Mathematisches Forschungsinstitut Oberwolfach

Report No. 7/2008

Mini-Workshop: Mathematics of Solvency

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February 10th – February 16th, 2008

Abstract. The discussion on solvency regulation for financial institutions, in particular the demand for appropriate risk measures, led to an axiomatic approach (coherent risk measures, Artzner et a. (2004)) which is now generally accepted for one period risks. The mini-workshop was dedicated to the discussion of an extension of this concept to the multi period case as well as on the consequences of the resulting risk measures for decisions and control.

Mathematics Subject Classification (2000): 91B28, 91B30.

Introduction by the Organisers

The mini-workshop *Mathematics of Solvency*, organised by Freddy Delbaen (Zurich), Karl-Theodor Eisele (Strasbourg) and Christian Hipp (Karlsruhe) was held during the week February 10th–February 16th, 2008. The 15 participants from academia and practice – most of whom came from supervision authorities – discussed the foundation of mathematical concepts for risk and risk measurement as well as their consequences for various decision or control activities in financial services. Such concepts are of major importance for the definition of a necessary solvency capital – which takes into account diversification effects – for banks, investors, and insurers, from the perspective of supervision, of risk management, of rating, and of capital allocation. They help industry to develop consistent foundations for dealing with insurance and financial risks.

The investigation of risk concepts in the multi period setting leads to new mathematical questions in the theory of stochastic processes. A sensible definition of a risk measure requires the property of super-additivity with respect to different risks as well as a submartingale property with respect to time. Such a definition leads to a theory of non-linear pricing of risks, as well as to a better understanding of the time evolution of risk assessment. These two concepts were not treated in traditional finance.

The consistency of risk measurement over different time intervals leads to monotonicity properties of convex functionals. The representation of these time consistent valuations is related to backward stochastic differential equations and difference equations, respectively. The information available to agents is modeled with filtrations and the difference in information leads to problems known as enlargement of filtrations.

In the talks, methods and concepts from various fields were used: stochastic processes, game theory, functional analysis, and stochastic control. In addition, experience with existing and operating real world risk valuation concepts has been reported and discussed.

Mini-Workshop: Mathematics of Solvency

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Abstracts

The meaning of solvency

Philippe Artzner

Industry and, to a lesser extent, academia have two types of definition of solvency, based on cash-flows or on balance sheet. The second type depends on "market-consistent" or "supervisory- oriented" accounting for liabilities.
Stochastic processes are the tools for handling risk assessment and solvency assessment at the same time rather than having two different horizons for the two assessments. The relation between risk measurement and acceptancy decisions is discussed.
The novelty of the cost of capital approach is examined as well as its implementation in the Swiss Solvency Test.
Finally, the distinction between reserves (shareholders' money) and provisions (policyhoders' money) is taken into account and compared to the distinction between short term and run-off risks.

One-period risk measures revisited

Philippe Artzner

The extended use of one-step risk measures in the construction of (time consistent) multi-period risk measures (see the talks by M. Hardy, M. Kupper) would already justify a reexamination of one-period risk measures.
In a first part we carefully distinguish the accounting at dates 0 and 1 respectively, from the investment at date 0 into one (see Artzner et al., Mathematical Finance, 1999) or several (see Artzner-Delbaen-Koch, Afir Colloquium, Zürich, 2005, Frittelli and Scandolo, Mathematical Finance, 2006) eligible assets (often called *numéraires*).
In the case of a single asset available for investment of extra capital there is no compelling reason for it to be identical to the numéraire chosen for expressing future values. We moreover suspect that a supervisor prefers all accounting to be done in (date 0 as well as date 1) cash, "legal tender of all debts, public or private". By contrast we sort of "deny" to the supervisor the choice of eligible asset(s) to invest into, leaving him the responsibility of the choice of the acceptance set. Finally we state how acceptance set and investment vehicle are characterized by the risk measure.
We extend the theory to study the advantages of investing extra capital in several eligible assets.
Finally we address the problem which initiated this research: for a business dealing with different currencies, choose an investment of extra capital in a basket of assets in different currencies. The accounting question examined in the first part translates here in the notion of compatible "domestic" and "foreign" risk measures.

