbooks, entrance examinations and teachers' and students' experience. Our textbooks are edited with the rule-example-exercise format. The meaning of concepts is embedded through calculations and algorithms. The University Entrance Examination strongly influenced what and how teachers design to teach and what and why students anticipate learning in our culture of diploma first. Although applied problems are not really excluded in this examination, teachers and students believe that to acquire basic concepts and skills is necessary to solve applied or modeling problems. Under the above background, teachers' and students' experience make the notions of modeling unfamiliar.

For encouraging teachers and students to appreciate mathematics modeling activities Lesh et al. [3], Gravemeijer [2], we invited Huu to try math modeling activities. Before providing activities for Huu, we talked about the view of good teaching, the benefits of understanding students' thinking, teachers' reflection and process of reflective practice with Huu. He agreed that to engage students into productive learning is good teaching, exploring students' thinking is beneficial for instructional design and teachers' development of professional proficiency, and teachers should reflect on problem solving, teaching and learning based on both practical and research fields. Moreover, we proposed that producing, clarifying and dissolving doubt Cooney [1] or discontent are important for teachers' reflections. Therefore, Huu is encouraged to produce his doubt or discontent, and we can clarify and dissolve the doubt or unsatisfaction together.

Five supporting activities are provided for Huu to teach mathematical modeling problems: Understanding Teaching Material, Analyzing Teaching of Mathematical Modeling, Observing One Teacher's Inexperienced Teaching of mathematical modeling, Conceptualizing Students' Thinking of Mathematical Modeling, and Reformulating Teaching Interventions. The sixth activity is not pre-planned. We just wait and see if Huu actively design new mathematical modeling activities.

We reflect how Huu change regarding his role in the teaching materials, the types of questions posed by him, and his belief. In sum, it is confirmed that providing the source of teaching material is helpful for establishing the teachers' confidence in teaching practice, inspecting and learning from another teacher's teaching can break through the limitation of teachers' reflection, and openly discussing the analysis framework of students' modeling thought encourages teachers to design new activities. Three mechanisms are proposed to interpret Huu's change: multiple experiences as interactive evidence, projection of empathy and encouragement of uncertainty. Multiple experiences denote Huu's or other teacher's divers experiences. Interactive evidence denotes these experiences can be selected as evidence to compare or contrast different behaviors or beliefs. The projection of empathy from teachers' experience of solving modeling problems to students' thinking drives teacher to reflect students' difficulty. The encouragement of uncertainty inspires the feeling of doubt. The first two mechanisms are rooted in teachers' experience and knowledge. The third mechanism is originated from unknown.

References

[1] Cooney, T. (2001). *Considering the paradoxes, perils, and purposes of conceptualizing teacher development.* In F. L. Lin & T. Cooney (Eds.), Making sense of mathematics teacher education (pp.9-31). Dordrecht: Kluwer Academic Publishers.

[2] Gravemeijer, K. (1994). *Developing realistic mathematics education.* Utrecht: CD β.

[3] Lesh, R. & Doerr, H. M. (Eds.) (2003). *Beyond constructivism: Models and modeling perspectives on mathematics teaching, learning, and problem solving.* Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Mathematics Teacher Education: A practical field and an Emerging Field of Research

Konrad Krainer

Mathematics teachers' learning is a lifelong learning process. It starts with one's own experiences of mathematics teaching from the perspective of a student, or even with mathematical activities before schooling. Student teachers, teachers and teacher educators are all regarded as *teachers*. They are seen as active constructors of their knowledge, embedded in a variety of social environments which influence and shape teachers, and which at the same time are influenced and shaped by them. Therefore, teachers are expected to continuously reflect in and on their practice and to change it where it is appropriate. *Teacher education* is understood as a goal directed *intervention* in order to promote teachers' learning, including all formal kinds of teacher preparation and professional development as well as informal activities, eventually organized by teachers themselves. *Mathematics teacher education* can aim at improving teachers' (different kinds of) *knowledge*, their *beliefs*, their *practice* and finally at contributing to their *students' affective and cognitive growth*. It is a challenge to find answers to the questions of where, under which conditions, how and why mathematics teachers learn, and how important the domain-specific character of mathematics is. The major sources of this contribution are recent analyses on research in the field of mathematics teacher education, papers written in the Journal of Mathematics Teacher Education (JMTE) and various books and proceedings focussing on mathematics teacher education (see references).

