Mathematisches Forschungsinstitut Oberwolfach

Report No. 15/2008

The Mathematics and Statistics of Quantitative Risk Management

Organised by
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Paul Embrechts, Zürich
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March 16th – March 22nd, 2008

Abstract. Over the last 20 years risk management has become one of the more challenging tasks in the financial and insurance industries. With the current uncertainty in the financial institutions and markets, risk management is a major and pressing topic of interest. Risks in insurance and finance are often described by stochastic models such as stochastic differential equations, which describing the evolution of prices of risky assets (i.e., stock shares, interest rates, foreign exchange rates, etc.) or by difference equations for time series. In order for these models to be useful, optimal statistical methods have to be utilized to fit the models to data. This workshop drew together researchers from a myriad of areas related to risk management including statistics, econometrics, applied probability theory, and econometrics. The main objective was to account for the state of the art of statistical and probabilistic modeling in risk management and, in particular, to collect problems which need an urgent theoretical solution.

Mathematics Subject Classification (2000): 62 Statistics: 60 Probability.

Introduction by the Organisers

The Mathematics and Statistics of Quantitative Risk Management Workshop, organized by Thomas Mikosch (Copenhagen), Richard A. Davis (New York), and Paul Embrechts (Zürich), was held March 16th–March 22nd, 2008. This meeting was well attended with over 40 participants from four continents. This workshop was a blend of researchers with various backgrounds in mathematical finance, statistics, econometrics, extreme value theory, applied probability, and insurance.

Modern quantitative risk management integrates a wide range of sophisticated mathematical techniques and tools. An overview from the statistical side is given in the recent monograph by McNeil, Frey, Embrechts. Relevant areas of research include the theory of high-dimensional data structures; rare event simulation; theory of risk measures; (multivariate) time series analysis; extreme event modeling and extreme value statistics; optimization; and linear, quadratic, and convex programming. Recent questions related to multi-period risk measures involve deep results from a variety of fields. Functional data analysis is instrumental for designing and analyzing risk measures, a geometric theory of extremes is useful for the analysis of generalized risk scenarios, Malliavin calculus has become important for the calculation of risk measure sensitivities, functional regular variation is a relevant concept for analyzing stochastic processes exhibiting extreme behavior, advanced rare event simulation techniques, numerical and optimization methods, Lévy processes and more general diffusions are the building blocks for constructing dynamic stochastic models in finance and econometrics.

As evidenced by the recent upheavals in the markets and financial institutions, there is a pressing and critical need to develop and refine tools and methods in quantitative risk management. Expanding on the theory in quantitative risk management should have immediate impact for the financial and insurance industries as well as for supervisory authorities. The objective is to design mathematically tractable, practically relevant and statistically estimable risk measures. An advanced theory also allows one to critically study the present use of tools and methods in quantitative risk management.

Risks in insurance and finance are described by mathematical and probabilistic models such as partial differential equations and stochastic differential equations describing the evolution of prices of risky assets — price of stock, composite stock indices, interest rates, foreign exchange rates, commodity prices — or difference equations describing the evolution of financial returns. The 2003 Nobel prize winning ARCH model is an outstanding example. Applications of these models require advanced simulation and numerical methods and statistics plays a vital role in the estimation of unknown parameters (possibly infinite dimensional) from historical data.

Due to their complexity, problems of quantitative risk management require multidisciplinary solutions. They involve functional analysts who design and analyze risk measures, probabilists who model with stochastic differential equations and time series, applied probabilists who solve the simulation problems, numerical analysts who deal with high-dimensional integration and optimization problems, and statisticians who fit stochastic models to the data and predict future values of risky assets.

Among the challenging problems which were discussed at the meeting are:

- Risk problems are often high-dimensional: a portfolio typically consists of several hundred assets. Modern mathematics and statistics does not offer immediate solutions. For example, the number of historical observations is often smaller than the number of parameters in the model. Techniques from function data analysis (FDA) may prove useful in this context. FDA methods are designed to deal with panel data in which the number of panels, which consist of time series, can be large.

- Risks are dependent across the assets and through time. A key problem is the sensitivity of a particular modeling paradigm to model miss-specification of multivariate models. Robustness to parameter estimation does not quite fit the bill, since, for example, parameters coming from a particular copula (arising from a multivariate distribution) may be completely meaningless if the true model does not involve such quantities. Emphasizing this aspect of sensitivity to model miss-specification encompasses a number of the issues that were ultimately addressed at this workshop.
- Financial and insurance data are not stationary. They contain structural breaks due to changes in the economic or social environments. A relevant question is how such changes can be incorporated in theoretical models and in the corresponding statistical analysis of data. Given one accepts structural breaks, a natural questions arises as to the range of data on which one may conduct reliable inferences.
- Various popular models for risk management are based on statistical ideas and techniques (copulas, variance-covariance models, historical simulation,...). Although these methods are popular, their limitations have not been theoretically studied. For example, it is unclear what sense popular classes of copulas (Gaussian, student, Archimedean, etc.) achieve in a universe of multivariate distributions where the classes of distributions described by them are far from being dense in the class of all multivariate distributions. The discussions at the workshop did not solve the problem, but the talks given brought more theoretical clarity as regards the estimation of certain types of copulas such as Archimedean, extreme value, and Paretean copula.
- Modern risk management asks for the determination/estimation and aggregation of risk measures calculated at high quantiles (99.9% and above) and across different time periods, from ten days to one year. This requires careful statistical analysis. The discussions showed that multivariate extreme value theory comes close to its boundaries of applicability and techniques. Rare event simulation using importance sampling can be useful, but may break down when heavy-tailed risks are involved.
- It was also pointed out where mathematical theory reaches its limits. For instance, the non-existence of useful risk measures on spaces of random variables with infinite mean (as a consequence of results in functional analysis) was shown. The numerical calculation of risk measures and the solution of related optimization questions (capital allocation, calculation of worst case scenarios) leads to challenging mathematical problems which can be hard to solve.
- A natural topic of the workshop was the recent worldwide crisis of credit portfolios. In the past, mathematical models have been designed to avoid the present situation and they are implemented in the framework of the Basel II accord. But they obviously have not been used successfully. Both

formal and informal reasons for the present situation were discussed. Although it would be inappropriate to blame a mathematical model for its failure, there is evidence that various models are too simplistic and do not incorporate market information sufficiently fast. Further, it appears that the statistical analysis of the data was not conducted with sufficient care.