We notice that two expected-shortfall based risk measures in different currencies (i.e. with a genuinely random exchange rate) fail to be compatible.
This is joint work with Freddy Delbaen, ETHZ and Pablo Koch-Medina, SwissRe.

Current problems in insurance solvency

Allan Brender

The current international approach to studying and providing for solvency of insurance companies is described in A Global Framework for Insurer Solvency Assessment, a publication of the International Actuarial Association (available from www.actuaries.org). The framework involves the testing of the financial condition of the institution for a fixed time period (the time horizon) under various risks, each at a critical stress level (such as the 99th percentile). Distributions for the outcome of various risks are assumed to be available. The insurer is supposed to hold an appropriate terminal provision for each policy/contract at the end of the horizon. Required capital for each risk is essentially the difference between the terminal provision at the end of the time horizon for the critical stress level and the liability held for the policy/contract at time zero.
Although the object is to study solvency, the accounting regime must be taken into account. There must be consistency between the calculation of liabilities (the province of accounting) and required capital. For example, in Europe the Solvency II project involves the definition of both the regulatory financial reporting regime and the regulatory capital requirement. In Canada, where there is a single financial reporting system as defined by the Canadian Accounting Standards Board independent of financial regulators, the approach is to define an insurer's total asset requirement and subtract financial reporting liabilities to arrive at required capital.
Several difficulties presented by the current approach are discussed:

(1) **Time horizon**: The prevailing approach is to employ a time horizon of one year. For life insurance, this may be too short since the recognition of a significant change in underlying experience (as opposed to a random fluctuation) as well as the introduction of compensatory management action may require several years, during which time the insurer is at financial risk. For non-life or general insurance, a one-year time horizon fails to recognize the effects of the multi-year business cycle; the insurer's position within the cycle can have a significant impact on its solvency position but is not taken into account.
(2) **Critical level**: Little is known about the appropriate and consistent choices of critical stress levels for different time horizons.

(3) **Policyholder Behaviour**: This is an important element of an insurer's experience. However, the effects of other significant risk variables (in particular, economic variables) on policyholder behaviour are largely anecdotal. Little is known about appropriate stochastic distributions to represent aspects of policyholder behaviour such as lapse rates.
(4) **Economic Variables**: Models for interest rates or equity indexes are often taken from financial economics. However, these are often intended to be used over relatively short time periods. These models may not be appropriate for studying insurance solvency, particularly for the very long terms considered in life insurance where it may be necessary to consider terms that extend beyond the length of an interest rate cycle.
(5) **Liquidity**: It would appear that liquidity poses a different type of risk for insurers, mostly company-based, than it does for banks where it has a significant market component. There is little agreement on how to treat liquidity risk. Measurement or quantification is difficult. Some would say liquidity is a risk to be managed but one that cannot be quantified and covered in a capital requirement. Much more work must be done here.
(6) **Mortality and Morbidity**: Although actuaries have studied changes in mortality and morbidity rates for many years, not much work has been done to study the probability distributions for these rates. These distributions are essential for the current approach to solvency; again, more work is required.

An assessment of an insurer's solvency requires one to know something of the interactions between various risks to which the insurer is exposed. To date, there are three approaches taken to this problem:

- Correlation. This is the approach used in the US RBC requirement and in Solvency II. However, the various correlations used appear to be fairly arbitrary and based upon a collective but essentially subjective judgement. This may not be acceptable in a solvency system that purports to have a high degree of capital adequacy. In fact, another talk in this seminar (cf. D. Filipovic) showed that the correlation structure being proposed for Solvency II is fundamentally flawed.
- Copulas. Copulas present a promising approach to the handling of relationships between risks. However, this is a young field and there is little available guidance as to the appropriate choice of copulas in a particular situation. Since the interactions between risks will strongly depend upon the particular copula, this lack of guidance is a cause for concern.
- Algorithms in Modelling. Computer projection models may be used to study an insurer's financial condition. Models may incorporate relationships between various risks (for example, interest rates and policy lapses). These are usually expressed by algorithms. There is little or no literature available on the appropriate construction of such algorithms.

