Mathematisches Forschungsinstitut Oberwolfach

Report No. 2/2008

Set Theory

Organised by
Sy-David Friedman, Vienna
Menachem Magidor, Jerusalem
W. Hugh Woodin, Berkeley

January 13th – January 19th, 2008

Abstract. This lively workshop presented some of the most exciting recent developments in set theory, including major new results about ordered sets, Banach spaces, determinacy, Ramsey theory, pcf theory, inner models, forcing, descriptive set theory, cardinal characteristics as well as other topics in combinatorial set theory and applications of set theory.

Mathematics Subject Classification (2000): 03Exx.

Introduction by the Organisers

This was a highly successful workshop on some of the exciting recent developments in set theory. There were 45 participants. By limiting the number of talks to 25, there was ample time for research collaboration in small groups, which rendered the meeting very relaxed, enjoyable and productive.

Among the highlights of the workshop are the following:

Justin Moore presented some dramatic new work on the existence of a universal Aronszajn line, demonstrating that the existence of a canonical, universal linear ordering is not limited to the countable case. There were several beautiful presentations in the area of descriptive set theory: Christian Rosendal provided a rough classification of Banach spaces, Slawomir Solecki described G-delta ideals of compact sets, Simon Thomas drew some striking connections between set theory and geometric group theory and John Clemens discussed Borel homomorphisms of smooth Sigma-ideals. A major new result was presented by David Schrittesser, who showed that projective measurability does not imply the projective Baire property.

There were two major results in the related areas of large cardinals and determinacy: John Steel calculated the consistency strength of determinacy for games on reals and he together with Ronald Jensen discovered a way of building the core model for a Woodin cardinal in ZFC without additional large cardinal hypotheses. Several talks presented important recent work related to the pcf theory: Todd Eisworth discussed combinatorial properties at the successor of a singular cardinal, Matteo Viale told us about reflection and approachability and Moti Gitik brought us up to date on consistency results regarding the behaviour of the generalised continuum function.

In combinatorial set theory we heard lectures on induced Ramsey theory, MAD families, weak diamonds and simple cardinal invariants. And in the theory of forcing we heard about Laver products, p-points, indestructibility of weak compactness, mixed support iterations and reflection.

Paul Larson's very nice talk on Martin's Maximum and definability rounded out a very successful meeting.

Sy-David Friedman
Menachem Magidor
Hugh Woodin

Workshop: Set Theory

Table of Contents

Abstracts

MAD families on singular cardinals

Jörg Brendle

Let κ be an infinite cardinal. A family $\mathcal{A} \subseteq [\kappa]^\kappa$ of κ-sized subsets of κ is called *almost disjoint* if $|A \cap B| < \kappa$ for all distinct $A, B \in \mathcal{A}$. It is a *maximal almost disjoint family* (a *mad family*, for short) if it is maximal with this property, that is, for every $C \in [\kappa]^\kappa$ there is $A \in \mathcal{A}$ such that $|A \cap C| = \kappa$. Obviously, a partition of κ into $\lambda < cf(\kappa)$ many sets is a mad family of size λ. Accordingly, we define the *almost disjointness number* on κ by $\mathfrak{a}_\kappa := \min\{|\mathcal{A}| : \mathcal{A} \subseteq [\kappa]^\kappa$ is mad and $|\mathcal{A}| \geq cf(\kappa)\}$. An easy diagonal argument shows that $\mathfrak{a}_\kappa > cf(\kappa)$.

While $\mathfrak{a} := \mathfrak{a}_\omega$ has been intensively studied in the past, and, in particular, a number of deep consistency results concerning the order-relationship between $\mathfrak{a}$ and other cardinal invariants of the continuum have been obtained (see, for example, [6], [2], and [3]), very little is known about $\mathfrak{a}_\kappa$ for uncountable κ. For regular uncountable κ, $\mathfrak{a}_\kappa$ has recently been investigated by Blass, Hyttinen, and Zhang [1]; among others they obtained the interesting result saying that $\mathfrak{d}_\kappa = \kappa^+$ implies $\mathfrak{a}_\kappa = \kappa^+$. For singular κ, a straightforward stretching argument shows that given a mad family on $cf(\kappa)$, there is also a mad family on κ of the same size; in particular,

(1) $\mathfrak{a}_\kappa \leq \mathfrak{a}_{cf(\kappa)}$.

Furthermore, Kojman, Kubiś, and Shelah [5] have obtained two important results, namely,

(2) $\mathfrak{a}_\kappa \geq \min\{\mathfrak{b}_{cf(\kappa)}, \mathfrak{b}_\kappa\}$ where $\mathfrak{b}_{cf(\kappa)}$ is the *unbounding number* on $cf(\kappa)$, and $\mathfrak{b}_\kappa$ is a cardinal defined in terms of pcf (in most situations, it is max pcf);

(3) if $\mathfrak{a}_\kappa \leq \lambda < \mathfrak{b}_\kappa$, then there is a mad family on κ of size λ.

In particular, if $\mathfrak{a}_\kappa < \kappa$, then $\mathfrak{a}_\kappa \geq \mathfrak{b}_{cf(\kappa)}$.

In view of (1) above it is natural to ask

Problem 1. *Is* $\mathfrak{a}_\kappa < \mathfrak{a}_{cf(\kappa)}$ *consistent?*

This is part of a much more general problem: can one construct, either in ZFC or at least consistently, mad families on κ which are not "induced" by mad families on $cf(\kappa)$. The point is that every "known" mad family on κ is obtained by stretching a mad family on $cf(\kappa)$ and then possibly increasing size by using (the proof of) (3).

We consider Problem 1 for κ of countable cofinality. Assume κ_n is a strictly increasing sequence of regular cardinals with limit κ. Our approach is as follows:

- force a generic mad family on κ of size μ where μ is regular with $\aleph_2 \leq \mu < \kappa$ which does *not* come from a mad family on ω (in fact: such that an isomorphism-of-names argument shows there is no mad family on ω of size μ);

- embed this forcing into the template framework (see [6], [2], and [3]) which forces $\mathfrak{b} = \mathfrak{d} = \mu$ and preserves the madness of the generic mad family on κ (and such that the isomorphism-of-names argument still works).

Since $\mathfrak{a} \geq \mathfrak{b}$ in ZFC, the isomorphism-of-names argument will yield $\mathfrak{a} > \mu$. On the other hand, the generic mad family on κ will witness $\mathfrak{a}_\kappa \leq \mu$; together with $\mathfrak{a}_\kappa \geq \mu$ which follows from (2), this gives $\mathfrak{a}_\kappa = \mu$.

More explicitly, given a set L and a family $\mathcal{K} \subseteq \{K_a : a \in L\} \subseteq \mathcal{P}(\kappa)$, we devise a ccc forcing $\mathbb{P}_\mathcal{K}$ which adds $F_a \subseteq K_a$ such that for distinct $a, b \in L$, $F_a \cap F_b \subseteq \kappa_n$ for some n. Setting $L^0 := \{a \in L : K_a = \kappa\}$ and assuming $|K_a| < \kappa$ for $a \notin L^0$, we show that if $\mu := |L^0| \geq \aleph_1$, then $\mathbb{P}_\mathcal{K}$ adds a mad family $\mathcal{F} = \{F_a : a \in L^0\}$ on κ of size μ. If, furthermore, μ is measurable, then, after a preparatory forcing which may involve collapsing μ to a cardinal $\geq \aleph_2$, $\mathbb{P}_\mathcal{K}$ forces there is no mad family on ω of size μ. Embedding $\mathbb{P}_\mathcal{K}$ into the template framework, a procedure which is standard though technical (and much more so than the earlier similar [3]), we obtain for example

Theorem 2. [4] *If the existence of a measurable cardinal is consistent, then so is* $\mathfrak{a}_{\aleph_\omega} = \aleph_2$ *and* $\mathfrak{a} = \aleph_3$.

Many related problems are still open. For example:

Problem 3. *Is* $\mathfrak{a}_{\aleph_\omega} < \mathfrak{a}$ *consistent with* $\mathfrak{a} > \aleph_\omega$*?*

Problem 4. (Kojman, Kubiś, and Shelah [5]) *Is* $\mathfrak{a}_{\aleph_\omega} = \aleph_\omega$ *consistent?*

Problem 5. *Is* $\mathfrak{a}_{\aleph_{\omega_1}} < \mathfrak{a}_{\aleph_1}$ *consistent?*

References

[1] A. Blass, T. Hyttinen, and Y. Zhang, Mad families and their neighbors, preprint.
[2] J. Brendle, Mad families and iteration theory, in: Logic and Algebra (Y. Zhang, ed.), Contemp. Math. 302 (2002), Amer. Math. Soc., Providence, 1–31.
[3] J. Brendle, The almost disjointness number may have countable cofinality, Trans. Amer. Math. Soc. 355 (2003), 2633–2649.
[4] J. Brendle, Mad families on singular cardinals, in preparation.
[5] M. Kojman, W. Kubiś, and S. Shelah, On two problems of Erdős and Hechler: new methods in singular madness, Proc. Amer. Math. Soc. 132 (2004), 3357–3365.
[6] S. Shelah, Two cardinal invariants of the continuum $\mathfrak{d} < \mathfrak{a}$ and FS linearly ordered iterated forcing, Acta Math. 192 (2004), 187–223.

Borel homomorphisms of smooth σ-ideals

John D. Clemens
(joint work with Clinton Conley and Benjamin Miller)

We study the relationship between the quasi-order of Borel reducibility on the class of countable Borel equivalence relations and the quasi-order of Borel homomorphism on smooth sigma-ideals. For Borel equivalence relations E and F on the Polish spaces X and Y, E is *Borel-reducible* to F, $E \leq_B F$, if there is a Borel function $f : X \to Y$ such that for all $x_1, x_2 \in X$ we have $x_1 E x_2$ iff $f(x_1) F f(x_2)$. This quasi-ordering has been well-studied, and provides a comparison of the relative complexity of the equivalence relations E and F.

An equivalence relation is said to be *countable* if every equivalence class is countable; a good introduction to the theory of countable Borel equivalence relations may be found in [3]. Even among the countable Borel equivalence relations, the quasi-order $\leq_B$ is very complicated, as shown by the work of Adams-Kechris ([1]) and Hjorth-Kechris ([4]).

Here we consider a different notion of comparison between countable Borel equivalence relations. For a countable Borel equivalence relation E on X, we define the *smooth σ-ideal of E*, $\mathcal{I}_E$, to be the σ-ideal generated by the Borel partial transversals for E, where a partial transversal is a set $B \subseteq X$ meeting each equivalence class in at most one point. Then $\mathcal{I}_E$ consists of all Borel sets $B \subseteq X$ such that $E \restriction B$ is smooth, i.e., Borel-reducible to the identity relation on some Polish space. Note that $\mathcal{I}_E$ is a Π^1_1-on-Σ^1_1 σ-ideal. Our primary goal is to determine to what extent the smooth σ-ideal $\mathcal{I}_E$ captures the complexity of the countable Borel equivalence relation E.

For σ-ideals $\mathcal{I}$ and $\mathcal{J}$ on the Polish spaces X and Y, a *Borel homomorphism* from $\mathcal{I}$ to $\mathcal{J}$ is a Borel function $\pi : X \to Y$ such that for all $B \in \mathcal{J}$ we have $\pi^{-1}[B] \in \mathcal{I}$. We write $\mathcal{I} \preccurlyeq_B \mathcal{J}$ when there is a Borel homomorphism from $\mathcal{I}$ to $\mathcal{J}$.

When π is a Borel reduction from E to F, then π is a Borel homomorphism from $\mathcal{I}_E$ to $\mathcal{I}_F$, but the converse is false. There are countable Borel equivalence relations E and F such that there is a Borel homomorphism from $\mathcal{I}_E$ to $\mathcal{I}_F$ but E is not Borel reducible to F (see [5] for examples of such relations). We establish the precise relationship between $\preccurlyeq_B$ and a weakening of $\leq_B$, and determine the complexity of the quasi-order $\preccurlyeq_B$ on the Π^1_1-on-Σ^1_1 σ-ideals.

Let E and F be countable Borel equivalence relations on X and Y. A Borel map $\pi : X \to Y$ is a *Borel homomorphism* from E to F if the image of each E-class under π is contained in a single F-class. We say that π is an *almost homomorphism* from E to F if the image of each E-class is contained in the union of finitely many F-classes. For a countable Borel equivalence relation E on X, we say that a map $\pi : X \to Y$ is *smooth-to-one* if for all $y \in Y$ the inverse image $\pi^{-1}(y)$ is in $\mathcal{I}_E$, i.e., $E \restriction \pi^{-1}(y)$ is smooth.