Some of the main objectives of the workshop are summarized here:

- Theory and statistical practice of risk management bear a multitude of contradictory problems which were discussed in a rigorous way.
- The workshop emphasized some of the major problems in this area. The critique mainly concerns statistical problems although modeling problems (called "model risk" in practice) were given serious consideration.
- The workshop brought together some of the leading academic researchers to discuss successes, failures and limitations of present statistical technology in risk management.
- The mixture of researchers from different fields who often do not go to the same conferences, was viewed as a successful experiment by all participants.
- The workshop set the stage for future statistical and mathematical research in the area of quantitative risk management. At present there seem to exist more problems than solutions. Therefore a future meeting (perhaps in 2011) to address these issues would be useful.

Workshop: The Mathematics and Statistics of Quantitative Risk Management

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Abstracts

Weighted Empirical Processes for Lévy Processes

Boris Buchmann
(joint work with Alexander Szimayer)

Consider a Lévy process $L = (L_t)_{t\geq 0}$ on $\mathbb{R}$, i.e., a process defined on a probability space $(\Omega, \mathcal{F}, P)$ with independent stationary increments and càdlàg paths. The finite dimensional distributions of L are completely determined by the distribution of L_1 and its characteristic function satisfies the Lévy-Khintchine formula:

$$E\exp(i\theta L_1) \;=\; \exp\left(i\theta\gamma - \frac{\sigma^2}{2}\,\theta^2 + \int_{\mathbb{R}\setminus\{0\}} (e^{i\theta x} - 1 - i\theta x 1_{|x|\leq 1})\,\Pi(dx)\right).$$

Here $\sigma \geq 0$, $\gamma \in \mathbb{R}$ and Π is a nonnegative Borel measure Π on $\mathbb{R}\setminus\{0\}$ satisfying

$$\int_{\mathbb{R}\setminus\{0\}} \left(x^2 \wedge 1\right) \Pi(dx) \;<\; \infty\,. \tag{1}$$

The measure Π is called the *Lévy measure* Π of L. It is uniquely determined by its Lévy tail $\overline{\Pi} : \mathbb{R}\setminus\{0\} \to \mathbb{R}_0^+$, defined by

$$\overline{\Pi}(x) = \Pi([x,\infty))\,, \quad x > 0\,, \qquad \overline{\Pi}(x) = \Pi((-\infty, x])\,, \quad x < 0\,.$$

The Lévy measure Π is finite if and only if L exhibits finitely many small jumps on any nonempty open interval almost surely. This phenomenon is called *finite activity.* Vice versa, processes with infinite Lévy measures are called processes of *infinite activity.* Lévy processes of finite activity include compound Poisson processes and are important, for instance, in the standard model of risk and and queuing theory [e.g. Daykin, Pentikäinen & Pesonen (1994), Grandell (1991) or Asmussen (1987)]. In the past years processes with infinite activity have attracted many researchers and occur in such diverse applications as storage modeling [Moran (1959), Brockwell & Chung (1975)], turbulence [Barndorff-Nielsen (1998)] and finance [Madan & Seneta (1987), Eberlein & Keller (1995), Barndorff-Nielsen (1998), Rydberg (1999), Eberlein & Raible (1999), Barndorff-Nielsen & Sheppard (2001)]. Typical examples are gamma-processes, stable processes, hyperbolic Lévy motions, generalized Lévy motions, normal inverse Gaussian processes and variance gamma processes, but more general processes are also of interest [Klüppelberg, Kyprianou & Maller (2004)]. Recently, Lévy processes entered time series modeling in continuous-time [Brockwell (2001), Klüppelberg, Lindner & Maller (2004) among others].

In this talk we are concerned with the insufficiently resolved question how the Lévy measure should be estimated non-parametrically. Many authors have considered the parametric inference of Lévy processes [Akritas (1982), Höpfner & Jacod (1993), Höpfner (1997) and Woerner (2001), (2003), (2004) and references therein]. Recently, Figueroa-Lopez & Houdre (2006) studied nonparametric inference for Lévy densities. Jongbloed, van der Meulen & van der Vaart (2005) proposed density estimators within the subclass of self-decomposable distributions. For compound Poisson processes, Buchmann & Grübel (2003) have constructed estimators for the distribution function of the jumps [cf. also Buchmann & Grübel (2004)]. Van Es, Gugushvili & Spreij (2006) investigated nonparametric density estimation for compound Poisson processes.

Although considerations of the nonparametric inference for general Lévy processes goes back to Rubin & Tucker (1959) and Basawa & Brockwell (1982), it is advocated by them to estimate the distribution function of the *finite* measure $dK(x) = x^2/(1+x^2)\, d\Pi(x)$. As this approach does not provide us with any insight about the nature of a possible singularity of Π, we depart from it and estimate the tail of Π itself (with no restrictions other than (1)). We employ weight functions to deal with possible singularities at zero and study the asymptotic properties of the corresponding weighted empirical processes. Within a special class of weight functions, we give necessary and sufficient conditions that ensure strong consistency and asymptotic normality of the weighted empirical processes, provided that complete information on the jumps is available. At the right end-point of Π, such conditions follow easily from the classical results by O'Reilly (1974) and Lai (1974). At zero this is no longer true due to a possible singularity of the Lévy measure. To cope with infinite activity processes, we also depart from the assumption, that trajectories are observed in full by analyzing sampling schemes where the possibly infinitely many small jumps are neglected. We establish a bootstrap principle and provide a simulation study for some prominent Lévy processes, such as α-stable processes and variance-gamma.

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A Market Model for Stochastic Mortality

Andrew Cairns

Recent years have seen the development of a number of models for the future development of aggregate mortality rates. Amongst these the Olivier and Smith model (Olivier and Jeffery, 2004, and Smith, 2005) was developed within the forward-rate framework discussed by Cairns et al. (2006) and Miltersen and Persson (2005). This model has a number of useful properties that make it a very good model for use in the valuation of life insurance contracts that incorporate embedded options. We discuss here a generalization of the Olivier and Smith model. Dynamics of the model in its published form are driven by a sequence of *univariate* gamma random variables.We demonstrate that the model in this form does not adequately match historical data. We discuss a generalization of the model that uses multivariate Gamma random variables as drivers.This approach potentially gives us much greater control over the term structure of volatility of spot survival probabilities and over the correlation term structure. We introduce a possible approach for simulation of multivariate gamma random variables.