The last point also leads to the suggestion that there is a need for development of a science of corporate modelling.

Finally, we turn to the problem of extreme events. Experience has shown that in many cases the distributions used in solvency work fail to capture extreme experience that occurs from time to time in real life. Moreover it can often be observed that it is a common human failing to underestimate the likelihood that various extreme events will actually happen. This leads to the observation that most distributions of risk factors that are commonly used in solvency work do not have sufficiently thick tails. Since these are rare events, we are not likely to have sufficient experience to modify these tails using, for example, extreme value theory. It is suggested that it is necessary to supplement solvency studies with disciplined scenario and stress testing. Further development is also needed here. In fact, this is likely to be a future project of the Solvency Sub-committee of the International Actuarial Association.

The allocation of risk based capital

Freddy Delbaen

The space $(\Omega, \mathcal{F}, \mathbb{P})$ denotes a probability space. We supppose that there are N business lines that have a random outcome $X_1, \ldots, X_N$. We suppose that these outcomes are in L^∞. Risk adjusted values and risk measures are evaluated via a coherent utility function

$$u \colon L^\infty \to \mathbb{R}.$$

A coherent utility function satisfies the following properties

- $u(\xi) \geq 0$ for all $\xi \geq 0$, also $u(0) = 0$
- u is monetary, i.e. for $a \in \mathbb{R}$, $u(\xi + a) = u(\xi) + a$
- u concave
- u is positively homogeneous

The first three properties are standard and lead to concave monetary utility functions. We are interested in solving the following set of inequalities: find $k_1, \ldots, k_N$ so that

- for all $0 \leq \lambda_1, \ldots, \lambda_N \leq 1$ we have $u(\sum_j \lambda_j X_j) \leq \sum_j \lambda_j k_j$
- $\sum_j k_j = u(\sum_j X_j)$

Such a system of numbers is called a fair allocation. It gives all kind of coalitions more than what they would get on a stand-alone basis. It is a way to distribute the gains coming from diversification.

A necessary condition to solve this system leads to positive homogeneity. That's why we included it from the beginning. A coherent utility function is represented via its Fenchel-Legendre transform. This means that there is a set of scenarios $\mathcal{S} \subset ba$ such that $\mathcal{S}$ is weak* compact convex in the dual ba of L^∞. The set $\mathcal{S}$ only contains finitely additive probability measures. The representation of u is

$$u(\xi) = \inf_{\mu \in \mathcal{S}} \mathbb{E}_\mu[\xi].$$

The following results hold

- if $\mu \in \mathcal{S}$ satisfies $\mathbb{E}_\mu[\sum_j X_j] = u(\sum_j X_j)$ then $k_j = \mathbb{E}[X_j]$ defines a solution of the set of inqualities. Also the converse holds. So this procedure gives *all* fair allocations.
- k_j is given by the derivative of u if and only if the measure μ is unique
- in case u is Fatou, the derivative is necessarily sigma-additive.

The proof is standard duality theory, except for the last statement where we use that a Borel measurable linear function on L^∞ is automatically given by an element of L^1. This automatic continuity theorem (due to J.P.R. Christensen) is a non trivial result from descriptive set theory.