Our main result is the following characterization of $\preccurlyeq_B$:

Theorem 1. *For countable Borel equivalence relations E and F on the Polish spaces X and Y, the following are equivalent:*

(1) *There is a Borel homomorphism from $\mathcal{I}_E$ to $\mathcal{I}_F$.*
(2) *There is a smooth-to-one Borel almost homomorphism from E to F.*

Using this and results about the complexity of the Borel reducibility quasi-order on the class of countable Borel equivalence relations, we show that the relation $\preccurlyeq_B$ is complicated both as a quasi-order and descriptively:

Theorem 2. *Every Σ^1_1 quasi-order on a Polish space is reducible to $\preccurlyeq_B$. Every Σ^1_2 subset of a Polish space is reducible to $\preccurlyeq_B$.*

Finally, we derive a corollary about homogeneous σ-ideals. Following Zapletal ([6]), we say that a σ-ideal $\mathcal{I}$ on a Polish space X is *homogeneous* if for every Borel set $B \subseteq X$ there is a Borel homomorphism from $\mathcal{I}$ to $\mathcal{I} \restriction B$. An equivalence relation E is *hyperfinite* if E is the increasing union of a countable sequence of Borel equivalence relations each of which has finite equivalence classes. We then have the following characterization of when $\mathcal{I}_E$ is homogeneous:

Theorem 3. *For a countable Borel equivalence relation E, the following are equivalent:*

(1) *$\mathcal{I}_E$ is homogeneous.*
(2) *E is hyperfinite.*

Zapletal asked whether there were natural examples of σ-ideals whose associated forcing is proper but which are not homogeneous. The forcing associated to $\mathcal{I}_E$ is proper for any countable Borel equivalence relation E, so $\mathcal{I}_E$ satisfies these conditions whenever E is a non-hyperfinite countable Borel equivalence relation; however, these are imperfect examples as there is a dense subset of the associated forcing which is homogeneous.

Proofs of the results announced here may be found in [2].

References

[1] S. Adams and A. S. Kechris, *Linear algebraic groups and countable Borel equivalence relations*, **J. Amer. Math. Soc., 13(4)**, (2000), pp. 909–943.
[2] J. D. Clemens, C. Conley, and B. Miller, *Borel homomorphisms of smooth σ-ideals*, preprint (2007), available at: http://www.math.psu.edu/clemens/Papers/homomorphism.pdf
[3] S. Jackson, A. S. Kechris, and A. Louveau, *Countable Borel equivalence relations*, **J. Math. Log., 1(2)**, (2002), pp. 1–80.
[4] G. Hjorth and A. S. Kechris, *Rigidity theorems for actions of product groups and countable Borel equivalence relations*, **Mem. Amer. Math. Soc., 177(833)**, (2005), viii+109.
[5] S. Thomas, *Some applications of superrigidity to Borel equivalence relations*, **Set theory (Piscataway, NJ, 1999)**, pp. 129–134, DIMACS Ser. Discrete Math. Theoret. Comput. Sci., 58, Amer. Math. Soc., Providence, RI, 2002.
[6] J. Zapletal, **Forcing idealized**, preprint.

Stationary reflection

James Cummings
(joint work with Dorshka Wylie)

Let S, T be stationary subsets of a regular uncountable cardinal κ. Then we say $S < T$ if S reflects almost everywhere in T, and $S <^* T$ if $U < T$ for all stationary $U \subseteq S$. It is easy to see that $S <^* T$ is only possible when $\kappa = \mu^+$ for μ regular, $S \subseteq \mu^+ \cap \operatorname{cof}(< \mu)$, and $T \subseteq \mu^+ \cap \operatorname{cof}(\mu)$ (modulo NS).

Starting with ω many supercompact cardinals, we show how to produce a model in which there are stationary sets $B_n \subseteq \omega_{n+2} \cap \operatorname{cof}(\omega_{n+1})$ such that $\omega_{n+2} \cap \operatorname{cof}(\omega_n) <^* B_n$ for all n. This is in some sense an optimal result, since by work of

Jech and Shelah the assertion $\omega_{n+2} \cap \mathrm{cof}(\omega_n) <^* \omega_{n+2} \cap \mathrm{cof}(\omega_{n+1})$ can't hold for successive values of n.

Some representation theorems

Mirna Džamonja

We present two theorems which can be used to represent compact connected Hausdorff spaces in an algebraic context, using a Stone-like representation. The first theorem stems from the work of Wallman and shows that every distributive disjunctive normal lattice is the lattice of closed sets in a unique up to homeomorphism connected compact Hausdorff space. The second theorem stems from the work of Jung and Sünderhauf. Introducing the notion of strong proximity involution lattices, it shows that every such lattice can be uniquely represented as the lattice of pairs of compact and open sets of connected compact Hausdorff space. As a consequence we easily obtain a somewhat surprising theorem birepresenting distributive disjunctive normal lattices and strong proximity involution lattices.

Some Open Problems in Combinatorial Set Theory

Todd Eisworth

In this talk, we will isolate several open questions that have arisen in our recent research on square-brackets partition relations at successors of singular cardinals.
Question 1: Suppose $\lambda = \mu^+$ for μ singular, and S is a stationary subset of $\{\delta < \lambda : \mathrm{cf}(\delta) = \mathrm{cf}(\mu)\}$. Must there necessarily exist a sequence $\langle C_\delta : \delta \in S \rangle$ such that

- C_δ is club in δ,
- $\mathrm{otp}(C_\delta) = \mu$, and
- for every closed unbounded $E \subseteq \lambda$, the set of $\delta \in S$ with $C_\delta \setminus E$ bounded in δ is stationary.

A positive answer would establish that $\lambda \nrightarrow [\lambda]^2_\lambda$ holds outright in ZFC, while a consistent negative would appear to be quite difficult to obtain.
Question 2 Suppose κ is a regular Jonsson cardinal. Can there be a function $F : [\kappa]^{<\kappa} \to \kappa$ and a uniform filter $\mathcal{F}$ on κ such that for any $A \in [\kappa]^\kappa$, the range of $F \restriction [A]^{<\omega}$ is in $\mathcal{F}$?

If κ is the successor of a singular cardinal, there there such F and $\mathcal{F}$. However, the existence of such a coloring at a Jonsson cardinal seems quite strong indeed. This question has connections with the theory of minimal walks:
Question 3 Suppose κ is a regular Jonsson cardinal. Can there exist a C-system on κ with a non-trivial trace filter?
Question 4 Suppose $\lambda = \mu^+$ for μ singular strong limit. Is the existence of an indecomposable ultrafilter on λ compatible with $2^\mu > \mu^+$?

Aspects of Indestructible Weak Compactness

Gunter Fuchs

I came across the concept of indestructible weak compactness when working on Maximality Principles for closed forcings. There are many variations of these principles — let me concentrate on the lightface principles for $<\kappa$-closed forcing, where κ is a regular cardinal. The Maximality Principle for $<\kappa$-closed forcing, $\mathsf{MP}_{<\kappa-\mathrm{closed}}$, says that any statement about κ that can be forced to be true by $<\kappa$-closed forcing in such a way that it stays true in any further forcing extension obtained by $<\kappa$-closed forcing, is already true. This is a very natural principle that has many interesting consequences, see [4]. The idea of formulating maximality principles in this way is due to Stavi and Väänänen, and was rediscovered by Joel Hamkins. Say that a weakly compact cardinal κ is *indestructibly* weakly compact if it stays weakly compact in any forcing extension obtained by $<\kappa$-closed forcing. When investigating the consistency strength of $\mathsf{MP}_{<\kappa-\mathrm{closed}}$ at a large cardinal κ, it turned out that for large cardinal properties that are $\Pi^1_1(H_\kappa)$, the addition of the maximality principle to the large cardinal property doesn't increase the consistency strength of the theory. But for weak compactness, the addition of the maximality principle makes it indestructible. More precisely:

Theorem. *The following theories are equiconsistent:*

(1) ZFC + $\mathsf{MP}_{<\kappa-closed}$ + κ *is weakly compact,*
(2) ZFC + κ *is indestructibly weakly compact.*

So this raises the question how strong indestructible weak compactness is. It turns out to be surprisingly strong: Apter and Hamkins ([1]) have established a connection to supercompactness, namely that if κ is indestructibly weakly compact in a model of set theory which is a forcing extension of an inner model by a forcing which has a closure point below κ, then κ was supercompact in the ground model. Also, Jensen, Schimmerling, Schindler and Steel ([5]), using an observation of mine, have shown that if κ is indestructibly weakly compact (and only indestructibility under the collapse of $\kappa^{+\mathsf{HOD}}$ to κ is needed), then there is a non-domestic premouse (i.e., a mouse in which there is an externally measurable cardinal which is simulteneously a limit of Woodin cardinals, and a limit of cardinals strong up to that cardinal — this is stronger than the $\mathsf{AD}_{\mathbb{R}}$ hypothesis).

The latter argument exploits the failure of weak covering at the indestructible weakly compact cardinal in the stack over K^c, built up to that cardinal. I looked more directly at the generic embeddings that one gets from an indestructibly weakly compact cardinal κ: By collapsing $\mathcal{P}(\mathcal{P}_\kappa(\lambda))$ to size κ, one finds in the generic extension a V-normal fine measure on $\mathcal{P}_\kappa(\lambda)$ in the sense of V (since κ is still weakly compact in the extension, one can derive such a measure from a sufficiently nice weakly compact embedding). One can now take the ultrapower of V by that ultrafilter. This ultrapower is well-founded, and it has a property that's slightly resembling supercompact embeddings: If $j : \mathsf{V} \longrightarrow M$ is the generic ultrapower, then $j \restriction \lambda \in M$. I use this property to prove that the countably stationary tower, $\mathbb{Q}_{<\kappa}$ is well-founded and the corresponding ultrapower is $< \kappa$-closed in the forcing extension. As an application, if κ in addition is completely Jónsson, then every set of reals in the Chang model has the regularity properties (using standard stationary tower arguments). Another application is that if κ is indestructibly weakly compact and $\mu < \kappa$ is a regular cardinal, then after collapsing κ to be μ^+, $\mathsf{MA}^+(<\mu-\text{closed})$ holds. The version for $\mu = \omega_1$ is a well-known strengthening of Martin's axiom which was introduced by Foreman, Magidor and Shelah, and has many consequences known to follow from Martin's Maximum. For an exposition of these results and methods, see [3].

The overall question that remains is: What exactly is the strength of indestructible weak compactness? It would be interesting to see what else can be done with the generic embeddings induced by indestructible weak compactness. One has to be careful, though, when working with this concept: It is a brittle large cardinal property, for indestructibility is destroyed by small forcing. And it has almost no reflection properties: It is possible (from rather large cardinals) that the least weakly compact cardinal is indestructibly so — this is shown in [2].

References

[1] Arthur W. Apter and Joel David Hamkins. Indestructible weakly compact cardinals and the necessity of supercompactness for certain proof schemata. *Mathematical Logic Quarterly*, 47(4):563–571, 2001.
[2] Gunter Fuchs. Combined maximality principles up to large cardinals. In preparation.
[3] Gunter Fuchs. Generic embeddings associated to an indestructibly weakly compact cardinal. In preparation.
[4] Gunter Fuchs. Maximality principles for closed forcings: Implications, separations and combinations. *Journal of Symbolic Logic*, 73(01):276–308, March 2008.
[5] Ronald B. Jensen, Ernest Schimmerling, Ralf Schindler, and John R. Steel. Stacking mice. 2007. In preparation.

Cardinal Arithmetic and dropping cofinalities on a stationary set

Moti Gitik

Variations of Short Extenders Forcings seem to be relevant for attempts to construct models with a countable set of regular cardinals a having $\mathrm{pcf}(a)$ uncountable (version of a negation of the PCF-conjecture) or models with a cardinal λ such that the set

$$\{\kappa < \lambda \mid \kappa \text{ is a singular cardinal and } \mathrm{pp}(\kappa) > \lambda\}$$

is uncountable (the negation of the Shelah Weak Conjecture).
Shelah's No Hole and Localization principles pose strict limitation on possible cardinal (or pcf) configurations.
We continue to study cardinal configurations compatible with such limitations and prove the following theorem about dropping cofinalities.

Theorem 1. *Let $\lambda_0 < \kappa_0 < \cdots < \lambda_n < \kappa_n < \cdots$, $\kappa = \bigcup_{n<\omega} \kappa_n = \bigcup_{n<\omega} \lambda_n$. Suppose that for each $n < \omega$ there are E_{λ_n} a λ_n^{+n+1}-extender over λ_n and E_{κ_n} a κ_n^{+n+2}-extender over κ_n.*
Let $A \subseteq \kappa^{+2} \cap \mathrm{cof}(\kappa^+)$. Then in a cofinality preserving generic extension the

following hold:
there is a sequence of regular cardinals $\langle \rho_n \mid n < \omega \rangle$ *such that*

(1) $\lambda_n < \rho_n < \kappa_n$, *for each* $n < \omega$
(2) $tcf(\prod_{n<\omega} \rho_n^{n+2}/\mathit{finite}) = \kappa^{++}$
(3) *if* $\langle f_\alpha \mid \alpha < \kappa^{++} \rangle$ *is a continuous scale in* $\prod_{n<\omega} \rho_n^{n+2}$, *then for a club* $\cap \operatorname{cof}(\kappa^+)$ *many* α*'s*

$$\alpha \in A \rightarrow \operatorname{cof}(f_\alpha(n)) = \rho_n^{n+1}$$

$$\alpha \notin A \rightarrow \operatorname{cof}(f_\alpha(n)) < \lambda_n.$$

Constructing K in ZFC

Ronald Jensen
(joint work with John Steel)

We assume: ZFC + "No Woodin Cardinal in K^c" and construct K which

- satisfies the standard first order definition;
- is absolute in any inner model containing K;
- is absolute in set forcing extensions;
- is universal and satisfies the weak covering lemma.