In the last twenty years, researchers in mathematics education put *increasing attention to mathematics teacher education*. This shift is reflected in the emergence of international handbooks. In 1996, the first *International Handbook of Mathematics Education* was published. The first *International Handbook of Mathematics Teacher Education* will be published in 2008. Till about 1990, mathematics teacher education was mainly a *field of practice* (mostly presenting "success stories", more or less theory-grounded and evidence-based), since then it increasingly became also a *field of research*. In 1998, the Journal of Mathematics Teacher Education (JMTE) was launched. Parallel to the start of JMTE, also numerous books and articles on problems and progress in describing and interpreting learning processes by mathematics teachers were published.

The genesis and further development of mathematics (teacher) education can be seen as an *extension of research questions* (of course, overlapping and deviations concerning different countries and contexts exist; a first version of this model was described in Zehetmeier & Krainer, 2005):

- *Phase 0 - Content-related, system-internal questions:* Focus on mathematical contents and curriculum. *Teacher education: introduction to calculus, powerful tasks, fundamental ideas.*
- *Phase 1 - Content-related extension of the system:* Focus on applications and history of mathematics. *Teacher education: sense making.*

- *Phase 2 - Pedagogical and psychological extension:* Focus on the mathematical learning of students. *Teacher education (research): study of students' mathematical thinking (errors ...), emphasis on problem solving (and problem posing).*
- *Phase 3 - Sociological and epistemological extension:* Focus on interactions between students and teachers in classrooms and on the epistemological status of concepts. *Teacher education (research): reflections on mathematics teaching (e.g. transcripts of videos).*
- *Phase 4 - Interdisciplinary extension:* Focus on the learning of teachers and on the impact of teacher education and school development. *Teacher education (research): study of teacher education processes and/or development of new programmes or material (e.g. CD-ROMs).*
- *Phase 5 - Self-reflection-based extension:* Focus on the learning of teacher educators. *Teacher education (research): understanding and improving our own practice as teacher educators (particular and general insights).*
- *Phase 6 - Society- and policy-directed extension:* Focus on mathematical abilities and potentials of education systems, the economy and of the society as a whole; mathematical competencies are regarded as key parts of a "learning system", e.g. putting an emphasis on students' and teachers' mathematical knowledge and beliefs. *Teacher education (research): Closer focus on impact (knowledge, beliefs, practices, motivation ...). Increased external studies on mathematics teacher education and its use as steering knowledge for educational policy (e.g. for defining standards for teacher education).*

There are *three common trends* that can be sifted out of the literature on student teachers', teachers' and teacher educators' learning in mathematics in the last twenty years. These *three trends* refer to the intervention goals and designs of teacher education activities as well as to the research focus on the processes and results of these activities. With regard to two of these trends, namely a) teacher educators' and researchers' *increasing attention* to the *social dimension* and b) to *teachers' reflections*, there is sufficient evidence including examples that indicate the variety of corresponding activities (see e.g. Llinares & Krainer, 2006). The third trend, the increasing attention to the *general conditions* of teacher education (e.g. time, structure, institutional settings, human resources, curriculum), is newer and can be seen as an influence of work done in other fields (e.g. organisational development) on the practice and research in mathematics teacher education.

It is worth taking the three trends seriously and to regard them as *challenges for the future*. Firstly, looking at mathematics teacher education as a *field of practice*, it makes sense to reflect critically the *balances of the individual and the social* (dimensions of teaching and learning), of *action and reflection* (in and on practice) and of *internal and external* (resources and support); it is equally important to concentrate at specific challenges of mathematics education, for example the insufficient image of mathematics teaching or the change that new means (e.g. technology) bring to it. Secondly, *more research* is needed in mathematics teacher education. Many studies on these phenomena so far used qualitative research methods, and it makes sense to go further there in order to generate new explanations and assumptions. It is desirable to use the synergy of teachers' expertise and therefore to engage them in research activities and to support action research, among others with the goal that some of them might develop deeper interest in research and to enlarge the scientific community. However, also more external and quantitative research is needed, in particular looking at the outcomes of different types of teacher education or at longitudinal studies of mathematics teachers' learning and career. Overall, there is a future challenge to combine qualitative and quantitative research methods and to integrate teachers' systematic reflections into research projects. Thirdly, the fact that mathematics is a major subject at schools and in comparative studies can be taken as a chance to see mathematics teacher education as *collaborative endeavour of researchers, practitioners and educational policy.*

References

[1] Adler, J. Ball, D., Krainer, K., Lin, F.-L. & Novotna, J. (2005). Mirror images of an emerging field: Researching mathematics teacher education. Educational Studies in Mathematics 60(3), 359-381.

[2] Ball, D. & Even, R. (Eds.) (In preparation). The International Commission on Mathematical Instruction (ICMI) - The Fifteenth ICMI Study: The Professional Education and Development of Teachers of Mathematics.