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Quanto Options and Mixture Exponential Jump-Diffusions

Ngai Hang Chan

A foreign equity option (or quanto option) is a derivatives security whose value depends on an exchange rate and a foreign equity. In this talk, the valuation of quanto options is studied when the foreign equity prices and the exchange rates follow double exponential jump diffusions (DEJD). In particular, the two underlying assets are allowed to have common jumps and dependent jump sizes. The jump sizes are modeled by the multivariate exponential distribution of Marshall and Olkin (1967). Analytical pricing formulas are obtained for various types of quanto options. When the exchange rate and foreign asset evolve as DEJD, it is shown that the domestic equivalent asset follows a mixture exponential jump diffusion (MEJD). Laplace transforms of various forms under MEJD are derived and the corresponding Laplace inversions are implemented. The proposed approach is applied to options on two assets such as the quanto options and path-dependent options under MEJD. Numerical results demonstrate the usefulness of the proposed approach.

Ruin estimates for certain risk processes driven by Markov chains with general state space

Jeffrey F. Collamore

We study stochastic recurrence equations for certain Markov-driven processes which arise in insurance and financial mathematics, as well as other applied areas. Our primary objective is to consider an insurance company in a Markov-driven stochastic economic environment, and to develop sharp asymptotics for the probability of ruin. Thus, for example, the investment returns may be modeled according to an ARMA process or stochastic volatility model. Such investment processes can be viewed as Markov chains in a general state space. Our main result asserts that the probability of ruin decays at a certain polynomial rate, which we characterize, and thereby extend results of Goldie (Ann. Appl. Probab., 1991). Also, we develop corresponding asymptotics for the tail of a GARCH(1,1) process, or related process, but with a Markov- dependent driving sequence, as may arise, e.g., under regime switching. In establishing the above asymptotics we uncover, moreover, a close connection with geometric recurrence for certain Markovian operators which arise, e.g., in large deviations theory. Here we develop recurrence properties for these operators under a nonstandard Gärtner-Ellis-type assumption on the underlying driving process.

The asymptotic behaviour of the density of the supremum of a stable process

Ron A. Doney
(joint work with M. Savov)

The asymptotic behaviour of both the lower tail and the upper tail of the distribution of the maximum of a stable process at a fixed time has been known since the work of Bingham, [6]. (A simpler derivation can be found in Chapter VII of [4].) This talk is devoted to the solution of the analogous question for the corresponding density. This turns out to be considerably more difficult, but the result may have important implications for some areas, such as optimal stopping.

Let X be a strictly stable process of index $\alpha \in (0,2)$, which has positive jumps, so that its Lévy measure has density

$$\nu(x) = \begin{cases} c_+ x^{-(\alpha+1)}, x > 0 \\ c_- |x|^{-(\alpha+1)}, x < 0 \end{cases},$$

where $c_+ > 0, c_- \geq 0$. Assume also that X is not a subordinator. Then if S is its maximum process it is known that both X_t and S_t have continuous density functions, f_t and m_t say, and by scaling we have $f_t(x) = t^{-\eta} f(x/t^\eta)$ and $m_t(x) = t^{-\eta} m(x/t^\eta)$, where $\eta = 1/\alpha$ and f and m stand for f_1 and m_1. From (14.37), p 88 in [9] we know that

$$f(x) \sim A x^{-(\alpha+1)} \text{ as } x \to \infty, \tag{1}$$

where the constant A is known explicitly, and from Proposition 4, p 221 in [4]

$$P(S_1 > x) \sim P(X_1 > x) \sim A\alpha^{-1} x^{-\alpha} \text{ as } x \to \infty. \tag{2}$$

We show that in fact we can improve this to

$$m(x) \sim f(x) \sim A x^{-(\alpha+1)} \text{ as } x \to \infty. \tag{3}$$

We also consider the corresponding small time problem : here the known result is that

$$P(S_1 \leq x) \sim B x^{\alpha\rho}, \text{ as } x \downarrow 0, \tag{4}$$

where $\rho = P(X_1 > 0)$, and we establish the obvious conjecture that

$$m(x) \sim B' x^{\alpha\rho - 1} \text{ as } x \downarrow 0, \tag{5}$$

with $B' = \alpha\rho B$.

Our method of proof involves relating m to several other quantities, such as the the density of the position of a standard *stable meander at time 1,* the density of the value of the stable process *conditioned to stay positive at time 1,* and the bivariate density of the *ladder height process* of X at time 1. As a byproduct, we get the asymptotic behaviour of these quantities.

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Systemic risk

Mark Davis

This meeting on financial risk management started on the day that a major Wall Street bank, Bear Stearns, was sold to J.P. Morgan for $2 a share, having collapsed in a few days after an illustrious 85-year history. This talk was an attempt to see what the implications of this event might be for mathematical modelling of inter-bank financial risk.

As one commentator put it, Bear Stearns was not “too big to fail”, but was “too interrelated to fail suddenly”. This puts the spotlight on the modelling of interactions between market counterparties. Traditionally, ‘correlation’ in credit risk has been handled either by ‘latent variable’ models or by ‘contagion’ models. An instance of the latter is the ‘infectious defaults’ model of Davis and Lo (2001) which posits a simple mechanism whereby default by one obligor may trigger off default by others. However, this model and others like it are too schematic to mirror the complex interactions existing in reality. Additionally, most credit risk models calculate only the distribution of actual losses, while in many cases the most significant risk is loss of mark-to-market value due to changing asset prices or defaults in some other part of the credit spectrum.

A one-period model of n interacting agents in a financial system has been provided by Hyun Song Shin in a 2006 BIS Working Paper. In this model, the agents have random income, hold some quantity of real assets and leverage themselves by issuing debt in the form of zero-coupon bonds. Settlement of all payments at the end of the period, taking into account the seniority ranking of the bonds, leads to a fixed-point problem which can be shown to have a unique solution under quite general conditions. Combining this with risk-neutral valuation leads to simultaneous valuation of all the debt in the system. None of it can be valued in isolation.

Shin's model is undoubtedly a major step in the right direction but, like most good research, it opens up as many questions as it answers. Among them are

- Extension to dynamic models, with consequent questions about design of trading strategies to manage risk.
- The role of 'information': obviously it is unrealistic to suppose that the exact structure of debt across the market is known to all participants, and lack of such knowledge affects asset prices. It would be valuable to have a model that quantifies these effects.
- Scalability: at the moment, network models like Shin's are only implementable for very small n. Some way of scaling them up needs to be found, perhaps distinguishing between 'local' interactions and some 'global' picture.