The above results show that for allocating risk based capital, the risk measure needs to be coherent. This shows that Value at Risk (or quantile) cannot be used for a consistent risk capital allocation. Basically the non convexity/concavity of VaR is in contradiction with the argument that diversification pays. It is not difficult to find that the "coherence" conditions are necessary and sufficient. The Fatou property allows for the representation of the risk measure via sigma-additive measures. The set-theoretic result shows that in such a case the capital allocation problem has unique solutions when $\mathcal{S}$ is weakly compact (in L^1) and is "strict-convex". The kind of strict convexity that is needed is not easy to describe since for constant functions a you can use any element $\mu \in \mathcal{S}$ to calculate $u(a) = a$. The need for weak compactness is based on the famous (and difficult) James' theorem from functional analysis.

The case where $\mathcal{S} = \{\mathbb{Q} \ll \mathbb{P} \mid \frac{d\mathbb{Q}}{d\mathbb{P}} \leq K\}$ is particularly interesting since it gives as a risk measure *TailVar*. In case the distribution function of $X = \sum_j X_j$ is continuous we find a unique fair allocation given by

$$k_j = \mathbb{E}[X_j \mid X \leq q],$$

where q is a $1/K$ quantile of X (the reader can see that we only need an exact quantile, i.e. $\mathbb{P}[X \leq q] = 1/K$. This is sufficient to get the uniqueness of $\mathbb{Q} \in \mathcal{S}$. It does not matter what quantile we use, the associated measure is always the same. In case there is no exact quantile, the measure $\mathbb{Q}$ is (at least in atomless spaces) non-unique and there can be more fair allocations. This situation is not relevant for practical applications since the set of X for which we have uniqueness is norm dense in L^∞, so a slight change in one of the X_j is sufficient to gurantee the uniqueness.

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The standard model in Solvency II: aggregation and diversification

Damir Filipović

This talk consists of three parts: first I give a short overview of the Solvency II process. We are currently in the consultancy phase for Quantitative Impact Study 4 (QIS4). A benchmarking study carried out on behalf of the Chief Risk Officer Forum in 2007 showed that diversification is a major issue for solvency capital requirements in the standard model.
In the second part, I describe the modular structure of the QIS3 and QIS4 standard model. We then discuss the conceptual fallacy of multi-level aggregation. An example illustrates that implied level 1 correlation is a function of the portfolio weights. A direct comparison of standard and implied internal level 1 correlation - and thus diversification - is meaningless. The best solution would be that the standard model provides the full level 2 correlation matrix.
In the third part, I present a realistic approach to realize diversification effects across legal entities on the group level of an insurance conglomerate. Diversification requires capital mobility. This mobility, however, may be constrained by the local regulators. In a new approach we suggest that diversification benefits be realised by means of standardized capital and risk transfer instruments. A numerical example illustrates the potential usefulness of this approach in practice.

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The iterated CTE – a dynamic risk measure

Mary R. Hardy

In this paper we start with a single period risk measure, the Conditional Tail Expectation (or TailVaR) . We extend the measure to a multi-period framework, and then explore the characteristics of the resulting dynamic risk measure. Our approach to the multi-period extension of the single period problem may be applied to any static risk measure. We focus on the CTE measure, as it is well known, coherent and commonly used for equity-linked insurance (e.g. Hardy (2003)).

Let $E_D[\,]$ represent the expectation under the distorted probability measure. Let $H_{t,T}(L)$ denote the risk measure at t for a loss L at $T > t$. Then, allowing for discounting at continuous force of interest r per year, and denoting by $\mathcal{F}_t$ the information on the stock price process up to time t, the risk measure to maturity for an T-year contract at some time t, where $0 \leq t \leq T$ is:

$$H_{t,T}(L) = E_D\left[L\, e^{-r(T-t)} \middle| \mathcal{F}_t\right] \tag{1}$$

Let $J_{t,T,k}[L]$ represent the iterated risk measure at t for a loss L at T, assuming iteration at intervals of k years. Assume $k\,n = T - t$ for some integer n, which represents the number of time steps. We assume equal length of time steps here for convenience, though it is not necessary.