[3] Jaworski, B., Wood, T. & Dawson, S. (Eds.) (1999). Mathematics Teacher Education. Critical International Perspectives. London: Falmer Press. Krainer, K., Goffree, F. & Berger, P. (Eds.) (1999). On Research in Mathematics Teacher Education. European Research in Mathematics Education I.III. Osnabrück: Forschungsinstitut für Mathematikdidaktik.

[4] Lerman, S. & Tsatsaroni, A. (2004). Surveying the Field of Mathematics Education Research. Regular Lecture at ICME-10. Copenhagen, Denmark. `http://www.icme-organisers.dk/dg10/Lermanpaper.pdf`

[5] Llinares, S. & Krainer, K. (2006). Mathematics (student) teachers and teacher educators as learners. In A. Gutiérrez & P. Boero (Eds.), Handbook of Research on the Psychology of Mathematics Education. Past, Present and Future (pp. 429-459). Rotterdam: Sense Publishers.

[6] Ponte, J. P. & Chapman, O. (2006). Mathematics teachers' knowledge and practices. In A. Gutiérrez & P. Boero (Eds.), Handbook of Research on the Psychology of Mathematics Education. Past, Present and Future (pp. 461-494). Rotterdam: Sense Publishers. Sowder, J. (2007). The Mathematical Education and Development of Teachers. In F. Lester (Ed.), Second Handbook of Research on Mathematics Teaching and Learning (pp. 157-223). Greenwich, CT: NCTM.

[7] Zehetmeier, S. & Krainer, K. (2005). Handbuch Lehrerbildung [Rezension]. Journal für Mathematik-Didaktik, 26(3/4), 282-294.

Design-based Research and its impact on the beliefs and practices of teachers

Malcolm Swan

Design-based research is concerned with *transforming* educational practices in authentic situations. This approach is characterised as: interventionist, iterative; theory driven; process and utility oriented (e.g. [1]).

In England, the GCSE is the major end qualification for the compulsory phase of education. Each year, 16-19 year-old students that fail to attain the minimum grade required for planned careers or entry into higher education embark on re-sit courses within further education (FE) institutions. Teaching is teacher-centred and transmission-oriented. Colleges over-recruit to compensate for high drop-out rates, attendance is irregular, learning strategies are passive and pass rates are poor. The challenge of the research was to see how far the design of professional development (PD) could enable a transformation from transmission to collaborative teaching practices and from passive to active learning behaviours.

The paper describes three iterations of the research. The first involved just 4 teachers (over 2 years), the second a sample of 45 teachers (1 year); the third, 90 teachers (2 years). Throughout, underlying theories, materials and the PD activities were iteratively refined and revised. Research-based principles for design were elicited from the literature and from earlier empirical studies (e.g. [2], [3]). The design of the PD itself rested on four principles: recognise and make explicit existing beliefs and discuss these in a non-judgmental atmosphere; illustrate vivid, contrasting practices using video (thus providing 'challenge'); invite teachers to 'suspend' disbelief and act in new ways, 'as if they believed differently'; encourage teachers to meet together and reflect on their new experiences. The final design of the PD required 4 days with opportunities in between for classroom experimentation. An extensive collection of classroom activities was developed to illustrate how students may be encouraged to think mathematically. These involved students: *classifying mathematical objects; interpreting multiple representations; evaluating mathematical statements; creating and solving problems - learners; and analysing reasoning and solutions.*

Teachers and students' beliefs, practices and learning were monitored through pre-and post questionnaires, tests and through classroom observation. Students were also asked to report on their teacher's behaviours to provide additional triangulation.

The professional development intervention resulted in: students reporting changes to teachers' practices; teachers reporting changes to their own beliefs about teaching and learning; student learning associated with greater use of the activities and also with student-centred implementations; a significant (but small) improvement in self-efficacy of students. When discussion activities were not used students continued to show a significant regression in confidence, effectance motivation, algebra anxiety and an increase in passive learning behaviours. The outcomes of the research have been an extensive collection of classroom resources that have been circulated to every post-16 institution in England ([4], [5]).

References

[1] Kelly, A., Theme issue: The role of design in educational research. Educational Researcher, 2003. 32(1): p. 3-4.
[2] Bell, A., Some experiments in diagnostic teaching. Educational Studies in Mathematics, 1993. 24(1).
[3] Brousseau, G., Theory of Didactical Situations in Mathematics. Mathematics Education Library. Vol. 19. 1997, Dordrecht: Kluwer.
[4] Swan, M., Collaborative Learning in Mathematics: A Challenge to our Beliefs and Practices. 2006, London: National Institute for Advanced and Continuing Education (NIACE); National Research and Development Centre for Adult Literacy and Numeracy (NRDC). 366.
[5] Swan, M., Improving Learning in Mathematics: Challenges and Strategies. 2005, Sheffield: Teaching and Learning Division, Department for Education and Skills Standards Unit.