The penalty function for time consistent utility functions

Freddy Delbaen

For a Brownian motion $W_t \in \mathbb{R}^d$, $t \leq T$ we characterize the time consistent utility functions as follows: for $\xi \in L^\infty : u_t(\xi) = \inf\{E_Q[\xi + \int_t^T f_n(q_n)du|F_t]|Q \sim P\}$ where $dQ/dP = \xi(q, W)$ and $f : [0,T] \times \Omega \times \mathbb{R}^d \to \overline{\mathbb{R}}_t$ is a function convex in q and predictable in (t, w). Furthermore the normalization $f_u(0) := 0$ comes from the assumption that $u_=(\xi) \geq 0$ implies $E_P[\xi] \geq 0$. The method of proof is via a truncation and essentially based on Rao's representation of supermartingales of class D. In the case where f only depends on q and if g is the Legendre transform of f, the $u_t(\xi)$ process relates to solutions of BSDE $dY_t = g(Z_t)dt - Z_t dW_t$ where $Y_T = \xi$ and Y remains bounded. The case $\limsup(g(\xi)/|\xi|^2) < \infty$ identifies $u_t(\xi)$ with Y_t but if $\limsup(g(\xi)/|\xi|^2) = +\infty$ then this relation breaks down.

Empirical Processes of Extreme Cluster Functionals

Holger Drees
(joint work with Holger Rootzén)

In the literature on extreme value statistics for time series, quite general results are known about estimators of the marginal tails. Usually it is assumed that the suitably standardized exceedances $X_{i,n} := a_n(X_i - u_n)$ converge in distribution to a generalized Pareto distribution, i.e. $P(X_{1,n} > x | X_{1,n} > 0) \to (1 + \gamma x)_+^{1/\gamma}$ for some extreme value index $\gamma \in \mathbb{R}$; here $(u_n)_{n\in\mathbb{N}}$ denotes a sequence of thresholds tending to the right endpoint of the range of the marginal d.f. F. Rootzén (2007) analyzed the asymptotic behavior of the tail empirical process

$$e_n(x) := \frac{1}{\sqrt{n\overline{F}(u_n)}} \sum_{i=1}^{n} \left(1_{\{X_{i,n}>x\}} - \overline{F}(u_n + a_n x)\right), \quad x \in \mathbb{R},$$

for strong mixing time series and for absolutely regular time series; here $\overline{F}$ denotes the survival function of X_1. From a previous version of that paper, Drees (2000, 2003) derived a weighted approximation for this tail empirical process under absolute regularity and discussed statistical applications, like the analysis of estimators of the extreme value index or extreme quantiles. However, the tail empirical process does not describe the extreme value dependence structure of the times series. By and large, results on the asymptotic behavior of estimators of the extremal dependence structure under suitable mixing conditions are restricted to estimators of the extremal index and, more general, the distribution of the size of clusters of extreme observations. Unfortunately, these estimators are of very limited value in quantitative risk management. For instance, the distribution of the total sum of losses exceeding a high threshold in a period of given length cannot be described in terms of the cluster size distribution. We discuss empirical processes which capture more general aspects of the dependence between extreme observations of an absolutely regular stationary time series $(X_i)_{i\in\mathbb{N}}$. To this end, the time series is split into m_n blocks of length r_n, say, and the core of a cluster is defined as the minimal sequence of consecutive standardized observations $X_{i,n}$ which contains all positive values in one block. Now, following an approach by Yun (2000) (see also Segers, 2003), let $\mathcal{F}$ be a family of cluster functionals, i.e. measurable functionals $f(Y_{j,n})$ of a block $Y_{j,n} := (X_{i,n})_{(j-1)r_n<i\le jr_n}$ which only depend on the pertaining core. We give sufficient conditions for the convergence of the empirical process

$$Z_n(f) := \frac{1}{\sqrt{n\overline{F}(u_n)}} \sum_{i=1}^{m_n} \left(f(X_{i,n}) - E(f(X_{i,n}))\right), \quad f \in \mathcal{F},$$

to a Gaussian process with continuous sample paths. The results are obtained by first applying a 'big blocks - small blocks'-technique and then general functional limit theorems for empirical process of i.i.d. observations. Since these general conditions are quite abstract and involved, we also discuss important special cases, like (generalized) tail array sums (including the process of upcrossings over extreme intervals) and functionals describing the distribution function of order statistics in a cluster. In particular, it turns out that the general approach leads to conditions for the convergence of the tail empirical process which are usually easier to verify than the conditions established by Rootzén (2007).

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Advanced credit portfolio modeling and CDO pricing

Ernst Eberlein
(joint work with R. Frey and E. A. von Hammerstein)

Modeling dependence is a key issue when one derives the loss distribution of a portfolio of credit instruments. We extend the factor model approach of Vasiček by using more sophisticated distributions for the factors. Completely different distributions from the class of generalized hyperbolic distributions and their limits can be chosen for the systematic and the idiosyncratic factor in this approach. As a result an almost perfect fit to market quotes of DJ iTraxx Europe standard tranches is achieved. The correlation structure remains flat over all CDO tranches and maturities. No base correlation framework is needed.

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Modelling the term structure of CDO losses

Damir Filipovic
(joint work with Ludger Overbeck and Thorsten Schmidt)

We consider the loss process L_t on a pool of credits (CDO) with overall nominal normalized to 1. Define the (T,x)-bond as the instrument which pays $1_{\{L_T\le x\}}$ at maturity T. Its price at $t \le T$ is given by risk neutral expectation

$$P(t,T,x) = E[e^{-\int_t^T r_s} 1_{\{L_T\le x\}} \mid \mathcal{F}_t],$$

where r_t denotes the short rates. The risk free T-bond is then simply $P(t,T) = P(t,T,1)$.

It turns out that all contingent claims with payoff $F(L_T)$ can be written as linear combination of (T,x)-bonds. This makes them so fundamental.

The ultimate goal is to provide a term structure model of Heath-Jarrow-Morton type for $P(t,T,x)$, that is,

$$P(t,T,x) = 1_{\{L_t\le x\}} \exp\left(-\int_t^T (f(t,u)+\phi(t,u,x))du\right)$$

for some Itô processes

$$f(t,T) = f(0,T) + \int_0^t a(s,T)ds + \int_0^t b(s,T)dW_s$$
$$\phi(t,T,x) = \phi(0,T,x) + \int_0^t \alpha(s,T,x)ds + \int_0^t \beta(s,T,x)dW_s.$$

We provide necessary conditions on the joint dynamics of $f(t,T)$, $\phi(t,T,x)$ and L_t such that the (T,x)-bond market is arbitrage-free. Conversely, we give sufficient conditions on the stochastic basis such that for any given volatility specification $b(t,T)$ and $\beta(t,T,x)$, there exists a loss process L_t and $a(t,T)$, $\alpha(t,T,x)$ such that $P(t,T,x)$ above defines an arbitrage-free collection of (T,x)-bonds.

Finally, we provide an affine specification of the above generic model, which allows for efficient computation of CDO derivatives such as credit default swaps (CDS).