The background condition for the construction of K^c is robustness.

Even more simple cardinal invariants

Jakob Kellner

Blass [1] introduced a classification of cardinal characteristics, and in particular defined Π^0_1 characteristics. In the remarkable paper *Many simple cardinal invariants* [2], Goldstern and Shelah showed that — in a very strong sense — there are many Π^0_1 characteristics:

Theorem 1. *(Goldstern and Shelah) Assume CH. Assume that* ${\kappa_\epsilon}^{\aleph_0} = \kappa_\epsilon$ *for all* $\epsilon \in \omega_1$ *and that the functions* $f_\epsilon, g_\epsilon : \omega \rightarrow \omega$ *(*$\epsilon \in \omega_1$*) are suitably chosen. Then there is a partial order* P *preserving cardinals which forces that* $c^\forall(f_\epsilon, g_\epsilon) = \kappa_\epsilon$ *for all* $\epsilon \in \omega_1$.

The Π^0_1 cardinal characteristics $c^\forall(f, g)$ are defined as follows:

Definition 2. Let $f, g : \omega \rightarrow \omega \setminus 1$ be such that $f(n) > g(n)$ for all n.

- $B : \omega \rightarrow \mathfrak{P}(\omega)$ is an (f, g)-slalom if $B(n) \subseteq f(n)$ and $|B(n)| < g(n)$ for all $n \in \omega$.
- A family $\mathfrak{B}$ of (f, g)-slaloms $\forall$-covers, if for all $\nu \in \prod f$ there is a $B \in \mathfrak{B}$ such that $\nu(n) \in B(n)$ for all $n \in \omega$.
- $c^\forall(f, g)$ is the minimal size of a $\forall$-covering family of (f, g)-slaloms.

Note:

- There are some variants of this definition. For example a $\exists$-cover contains for each $\nu \in \prod f$ a slalom B such that $\nu(n) \in B(n)$ infinitelz often. This defines the invariant $c^{\exists}(f,g)$.
- If the κ_ϵ are chosen to be different, then we get a universe with $\aleph_1$ many pairwise different simple cardinal characteristics (so the continuum has to be at least $\aleph_{\omega_1}$).
- From the point of view of forcing theory, this poses an interesting problem: One cannot use countable support iterations (otherwise the continuum is collapsed), nor finite support iterations (which add Cohen reals).
- In this particular instance, it is sufficient to use a coutable support product. Generally, it is hard to control the behaviour of such product, but in this particular case finite splitting lim-sup forcings can be used. These forcings (and their countable support products) all satisfy fusion and pure decision and therefore continuous reading of names, properness and are ω^ω-bounding. Together with CH we get perservation of all cardinals.

So in the Goldstern Shelah model there are $\aleph_1 < \aleph_{\omega_1} \leq \mathfrak{c}$ many different characteristics. It is natural to ask whether we can construct a universe with continuum many cardinal invariants. The answer is yes [3]:

Theorem 3. *Consistenly there are pairwise different κ_ϵ and (f_ϵ, g_ϵ) for$\epsilon \in \mathfrak{c} = \aleph_{\mathfrak{c}}$ such that $c^{\forall}(f_\epsilon, g_\epsilon) = \kappa_\epsilon$.*

The proof is a modification of Goldstern and Shelahs proof. Again, we can use finite splitting lim-sup tree forcings. However, instead of a product one has to use a mixture of product and iteration.

In joint work with Shelah we are investigating further generalizations of similar effects: We investigate forcing constructions (variants of products or mixtures between products and iterations) to get large continuum, using the theory of creature forcing [5] as framework. This way we can deal with $c^{\exists}$ [4], and hopefully will be able to construct universes in which several of the well known invariants are pairwise different. However, in these models $\mathfrak{d}$ will be $\aleph_1$, since the constructions work for ω^ω-bounding forcings only.

References

[1] Andreas Blass. Simple cardinal characteristics of the continuum. In *Set theory of the reals (Ramat Gan, 1991)*, volume 6 of *Israel Math. Conf. Proc.*, pages 63–90. Bar-Ilan Univ., Ramat Gan, 1993.

[2] Martin Goldstern and Saharon Shelah. Many simple cardinal invariants. *Arch. Math. Logic*, 32(3):203–221, 1993.

[3] Jakob Kellner. Even more simple cardinal invariants. to appear in Arch. Math. Logic.

[4] Jakob Kellner and Saharon Shelah. Decisive creatures and large continuum. math.LO/0601083.

[5] Andrzej Rosłanowski and Saharon Shelah. Norms on possibilities. I. Forcing with trees and creatures. Mem. Amer. Math. Soc. **141** (1999), no. 671, xii+167.

Canonical, induced, continuous Ramsey theorems.

Menachem Kojman
(joint work with Stefanie Frick)

An induced Ramsey theorem is a partition theorem for graphs in which the monochromatic object is required to be an induced subgraph of the colored graph. The finite induced Ramsey theorem by Erdős, Hajnal, Posa, Nesetril, Rödle and Deuben states that for every finite graph H there is a finite graph G such that for every coloring of the edges of H by two colors there is an induced monochromatic copy of H. This theorem does not hold with "countable" instead of finite (Erdős and Pósa).

The theorem presented here is a generalization of the finite induced Ramsey theorem and of Blass' theorem about contiuous partitions of the Cantor space. The object which is partitioned is the continuous graph $G_{\max}$, obtained as an inverse limit of all finite random graph. The theorem states that for every finite ordered graph H there exists a finite number r such that for all n and a continuous coloring of all copies of H in $(G_{\max}, <_{lx})$ by n colors, there is an induced copy of $G_{\max}$ on which at most r colors occur. In fact, there is a basis for all finite cotinuous partitions of H in G consisting of a single "canonical" coloring.

Mixed Support Iterations and Applications

John Krueger

In my talk, I consider generalizations of the following properties to cardinals larger than ω_2, using mixed support iterations.

(I) There are no Aronszajn trees on ω_2 ([8]).

(II) There is a disjoint stationary sequence on ω_2; that is, there is a sequence $\langle \mathcal{S}_\alpha : \alpha \in S \rangle$, where $S \subseteq \omega_2 \cap \text{cof}(\omega_1)$ is stationary, $\mathcal{S}_\alpha$ is a stationary subset of $P_{\omega_1}(\alpha)$ for α in S, and $\mathcal{S}_\alpha \cap \mathcal{S}_\beta$ is empty for $\alpha < \beta$ in S ([2], [6]).

(III) For all regular $\lambda \geq \omega$, there are stationarily many N in $[H(\lambda)]^{\aleph_1}$ which are internally stationary but not internally club ([4], [1]).

(IV) For all regular $\lambda \geq \omega_2$, there are stationarily many N in $[H(\lambda)]^{\aleph_1}$ which are internally club but not internally approachable ([7], [1]).

Suppose that $\mu^{<\mu} = \mu$, $\mu < \kappa$, and for all $\zeta < \kappa$, $\zeta^{<\mu} < \kappa$. Let λ be a strongly inaccessible cardinal larger than κ.

Let $\langle \mathbb{P}_i, \dot{\mathbb{Q}}_j : i \leq \lambda, j < \lambda \rangle$ be a forcing iteration satisfying the following properties:

(1) each $\dot{\mathbb{Q}}_i$ is forced to be μ-closed,

(2) for each even i, $\dot{\mathbb{Q}}_i$ is a name for Cohen forcing $\text{ADD}(\mu)$,

(3) supports on the even coordinates are of size less than μ, and supports on the odd coordinates are of size less than μ^+.

In addition, define a weak ordering on any two step iteration $\mathbb{P} * \dot{\mathbb{Q}}$ by letting $p_1 * \dot{q}_1 \leq^* p_0 * \dot{q}_0$ if $p_1 * \dot{q}_1 \leq p_0 * \dot{q}_0$ and $p_0 = p_1$. Now assume:

(4) for each even i, the two-step iteration $\langle \dot{\mathbb{Q}}_i * \dot{\mathbb{Q}}_{i+1}, \leq^* \rangle$ with the weak ordering is κ-strategically closed.

Then the iteration satisfies a variety of nice properties, including: (a) preserving all cardinals and cofinalities less than or equal to κ, (b) (κ, ∞, μ)-distributive, (c) proper for elementary substructures of size less than κ which are internally approachable of length μ, and (d) any set of ordinals is in the ground, provided that its intersection with any set of size less than κ in the ground model is also in the ground model ([3]).

Using such an iteration, we are able to obtain models which satisfy versions of properties (I) through (IV), where ω, ω_1, and ω_2 are replaced by μ, κ, and $\kappa^+ = \lambda$ ([6], [5]).

References

[1] M. Foreman and S. Todorčević, *A new Löwenheim-Skolem theorem*, Trans. Amer. Math. Soc. **357** (2005), 1693–1715.

[2] S. Friedman and J. Krueger, *Thin stationary sets and disjoint club sequences*, Trans. Amer. Math. Soc. **359** (2007), 2407–2420.

[3] J. Krueger, *A general Mitchell style iteration*, Submitted.

[4] ———, *Internal approachability and reflection*, Submitted.

[5] ———, *Internally club and approachable for larger structures*, Submitted.

[6] ———, *Some applications of mixed support iterations*, In Preparation.

[7] ———, *Internally club and approachable*, Adv. Math. **213** (2007), no. 2, 734–740.

[8] W. Mitchell, *Aronszajn trees and the independence of the transfer property*, Ann. Math. Logic **5** (1972/73), 21–46.

Martin's Maximum and definability in $H(\aleph_2)$

Paul B. Larson

The following maximal version of Martin's Axiom was introduced by Foreman, Magidor and Shelah in [2].

Definition 1. Martin's Maximum (MM) is the statement that if $\mathbb{P}$ is a partial order such that forcing with $\mathbb{P}$ preserves stationary subsets of ω_1, and $\langle D_\alpha \mid \alpha < \omega_1 \rangle$ is a collection of dense subsets of $\mathbb{P}$, then there is a filter $G \subset \mathbb{P}$ meeting each D_α.

By convention, MM^+ is MM with the further requirement that if τ is a $\mathbb{P}$-name for a stationary set, then $\{\alpha < \omega_1 \mid \exists q \in G \; q \Vdash \alpha \in \tau\}$ is stationary, and MM^{++} is MM^+ but with $\aleph_1$ many names for stationary subsets of ω_1. We let $\mathrm{MM}^{+\omega}$ denote the version of MM^+ with countably many names.

The $\mathbb{P}_{\max}$ axiom $(*)$ (Definition 5.1 of [7]) says that the Axiom of Determinacy (AD) holds in the inner model $L(\mathbb{R})$ and that $L(\mathcal{P}(\omega_1))$ is a $\mathbb{P}_{\max}$-generic extension of $L(\mathbb{R})$ (the partial order $\mathbb{P}_{\max}$ is introduced in [7] and is not used in this work, though some parts use some of the $\mathbb{P}_{\max}$ machinery). Woodin showed [7] that $\mathrm{MM}^{++}(\mathfrak{c})$ (MM^{++} restricted to posets of cardinality the continuum or less) and $(*)$ are independent over ZFC, assuming the consistency of a strong form of determinacy (see Theorem 10.69 in [7]; that $(*)$ does not imply $\mathrm{MM}^{++}(\mathfrak{c})$ follows from arguments in [4] but was known previously by Woodin). In [3], we used some of the ideas from Woodin's proof that $\mathrm{MM}^{++}(\mathfrak{c})$ does not imply $(*)$ to show that $\mathrm{MM}^{+\omega}$ does not imply $(*)$. In this work we modify the argument from [3] to produce a model of $\mathrm{MM}^{+\omega}$ in which there is a wellordering of the reals definable in $H(\aleph_2)$ without parameters. This answers a question asked privately by Todorcevic shortly after the results of [3] were announced. Our interest in the question was reawakened by recent work of Asperó ([1], for instance). We note that $(*)$ implies that there is no wellordering of the reals definable in $H(\aleph_2)$ without parameters (this follows immediately from the fact that the $\mathbb{P}_{\max}$ extension is a homogeneous extension of a model of AD containing the reals), so $(*)$ fails in the model here also.