Prospective mathematics teachers' professional knowledge

Gabriele Kaiser

(joint work with Sigrid Blömeke, Rainer Lehmann, Anja Felbrich, Christiane Müller, Björn Schwarz)

Teacher education has already been criticised for a long time without its effectiveness ever being analysed empirically on a broader base. The International Association for the Evaluation of Educational Achievement (IEA) therefore launched in 2006 an international study on teacher education (TEDS-M) that focus on policy, practice and teachers' readiness to teach mathematics. Until now 20 countries have decided to participate. The study will be carried out fall 2007 in the Southern hemisphere and spring 2008 in the Northern hemisphere, the results will be released December 2009. In order to facilitate the development of the framework and the instruments IEA carried out a preparatory study - Mathematics Teaching in the 21st Century (MT21) -, whose results are already available and which will be described in this paper.

In Germany as well as in many other countries systematic information about how teachers are trained and how they actually perform at the end of their education is almost non-existent. Program descriptions are usually highly idiosyncratic, and the testing of teachers is almost a taboo. MT21 is the first study that tries to shed light on this important field of education.

In an attempt to fill this gap, the knowledge and belief systems of future teachers are investigated as key factors influencing student achievement. In Germany, middle-school teachers are prepared in two different ways. On the first route, future primary and lower-secondary teachers (grade 1 through 10) are trained, on the second future lower- and higher-secondary teachers (grade 5 through 13). For the study four German regions were selected to take part in MT21. In these four regions all teacher-training institutions were sampled. Within these institutions a complete census was taken. The overall sample size was 849.

The German MT21 data lead to the following conclusions (for details see [1]):

Future teachers' knowledge and beliefs depend heavily on how they are trained. They gain knowledge in those fields emphasized in teacher education and their beliefs change in accordance with the curriculum taught at their institutions.

Regarding mathematical knowledge, future primary/ lower-secondary teachers are at a disadvantage in almost all dimensions measured compared to lower/ higher-secondary teachers. This reflects a significantly lower amount of subject-related learning opportunities.

There are, however, noteworthy relative strengths and weaknesses associated with the respective routes. Primary/lower-secondary teachers show better results in areas in which they are trained, in particular geometry and data-based activities. As opposed to this observation, lower/higher-secondary teachers perform very well in the sub-domain of functions which is a field strongly emphasized in this version of German teacher education. In a cognitive perspective primary/lower-secondary teachers perform relatively well in modelling, an area where lower/higher-secondary teachers display unexpectedly low performance levels. Again, this reflects the moment of emphasis in the corresponding teacher-education programs.

Mathematical knowledge and pedagogical content knowledge are highly correlated in the sense that the ability named first is an important pre-condition for the second. However, mathematical knowledge is by no means sufficient for high achievement!

Primary/lower-secondary teachers outperform lower/higher-secondary teachers as far as pedagogical knowledge is concerned. Taking into account that both mathematical knowledge and pedagogical knowledge are required to teach well, deficits may exist in both types of teacher preparation. Multilevel analyses show that it makes a difference in which federal state a candidate is trained. Future teachers educated in states which provide a longer training do better than teachers elsewhere.

Student recruitment - or academic selectivity - is another issue that matters. In states where training institutions can select between a high number of applicants, future teachers perform better. This finding has obvious policy implications: Not all states or institutions are able to attract a sufficient number of applicants to fill the available slots in their programs - which would be necessary to meet the foreseeable demand! It appears somewhat contradictory to have strong academic selection standards at the entrance of the respective teacher education programmes and then to hire untrained staff because the training institutions cannot provide enough teachers.

Indications are that future teachers' beliefs are changing substantially during teacher education. At the beginning, students show relatively traditional beliefs, apparently seeing little value in teaching methods that provide individual learning opportunities for a heterogeneous group of students. At the end of teacher education, they seem to have a more constructivist-orientated view of instruction, combined with a willingness to support demanding cognitive learning processes.

This result challenges existing research claiming that beliefs are relatively stable over time and little susceptible to external influence.

References

[1] S. Blömeke; G. Kaiser; R. Lehmann (Eds.) (2008). Professionelle Kompetenz angehender Lehrerinnen und Lehrer. Wissen, Überzeugungen und Lerngelegenheiten deutscher Mathematik-Studierender und Referendare - erste Ergebnisse zur Wirksamkeit der Lehrerausbildung. Münster, Waxmann.

Upper elementary school teachers' conceptions about and approaches towards mathematical modelling and problem solving: How do they cope with reality?