Rank-Based Inference for Bivariate Extreme-Value Copulas

Christian Genest
(joint work with Johan Segers)

Several nonparametric estimators are available for the Pickands dependence function of a bivariate extreme-value copula. All of them, however, require knowledge of the univariate margins. In this paper, rank-based versions of some of these estimators are proposed for the case where the margins are unknown. In particular, the asymptotic distributions of rank-based versions of the estimators of Pickands [*Bull. Inst. Internat. Statist.* **49** (1981) 859-878] and Capéraà, Fougères and Genest [*Biometrika* **84** (1997) 567–577] are found using results on the limit behavior of a class of weighted bivariate empirical processes. Small- and large-sample comparisons indicate that even when the margins are known, a more efficient estimator arises if the information about the margins is ignored and ranks are used instead. This work is joint with Johan Segers, from Université catholique de Louvain.

Multivariate risk processes: interaction and destabilization

Rudolf Grübel
(joint work with Nicole Bäuerle)

Suppose that $N(t)$ is the number of claims arriving at an insurance company up to time t and that the successive claim sizes are $U_1, U_2, \ldots$. We assume that there is a continuous premium income with rate c and that the initial capital is r_0. Then the company surplus at time t is given by

$$R(t) \;=\; r_0 + c \cdot t - \sum_{k=1}^{N(t)} U_k.$$

In the classical Cramér-Lundberg model of risk theory $N = (N(t))_{t \in \mathbb{R}}$ is a Poisson process with constant rate and claim sizes are independent and identically distributed; further, N and $(U_i)_{i \in \mathbb{N}}$ are assumed to be independent. A classical quantitative risk measure in this situation is the exponential rate of decrease of the probability of ruin,

$$P(R(t) < 0 \text{ for some } t > 0 \,|\, R_0 = r_0),$$

regarded as a function of the initial capital r_0, as $r_0 \to \infty$.

We consider an extension of this model to more than one business line. A crucial point is the modelling of the multivariate counting process $N = (N_1, \ldots, N_d)$, where N_i now counts the number of claims of type i, $1 \le i \le d$. We take N to be a multivariate birth process. These are specified by their birth rate function $\beta : \mathbb{N}_0^d \to \mathbb{R}_+^d$. We discuss four prototypical examples for $d = 2$: an independent case with constant β, a case with repelling intensities, a case with attracting intensities, and finally a 'binary' case, where claims of a specific type are only accepted if the number of claims of this type is smaller than the corresponding number for the other type.

Of particular interest is a class of models with repelling intensities that can be obtained from the independent case via h-transforms. We give the resulting β as a function of a distribution that is concentrated on the d-dimensional probability simplex, which leads to directional mixing. We show that these counting processes do not stabilize and obtain a fluid limit result. For the resulting surplus processes we investigate the asymptotic behavior of the (global) ruin probabilities. It turns out that rate of exponential decrease can be written as the infimum of all relevant rates of the associated one-dimensional mixture models.

Empirical log-optimal portfolio selection

Laszlo Gyorfi

Consider the problem of optimal trading for assets. The dynamic portfolio selection is a discrete time model for multi-period trading, where in each trading period there is a rebalancing between the assets. For a stationary and ergotic market process, we introduce an empirical portfolio selection, which achieves asymptotically the best possible growth rate. Our theoretical results are illustrated for NYSE data.

Ruin probabilities in the presence of general semimartingale investments and heavy-tailed claims

Henrik Hult

(joint work with Filip Lindskog)

In this paper we study the asymptotic decay of finite time ruin probabilities for an insurance company that faces heavy-tailed claims, uses predictable investment strategies and makes investments in risky assets whose prices evolve according to quite general semimartingales. We show that the ruin problem corresponds to determining hitting probabilities for the solution to a random perturbation of a stochastic integral equation. We derive a large deviation result for the hitting probabilities that holds uniformly over family of semimartingales and show that this result gives the asymptotic decay of finite time ruin probabilities under optimal investment strategies.

Operational risk, Pareto copulas and regular variation

Claudia Klüppelberg

(joint work with Sidney Resnick)

Introduction. Operational Risk is defined as the risk of losses resulting from inadequate or failed internal processes, people and systems, or external events. It is classified in different loss types and business lines, where we model the operational loss in each of these cells by a compound Poisson process model. More precisely, we model the vector of all such processes as a multivariate compound Poisson process and the bank's total aggregate operational loss process as the summation

$$X^+(t) = X_1(t) + X_2(t) + \cdots + X_d(t)\,, \qquad t \geq 0\,.$$

Denote its distribution function by $F_t^+(\cdot) = P(X^+(t) \le \cdot)$. As risk measure we use the total Operational Value-at-Risk up to time t at confidence level κ, which is defined as the quantile

$$\mathrm{VaR}_t^+(\kappa) = G_t^{+\leftarrow}(\kappa) = \inf\{z \in \mathbb{R} :\ G_t^+(z) \ge \kappa\}\,, \quad \kappa \in (0,1)\,,$$

for κ near 1 (e.g. 0.999). Under heavy tailed loss severities in at least one cell,

$$P(X^+(t) > z) \sim E[N^+(t)]\, P(\Delta X^+ > z) =: E[N^+(t)]\, \overline{G}^+(z), \quad z \to \infty,$$

which implies for regularly varying ($\mathcal{R}_{-\alpha}$), subexponential ($\mathcal{S}$) or rapidly varying ($\mathcal{R}_{-\infty}$) loss distributions that

$$\mathrm{VaR}_t^+(\kappa) := G^{+\leftarrow}(\kappa) \sim F^{\leftarrow}\Big(1 - \frac{1-\kappa}{E[N^+(t)]}\Big), \quad \kappa \uparrow 1.$$

For precise formulations with references we refer to Böcker and Klüppelberg (2006, 2007).

The Pareto Copula. We are interested in the influence of dependence between the compound Poisson process components on the total loss process $X^+(\cdot)$ and $\mathrm{VaR}_t^+(\cdot)$. We study this by embedding the problem into the more general Lévy process setting. We start with a triangular array of $\{\boldsymbol{X}_{n,j}, n \ge 1, j \ge 1\}$ in $\mathbb{R}^d$ with rows iid and $\boldsymbol{X}_{n,1} \sim F_n$. For $\boldsymbol{X}_{n,j} = (X_{n,j}^{(i)}, i = 1,\dots,d)$ we set $F_n^{(i)}(x) = P\{X_{n,1}^{(i)} \le x\}$ for $x \in \mathbb{R}$. Assume for simplicity the one dimensional marginal distributions $F_n^{(i)}$ are continuous. We introduce the marginal transformation

$$\boldsymbol{\mathcal{P}}_{n,j} = (\mathcal{P}_{n,j}^{(1)},\dots,\mathcal{P}_{n,j}^{(d)}) = \Big(\frac{1}{1 - F_n^{(1)}(X_{n,j}^{(1)})},\dots,\frac{1}{1 - F_n^{(d)}(X_{n,j}^{(d)})}\Big), \tag{1}$$

which yields standard Pareto marginals; i.e. $P\{\mathcal{P}_{n,j}^{(i)} > x\} = x^{-1}$, $x \ge 1$ for $i = 1,\dots,d$.