We use a variant of the set of reals $X^{\omega_1}_{(Code)}(\mathcal{S}, z)$ from [3], which is itself a variant of a set of reals from [7] (see Definition 10.22 of [7]). We call our set of reals $X^2_{(Code)}(\mathcal{S})$. As with the other variants, under $(*)$ this set is equal to $\mathcal{P}(\omega)$ (MM^{++} in conjunction with the existence of a Woodin cardinal below a measurable also implies that this set consists of all subsets of ω; modulo standard $\mathbb{P}_{\max}$ arguments, the proof in the $\mathbb{P}_{\max}$ context is the same). Our forcing construction is an iterated forcing which uses the construction from the consistency proof for MM from [2], adding forcings to make the set $X^2_{(Code)}(\mathcal{S})$ code the parameter $\mathcal{S}$. In the end $\mathcal{S}$ is the only suitable parameter coding itself via $X^2_{(Code)}$, and is thus definable in $H(\aleph_2)$. The parameter $\mathcal{S}$ is a partition of ω_1 into $\aleph_1$-many stationary sets, and there are numerous ways to define a wellordering of $\mathcal{P}(\omega_1)$ in $H(\aleph_2)$ from such a parameter under the assumption of MM (the axiom ψ_{AC} from [7] allows this, for instance). A ladder system on ω_1 (under PFA, [6]) and in fact any subset of ω_1 not constructible from a real (MM + "there exists a measurable cardinal", [5]) can also be used as parameters defining a wellordering of the reals definable in $H(\omega_2)$.

The following question is still open.

Question 2. Does Martin's Maximum^{++} imply $(*)$?

References

[1] D. Asperó, *Guessing and non-guessing of canonical functions*, Annals of Pure and Applied Logic 146 (2007) 2–3, 150–179

[2] M. Foreman, M. Magidor, S. Shelah, *Martin's Maximum, saturated ideals, and non-regular ultrafilters. Part I*, Annals of Mathematics 127 (1988), 1–47

[3] P. Larson, *Martin's Maximum and the* $\mathbb{P}_{\max}$ *axiom* $(*)$, Annals of Pure and Applied Logic 106 (2000) 1–3, 135–149

[4] P. Larson, *The size of* $\tilde{T}$, Archive for Mathematical Logic 39 (2000) 7, 541–568

[5] P.B. Larson, *The nonstationary ideal in the* $\mathbb{P}_{\max}$ *extension*, Journal of Symbolic Logic 72 (2007) 1, 138–158

[6] J.T. Moore, *Set mapping reflection*, Journal of Mathematical Logic 5 (2005) 1, 87–97

[7] W.H. Woodin, **The axiom of determinacy, forcing axioms, and the nonstationary ideal**, Walter de Gruyter & Co., Berlin, 1999

Rudimentary recursion, provident sets and forcing.

Adrian R. D. Mathias

Many important set-theoretical functions, such as rank or transitive closure, are defined by a recursion on the epsilon relation of the form

$$F(x) = G(F \restriction x)$$

where G is a rudimentary function, and $F \restriction x$ denotes the restriction of F to (the members of) x.

Others become similarly definable when we permit parameters. If p is a set, call F *p-rud-rec* if there is a rud function G such that for all x,

$$F(x) = G(p, F \restriction x),$$

and call a set A *provident* if it is transitive and closed under all p-rud-rec functions with p a member of A.[1]

If ζ is the least ordinal not in a provident set A, then ζ is indecomposable, that is, that the sum of two ordinals less than ζ is less than ζ. Conversely, if ζ is indecomposable, $\eta \geqslant \zeta$ and $p \in J_\zeta$, then the Jensen set J_η is closed under all p-rud-rec functions; in particular J_ζ is provident.

Let c be a transitive set. A modification of the usual hierarchy defining the constructible closure $L(c)$ of c proves desirable: define, by a simultaneous rudimentary recursion on ordinals, sets c_ν, P^c_ν thus:

$$c_0 = \varnothing \qquad c_{\nu+1} = c \cap \{x \mid x \subseteq c_\nu\} \qquad c_\lambda = \bigcup_{\nu<\lambda} c_\nu$$

$$P^c_0 = \varnothing \qquad P^c_{\nu+1} = \{c_\nu\} \cup c_{\nu+1} \cup \mathbb{T}(P^c_\nu) \qquad P^c_\lambda = \bigcup_{\nu<\lambda} P^c_\nu$$

$\mathbb{T}$ being the rudimentary function introduced in [4] such that for each transitive set u, $u \subseteq \mathbb{T}(u) \subseteq \mathcal{P}(u)$, $u \in \mathbb{T}(u)$, and the rudimentary closure of $u \cup \{u\}$ equals $\bigcup_{n\in\omega} \mathbb{T}^n(u)$. Then P^c_θ will be provident whenever θ is indecomposable and > 1.

Let $\theta \geqslant \omega$ be indecomposable, and C a collection of transitive sets, each of rank strictly less than θ, with the property that any two elements of C are members of a third. Then $\bigcup_{c\in C} P^c_\theta$ is provident, and every provident set is of this form.

Another characterization is that a provident set is a transitive set A closed under ordinal addition and pairing and under the functions rank, transitive closure and $\mathbb{T}$, and with the property that whenever c is a transitive member of A and ν is an ordinal in A, the set P^c_ν is in A. This remark gives a finitely axiomatisable set theory, which we call MSF, for "minimal for set forcing", of which the transitive models are precisely the provident sets.

The following theorem suggests that the class of rudimentarily recursive functions (with parameters) might be expected to provide a fine analysis of forcing much as the class of rudimentary functions has provided a fine analysis of constructibility.

[1]At Oberwolfach I called such sets *progressive*, in ignorance of that term's established use in PCF theory.

Theorem. *Let A be provident and $\mathbb{P}$ a separative poset which is a member of A. Then forcing over A with $\mathbb{P}$ can be defined in A, and the forcing relation for Δ_0 formulae is in a precise sense close to $\mathbb{P}$-rud-rec; if A is provident and if X is $(A, \mathbb{P})$ generic, then the generic extension $A[X]$ is provident.*

Versions of the above for certain J-like fragments were known to Hauser [2] and to Steel, cf. [5]; our more general result shows that as each of the computations required for establishing properties of forcing and building generic extensions can be done in a local J-like fragment, no global J-like structure is needed.

Each p-rud-rec function is primitive recursive in the sense of Jensen and Karp [3]. Many years ago Jensen (unpublished) verified that set forcing over a primitive recursively closed set is definable and that the extension will be primitive recursively closed. Presumably there are many intermediate function classes closure under which is similarly preserved under set-generic extension.

Finally, Gandy in [1] mentions an unsatisfactory attempt to study fragments of the infinitary language $L_{\omega_1\omega}$ using the class of rudimentary functions; it appears that the fragments $A \cap L_{\omega_1\omega}$, where A is provident, will have the properties he was seeking.

References

[1] R. O. Gandy. Set-theoretic functions for elementary syntax. In *Proceedings of Symposia in Pure Mathematics*, **13**, Part II, ed. T. Jech, American Mathematical Society, 1974, 103–126.
[2] K. Hauser, Generic Relativizations of Fine Structure. *Arch. Math. Logic* **39** (2000), no. 4, 227–251.
[3] R. B. Jensen and C. Karp, Primitive recursive set functions. In *Proceedings of Symposia in Pure Mathematics*, **13**, Part I, ed. D. Scott, American Mathematical Society, 1971, 143–176.
[4] A. R. D. Mathias. Weak systems of Gandy, Jensen and Devlin. in *Set Theory*, Centre de Recerca Matemàtica, Barcelona 2003–4, edited by Joan Bagaria and Stevo Todorcevic, Trends Math., Birkhäuser, Basel, 2006, 149–224.
[5] J. Steel. Scales in $K(\mathbb{R})$. Preprint dated September 11 2003.

More results on weak diamonds and clubs

Heike Mildenberger

This is work on a relative to Juhász' question [5, Question 15.3], whether Ostaszewski's club principle implies the existence of a Souslin tree. This question is still open. The following is a double weakening of Ostaszewski's principle:

Definition 1. Let $\clubsuit_{\mathrm{w}^2}$ be the following statement: There is some $\langle A_\alpha : \alpha < \omega_1, \alpha \text{ limit}\rangle$ such that for every α, A_α is cofinal in α and for every $X \subseteq \omega_1$ the set $\{\alpha \in \omega_1 : A_\alpha \subseteq^* X \vee A_\alpha \subseteq^* \alpha \setminus X\}$ is stationary.

We prove

Theorem 2. $\clubsuit_{\mathrm{w}^2}$ *and CH does not imply the existence of a Souslin tree.*

We reduce this to a weak diamond, more specifically to the weak diamond of the reaping relation, $\diamondsuit$(reaping).

Lemma 3. $\diamondsuit$(*reaping*) *implies* $\clubsuit_{w^2}$.

We recall the definition of the weak diamond for the reaping relation:

Definition 4. (Special case of Definitions 4.3/4.4 of [6])

(1) A function $F\colon 2^{<\omega_1} \to A$ is called a Borel function if each part $F \restriction 2^\alpha$, $\alpha < \omega_1$, is a Borel function. The complexity of the set $\{F \restriction 2^\alpha \,:\, \alpha < \omega_1\}$ can be high.

(2) $\diamondsuit$(reaping) is the following statement: For every Borel map $F\colon 2^{<\omega_1} \to 2^\omega$ the statement $\diamondsuit_F(2^\omega, [\omega]^\omega,$ is almost constant on) holds, i.e., there is some $g\colon \omega_1 \to [\omega]^\omega$ such that for every $f\colon \omega_1 \to 2$ the set

$$\{\alpha \in \omega_1 \,:\, F(f \restriction \alpha) \text{ is almost constant on } g(\alpha)\}$$

is stationary.

So the first theorem will be a corollary of the following:

Theorem 5. $\diamondsuit$(*reaping*) *together with CH and "all Aronszajn trees are special" is consistent relative to ZFC.*

The forcing used in the proof is from [7, 1, 4]. Aronszajn trees are specialised in a very careful way with forcings with side conditions that have simple $\mathbb{D}$-completeness systems. This is an application of Shelah's theory of iterating proper forcing without adding reals [7, Chapter 7]. Jensen [2] constructed the first model of CH where all Aronszajn trees are special. The main technical result from [4], that for the forcings from [7, Chapter 7, §7] and the ones from [1] many (M, P)-generic filters can be computed in a Borel manner from arguments given by a game played in α rounds for some $\alpha < \omega_1$, is adapted to give a different game: The second player imitates Miller forcing and the preservation of P-points to find infinite realms of constancy. The first player finds a real coding the second order parameters in the completeness system such that all other reals not eventually below it (so for example the Miller reals) are equally suitable for coding these parameters. The transition from the game and from many guessed countable elementary substructures with little forcing scenarios to the weak diamond for the reaping relation in $V^{\mathbb{P}_{\omega_2}}$ is analogous to [4]. An article on these results [3] is in preparation.

Since many parts of the coding technique are based on the existence of a simple countably closed completeness system and on the diamond in the ground model and of course on the properness of the forcing, it is an open question how to get analogous models with $2^{\aleph_0} \geq \aleph_2$.

References

[1] Uri Abraham and Saharon Shelah. A Δ^2_2 well-order of the reals and incompactness of $L(Q^{MM})$. *Annals of Pure and Applied Logic*, **59**:1–32, 1993.

[2] Keith Devlin and Howard Johnsbråten. *The Souslin Problem*, volume Lecture Notes in Mathematics. Springer, 1974.

[3] Heike Mildenberger. Finding generic filters by playing games. *Preprint*, 2008.

[4] Heike Mildenberger and Saharon Shelah. Specialising Aronszajn trees and preserving some weak diamonds. *Submitted*, 2005.

[5] Arnold Miller. Arnie Miller's problem list. In Haim Judah, editor, *Set theory of the reals (Ramat Gan, 1991)*, Israel Math. Conf. Proc. **6**, pages 645–654. Bar-Ilan Univ., Ramat Gan, 1993.
[6] Justin Tatch Moore, Michael Hrušák, and Mirna Džamonja. Parametrized $\diamondsuit$ principles. *Trans. Amer. Math. Soc.*, **356**:2281–2306, 2004.
[7] Saharon Shelah. *Proper and Improper Forcing, 2nd Edition*. Springer, 1998.