Lieven Verschaffel

(joint work with Fien Depaepe, and Erik de Corte)

In the past, ample research evidence was found that in traditional (elementary) school contexts word problems do not foster in students a genuine disposition for realistic mathematical modelling and problem solving. More specifically, several studies (for an overview see Verschaffel et al. [3]) revealed that students often incorrectly neglect their common-sense knowledge and experience about the real world when confronted with word problems in a scholastic context. Current reform-based approaches to (elementary) mathematics education are characterized by a strong(er) orientation towards authentic mathematical modelling. Also in Flanders, new standards and goals related to this topic have been formulated and have led to a new generation of textbooks for mathematics education wherein this new modelling perspective is (at least to some extent) reflected. However, the question remains to what extent these reform-based ideas and textbooks also effectively impact the classroom culture and practice in today's regular Flemish classes.

In the present paper we report on the results of a seven-month-long video-based study in two sixth-grade classrooms in Flanders (September 2005 - April 2006) which used the same popular reform-based textbook called Eurobasis. An average of one problem-solving lesson a week was videotaped for each of the two teachers involved. Shortly after the videotaped lessons we scheduled an in-depth interview with the teachers. The analysis focused on the nature of the problems selected and created by the teachers as well as on the level of "realism" the teachers allowed in the different stages of the word problem solving process. For the analysis of the tasks, we relied on (a reduced version of) Palm's [2] classification scheme of aspects of "realism" of word problem, comprising dimensions such as: 1. Context (is the problem embedded in a potentially real and meaningful context for the students), 2. Data (are the values provided in the problem identical or very close to values in real life outside school), and 3. Form (is the problem presented in a form that resembles a form used in the real world). For the analysis of the teacher's instructional approach, we used an adapted version of Chapman's

[1] analytic frame distinguishing between a paradigmatic-oriented (focusing on the underlying logico-mathematical structure) vs. a narrative-oriented (focusing on the contextual embedding) instructional treatment during the entry into the word problem (understanding and mathematization phase) and the exit out of the problem (interpretation and evaluation phase). Teachers' actual teaching behavior as observed in the videotaped lessons was confronted with their proclaimed conceptions, beliefs and attitudes as verbalized during the interview.

First, the analysis of the tasks used in the five lessons analyzed so far revealed (a) that one teacher (named Ana) used considerably fewer problems than the other teacher (named Pete), (b) that this first teacher used much more self-generated problems, and (c) that the nature of these additional self-generated problems was somewhat more realistic than the problems offered in the textbook.

Second, the analysis of teachers' instructional approaches in these five lessons showed (a) in general, considerably more attempts of both teachers to focus on the problems' prototypical nature than on their contextual embedding, (b) no instructional attention by any of the teacher to the complex relationship between the real-world situation to be modelled and the mathematic model, before the actual computational activities, (c) slight differences in the instructional approaches of the two teachers, in the sense that teacher Ana followed a more narrative approach in the very first stage of the exploration of the broad exploration of the problem context as well as during the very final stage of exploring the relevance of the problem being solved for out-of-school situations.

Finally, the provisional analysis of the interview data revealed considerable differences between the two teachers' conceptions of and attitudes towards realism in word problem solving, which reflected to a large extent the observed differences in their actual preference for certain types of tasks and their instructional approach of the selected tasks.

References

[1] Chapman, O. (2006). *Classroom practices for context of mathematics word problems.* Educational Studies in Mathematics, 62, 211-230.

[2] Palm, T. (2002). *The realism of mathematical school tasks. Features and consequences.*[Unpublished doctoral dissertation] University of Umea, Sweden.

[3] Verschaffel, L., Greer, B., & De Corte, E. (2000). *Making sense of word problems.* Lisse: Swets & Zeitlinger B.V.

Revisiting pedagogical content knowledge: After 20 years

Dina Tirosh and Tommy Dreyfus

The main question that we addressed in the workshop was: Are there specific types of knowledge needed for teaching? We started by arguing that the separation of 'mathematics' and 'education' in professional development is detrimental and that their combination is non-trivial. We then described three attempts to define the specific types of mathematics that are needed for teaching. The first, Pedagogical Content Knowledge (PCK), was proposed by Shulman (1986, 1987).

The second, Mathematical Knowledge for Teaching (MKT), was described and discussed by Ball and Bass (2003). The third, Mathematics for Teaching, was described by Cuoco (2001). The participants in the workshop were then asked to form groups, and to work, together on the following activity:

(1) Write a multiple-choice item dealing with mathematical content (derivative, multiplication, proof, . . .), which teachers will be able to answer but which will be a challenge to those in other professions, including mathematicians;
(2) Write down your reasons for choices you made in designing the item: What are the task features that "favor" teachers?
(3) Hand in your work to us: Task and Reasons;
(4) Present your choice to the other groups.