In the final k-year step of the contract,

$$J_{T-k,T,k}(L) = H_{T-k,T}(L) \tag{2}$$

Then, for $t \leq T - 2k$

$$J_{t,T,k}(L) = E_D\left[J_{t+k,T,k}(L)\, e^{-r} \middle| \mathcal{F}_t\right] = H_{t,t+k}(J_{t+k,T,k}(L)) \tag{3}$$

So, any single period risk measure can be used to construct a multi-period risk measure by iteration. If we start with a risk measure having some attractive features, such as the CTE, then we retain those features after iteration. We use the iterated CTE (ICTE) through the remainder of this paper.

The iterated CTE risk measure is a special case of the CTE risk measure. Since the CTE is coherent, (Artzner et al, 1999) so is the iterated CTE, $J_{t,T,k}(L)$, for any fixed t. The proof of this is simple through backwards induction on t.

The iterated risk measure satisfies dynamic consistency (Riedel (2003)) since the risk measure at $t-k$ is fully determined by the set of possible values of the measure at t. More formally, we have

$$J_{t,T,1}(L) = H_{t,t+1}(J_{t+1,T,1}(L))$$

If for two random variables L_1 and L_2, $J_{t+1,T,1}(L_1) = J_{t+1,T,1}(L_2)$, for all $\mathcal{F}_{t+1}$, then clearly $J_{t,T,1}(L_1) = J_{t,T,1}(L_2)$ also.

A dynamic risk measure is *relevant* (Riedel (2003)) if every loss which is not excluded by the current history carries positive risk and losses which are excluded carry zero risk. The Iterated CTE satisfies this criterion.

We illustrate the ICTE with a 10-year single premium contract with a guaranteed minimum death benefit and a guaranteed minimum maturity benefit. The premium is invested into a mutual-type fund, from which a regular charge is deducted to cover expenses. The policyholder receives the fund balance on death or maturity, subject to a minimum of 100% of the initial premium. There is no guarantee on surrender. We assume no hedging, and that the risk is covered entirely by holding capital in bonds. We use a Regime Switching Lognormal model, with 2 regimes for projecting the fund value. The required capital for the contract is assumed to be the 95% CTE of the present value of the guarantee liability, net of management charge income.

The current approach to the economic capital requirement is to recalculate the single period CTE at each valuation date.

We have run 10,000 independent projections of the policy cashflows using the current approach and using the ICTE. Each run involves projecting the fund 10 years, in monthly steps, and at each year end interpolating CTE factors to determine the capital required.

Using the current approach, there is an estimated 4.5% probability that, at some point during the term of the contract additional capital will be required of more than 10% of the initial premium.

Using the ICTE, the estimated probability of requiring additional capital of more than \$10, per \$100 initial premium, is estimated at less than 0.2%. However, the price, in terms of carrying additional capital through the term of the contract, is very heavy.

An alternative approach is to change the α standard as the contract moves towards maturity. To illustrate we have set $\alpha = 55\%$ for the first year of the contract, increasing by 5% per year to a maximum of 95% for the final two years of the 10 year contract. The risk of major additional cashflow requirements are very much lower than we found using recalculated CTEs, and are similar to the 95% ICTE plot. The volatility of the cashflows is very much mitigated also, and the probability of requiring additional capital of more than 10% of the initial premium is, as with the full 95% ICTE simulations, less than 0.2%.

Details are available in Hardy and Wirch (2004).

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Pricing of insurance risk

Christoph Hummel

The focus of this talk was the treatment of cost of capital in pricing insurance risk. To this end, different approaches used in practice were illustrated by means of a stylized example. The main issue raised and discussed was the following: Using the approach deduced from solvency/SST-requirements, the price of a risk could not depend on its maturity.

Market consistent valuation in non-deep and liquid markets

Philippe Keller

We discuss the non-uniqueness and the different views taken (e.g. production cost, transfer value,...) of market consistent valuation of insurance liabilities in situations where cash flows cannot be replicated perfectly. We then show examples how associated risk margins differ. We also discuss the basis and calibration of cost of capital rates.