Definition 1. Suppose $\boldsymbol{X}_{n,1}$ has distribution F_n with continuous marginals. Define $\boldsymbol{\mathcal{P}}_{n,j}$ as in (1). Then we call the distribution ψ_n of $\boldsymbol{\mathcal{P}}_{n,j}$ a *Pareto copula.*

The Pareto Lévy Copula.

Proposition 2 (de Haan and Resnick (1977), Resnick (1987,2007))**.** *Let $\boldsymbol{X}_{n,1}$ be a random vector with distribution F_n such that $nF_n(\cdot) \xrightarrow{v} \nu(\cdot)$ holds. Let ψ_n be its Pareto copula. Then the following holds.*
(a) *There exists a Radon measure ψ_∞ on the Borel subsets of $[\mathbf{0},\boldsymbol{\infty}] \setminus \{\mathbf{0}\}$ such that $n\psi_n(n\cdot) \xrightarrow{v} \psi_\infty(\cdot)$.*
(b) *For $i = 1,\dots,d$ we have $\psi_\infty^{(i)}((x,\infty]) = \psi_\infty\big([0,\infty]^{i-1} \times (x,\infty] \times [0,\infty]^{d-i}\big) = x^{-1}$ for $x > 0$.*
(c) *ψ_∞ is a Lévy measure on $\mathbb{R}_+^d$.*

Definition 3. We call the Lévy measure ψ_∞ a *Pareto Lévy copula* and a Lévy process $(\boldsymbol{X}_\infty(t))_{t \ge 0}$ a *Pareto Lévy process.*

Remark 4. The marginal processes $\{X_\infty^{(i)}(t)\}_{t\geq 0}$ are 1-stable processes with only positive jumps. However, the multivariate process $\{\boldsymbol{X}_\infty(t)\}_{t\geq 0}$ is not stable unless ψ_∞ has the homogeneity property $\psi_\infty(t\cdot) = t^{-1}\psi_\infty(\cdot)$.

The following marginal transformation of an arbitrary Lévy measure ν on $[\boldsymbol{0},\boldsymbol{\infty}]\setminus\{\boldsymbol{0}\}$ yields a Pareto Lévy process. Assume for simplicity that the one-dimensional marginal Lévy measures are continuous. Define $\overline{\nu}(\boldsymbol{x}) = \nu((x_1,\infty]\times\cdots\times(x_d,\infty])$ for $\boldsymbol{x}\in[\boldsymbol{0},\boldsymbol{\infty}]\setminus\{\boldsymbol{0}\}$. Then

$$\overline{\nu}(\boldsymbol{x}) = \psi\Big(\frac{1}{\overline{\nu}^{(1)}(x_1)},\ldots,\frac{1}{\overline{\nu}^{(d)}(x_d)}\Big), \quad \boldsymbol{x}\in[\boldsymbol{0},\boldsymbol{\infty}]\setminus\{\boldsymbol{0}\},$$

and ψ defines a Pareto Lévy copula.

Example 5. (1) Independence Pareto Lévy-copula:
$$\psi_\perp(\boldsymbol{x}) = x_1^{-1}I_{\{x_2=\ldots=x_d=0\}} + \cdots + x_d^{-1}I_{\{x_1=\ldots=x_{d-1}=0\}}.$$
(2) Complete (positive) dependence Pareto Lévy copula:
$$\psi_\|(\boldsymbol{x}) = \min(x_1^{-1},\ldots,x_d^{-1})$$
(3) Archimedian Pareto Lévy copula: $\psi(\boldsymbol{x}) = \phi^{\leftarrow}(\phi(x_1^{-1})+\cdots+\phi(x_d^{-1}))$ for a so-called generator $\phi:[0,\infty]\to[0,\infty]$ with certain regularity properties.
(4) Clayton Pareto Lévy copula: $\psi_\theta(\boldsymbol{x}) = (x_1^\theta+\cdots+x_d^\theta)^{-1/\theta}$.
Note that $\lim_{\theta\to\infty}\psi_\theta(\boldsymbol{x}) = \psi_\|(\boldsymbol{x})$ and $\lim_{\theta\to 0}\psi_\theta(\boldsymbol{x}) = \psi_\perp(\boldsymbol{x})$

Estimating Total OpVar. Denote by $\mathrm{VaR}_t^i(\cdot)$ the stand alone OpVaR of cell i and by $\mathrm{VaR}_t^+(\cdot)$ the total OpVaR.

Theorem 6. *Assume $X_1,\ldots,X_d$ are completely dependent cell processes and the loss severity distributions F_i are strictly increasing. If $F^+\in\mathcal{S}\cap\mathcal{R}_{-\alpha}$ for $\alpha\in(0,\infty]$, then*

$$\mathrm{VaR}_t^+(\kappa)\sim\sum_{i=1}^d \mathrm{VaR}_t^i(\kappa), \quad \kappa\uparrow 1,$$

Theorem 7. *Assume $X_1,\ldots,X_d$ are independent cell processes. If $F_1\in\mathcal{R}_{-\alpha}$ for $\alpha\in(0,\infty)$ and*
$\lim_{x\to\infty}\overline{F}_i(x)/\overline{F}_1(x) = c_i\in[0,\infty)$ for all $i=2,\ldots,d$, then

$$\mathrm{VaR}_t^+(\kappa)\sim\mathrm{VaR}_t^1\left(\frac{(\lambda_1+c_2\lambda_2+\cdots+c_d\lambda_d)\,t}{1-\kappa}\right), \qquad \kappa\uparrow 1.$$

Definitions and relations, which yield the following lemma can be found in Klüppelberg and Resnick (2007).