Each group presented its activity. The discussions focused on several issues, including:

(1) The role of culture in assessing teachers' mathematical knowledge for teaching;
(2) The role of beliefs and values in assessing teachers' mathematical knowledge for teaching;
(3) The question what are improved methods for assessing teachers' mathematical knowledge for teaching.

We summarized by raising several issues that are to be discussed at ICME 11 (TSG 27):

Q1. What are (and have been) different perspectives about mathematical knowledge for teaching? What are the bases–theoretical or empirical–for these perspectives? Where are areas of overlap and agreement? Where and what are major differences? Do these represent substantially conflicting views, or do they simply reflect giving attention to different aspects of the question?
Q2. What are (and have been) different methods of studying mathematical knowledge for teaching –what teachers know and use, or what they need to know and know how to use? What are key common and distinct aspects of the methods used to answer these questions?
Q3. What are (and have been) different empirical research fields or contexts e.g. in primary or secondary classrooms, with or without digital technologies, with or without teachers as co-researchers, in mathematics or mathematics methods classes for teachers? How have these shaped the research questions pursued?
Q4. How could research results gained by different research methodologies (quantitative, qualitative, interpretative research methods) support teachers developing their mathematical knowledge and their professional view on the nature of mathematics in teaching?

Q5. What are some distinct approaches to helping teachers develop the mathematical knowledge they need to know and know how to use? What kinds of evidence are there about how these functions and with what effects?

The teaching of proof as a gateway to teaching models

Fulvia Furinghetti
(joint work with Francesca Morselli)

Introduction

There is a big amount of literature on proof in math education, see the issues of the Newsletter on proof (`http://www.lettredelapreuve.it/`). Research focuses mostly on students, less on teachers and even less on teachers' beliefs. Our contribution is set in this last strand of research: we study the way teachers deal with proof in secondary school, by means of the theoretical lens provided by the theory of beliefs. In particular, here we discuss the role of context in decisions concerning the teaching of proof. Though our focus is on proof, our findings have links with various aspects of teaching practice.

Theoretical framework

Our theoretical framework goes across three fields of research: research on proof, research on beliefs, research on teachers. Concerning proof we take as a reference the work of Barbin (see Barbin [1]), which is an accurate outline of the crucial moments in the historical evolution of the concept of proof. These moments have strong links with didactic practice, since they allow to stress the two main roles of proof, that is:

- The proof as a social act aimed at convincing
- The proof as an act that *illuminates*, since explains the reasons which lead to the results and fosters the understanding.

Beliefs theory about mathematics and its teaching is central in mathematics education research (see Leder et al. [2], Thompson [3]) and even more in research on teacher professional development. Beliefs shape the view of mathematics and its teaching. According to Skemp (see Skemp [4]) mathematics may be viewed as Instrumental (as a set of recipes; in short, to know what to do) or as relational (giving emphasis on relations between objects; in short, to know what to do and why). In [5] three views of mathematics are outlined: Problem solving, Platonist, Instrumentalist. According to Ernest (see Ernest [5]) enacted models of teaching and learning come from the view of mathematics (which originate professed models of teaching and learning) via the mediation of the social context. Concerning enacted models we mainly refer to the models of mathematics teaching identified by Kuhs & Ball as quoted in Thompson's [3]: *learner-focused, content-focused with an emphasis on conceptual understanding, content-focused with an emphasis on performance, classroom focused.*

Methodology

Ten teachers, with a long-lasting experience, were selected for our study. They were working in upper secondary schools with different orientations (scientific lyceum, humanistic lyceum, artistic lyceum, technological secondary school,...). We carried out semi-structured interviews. Some recurrent grounding themes emerged from the first reading of the interviews. A qualitative analysis of the interviews was performed according to these grounding themes. A further qualitative analysis led to outline different profiles, according to the different teachers' positions as regards all the themes.

Outline of our results
Proof appeared as a catalyst of important factors in teaching and the following grounding themes emerged: textbooks, programs, context, students' autonomy, student seen as a future citizen. The permanencies and variations according to the grounding themes led us to outline five profiles of teachers: Pragmatic teacher, Contextualizing teacher, Instrumental teacher, Reflective teacher, Ethical teacher. These profiles evidence the presence of a dominant belief in their espoused models of teaching and learning and the crucial role of the context in shaping the enacted models.