On a Question of Hamkins and Löwe

William Mitchell

The question, which Hamkins and Löwe raised in connection with their work in [1] on the modal logic of forcing, is as follows:

Question 1. *Is there a model N of ZFC with the property that if λ is any cardinal of N and H is a generic subset of* $\mathrm{col}(\omega, \lambda)$*, then $N[H] \equiv N$?*

Here $\mathrm{col}(\omega, \lambda)$ is the forcing to collapse λ onto ω. I present an answer to the simpler problem:

Question 2. *Is there a model N as above, except that $N[H] \equiv N$ is only required when H is the collapse of a* regular *cardinal of N?*

Theorem 3 gives an upper bound for the consistency strength of a positive answer to Question 2:

Theorem 3. *Suppose that $V \models o(\kappa) = \kappa^+$, $C \cap \kappa$ is the club subset given by Radin forcing at κ, and G is an Easton product of Levy collapses so that the cardinals of $V[C][G]$ below κ are the members of C together with the successors of the singular limit points of C. Then $N = V_\kappa[C][G]$ satisfies Question 2.*

The best known lower bound for such a model is substantially weaker:

Theorem 4. *Assume that $N = V$ satisfies Question 2. Then there is a closed unbounded class D of cardinals of V which are regular in K. Hence, by a result of* [3]*, the set of cardinals λ such that $o^K(\lambda) \geq \alpha$ is stationary in K for all ordinals α.*

Furthermore, it is possible to assign to each limit point of D a normal measure U_λ in K in such a way that if $\mathrm{cf}(\lambda) > \omega$ *then there is a club subset $C_\lambda \subseteq \lambda$ such that if $x \in U_\lambda$ then $x \cap \lambda' \in U_{\lambda'}$ for all sufficiently large $\lambda' \in C_\lambda$.*

The construction of N given in Theorem 3 can be modified by choosing the Levy collapses so that the cardinals of $V[C][G]$ are the limit points of C together with the successors of members of C. I had thought of this construction as one of a series of rather pathological models which show that Theorem 4 cannot be strengthened to assert that the ultrafilters U_λ are the filters directly generated by the co-bounded sets of successor cardinals below λ. However in a conversation with Philip Welch here at Oberwolfach we realized that this modified construction avoids the difficulty which prevents the model constructed in Theorem 3 from giving a solution to Question 1: the fact that this construction would involve collapsing the successors of singular cardinals, which would require a much larger cardinal in the ground model. We believe that we have a proof of the following lower bound for a positive answer to Question 1:

Theorem 5. *If $N = V$ satisfies Question 1 then there is a closed unbounded set of cardinals of N which have a weak repeat point in K.*

I conjecture that the actual consistency strength of positive answer to Question 1 is approximately the same as the hypothesis which I used in [2] to obtain a model in which the club filter on ω_1 is an ultrafilter. In particular, this consistency strength would be less than $o(\kappa) = \kappa^{++}$.

References

[1] J. D. Hamkins and B. Löwe. The modal logic of forcing. *Transactions of the American Mathematical Society*, 2008? to appear.
[2] W. J. Mitchell. How weak is a closed unbounded ultrafilter? In M. Boffa, D. V. Dalen, and K. McAloon, editors, *Logic Colloquium '80*, pages 209–230. North-Holland, Amsterdam, 1982.
[3] W. J. Mitchell. Applications of the covering lemma for sequences of measures. *Transactions of the American Mathematical Society*, 299(1):41–58, 1987.

A universal Aronszajn line

Justin Tatch Moore

An uncountable linear order is *Aronszajn* if it has no uncountable separable suborders and does not contain a copy of ω_1 or $-\omega_1$. These linear orders were considered and proved to exist by Aronszajn and Kurepa [3] in the course of studying Souslin's Problem [8]. The purpose of this abstract is to announce the following result:

Theorem 1. *(*PFA*)*[1] *There is a universal Aronszajn line.*

In fact, the universal Aronszajn line η_C of Theorem 1 has a very simple description in terms of *Countryman lines*: if C is Countryman, then η_C is the direct limit of the products

$$C,\ C \times (-C),\ C \times (-C) \times C, \ldots$$

Recall that a linear order is Countryman if it is uncountable and its Cartesian square can be covered by countably many non decreasing relations. Such linear orders are necessarily Aronszajn and have the property that no uncountable linear order can embed into both a Countryman line and its reverse. Such orders were first constructed by Shelah in [7] and are canonical in the presence of PFA in light of the following result which is essentially due to Todorcevic (see [12, 2.1.12]).

[1]Here PFA is a strong Baire category assumption due to Baumgartner. It extends $\mathrm{MA}_{\aleph_1}$ to a broader class of partial orders and is a natural assumption in this context (see [1], [2], [9, §8], [10], [11]).

Theorem 2. *($MA_{\aleph_1}$) Every two $\aleph_1$-dense non stationary Countryman lines are either isomorphic or reverse isomorphic. In particular if C is a Countryman line, then C and $-C$ form a two element basis for all Countryman lines.*

Theorem 1 builds on the following result, which demonstrates the special role Countryman lines play within the class of all Aronszajn lines.

Theorem 3. [6] *(PFA) Every Aronszajn line contains a Countryman suborder.*

Notice that if we let η denote the ordertype of $\mathbb{Q}$, then we have a rather strong analogy between the relationship of $\mathbb{N}$ and $-\mathbb{N}$ to η and the relationship of C and $-C$ to η_C under the assumption of PFA. In particular, η is the order type of the direct limit

$$\omega,\ \omega \times (-\omega),\ \omega \times (-\omega) \times \omega, \ldots$$

Notice that, just as any linear order which does not contain η must contain a non empty interval which is embeddable into ω or $-\omega$, we have the following proposition.

Proposition 4. *(PFA) If A is an Aronszajn line, then either A is bi-embeddable with η_C or else A contains a non empty interval which is bi-embeddable with C or $-C$.*

It turns out that characterizing when an Aronszajn line is Countryman plays an important role in the proof of Theorem 1.

Theorem 5. *(PFA) If A is an Aronszajn line which is not Countryman, then A contains an isomorphic copy of both C and $-C$ for some (equivalently any) Countryman line C.*

If we define an Aronszajn line to be *fragmented*[2] if it does not contain a copy of η_C, then the above analogy extends to give a strong connection between the (countable) scattered linear orders and the fragmented Aronszajn lines. In particular, one can associate a rank to a fragmented Aronszajn line which corresponds to how many applications of a derivative operation are necessary in order to trivialize it.

Laver has shown the countable linear orders are well quasi-ordered by embeddability [4] and the above analogy suggests the following conjecture.

Conjecture 6. *(PFA) The Aronszajn lines are well quasi-ordered by embeddability.*

This could be verified using the methods of [4] if one could prove the following from PFA (see [4] for undefined notions): If Q is a better quasi-order and C is a Countryman line, then Q^C is a better quasi-order.

It is interesting to note that the consequences of PFA needed in the proof of Theorem 1 are closely related to those used in [5] to deduce $|\mathbb{R}| \leq \aleph_2$ from PFA. It is an open problem whether the universality of η_C or the assertion the Aronszajn lines are well quasi-ordered themselves imply $|\mathbb{R}| \leq \aleph_2$.

[2]This definition is only appropriate under a hypothesis such as PFA.

The author's research presented in this abstract, as well as in part the travel to the workshop, was supported by NSF grant DMS-0401893.

References

[1] U. Abraham and S. Shelah. Isomorphism types of Aronszajn trees. *Israel J. Math.*, 50(1–2):75–113, 1985.

[2] Martin Goldstern and Saharon Shelah. The Bounded Proper Forcing Axiom. *J. Symbolic Logic*, 60(1):58–73, 1995.

[3] Dj. Kurepa. Ensembles ordonnés et ramifiés. *Publ. Math. Univ. Belgrade*, 4:1–138, 1935.

[4] Richard Laver. On Fraïssé's order type conjecture. *Ann. of Math. (2)*, 93:89–111, 1971.

[5] Justin Tatch Moore. Set mapping reflection. *J. Math. Log.*, 5(1):87–97, 2005.

[6] Justin Tatch Moore. A five element basis for the uncountable linear orders. *Annals of Mathematics (2)*, 163(2):669–688, 2006.

[7] Saharon Shelah. Decomposing uncountable squares to countably many chains. *J. Combinatorial Theory Ser. A*, 21(1):110–114, 1976.

[8] M. Souslin. Problème 3. *Fund. Math.*, 1:223, 1920.

[9] Stevo Todorcevic. *Partition Problems In Topology.* Amer. Math. Soc., 1989.

[10] Stevo Todorcevic. A classification of transitive relations on ω_1. *Proc. London Math. Soc. (3)*, 73(3):501–533, 1996.

[11] Stevo Todorcevic. Basis problems in combinatorial set theory. *Doc. Math.*, Extra Vol. II:43–52, 1998.

[12] Stevo Todorcevic. *Walks on ordinals and their characteristics*, volume 263 of *Progress in Mathematics.* Birkhäuser, 2007.

Banach spaces without minimal subspaces

Christian Rosendal
(joint work with Valentin Ferenczi)

In a celebrated paper, W.T. Gowers initiated a classification theory for Banach spaces. Since the task of classifying all (even separable) Banach spaces up to isomorphism is extremely complicated (just how complicated is made precise by Ferenczi, Louveau, and Rosendal), one may settle for a *loose classification of Banach spaces up to subspaces*, that is look for a list of classes of Banach spaces such that:

(a) each class is *pure*, in the sense that if a space belongs to a class, then every subspace belongs to the same class, or maybe, in the case when the properties defining the class depend on a basis of the space, every block subspace belongs to the same class,

(b) the classes are *inevitable*, i.e., every Banach space contains a subspace in one of the classes,

(c) any two classes in the list are disjoint,

(d) belonging to one class gives a lot of information about operators that may be defined on the space or on its subspaces.

We shall refer to this list as the *list of inevitable classes of Gowers*. Many classical problems are related to this classification program, as for example the question whether every Banach space contains a copy of c_0 or ℓ_p, solved in the negative by B.S. Tsirelson in 1974, or the unconditional basic sequence problem, also solved negatively by Gowers and B. Maurey in 1993. Ultimately one would hope to establish such a list so that any classical space appears in one of the classes, and so that belonging to that class would yield most of the properties which are known for that space. For example any property which is known for Tsirelson's space is also true for any of its block subspaces, so Tsirelson's space is a pure space, and as such, should appear in one of the classes with a reasonable amount of its properties. Also, presumably the nicest among the classes would consist of the spaces isomorphic to c_0 or ℓ_p, $1 \leq p < \infty$.

After the discovery by Gowers and Maurey of the existence of *hereditarily indecomposable* (or HI) spaces, i.e., spaces such that no subspace may be written as the direct sum of infinite dimensional subspaces, Gowers proved that every Banach space contains either an HI subspace or a subspace with an unconditional basis. These were the first two examples of inevitable classes. We shall call this dichotomy the *first dichotomy* of Gowers. He then used his famous Ramsey or determinacy theorem to refine the list by proving that any Banach space contains a subspace with a basis such that either no two disjointly supported block subspaces are isomorphic (which, for reasons that will become apparent later on, we shall call *tight by support*), or such that any two subspaces have further subspaces which are isomorphic. He called the second property *quasi minimality*. This *second dichotomy* divides the class of spaces with an unconditional basis into two subclasses (up to passing to a subspace). Finally, recall that a space is *minimal* if it embeds into any of its subspaces. A quasi minimal space which does not contain a minimal subspace is called *strictly quasi minimal*, so Gowers again divided the class of quasi minimal spaces into the class of strictly quasi minimal spaces and the class of minimal spaces.

Obviously the division between minimal and strictly quasi-minimal spaces is not a real dichotomy, since it does not provide any additional information. The main result of this paper is to provide the missing dichotomy for minimality, which we shall call the *third dichotomy*.

A first step in that direction was obtained by A. Pełczar, who showed that any strictly quasi minimal space contains a further subspace with the additional property of not containing any subsymmetric sequence. The first author proved that the same holds if one replaces subsymmetric sequences by embedding-homogeneous sequences (any subspace spanned by a subsequence contains an isomorphic copy of the whole space).

We define a space Y to be *tight* in a basic sequence (e_i) if there is a sequence of successive intervals $I_0 < I_1 < I_2 < \dots$ of $\mathbb{N}$ such that for all infinite subsets $A \subseteq \mathbb{N}$, we have

$$Y \not\sqsubseteq [e_n | n \notin \bigcup_{i \in A} I_i].$$

In other words, any embedding of Y into $[e_i]$ has a "large" image with respect to subsequences of the basis (e_i).

We then define a *tight basis* as a basis such that every subspace is tight in it, and a *tight space* as a space with a tight basis.

Theorem 1 (3rd dichotomy). *Let E be a Banach space without minimal subspaces. Then E has a tight subspace.*

Our actual examples of tight spaces turn out to satisfy one of two stronger forms of tightness. The first is called *tightness with constants*. A basis (e_n) is tight with constants when for for every infinite dimensional space Y, the sequence of successive intervals $I_0 < I_1 < \dots$ of $\mathbb{N}$ witnessing the tightness of Y in (e_n) may be chosen so that $Y \not\sqsubseteq_K [e_n | n \notin I_K]$ for each K. This is the case for Tsirelson's space.

The second kind of tightness is called *tightness by range*. Here the range, range x, of a vector x is the smallest interval of integers containing its support, and the range of a block subspace $[x_n]$ is $\bigcup_n$ range x_n. A basis (e_n) is tight by range when for every block subspace $Y = [y_n]$, the sequence of successive intervals $I_0 < I_1 < \dots$ of $\mathbb{N}$ witnessing the tightness of Y in (e_n) may be defined by $I_k =$ range y_k for each k. This is equivalent to no two block subspaces with disjoint ranges being comparable. We show that tightness by range is satisfied by some HI spaces and also by a space with unconditional basis constructed by Gowers.