Lemma 8. *Let ν be a Lévy measure with regularly varying marginals $\nu^{(i)}$ and homogeneous Pareto Lévy copula of order -1: $\psi(t\cdot) = t^{-1}\psi(\cdot)$. Then the Lévy measure ν is multivariate regularly varying.*

Theorem 9. *Assume that the Lévy measure is multivariate regularly varying and that all marginal Lévy measures $\nu^{(i)}\in\mathcal{R}_{-\alpha}$ for $0<\alpha<\infty$. Assume further that* *the loss severity distributions F_i are strictly increasing and continuous. Then, X^+ is compound Poisson with frequency parameter λ^+ and loss severity tail*

$$\overline{F}^+(z) \sim \frac{\nu^+((1,\infty])}{\lambda^+}\overline{F}_1(z) \in \mathcal{R}_{-\alpha}\,,$$

where $\nu^+((z,\infty]) = \nu(\{\boldsymbol{x} \in \mathbb{R}^d_+ : \sum_{i=1}^d x_i > z\})$ for $z > 0$. Furthermore, total OpVaR satisfies given by

$$\mathrm{VaR}_t(\kappa) \sim F_1^{\leftarrow}\left(1 - \frac{1-\kappa}{t\,\nu^+((1,\infty])}\right), \quad \kappa \uparrow 1\,.$$

Example 10. [Clayton Lévy copula] For $i = 1,2$ let $\nu^{(i)} \in \mathcal{R}_{-\alpha}$ for $0 < \alpha < \infty$. Since the Clayton Lévy copula is homogeneous of order -1, the Lévy measure ν is multivariate regularly varying. By analytic nonsense,

$$\nu^+((1,\infty]) \quad =: \quad \lambda_1 + \lambda_2\, c(\alpha,\theta,\lambda_1,\lambda_2)\,,$$

with $c(\alpha,\theta,\lambda_1,\lambda_2) > 0$. Then we obtain for total OpVar

$$\mathrm{VaR}_t^+(\kappa) \sim F_1^{\leftarrow}\left(1 - \frac{1-\kappa}{\big(\lambda_1 + \lambda_2\, c(\alpha,\theta,\lambda_1,\lambda_2)\big)\, t}\right), \quad \kappa \uparrow 1\,.$$

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Importance sampling, diffusions and rare event simulation

Don McLeish

We consider Monte Carlo estimation of rare events using exponential families of importance sampling distributions. In various examples it is shown that the exponential tilt is suboptimal. For the uniform distribution the exponential tilt or the beta density have relative error which is bounded as the probability of the rare event approaches 0 and this with the inverse transform implies there is a tilt giving relative error that is bounded (and less than around 0.738) for any continuous distribution. It is suggested that the extreme value distribution determined by the tails of the loss (for example the Gumbel distribution when the loss is either exponentially distributed or Normally distributed) is a candidate for an efficient importance sampling distribution.

A New perspective on Archimedean Copulas

Alex McNeil

It is shown that a necessary and sufficient condition for an Archimedean copula generator to generate a d-dimensional copula is that the generator is a d-monotone function. Moreover the class of d-dimensional Archimedean copulas is shown to coincide with the class of survival copulas of d-dimensional L1-norm symmetric distributions that place no point mass at the origin. The d-monotone Archimedean copula generators may be characterized using a little-known integral transform of Williamson (1956) in an analogous manner to the well-known Bernstein-Widder characterization of completely monotone generators in terms of the Laplace transform. These insights allow the construction of new Archimedean copula families and provide a general solution to the problem of sampling multivariate Archimedean copulas.

Generalized Affine Models

Nour Meddahi
(joint work with Bruno Feunou)

Affine models are very popular in modeling financial time series as they allow for analytical calculation of prices of financial derivatives like treasury bonds and options. The main property of affine models is that the conditional cumulant function, defined as the logarithmic of the conditional characteristic function, is affine in the state variable. Consequently, an affine model is Markovian, like an autoregressive process, which is an empirical limitation. The paper generalizes affine models by adding in the current conditional cumulant function the past conditional cumulant function. Hence, generalized affine models are non-Markovian, such as ARMA and GARCH processes, allowing one to disentangle the short term and long-run dynamics of the process. Importantly, the new model keeps the tractability of prices of financial derivatives. This paper studies the statistical properties of the new model, derives its conditional and unconditional moments, as well as the conditional cumulant function of future aggregated values of the state variable which is critical for pricing financial derivatives. It derives the analytical formulas of the term structure of interest rates and option prices. Different estimating methods are discussed (MLE, QML, GMM, and characteristic function based estimation methods). The paper presents an empirical example where one models jointly the high-frequency realized variance and the daily asset return and provides the term structure of risk measures such as the Value-at-Risk, which highlights the powerful use of generalized affine models.

Inference for Continuous Semimartingales Observed at High Frequency: A General Approach

Per Mykland
(joint work with Lan Zhang)

The econometric literature of high frequency data usually relies on moment estimators which are derived from assuming local constancy of volatility and related quantities. We here show that this first order approximation is not always valid if used naively. We find that such approximations require an ex post adjustment involving asymptotic likelihood ratios. These are given. Several examples (powers of volatility, leverage effect, ANOVA) are provided. The first order approximations in this study can be over the period of one observation, or over blocks of successive observations. The theory relies heavily on the interplay between stable convergence and measure change, and on asymptotic expansions for martingales. Practically, the procedure permits (1) the definition of estimators of hard to reach quantities, such as the leverage effect, of volatility, (2) the improvement in efficiency in classical estimators, and (3) easy analysis.

Aggregating Risk Capital, with an application to Operational Risk

Giovanni Puccetti
(joint work with Paul Embrechts)

In the Basel II regulatory setup for operational risk in banking, banks are requested to set aside capital for the purpose of offsetting Operational Risk. We analyze how interdependencies between individual random losses and the risk calculation procedure may influence different estimates for the minimum capital charge required.

Some functional central limit theorems for linear processes

Alfredas Račkauskas
(joint work with Ch. Suquet and R. Norvaiša)

Our contribution to Donsker-Prohorov invariance principle involves three directions of extension, dealing with:

- other topological frameworks for the weak convergence of partial sum processes;
- dependent random variables;
- infinite dimensional random elements.

We consider the space $BV_p[0,1]$ of functions having finite p variation, $p > 2$, and the space $\mathcal{H}_\alpha[0,1]$ of functions having continuity as t^α, $0 < \alpha < 1/2$. Necessary and sufficient conditions for weak convergence of partial sum processes shall be discussed for both functional frameworks.

Some applications of the results to multiple change problems in data shall be discussed.

Contracting for optimal investment under risk constraints

Chris Rogers

This talk considers the problem of a principal who wishes to maximize his expected utility of wealth at some fixed future time, and employs an agent to trade his portfolio. The utilities of principal and agent will typically be different, but we show that the principal can offer the agent a contract under which the agent acting in his own best interests will achieve the principal's optimum. We then show how in a simple complete market setting a principal constrained to satisfy a law-invariant coherent risk measure constraint on his terminal wealth can construct a contract with his agent (trader) such that the agent acting without any constraints in his own self-interest will trade to the principal's optimum.