Dynamic risk measures, valuations and optimal dividends for insurance

Michael Kupper

(joint work with Patrick Cheridito and Damir Filipović)

A dynamic risk measure in discrete time is a family of functions $\rho_t : E_{t+1} \times \cdots \times E_T \to E_t$, $t = 0, \ldots, T$, for a finite time horizon T and some model spaces of $\mathcal{F}_t$-measurable random variables E_t. $(X_{t+1}, \ldots, X_T) \in E_{t+1} \times \cdots \times E_T$ can be viewed (there are different interpretations) as the future discounted cash-flows of a financial position or the profit and losses of an insurance company, whereas $\rho_t(X_{t+1}, \ldots, X_T)$ is the risk capital needed at time t to be in an acceptable position. Using the concept of (strong) time-consistency we show that such a dynamic risk measure can be decomposed into one-step risk measures of the form $H_t : E_{t+1} \to E_t$, which we call the generators. On the other hand, starting with the local risk preferences $(H_t)_{t=1}^T$, we get a time-consistent dynamic risk measure through backward induction $\rho_t = -H_{t+1} \circ \cdots \circ H_T$. We then discuss reasonable properties of the generators. H_t typically has the properties of a classical one-step risk measure, however, we weaken the cash invariance to a sub-cash invariance property, allowing to define risk measures which are path dependent. We also discuss dual representations for dynamic risk measures and the link to the dual representations of the respective generators.

In the second part of the talk, we consider an insurance company with the objective of optimizing its dividends under exogenously given solvency constraints. We assume that the insurance company can invest in a financial market with $m + 1$ securities. The insurance company has to pay the (random) liabilities $L_1, \ldots, L_T$ at time $t = 1, \ldots, T$. In order to give these future liabilities a value today, we introduce and discuss market-consistent valuations. At each time t, the insurance company chooses an investment strategy for the next year and decides how much dividends are paid to the shareholders. Each strategy defines an asset-liability portfolio which typically has to satisfy some solvency constraints, imposed by the regulators. Here, we model the solvency constraints with a dynamic risk measure. In case of the dividends being measured by a dynamic utility function with sub-cash invariant generators (the negative of a dynamic risk measure), we show how the optimal strategy and the optimal dividends can be computed via backward induction.

Dynamic convex risk measures: time consistency, prudence, and sustainability

Irina Penner

In the talk we characterize and discuss various properties of a dynamic convex risk measure for bounded random variables. A dynamic risk measure $(\rho_t)_{t=0,1...}$ induces for each financial position X a risk process $(\rho_t(X))_{t=0,1...}$ describing the conditional risk assessments associated to X over the time. A key question in the dynamic setting is how these risk assessments in different periods of time are interrelated. This question has led to several notions of time consistency in the literature. We give a short overview of various time consistency notions, characterizing them in the way that was suggested in Tutsch [7]. The idea is that the degree of time consistency is determined by a sequence of benchmark sets. If a financial position at some future time is always preferable to some element of the benchmark set, then it should also be preferable today. The bigger the benchmark set, the stronger is the resulting notion of time consistency. Using various benchmark sets we obtain in particular strong time consistency, middle and weak rejection and acceptance consistency. In the following we study in more detail two notions: strong time consistency and middle rejection consistency, that we call prudence.
First we focus on the the strong notion of time consistency, which amounts to the recursion

$$\rho_t(-\rho_{t+1}) = \rho_t.$$

This form of time consistency was studied among others in Arztner et al. [1], Delbaen [4], Detlefsen and Scandolo [5], Cheridito et al. [2], Föllmer and Penner [6], Cheridito and Kupper [3]. We characterize this property in terms of penalty functions, acceptance sets and a joint supermartingale property of the risk measure and its penalty function. The characterization in terms of penalty functions provides the explicit form of the Doob and of the Riesz decomposition of the penalty function process for a time consistent risk measure. The supermartingale property of a strongly time consistent dynamic risk measure with an infinite time horizon ensures the existence of the limit