It turns out that there are natural dichotomies between each of these strong forms of tightness and respective weak forms of minimality. For the first notion, we define a space X to be *locally minimal* if for some constant K, X is K-crudely finitely representable in any of its subspaces. Notice that local minimality is easily incompatible with tightness with constants. Using an equivalent form of Gowers' game as defined by J. Bagaria and J. López-Abad we prove:

Theorem 2 (5th dichotomy). *Any Banach space E contains a subspace with a basis that is either tight with constants or is locally minimal.*

There is also a dichotomy concerning tightness by range. This direction for refining the list of inevitable classes of spaces was actually suggested by Gowers. P. Casazza proved that if a space X has a shrinking basis such that no block sequence is *even-odd* (the odd subsequence is equivalent to the even subsequence), then X is not isomorphic to a proper subspace. So any Banach space contains either a subspace, which is not isomorphic to a proper subspace, or is saturated with even-odd block sequences, and, in the second case, we may find a further subspace in which Player II has a winning strategy to produce even-odd sequences in the game of Gowers associated to his Ramsey theorem. This fact was observed by Gowers, but it was unclear to him what to deduce from the property in the second case.

We answer this question by using Gowers' theorem to obtain a dichotomy which on one side contains tightness by range, which is a slightly stronger property than the Casazza property. On the other side, we define a space X with a basis (x_n) to be *subsequentially minimal* if every subspace of X contains an isomorphic copy of a subsequence of (x_n). This last property is satisfied by Tsirelson's space and is incompatible with tightness by range.

Theorem 3 (4th dichotomy)**.** *Any Banach space E contains a subspace with a basis that is either tight by range or is subsequentially minimal.*

Projective measurability does not imply projective Baire-ness

David Schrittesser
(joint work with Sy Friedman)

We study the possibility of projective sets being regular. To be more precise, we obtain a model where every projective set is Lebesgue-measurable, but there is a Δ^1_3 set lacking the Baire-property.

Lebesgue-measurability and the property of Baire are examples of regularity properties of sets of reals. Other examples include the Ramsey-property, the perfect set property, or the property of being K_σ-regular.

It has long been known that the question of whether all sets in the projective hierarchy up to some level (say, $\boldsymbol{\Sigma}^1_n$) are regular in some given sense, is (in all interesting cases) independent of ZFC.

(1) For example in L, there is a Δ^1_2 well ordering, yielding a set of the least possible complexity which is not Lebesgue-measurable and has neither the Baire nor the perfect set property.
(2) In certain models of a fragment of PD, all sets are regular up to some level $\boldsymbol{\Sigma}^1_n$ (regular in every sense amenable to a game-theoretic treatment); at the same time, there is a Δ^1_{n+1} well-ordering, yielding an irregular set at that level.
(3) Another example is Solovay's model:

Theorem 1 (Solovay's theorem)**.** *If there is an inaccessible, you can force all projective sets to be measurable and have the Baire property (and the perfect set property).*

Observe that in all these examples, all regularity properties share the same type of behavior. The least level in the projective hierarchy at which an irregular set can be found is the same for many different interpretations of the word "regular". It would be more satisfying to prove a theorem as the one given below. Also, the large cardinals assumed, e.g. in example (2) are much too large. One would expect to obtain these theorems from much weaker assumptions, in the order of magnitude of an inaccessible. Let A and B be two notions of regularity, and assume they are in some sense "independent".

Pipe Dream. *The following is consistent, assuming* small large cardinals *(for any k,n):*

(1) *Every Σ^1_n set is A, but there is a non-A Δ^1_{n+1} set.*
(2) *Every Σ^1_k set is B, but there is a non-B Δ^1_{k+1} set.*

It should be mentioned that there can be implications:

Theorem 2 ([1]). *If all Σ^1_2 sets are Lebesgue-measurable, all Σ^1_2 sets have the Baire property.*

One pipe-dream come to true is the following, proved by Shelah.

Theorem 3 ([6]). *Assume that ZFC is consistent. Then it is consistent that all projective sets have the Baire property.*

Theorem 4 ([6]). *If all Σ^1_3 sets are Lebesgue-measurable, ω_1 is inaccessible in L.*

As a consequence, there is a model where all sets have the Baire-property, but with a non-measurable Σ^1_3 set. Using some of the same techniques, Shelah showed:

Theorem 5 ([7]). *Assume there is an inaccessible. In a forcing extension,*

- *every projective set is measurable,*
- *there's a set without the Baire-property.*

We improve this to get a theorem of our favorite type, making the set without the Baire property projective. See [5].

Theorem 6. *Assume there is a Mahlo and $V = L$. In a forcing extension,*

- *every projective set is measurable,*
- *there's a Δ^1_3 set without the Baire-property.*

Observe that by Theorem 5, the assumption of at least an inaccessible is necessary. We don't know if our assumption of a Mahlo is necessary, but in our proof, it is certainly vital, namely when we show that the Mahlo cardinal (which becomes ω_1 in our model) is not collapsed. By Theorem 2, the complexity of the none-Baire set is optimal.

The theorem uses the techniques of amalgamation and coding. Amalgamation allows for an iteration of forcing to have certain automorphisms, making all projective sets regular in the extension. We use a "full-support" variant of the technique of [6] which allows us to preserve some strong closure property of our iteration (in the sense of closure of partial orders, i.e. the existence of lower bounds).

Coding then provides the Δ^1_3 set. We use a variant of Jensen coding [2] via Suslin trees, from [3] (see also [4]); this is the main reason for forcing over L, i.e. starting with the assumption $V = L$. The difficulty is showing that iterating Jensen coding, interlaced with amalgamation, does not do unwanted damage, e.g. collapse κ.

Our method seems to be quite flexible in that it offers hope to be generalizable to other notions of regularity, such as the Ramsey property etc. We end by mentioning some open questions:

- Can we prove a variant of Theorem 6, where the set without the Baire property is Δ^1_{k+1}, and we have, in addition, Baire-property for all Σ^1_k sets, $k \geq 3$?
- For which σ-ideals can we substitute "Borel modulo I" for Baire property or measurable?
- Can we start from a larger ground model, possibly with large cardinals?
- Prove the Mahlo is necessary or get rid of it?

References

[1] T. Bartoszynski. Additivity of measure implies additivity of category. *Transactions of the American Mathematical Society*, 281:209–213, 1984.
[2] A. Beller, R. B. Jensen, and Ph. Welch. *Coding the universe*. Cambridge, 1982.
[3] R. David. A very absolute Π^1_2 real singleton. *Annals of Mathematical Logic*, 23:101–120, 1982.
[4] S. D. Friedman. David's trick. In *Sets and Proofs*, number 258 in London Mathematical Society Lecture Note Series, pages 67–71. Cambridge University Press, 1999.
[5] D. Schrittesser. *Projective measurability does not imply projective Baire-ness*. PhD thesis, Universität Wien, 2008. To be finished in spring 2008; until then check my homepage for a pre-print.
[6] S. Shelah. Can you take Solovay's inaccessible away? *Israel Journal of Mathematics*, 48:1–47, 1984.
[7] S. Shelah. On measure and category. *Israel Journal of Mathematics*, 52:110–114, 1985.

G_δ ideals of compact sets

Sławomir Solecki

I define a class of G_δ ideals of compact subsets of compact metric spaces that, on the one hand, avoids certain phenomena present among general G_δ ideals of compact sets (recently discovered by Mátrai) and, on the other hand, includes all naturally occurring G_δ ideals of compact sets (coming from the notions of meagerness, measure zero, topological dimension, etc).

I prove results on the structure of ideals in this class. I also show that there exists a natural rank on G_δ ideals of compact sets with values $\leq \omega_1$ and then give a result that the ideals in our class are exactly those with the highest possible rank.

On the other hand, I prove a theorem that the rank is unbounded below ω_1 among ideals not in our class. This construction involves block sequences, a type of combinatorial objects coming from the structural Banach space theory. Ideals obtained using this construction have been applied, jointly by Justin Moore and me, to answer a question of Louveau and Veličković on the existence of a G_δ ideal strictly above the nowhere dense ideal in the Tukey order.

Proper products

Otmar Spinas

Let $\mathbb{L}$ be Laver forcing, and let $\mathbb{M}$ be Miller forcing. We show the following:

Theorem 1. *The product forcing $\mathbb{L}^n \times \mathbb{M}^m$ is proper (actually Axiom A) for every $n, m < \omega$.*

For $n = 1$, $m = 0$ this is of course Laver's result in [1], for $m = 1$, $n = 0$ this is Miller's result in [2]. For $m = 2$, $n = 0$ this has been proved in [3]. The theorem remains true by adding as factors any finite number of virtually every classical tree forcing (say Sacks, Silver, Mathias, Steprans forcing).

References

[1] R. Laver, *On the consistency of Borel's conjecture*, Acta Math. 137 (1976), pp. 151–169.
[2] A. Miller, *Rational perfect set forcing*, Contemporary Mathematics 31 (1984), pp. 143–159.
[3] O. Spinas, *Ramsey and freeness properties of Polish planes*, Proceedings of the London Mathematical Society 82 (2001), no. 3, pp. 31–63.

The consistency strength of $\mathsf{AD}_{\mathbb{R}}$

John R. Steel

The $\mathsf{AD}_{\mathbb{R}}$ *hypothesis* is the assertion: there is an ordinal λ which is both a limit of Woodin cardinals, and of cardinals which are $< \lambda$-strong. It gets its name because Woodin showed around 15 years ago

Theorem 1 (Woodin)**.** *If the* ZFC *plus* $\mathsf{AD}_{\mathbb{R}}$*-hypothesis is consistent, then* ZF *plus* $\mathsf{AD}_{\mathbb{R}}$ *is consistent. Indeed, if λ witnesses the* $\mathsf{AD}_{\mathbb{R}}$ *hypothesis, then* $\mathsf{AD}_{\mathbb{R}}$ *holds in the derived model at λ.*

In the other direction, Woodin obtained from $\mathsf{AD}_{\mathbb{R}}$ a model in which OR is a limit of Woodins and $<$ OR strong cardinals (but Replacement fails; the order type of the strong cardinals is ω). We shall close the gap here:

Theorem 2. *If* ZF *plus* $\mathsf{AD}_{\mathbb{R}}$ *is consistent, then* ZFC *plus the* $\mathsf{AD}_{\mathbb{R}}$*-hypothesis is consistent. Indeed, if V is the minimal model of* $\mathsf{AD}_{\mathbb{R}}$*, then in some generic extension of V, there is a proper class extender model $\mathcal{M}$ satisfying the* $\mathsf{AD}_{\mathbb{R}}$*-hypothesis, and such that V is a derived model of $\mathcal{M}$.*

Woodin also produced a consistency-strength upper bound for $\mathsf{AD}_{\mathbb{R}} + \mathsf{DC}$, and our work shows this bound is exact.

Technically speaking, our work is a refinement of a proof, due to Neeman and the author, of Woodin's theorem that the Mouse Set Conjecture holds in the minimal model of $\mathsf{AD}_{\mathbb{R}} + \mathsf{DC}$. See [1], especially section 14, where the consistency strength of AD^+ plus "All Π^2_1 sets are Suslin" is computed.

References

[1] J. Steel, Derived models associated to mice, to appear in the proceedings of the Singapore 2005 workshop *Computational prospects of infinity*, World Scientific.

A Descriptive View of Geometric Group Theory

Simon Thomas

In this talk, I discussed some of the central notions of geometric group theory from the perspective of the theory of Borel equivalence relations and pointed out some intriguing connections with recent work of Louveau-Rosendal [5, 6] on K_σ equivalence relations. Throughout $\mathcal{G}$ denotes the Polish space of finitely generated groups introduced by Grigorchuk [1]; i.e. the elements of $\mathcal{G}$ are the isomorphism types of *marked groups* $\langle G, \bar{c} \rangle$, where G is a finitely generated group and $\bar{c}$ is a finite sequence of generators.

Definition 1. Let G, H be finitely generated groups with word metrics d_S, d_T respectively. Then G, H are said to be *quasi-isometric*, written $G \approx_{QI} H$, iff there exist constants $\lambda \geq 1$, $C \geq 0$ and a map $\varphi : G \to H$ such that:

(a) $\frac{1}{\lambda} d_S(x,y) - C \leq d_T(\varphi(x), \varphi(y)) \leq \lambda d_S(x,y) + C$ for all x, $y \in G$; and
(b) $d_T(z, \varphi[G]) \leq C$ for all $z \in H$.

A clear account of the basic properties of the quasi-isometry relation for finitely generated groups can be found in de la Harpe [2], including a proof of the following result.

Definition 2. Two finitely generated groups G_1, G_2 are said to be *virtually isomorphic* or *commensurable up to finite kernels*, written $G_1 \approx_{VI} G_2$, iff there exist subgroups $N_i \leqslant H_i \leqslant G_i$ for $i = 1, 2$ satisfying the following conditions:

(a) $[G_1 : H_1]$, $[G_2 : H_2] < \infty$.
(b) N_1, N_2 are finite normal subgroups of H_1, H_2 respectively.
(c) $H_1/N_1 \cong H_2/N_2$.