References

[1] É. Barbin, *La démonstration mathématique: significations épistémologiques et questions didactiques*, Bulletin APME 366 (1984), pp. 591-620.
[2] G. Leder, G., E. Pehkonen & Törner, G. (Eds.), *Beliefs: A hidden variable in mathematics education?*, Kluwer , Dordrecht / Boston / London (2002).
[3] A. G. Thompson, *Teachers' beliefs and conceptions: A synthesis of the research, in Grouws*, A. D. (Ed.), Handbook of research on mathematics learning and teaching, Macmillan, New York (1992), pp. 127-146.
[4] R. R. Skemp, *Relational understanding and instrumental understanding*, Mathematics teaching 77 (1976), pp. 20-27.
[5] P. Ernest, *The impact of beliefs on the teaching of mathematics*, in Bishop, A. et al. (Eds.), Mathematics, Education and Society, Unesco, Paris, Documents series 35 (1989), pp. 99-101.

Influence of lesson study on elementary mathematics instruction and curriculum development in Japan

Makoto Yoshida

International studies reveal that many US mathematics curricula lack focus, coherence, and/or rigor. In contrast, the Japanese curriculum has been found to exemplify the characteristics that US curricula lacked. In addition, classroom instruction in Japan is characterized as "structured problem solving that encourages students' active discovery and classroom discussions for developing solid student understanding". In Japan, curriculum and teaching methods are rigorously investigated through lesson study. In addition, the contents of mathematics textbooks, as well as the way they are presented in the textbooks are a reflection of many years of accumulated wisdom of experience gained through lesson study. In this session, we will explore some examples from the Japanese curriculum and learn how new math concepts are developed using what students have already learned in a structured problem solving setting. In addition, we will touch on how those ideas are developed through lesson study. The insights gained through this exploration will shed light on ways to improve curriculum currently in use by participants.

Learning to use mathematics in practice

Hyman Bass

This presentation describes the approach we have been using at the University of Michigan to prepare primary and middle grades teachers for the work of mathematics teaching. The tasks and instructional activities we have developed focus on developing mathematical knowledge for teaching the context of close attention to practice. In the session, participants will explore an example of the work and consider distinctive features of this approach.

Reporters: Bettina Rösken, Stefan Ufer

Participants

Prof. Dr. Alejandro Adem
Dept. of Mathematics
University of British Columbia
1984 Mathematics Road
Vancouver , BC V6T 1Z2
CANADA

Prof. Dr. Abraham Arcavi
Department of Science Teaching
Weizmann Institute of Science
76100 Rehovot
ISRAEL

Prof. Dr. Michele Artigue
Equipe DIDIREM
Case 7018
Universite Paris 7
2, place Jussieu
F-75251 Paris Cedex 05

Prof. Dr. Peter Baptist
Fakultät für Mathematik und Physik
Universität Bayreuth
95440 Bayreuth

Prof. Dr. Hyman Bass
Dept. of Mathematics
The University of Michigan
2074 East Hall
525 E. University Ave.
Ann Arbor , MI 48109-1109
USA

Prof. Dr. Alan J. Bishop
Faculty of Education
Monash University
VIC 3800
AUSTRALIA

Prof. Dr. Werner Blum
FB 17 - Mathematik/Informatik -
Universität Kassel
34109 Kassel

Prof. Dr. Chris Breen
School of Education
University of Cape Town
7701 Rondebosch
South Africa

Prof. Dr. David Clarke
Faculty of Education
University of Melbourne
109 Barry Street
Carlton , VIC 3053
Australia

Prof. Dr. Tommy Dreyfus
Department of Science Education
Joan & Jaime Constantiner School of
Education at Tel Aviv University
Ramat Aviv
Tel Aviv 69978
ISRAEL

Prof. Dr. Lianghuo Fan
Mathematics and Mathematics Education
National Institute of Education
Nanyang Technological University
1 Nanyang Walk
Singapore 637616
SINGAPORE

Prof. Dr. Sharon Friesen
Faculty of Education
University of Calgary
2500 University Drive NW
Calgary Alberta TN2 1N4
CANADA

Prof. Dr. Fulvia Furinghetti
Dipartimento di Matematica
Universita di Genova
Via Dodecaneso 35
I-16146 Genova

Prof. Dr. Peter Gritzmann
Zentrum Mathematik
Kombinatorische Geometrie (M9)
Technische Universität München
Boltzmannstr. 3
85747 Garching bei München

Prof. Dr. Gila Hanna
Ontario Institute for Studies in
Education of the University of
Toronto
252 Bloor Street West
Toronto ONT M5S 1V6
Canada

Prof. Dr. Aiso Heinze
Naturwissenschaftliche Fakultät I
Didaktik der Mathematik
Universität Regensburg
Universitätsstr. 31
93053 Regensburg