$$\rho_\infty(X) := \lim_{t\to\infty} \rho(X)$$

for all positions X. We briefly discuss the asymptotic behavior of a risk process, in particular the question whether the dynamic risk measure is asymptotically safe in the sense that the limiting capital requirement $\rho_\infty(X)$ covers the actual final loss $-X$. We also consider the case where $\rho_\infty(X)$ is exactly equal to $-X$, the property called asymptotic precision. The presented results for strongly time consistent dynamic risk measures were obtained in joint work with Hans Föllmer [6].

In the second part of the talk we study a weaker notion of time consistency, that we call prudence. Similar to the time consistent case, we characterize prudent dynamic risk measures in terms of acceptance sets, of penalty functions and by a certain supermartingale property. Prudent risk measures induce risk processes that can be upheld without any additional risk. We call such processes sustainable, and we give an equivalent characterization of sustainability in terms of a combined supermartingale property of a process and one-step penalty functions. This supermartingale property allows us to characterize the strongly time consistent risk measure arising from any dynamic risk measure by recursive construction as the smallest process that is sustainable and covers the final loss. Thus our discussion provides a new reason for using strongly time consistent risk measures.

Finally we illustrate the presented results by the example of the the entropic dynamic risk measure.

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The Swiss solvency test

Pascal Perrodo

The Swiss solvency test (SST) edicted by the Swiss Federal Office of Private Insurances is reviewed. This test is based on a market consistent valuation of companies at initial time as well as a stochastic modelling of economic quantities in the future. The way risk is measured is performed using the TVaR or expected shortfall.

This test is now running for almost four years and enters today in this obligatory phase of application for all companies. Some improvements have been since implemented and are presented as well as known problems or weaknesses.
The points emphasized in this talk are current developments needed with the solvency in presence of currencies, the problems related to modelling of groups as well as possible definitions for risk concentration.

Construction of the valuation portfolio in non-life insurance

Mario Wüthrich

We have constructed the valuation portfolio (VaPo) for a non-life insurance runoff. The VaPo construction is a first step towards analyzing insurance liabilities for pricing insurance products, for asset and liability matching and for solvency considerations. It maps the insurance liabilities onto financial instruments which are then valued by an accounting principle. In our contribution we have highlighted the difficulties in this construction and we have posted open modeling problems. Such open modeling problems cover dependence modeling between different risks and different accident years, claims inflation modeling, appropriate choices of underlying financial instruments, appropriate choices of time-consistent risk measures, as well as the question how one should combine information coming from different sources.

China's practice and experience of applying the EU SM models

Xie Zhigang

This study explores the calculation of statutory minimum capital requirement (MCR) in the solvency regime of general insurance industry in China. It provides a review and analysis on the principles for calculating MCR and then illustrates the process basing on insurance risk.
The insurance industry of China is currently reforming its solvency regime towards a risk oriented system. The commissioner, China Insurance Regulatory Commission (CIRC) has issued an exposure draft of Solvency Regulation (CIRC 2007, No.324) which proposes a comprehensive framework including both s static solvency capital requirements and dynamic solvency test. However, the draft regulation has not updated the calculation standard for the MCR which is a necessary and fundamental component for a solvency regime. The existing MCR calculation standard in China is a simplified version of the EU directives 73/239/EEC for non-life insurance and the 79/267/EEC for life insurance. It has been proved with about 5 year implementing experiences that the simplification of the EU Solvency 0 model (referring to its updated model Solvency I (2002) and the most recent exposure draft of the Solvency II directive proposal 2008) does not fit the China market well.
To reform the existing MCR model, it is acceptable that the calculation should be in a clear and simple manner such that it can be audited. But it may be

not practical for China to calculate MCR as a percentage of the total Solvency Capital Requirement (SCR) of Solvency II as currently tested in Europe. The SCR is much more comprehensively risk-based in principle and hence is with more volatility than the statutory MCR.