Theorem 3. *If G_1, G_2 are virtually isomorphic finitely generated groups, then G_1, G_2 are quasi-isometric.*

From now on, $\cong$, $\approx_{VI}$ and $\approx_{QI}$ will denote the isomorphism, virtual isomorphism and quasi-isometry relations on the space $\mathcal{G}$ of finitely generated groups. If E, F are Borel equivalence relations on the Polish spaces X, Y respectively, then we say that E is *Borel reducible* to F and write $E \leq_B F$ if there exists a Borel map $f : X \to Y$ such that $x \mathrel{E} y$ iff $f(x) \mathrel{F} f(y)$. We say that E and F are *Borel bireducible* and write $E \sim_B F$ if both $E \leq_B F$ and $F \leq_B E$. Finally we write $E <_B F$ if both $E \leq_B F$ and $F \nleq_B E$. The following result gives the precise Borel complexity of the isomorphism relation on $\mathcal{G}$.

Theorem 4 (Thomas-Velickovic [11])**.** *The isomorphism relation $\cong$ on $\mathcal{G}$ is a universal countable Borel equivalence relation.*

The following result shows that if we wish to understand the precise Borel complexity of the virtual isomorphism relation $\approx_{VI}$ (and also conjecturally of the quasi-isometry relation $\approx_{QI}$), then we must work within a strictly larger class of Borel equivalence relations than the relatively well-understood class of countable Borel equivalence relations.

Theorem 5 (Thomas [8]). $\cong <_B \approx_{VI}$.

It can be shown that the virtual isomorphism and quasi-isometry relations on $\mathcal{G}$ are both K_σ equivalence relations; and the recent work of Rosendal [6] suggests that the following conjecture should be true. (Here the equivalence relation E on the Polish space X is said to be K_σ iff E is the union of countably many compact subsets of $X \times X$.)

Conjecture 6. The quasi-isometry relation $\approx_{QI}$ on the space $\mathcal{G}$ of finitely generated groups is a universal K_σ equivalence relation.

Making use of the results of Rosendal [6], it is easy to prove the following weak form of Conjecture 6.

Theorem 7 (Thomas [10]). *The quasi-isometry relation on the space of connected 4-regular graphs is a universal K_σ equivalence relation.*

On the other hand, the following result strongly suggests that the quasi-isometry relation is strictly more complex than the virtual isomorphism relation.

Theorem 8 (Thomas [10]). *The virtual isomorphism relation $\approx_{VI}$ on the space $\mathcal{G}$ of finitely generated groups is* not *a universal K_σ equivalence relation.*

Corollary 9 (Thomas [10]). *The virtual isomorphism relation $\approx_{VI}$ on the space $\mathcal{G}$ of finitely generated groups is strictly less complex (with respect to Borel reducibility) than the quasi-isometry relation on the space of connected 4-regular graphs.*

An interesting feature of Theorem 8 is the key role which is played in its proof by Hjorth's notion of *turbulence* [3]. More specifically, we make use of the result of Kanovei-Reeken [4] that if G is a Polish group and X is a turbulent Polish G-space, then $E_G^X \not\leq_B E_1^+$.

Finally it should be pointed out that very little is known concerning the Borel complexity of the quasi-isometry relation $\approx_{QI}$ on the space $\mathcal{G}$ of finitely generated groups. In fact, the following result sums up the current state of knowledge regarding this problem.

Theorem 10 (Thomas [9]). *The quasi-isometry relation on the space $\mathcal{G}$ of finitely generated groups is not smooth.*

Here the Borel equivalence relation E on the Polish space X is said to be *smooth* iff there exists exists a Borel function $f : X \to Y$ into a Polish space Y such that $x \, E \, y$ iff $f(x) = f(y)$. By Silver [7], if E is a smooth Borel equivalence relation and F is a Borel equivalence relation with uncountably many F-classes, then $E \leq_B F$. Thus the smooth relations are the least complex Borel equivalence relations with respect to Borel reducibility.

References

[1] R. I. Grigorchuk, *Degrees of growth of finitely generated groups and the theory of invariant means*, Math. SSSR. Izv. **25** (1985), 259–300.

[2] P. de la Harpe, *Topics in Geometric Group Theory*, Chicago Lectures in Mathematics Series, The University of Chicago Press, Chicago, 2000.
[3] G. Hjorth, *Classification and Orbit Equivalence Relations*, Mathematical Surveys and Monographs **75**, Amer. Math. Soc., Providence, RI, 2000.
[4] V. G. Kanovei and M. Reeken, *Some new results on the Borel irreducibility of equivalence relations*, Izv. Math. **67** (2003), 55–76.
[5] A. Louveau and C. Rosendal, *Complete analytic equivalence relations*, Trans. Amer. Math. Soc. **357** (2005), 4839–4866.
[6] C. Rosendal, *Cofinal families of Borel equivalence relations and quasiorders*, J. Symbolic Logic **70** (2005), 1325–1340.
[7] J. H. Silver, *Counting the number of equivalence classes of Borel and coanalytic equivalence relations*, Ann. Math. Logic **18** (1980), 1–28.
[8] S. Thomas, *The virtual isomorphism problem for finitely generated groups*, Bull. Lond. Math. Soc. **35** (2003), 777–784.
[9] S. Thomas, *Cayley graphs of finitely generated groups*, Proc. Amer. Math. Soc. **134** (2006), 289–294.
[10] S.Thomas, *On the complexity of the quasi-isometry and virtual isomorphism problems for finitely generated groups*, preprint (2007).
[11] S. Thomas and B. Velickovic, *On the complexity of the isomorphism relation for finitely generated groups*, J. Algebra **217** (1999), 352–373.

Some consequences of reflection on the approachability ideal

Matteo Viale
(joint work with Assaf Sharon)

1. Approachability

There are two main results which motivate the introduction of the notion of approachability by Shelah: the original motivation is the characterization of the subsets a regular λ in the ground model which are stationary in the generic extension by a λ-closed forcing. Later, in the course of development of the pcf-theory of possible cofinalities Shelah used the approachability ideal to guarantee the existence of scales[1]. In this report our results are motivated by the possible application of this ideal to the combinatorial properties of the successor of a singular cardinal. Our aim is to get complete description of the approachabiltiy ideal for singular cardinal of countable cofinality in a model of MM.

Given $\mathcal{A} = \{a_\alpha : \alpha < \kappa^+\} \subseteq [\kappa^+]^{<\kappa}$, δ is weakly approachable with respect to $\mathcal{A}$ if there is H unbounded in δ of minimal order type such that $\{H \cap \gamma : \gamma < \delta\}$ is covered by $\{a_\alpha : \alpha < \delta\}$ and δ is approachable with respect to $\mathcal{A}$ if there is H unbounded in δ of minimal order type such that $\{H \cap \gamma : \gamma < \delta\} \subseteq \{a_\alpha : \alpha < \delta\}$.

Definition 1. Let κ be a singular cardinal. S is (weakly) approachable if there is a sequence $\mathcal{A} = \{a_\alpha : \alpha < \kappa^+\} \subseteq [\kappa^+]^{<\kappa}$ and a club C such that δ is (weakly) approachable with respect to α for all $\delta \in S \cap C$. $\mathcal{I}[\kappa^+]$ is the ideal generated by approachable sets, $\mathcal{I}[\kappa^+, \kappa]$ is the ideal generated by weakly approachable sets.

[1]We refer the reader to [2] for an exhaustive and clear exposition of the theory of approachability.

It is clear that $\mathcal{I}[\kappa^+] \subseteq \mathcal{I}[\kappa^+, \kappa]$. For many of the known applications of approachability, it is irrelevant whether we concentrate on the notion of weak approachability or on the apparently stronger notion of approachability. Moreover in the case that κ is strong limit $\mathcal{I}[\kappa^+] = \mathcal{I}[\kappa^+, \kappa]$ (section 3.4 and proposition 3.23 of [2]). For this reason we feel free to concentrate our attention on the notion of weak approachability which applies to a more general context. It is rather easy to show that $\mathcal{I}[\kappa^+, \kappa]$ is a normal κ^+-closed ideal which extends the non-stationary ideal. A main result of Shelah is that there is a stationary set in $\mathcal{I}[\kappa^+]$ for any singular cardinal κ (Theorem 3.18 [2]). There are several applications of this ideal to the combinatorics of singular cardinals, we remind the reader one of them and refer him to section 3 of [2] for a detailed account: the extent of this ideal can be used to size the large cardinal properties of κ. $\mathcal{I}[\kappa^+, \kappa]$ is trivial unless the cardinals below κ^+ have very strong combinatorial properties (in the range of supercompactness). Thus for example if square at κ holds $\mathcal{I}[\kappa^+] = \mathcal{I}[\kappa^+, \kappa] = P(\kappa^+)$ (Theorem 3.13 of [2]). On the other hand if λ is strongly compact and $\kappa > \lambda$ is singular of cofinality $\theta < \lambda$ then there is a stationary subset of κ^+ of points of cofinality less than κ which is not in $\mathcal{I}[\kappa^+, \kappa]$ (Shelah, Theorem 3.20 of [2]). In the same spirit if MM holds there is a stationary set of points of cofinality $\aleph_1$ which is not in $\mathcal{I}[\aleph_{\omega+1}, \aleph_\omega]$ (Magidor, unpublished). It is also consistent[2] that for unboundedly many $\alpha < \omega^2$ there is a stationary set of points of cofinality $\aleph_\alpha$ not in $\mathcal{I}[\aleph_{\omega^2+1}]$. It is an open problem whether it is consistent that there is a stationary set on $\aleph_{\omega+1}$ concentrating on cofinalities larger than $\aleph_1$ and not in $\mathcal{I}[\aleph_{\omega+1}]$ (see for example the introduction of [3] or the end of section 3.5 in [2]). We will give a partial answer to that showing that this is not the case in models of MM. Our results have however broader consequences than this and give serious constraints to the possible scenarios where this problem may have a positive solution. We briefly introduce some relevant concepts in our analysis. S^λ_κ denote the subset of λ of points of cofinality κ. A stationary subset of λ reflects on α if it intersects all the closed and unbounded subsets of α.

Definition 2. Let $\theta < \kappa$ be regular cardinals.

$R(\kappa, \theta)$ holds for infinite regular cardinals $\theta < \kappa$ if there is S stationary subset of κ such that for all families $\{S_i : i < \theta\}$ of stationary subsets of S there is $\delta < \kappa$ such that S_i reflects on δ for all $i < \theta$.

$R^*(\kappa)$ holds if if there is S stationary subset of κ such that for all families $\{S_i : i < \kappa\}$ of stationary subsets of S there is $\delta < \kappa$ such that S_i reflects on δ for all $i < \delta$.

It is clear that $R^*(\kappa)$ implies $R(\kappa, \lambda)$ which implies $R(\kappa, \theta)$ for all $\theta \leq \lambda < \kappa$. Moreover it is not hard to realize $R^*(\kappa)$ and $R(\kappa, \theta)$ since $R^*(\lambda)$ holds if λ is weakly compact and $R(\lambda, \aleph_1)$ follows from MM for all regular $\lambda > \aleph_1$. Given regular cardinals $\theta < \kappa$, κ is θ-inaccessible if $\lambda^\theta < \kappa$ for all $\lambda < \kappa$.

[2] See for example [5] where this is achieved in the presence of a very good scale on $\prod_{\alpha<\omega^2} \aleph_\alpha$.

Theorem 3 (Sharon, V.). *Assume:*

- κ *is singular of cofinality* θ,
- $\lambda < \kappa$ *is either* θ*-inaccessible or in* $[\theta^+, \theta^{+\omega})$,
- $R(\lambda, \theta)$ *holds.*

Then $S^{\kappa^+}_{\lambda} \in \mathcal{I}[\kappa^+, \kappa]$.

Immediate applications of Theorem 3 are the following:

Corollary 4. *Assume* λ *is weakly compact and* $\kappa > \lambda$ *is singular cofinality* $\theta < \lambda$. *Then* $S^{\kappa^+}_{\lambda} \in \mathcal{I}[\kappa^+, \kappa]$.

Proof. λ is θ-inaccessible and satisfy $R(\lambda, \theta)$. Now apply Theorem 3. □

Corollary 5. *Assume* MM. *Then club many points in* $S^{\aleph_{\omega+1}}_{>\aleph_1}$ *are approachable.*

Proof. MM implies $R(\aleph_n, \aleph_1)$ holds as witnessed by $S^{\aleph_n}_{\omega}$ for all $n > 1$ (see [4]). Now apply Theorem 3. □

We are also be able to prove:

Theorem 6. *Assume* PFA. *Then club many points in* $S^{\aleph_{\omega+1}}_{>\aleph_2}$ *are approachable.*

2. Chang conjectures for singular cardinals

Recall that the Chang conjecture $(\lambda, \kappa) \twoheadrightarrow (\theta, \nu)$ holds for $\lambda > \kappa \geq \theta > \nu$ if for every structure $\langle Y, \lambda, \kappa, \dots \rangle$ with predicates for λ and κ there is $X \prec Y$ such that $|X \cap \lambda| = \theta$ and $|X \cap \kappa| = \nu$. Cummings asked in [1]:

> *Is it consistent that* $(\kappa^+, \kappa) \twoheadrightarrow (\aleph_2, \aleph_1)$ *for a singular* κ *of countable cofinality?*

A simple outcome of our main result is the following:

Theorem 7. *Assume* MM. *Then* $(\kappa^+, \kappa) \twoheadrightarrow (\aleph_2, \aleph_1)$ *fails for all singular* κ *of cofinality at most* $\aleph_1$.