Prof. Dr. Klaus Hoechsmann
Dept. of Mathematics
University of British Columbia
1984 Mathematics Road
Vancouver , BC V6T 1Z2
CANADA

Prof. Dr. Barbara Jaworski
Dept. of Mathematics & Computer Science
Loughborough University
GB-Loughborough LE11 3TU

Prof. Dr. Gabriele Kaiser
Universität Hamburg
Fachbereich Erziehungswissenschaften
Von-Melle-Park 8
20146 Hamburg

Prof. Dr. Eckhard Klieme
Deutsches Institut für Internation.
Pädagogische Forschung
Schloßstraße 29
60486 Frankfurt

Prof. Dr. Konrad Krainer
Institut für Unterrichts- und
Schulentwicklung IUS
Universität Klagenfurt
Sterneckstrasse 15
A-9010 Klagenfurt

Prof. Dr. Jürg Kramer
Institut für Mathematik
Humboldt-Universität
Unter den Linden 6
10117 Berlin

Prof. Dr. Fou-Lai Lin
Department of Mathematics
National Taiwan Normal University
88 sec. 4, Ting Chou Road
Taipei
TAIWAN

Prof. Dr. Jan A. van Maanen
Freudenthal Instituut
Utrecht University
Aidadreef 12
NL-3561 GE Utrecht

Prof. Dr. John H. Mason
Faculty of Mathematics & Computing
The Open University
Walton Hall
GB-Milton Keynes , MK7 6AA

Prof. Dr. Joao Pedro da Ponte
Departamento de Educacao
Faculdade de Ciencias
Edificio C6, Campo Grande
P-Lisboa 1749-016

Prof. Dr. Kristina Reiss
Mathematisches Institut
Ludwig-Maximilians-Universität
München
Theresienstr. 39
80333 München

Prof. Dr. Jürgen Richter-Gebert
Zentrum Mathematik
TU München
Boltzmannstr. 3
85748 Garching bei München

Bettina Rösken
Fachbereich Mathematik
Universität Duisburg-Essen
Lotharstraße 65
47057 Duisburg

Prof. Dr. Alan H. Schoenfeld
University of California
Graduate School of Education
1670 Tolman Hall
Berkeley , CA 94720-1670
USA

Prof. Dr. Miriam G. Sherin
School of Education & Social Policy
Northwestern University
Evanston IL 60208
USA

Prof. Dr. Martin A. Simon
New York University
207 East Bldg., 239 Greene Street
New York , NY 10003
USA

Prof. Dr. Bharath Sriraman
Department of Mathematical Sciences
University of Montana
Missoula , MT 59812-1032
USA

Prof. Dr. Gernot Stroth
Institut für Mathematik
Naturwissenschaftliche Fakultät III
Universität Halle-Wittenberg
Theodor-Lieser-Str. 5
06120 Halle

Prof. Dr. Malcolm Swan
University of Nottingham
School of Education
Jubilee Campus
Wollaton Road
GB-Nottingham , NG8 1BB

Prof. Dr. Dina Tirosh
School of Education
Tel Aviv University
Ramat Aviv
Tel Aviv 69978
ISRAEL

Prof. Dr. Günter Törner
Fachbereich Mathematik
Universität Duisburg-Essen
47048 Duisburg

Dr. Stefan Ufer
Mathematisches Institut
Ludwig-Maximilians-Universität
München
Theresienstr. 39
80333 München

Prof. Dr. Ödön Vancso
Institute of Mathematics
Eötvös Lorand University
Mathematics Teaching & Education Centre
Pazmany Peter Setany 1/C
H-1117 Budapest

Prof. Dr. Lieven Verschaffel
CIPT
University of Leuven
Vesaliusstraat 2
B-3000 Leuven

Prof. Dr. Hans-Georg Weigand
Lehrstuhl für Didaktik der Mathematik
Universität Würzburg
Am Hubland
97074 Würzburg

Prof. Dr. Kai-Lin Yang
Department of Mathematics
Nat. Changhua University of Education
1, Chin-Teh Road
Changhwa City, Changhua 50007
TAIWAN

Prof. Dr. Makoto Yoshida
Dept. of Mechanical Engineering
Faculty of Engineering
Tokyo Metropolitan University
1-1 Minamiohsawa, Hachioji
Tokyo 192-0397
JAPAN

Prof. Dr. Rina Zazkis
Faculty of Education
Simon Fraser University
8888 University Dr.
Burnaby , B.C. V5A 1S6
CANADA

Prof. Dr. Günter M. Ziegler
Institut für Mathematik
MA 6-2
Technische Universität Berlin
Straße des 17. Juni 136
10623 Berlin