This study hence suggests that the MCR calculation should be partially and explicitly risk-based, rather than comprehensively risk based. This proposal can be realized in the following steps:

(1) The risk profile of China insurance industry, both of life and non-life industries, should be analyzed according to a consistently risk classification. A subjective and overall judgment should be given to each risk class, e.g. 70% of insurance risk for non-life insurance and 30% for life insurance and so on.
(2) Based on the risk classification and initial overall judgment, a decision should be made about what types of risk are significant and quantitatively measurable and hence should be taken into account for calculating solvency capital charges. Correspondingly, the time horizons and confidence levels are also decided for the calculation.
(3) The calculation starts on each main line of business among the selected sample insurers, and then the calculation results are further analyzed, including the comparisons to the existing MCR level and to the other regimes. The recommendations for industry standard may then be proposed.

It is understandable that there never be a "right standard" of MCR calculation. One of important criteria for appropriate MCR is its acceptability or applicability by the industry. It is therefore that the calculation will be based on insurers' quantitative risks only, selected from simple to complex, not once for all but step by step.

As an illustration of above proposal, this study provides the MCR calculation process for China general insurance industry, basing on insurance risk (divided into reserving risk and pricing risk) for about 70% of charge, the other part of about 30% MCR is given to investment risk.

In the study, seven sample companies are selected from 25 general insurers writing P&C business on China market, and their main lines of business include motor (vehicle damage, CTP, and others), commercial property, cargo, liability, household, casualty, and the others. Data records of business lines are collected from year 2000 to year 2005, with difference by quarterly, half year, or annual records. For motor insurance business, the outstanding reserve variable OS and the loss ratio variable LR are used to simulate reserving risk and pricing risk respectively, which are pre-assumed as Lognormal distribution, positively skewed and with heavy long tails (passed the one-sample Kolmogorov-Smirnov test and compared to Normal distribution). For the insufficient sample data, Bootstrapping methods are used to simulate OS and LR, 2000 times of simulation with Matlab software. The estimated results are $OS \sim Lognormal$ (13.1732, 0.05313) and $LR \sim Lognormal$ (0.738, 0.115). With confidence levels of 95%, 97.5% and 99%, the

reserving risk factors are calculated as 8.67%, 11.37% and 13.20% respectively, and the pricing factors corresponds to 19.03%, 22.29%, 26.23%. The factors for other business lines are similarly processed.
Since the calculation is processed among the selected sample companies individually. The results are further compared and analyzed with each line of business, and methods of simple average, weighted average, or individual adjustment are implemented to recommend industry risk factors.

line α	com. prop.	house-hold	liab.	motor CTP	motor dam.	motor other	cargo	cas.	other
reserving risk factors									
95%	58.37	29.21	24.10	11.07	11.12	13.61	72.87	32.00	46.73
99%	112.19	45.58	36.81	16.18	16.10	18.94	125.82	61.01	70.33
pricing risk factors									
95%	21.43	0.00	0.04	20.23	29.28	n.a.	28.73	0.00	27.36
99%	46.45	5.11	5.02	54.71	46.11	n.a.	60.45	0.00	29.62

Recommended industry factors for reserving and pricing risks (%)

It is understandable that both insurers and regulators are concerned about the proposed level of MCR calculation, particularly comparing to the current MCR standard in the Administration for Solvency Margin and Regulatory Indices (CIRC [No.1, 2003]). Our study shows that the capital charges of reserving and pricing risks, with a suggested (by NAIC) correlation coefficient ρ=0.26 of the two risk factors, will cover about 85% of the current standard. This means that it leaves only a small space for capital charge to investment risk, if the regulator, CIRC, does not expect to increase the current MCR level significantly.

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