The reader is referred to the forthcoming [6] for a detailed account on the material presented in this report.

References

[1] J. Cummings. Collapsing successors of singulars. *Proceedings of the American Mathematical Society*, 125(9):2703–2709, 1997.

[2] T. Eisworth. Successors of singualr cardinals. In M. Foreman, A. Kanamori, and M. Magidor, editors, *Handbook of Set Theory*. North Holland, to appear.

[3] M. Foreman and M. Magidor. A very weak square principle. *Journal of Symbolic Logic*, 1:175–196, 1997.

[4] M. Foreman, M. Magidor, and S. Shelah. Martin's Maximum, saturated ideals and nonregular ultrafilters. *Annals of Mathematics (2)*, 127(1):1–47, 1988.

[5] M. Gitik and A. Sharon. On SCH and the approachability property. 10 pages, To appear in Proceedings of the American Mathematical Society.

[6] A. Sharon and M. Viale. Reflection and approachability. In preparation.

P-points and definable forcing

Jindřich Zapletal

Shelah introduced the notion of P-point preservation. An ultrafilter U on ω is a P-point if every countable subset $\{a_n : n \in \omega\} \subset U$ has a pseudo-intersection in U, that is, there is a set $b \in U$ such that $b \setminus a_n$ is finite for every number $n \in \omega$. Under the Continuum Hypothesis P-points are plentiful, however their existence is not provable in ZFC alone. A forcing P preserves P-points if for every P-point U in the ground model and every set $a \subset \omega$ in the extension there is a set $b \in U$ in the ground model which is either a subset of or disjoint from a. The P-point preservation property is useful in ruling out Cohen reals and random reals from the extension as well for other reasons, and it is preserved under the countable support iteration of proper forcings.

In the context of definable proper forcing, this useful property has a serious drawback—it refers to undefinable objects such as ultrafilters. As a result, it is not so clear how difficult it is to verify it and how to go about its verification. In my talk, I will identify a simple property that for definable proper forcing is equivalent to P-point preservation.

Theorem 1. *(ZFC+LC+CH) The following are equivalent for a suitably definable proper forcing P:*

- *P preserves P-points*
- *P does not add a splitting real and (*) for every ground model $f \in \omega^\omega$ and every $g < f$ in the extension there are an infinite ground model set $a \subset \omega$ and a ground model function $h : a \to \mathcal{P}(\omega)$ such that for every $n \in a$ it is the case that $|h(n)| < 2^n$ and $g(n) \in h(n)$.*

In the case that $P = P_I$ for some $\mathbf{\Pi}^1_1$ on $\mathbf{\Sigma}^1_1$ ideal I, the theorem is provable without large cardinal assumptions.

The property (*) is a weakening of the Laver property. It appears for example in [1, Lemma 7.4.31] where it is parallel to, but not a part of, a proof that Blass-Shelah forcing preserves P-points. There is a definable proper forcing P that is bounding, preserves Baire category, outer Lebesgue measure, adds no splitting reals and fails (*).

The theorem can be applied to show that various forcings preserve P-points for which I previously only knew that they did not add splitting reals. This includes forcings with the combinatorial DiPrisco-Llopis-Todorcevic property of [2]. The theorem can be also applied to show that other forcings do not preserve P-points, such as any forcing that adds a bounded eventually different real. The Blass-Shelah forcing adds an unbounded eventually different real and preserves P-points.

The main ingredient of the proof is the following fact: if U is a P-point ultrafilter and I is a definable ideal on ω disjoint from it, then there is an F_σ-ideal $J \supset I$ still disjoint from U.

References

[1] Tomek Bartoszynski and Haim Judah. *Set Theory. On the structure of the real line.* A K Peters, Wellesley, MA, 1995.

[2] Jindřich Zapletal. Parametrized Ramsey theorems and proper forcing. submitted, 2007.

Suslin trees in $L[E]$

Martin Zeman

We generalize Jensen's characterisation of weak compactness in L to any extender model $L[E]$. For this purpose, we construct various kinds of global square sequences. SC is the class of all singular cardinals.

Theorem 1. *Assume $V = L[E]$. Let $A \subseteq \mathrm{SC}$ be a class. Then there is a class $A' \subseteq A$ and a sequence $\langle C_\alpha \mid \alpha \in \mathrm{SC}\rangle$ such that*

(1) *For every regular κ we have: $A \cap \kappa$ stationary implies $A' \cap \kappa$ stationary.*
(2) (a) *$C_\alpha \subseteq \alpha$ is closed unbounded and either $C_\alpha \subseteq \mathrm{SC}$ or else $\mathrm{otp}(C_\alpha) = \omega$.*
  (b) *$\bar\alpha \in \lim C_\alpha$ implies $\bar\alpha \notin A'$ and $C_{\bar\alpha} = C_\alpha \cap \bar\alpha$.*
  (c) $\mathrm{otp}(C_\alpha) < \alpha$.

Theorem 2. *Assume $V = L[E]$. Let κ be an inaccessible cardinal that is not weakly compact and $A \subseteq \kappa$ stationary. Then there is a sequence $\langle C_\alpha \mid \alpha \in \lim \cap \kappa\rangle$ such that*

(1) *C_α is a closed unbounded subset of α.*
(2) *If $\bar\alpha \in \lim C_\alpha$, then $C_{\bar\alpha} = C_\alpha \cap \bar\alpha$.*

Corollary 3. *Assume $V = L[E]$. Then exactly one of the following holds:*

(1) *κ is weakly compact and every stationary subset of κ has a reflection point.*
(2) *κ is not weakly compact and the nonreflecting stationary subsets of κ are dense in the family of all stationary subsets of κ.*

Theorem 4. *Assume $V = L[E]$. Then κ is weakly compact if and only if there is no Suslin tree.*

Reporter: Jakob Kellner

Participants

Prof. Dr. Alessandro Andretta
Dipartimento di Matematica
Universita degli Studi di Torino
Via Carlo Alberto, 10
I-10123 Torino

Prof. Dr. David Aspero
Departament de Logica, Historia i
Filosofia de la Ciencia
Universitat de Barcelona
Montalegre 6
E-08001 Barcelona

Prof. Dr. Joan Bagaria
Departament de Logica, Historia i
Filosofia de la Ciencia
Universitat de Barcelona
Montalegre 6
E-08001 Barcelona

Prof. Dr. Tomek Bartoszynski
The National Science Foundation
Division of Mathematical Sciences
4201 Wilson Boulevard
Arlington Virginia 22230
USA

Prof. Dr. Jörg Brendle
Group of Logic, Statistics & Theor.
Computer Science-Graduate School of
Eng.
Kobe University
Rokko-dai 1-1, Nada
Kobe 657 -8501
JAPAN

Prof. Dr. John D. Clemens
Department of Mathematics
Pennsylvania State University
University Park , PA 16802
USA

Prof. Dr. James W. Cummings
Department of Mathematical Sciences
Carnegie Mellon University
Pittsburgh , PA 15213-3890
USA

Prof. Dr. Carlos Augusto Di Prisco
Matematicas
IVIC
Apartado 20632
Caracas 1020-A
VENEZUELA

Dr. Mirna Dzamonja
School of Mathematics
University of East Anglia
GB-Norwich NR4 7TJ

Prof. Dr. Todd Eisworth
Department of Mathematics
Ohio University
321 Morton Hall
Athens , OH 45701-2979
USA

Prof. Dr. Qi Feng
Department of Mathematics
National University of Singapore
2 Science Drive
Singapore 117543
SINGAPORE

Prof. Dr. Matthew D. Foreman
Department of Mathematics
University of California at Irvine
Irvine , CA 92697-3875
USA

Prof. Dr. Sy-David Friedman
Kurt Gödel Research Center
for Mathematical Logic
Universität Wien
Währinger Str.25
A-1090 Wien

Dr. Gunter Fuchs
Institut für Mathematische
Logik und Grundlagenforschung
Universität Münster
Einsteinstr. 62
48149 Münster

Prof. Dr. Moti Gitik
Department of Mathematics
School of Mathematical Sciences
Tel Aviv University
Ramat Aviv, P.O. Box 39040
Tel Aviv 69978
ISRAEL

Prof. Dr. Ronald B. Jensen
Institut für Mathematik
Humboldt-Universität
10099 Berlin

Dr. Jakob Kellner
Kurt Gödel Research Center
for Mathematical Logic
Universität Wien
Währinger Str.25
A-1090 Wien

Prof. Dr. Peter Koepke
Mathematisches Institut
Universität Bonn
Beringstr. 1
53115 Bonn

Prof. Dr. Menachem Kojman
Dept. of Mathematics
Ben Gurion University of the Negev
PO BOX 653
84105 Beer Sheva
ISRAEL

Prof. Dr. John Krueger
Department of Mathematics
University of California
Berkeley , CA 94720-3840
USA

Prof. Dr. Jean Larson
Dept. of Mathematics
University of Florida
358 Little Hall
P.O.Box 118105
Gainesville , FL 32611-8105
USA

Prof. Dr. Paul B. Larson
Dept. of Mathematics and Statistics
Miami University
Oxford , OH 45056
USA

Prof. Dr. Richard Laver
Department of Mathematics
University of Colorado
Boulder , CO 80309-0395
USA

Prof. Dr. Alain Louveau
Equipe d'Analyse; Boite 186
Universite Pierre et Marie Curie
(Universite Paris VI)
4, Place Jussieu
F-75252 Paris Cedex 05

Prof. Dr. Adrian R.D. Mathias
Dept. de Mathematiques et Informat.
Universite de la Reunion
15, Avenue Rene Cassin
BP 7151
F-97715 St. Denis de la Reunion

Prof. Dr. Julien Melleray
Dept. of Mathematics, University of
Illinois at Urbana-Champaign
273 Altgeld Hall MC-382
1409 West Green Street
Urbana , IL 61801-2975
USA

PD Dr. Heike Mildenberger
Kurt Gödel Research Center
for Mathematical Logic
Universität Wien
Währinger Str.25
A-1090 Wien

Prof. Dr. William J. Mitchell
Dept. of Mathematics
University of Florida
358 Little Hall
P.O.Box 118105
Gainesville , FL 32611-8105
USA

Prof. Dr. Justin Tatch Moore
Department of Mathematics
Cornell University
Malott Hall
Ithaca , NY 14853-4201
USA

Prof. Dr. Christian Rosendal
Dept. of Mathematics, University of
Illinois at Urbana-Champaign
273 Altgeld Hall MC-382
1409 West Green Street
Urbana , IL 61801-2975
USA

David Schrittesser
Kurt Gödel Research Center
for Mathematical Logic
Universität Wien
Währinger Str.25
A-1090 Wien

Prof. Dr. Slawomir Solecki
Dept. of Mathematics, University of
Illinois at Urbana Champaign
273 Altgeld Hall
1409 West Green Street
Urbana , IL 61801
USA

Prof. Dr. Otmar Spinas
Mathematisches Seminar
Christian-Albrechts-Universität Kiel
Ludewig-Meyn-Str. 4
24118 Kiel

Prof. Dr. John R. Steel
Department of Mathematics
University of California
Berkeley , CA 94720-3840
USA

Prof. Dr. Simon Thomas
Department of Mathematics
Rutgers University
Hill Center, Busch Campus
110 Frelinghuysen Road
Piscataway , NJ 08854-8019
USA

Prof. Dr. Stevo Todorcevic
Equipe Logique Mathematique
Universite Paris VII
2, Place Jussieu
F-75251 Paris Cedex 05

Prof. Dr. Jouko Väänänen
Dept. of Mathematics
University of Helsinki
P.O. Box 68
FIN-00014 Helsinki

Prof. Dr. Boban D. Velickovic
Equipe Logique Mathematique
Universite Paris VII
2, Place Jussieu
F-75251 Paris Cedex 05

Matteo Viale
Kurt Gödel Research Center
for Mathematical Logic
Universität Wien
Währinger Str.25
A-1090 Wien

Prof. Dr. Philip D. Welch
Department of Mathematics
University of Bristol
University Walk
GB-Bristol BS8 1TW

Prof. Dr. W. Hugh Woodin
Department of Mathematics
University of California
Berkeley , CA 94720-3840
USA

Prof. Dr. Yasuo Yoshinobu
Graduate School of Information Science
Nagoya University
Chikusa-Ku
Nagoya 464-8601
JAPAN

Prof. Dr. Jindrich Zapletal
Dept. of Mathematics
University of Florida
358 Little Hall
P.O.Box 118105
Gainesville , FL 32611-8105
USA

Prof. Dr. Martin Zeman
Department of Mathematics
University of California at Irvine
Irvine , CA 92697-3875
USA

Prof. Dr. Stuart Zoble
Department of Mathematics
Wesleyan University
265 Church St.
Middletown , CT 06459-0128
USA