Extremes of stable random fields

Parthanil Roy

We consider a point process sequence induced by a stationary symmetric α-stable ($0 < \alpha < 2$) discrete parameter random field. It is easy to prove, following the arguments in the one-dimensional case in Resnick and Samorodnitsky (2004), that if the random field is generated by a dissipative group action then the point process sequence converges weakly to a cluster Poisson process. For the conservative case, no general result is known even in the one-dimensional case. We look at a specific class of stable random fields generated by conservative actions whose effective dimensions can be computed using the structure theorem of finitely generated Abelian groups. The corresponding point processes sequence is not tight and hence needs to be properly normalized in order to ensure weak convergence. This weak limit is computed using extreme value theory and some counting techniques.

Stochastic ordering and risk measures for portfolio vectors

Ludger Rüschendorf

The aim to introduce risk measures for portfolio vectors is to measure not only the risk from the variation of the components but also that arising from positive dependence between the components. We introduce some classes of portfolio risk measures as aggregation risk measures or generalized distortion risk measures and investigate their consistency w.r.t. dependence orderings like the supermodular, the directional convex or the Δ-monotone ordering. General convex risk measures are consistent w.r.t. the convex order. An analog of the Kusuoka representation result is given characterizing law invariant convex risk measures. It turns out that in the general case minimal correlation risk measures play the role in $d \geq 1$ that the expected shortfall plays in $d = 1$.

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The effect of memory on functional large deviations of infinite moving average processes and ruin probabilities

Gennady Samorodnitsky
(joint work with Souvik Ghosh)

We argue that in financial and other applications it is very useful to look at the phenomenon of long range dependence through lenses other than correlations. While concentrating on a doubly infinite moving average model with exponentially light tails we study functional large deviations, that are more informative than correlations.

The large deviations are very similar to those of an i.i.d. sequence as long as the coefficients decay fast enough. If they do not, the large deviations change dramatically. We study this phenomenon in the context of functional large, moderate and huge deviation principles. We also describe the change in the order of magnitude of ruin probabilities depending on the rate of decay of the moving average coefficients.

Conditional limit laws for multivariate excesses

Philippe Soulier
(joint work with Ann-Laure Fougeres)

The aim of this talk is the estimation of the conditional limit distribution of a random variable Y given that another random variable X exceeds an threshold x that tends to infinity. We focus on the framework proposed by Balkema and

Embrechts (2007), who assume that the level lines of the density of the pair (X, Y) are asymptotically locally elliptical. We propose estimators of the normalizing sequences and of the limiting distribution.

Open Problems in Financial Statistics

Michael Steele

This talk began with a discussion of a risk/reward paradox. We all agree that across assets, higher returns are typically associated with higher risks. Nevertheless, when we look at intertemporal returns we find the opposite effect; typically periods of higher volatility produce lower returns. One can explain some — but not all — of this phenomenon by Black's leverage effect, so some paradox remains. The other four problems concerned (a) a permutation technique for evaluating the uncertainty of trading strategies, (b) the use of "simulated verisimilitude" in model selection, (c) the contrast between effective mean reversion strategies (as in rebalanced portfolios) and momentum strategies (as in sector momentum), and (e) "Things Change" — the challenge of non-stationarity as illustrated by a half-dozen examples of major economic features that "changed" and never "changed back."

What Finance Has Done for Life Insurance - and Vice Versa

Mogens Steffensen

We present standardized methods for modeling and valuation of life insurance payment streams. These are based on assumptions about simple dependence structures and simple modeling of capital gains. We discuss how the mathematics of finance has influenced the view on these assumptions and how this influence has moved the industry concerning design and management. But the enlightenment is not one-way: We also provide an example of what life insurance can offer finance. The talk is based on the article 'Life insurance' to appear in Encyclopedia of Quantitative Finance.

Max-stable processes, representations and ergodicity

Stilian Stoev

Max-stable stochastic processes arise in the limit of component-wise maxima of independent processes, under appropriate centering and normalization. In this talk, we present necessary and sufficient conditions for the ergodicity and mixing of stationary max-stable processes.

The large classes of moving maxima and mixed moving maxima processes are shown to be mixing. Other examples of ergodic doubly stochastic processes and non-ergodic processes will be given. The developed ergodicity and mixing conditions involve a certain measure of dependence. We will address the statistical problem of estimating this measure of dependence.

Efficient estimation for ergodic diffusions sampled at high frequency

Michael Sørensen

A general theory of efficient estimation for ergodic diffusions sampled at high frequency is presented. High frequency sampling is now possible in many applications, in particular in finance. The theory is formulated in term of approximate martingale estimating functions and covers a large class of estimators including most of the previously proposed estimators for diffusion processes, for instance GMM-estimators and the maximum likelihood estimator. The asymptotic scenario considered is that the time between observations, Δ, goes to zero, while the number of observations, n, goes to infinity fast enough that nΔ, the observation time horizon, goes to infinity. The latter assumption is needed to ensure that drift parameters can be estimated consistently. This type of asymptotics for diffusions models has previously been considered by Prakasa Rao (1988), Yoshida (1992), Kessler (1997) and Gobet (2002). Simple conditions on the estimating functions are given that ensure rate optimality, where estimators of parameters in the diffusion coefficient converge faster than estimators of parameters in the drift coefficient, and efficiency. The conditions turn out to be equal to those implying small Δ-optimality in the sense of Jacobsen (2002) and thus give an interpretation of this concept in terms of the classical statistical concepts rate optimality and efficiency. Optimal martingale estimating functions in the sense of Godambe and Heyde are shown to be give rate optimal and efficient estimators under weak and natural conditions.

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Modelling conditional and unconditional heteroscedasticity with smoothly time-varying structure

Timo Teräsvirta
(joint work with Cristina Amado)

The modelling of time-varying volatility of financial returns has been a flourishing field of research for almost a quarter of a century following the introduction of the Autoregressive Conditional Heteroskedasticity (ARCH) model by Engle (1982) and the Generalized ARCH (GARCH) model developed by Bollerslev (1986). The increasing popularity of the class of GARCH models has been mainly due to their ability to describe the dynamic structure of volatility clustering of stock return series, specifically over short periods of time. However, one may expect that economic or political events or changes in institutions cause the structure of volatility to change over time. This means that the assumption of stationarity may be inappropriate under the evidence of structural changes in return financial series. In a recent paper, Mikosch and Starica (2004) argued that stylized facts in financial returns such as the long-range dependence and the 'integrated GARCH effect' can be well explained by unaccounted structural breaks in the unconditional variance; see also Lamoureux & Lastrapes (1990). Diebold (1986) was the first to suggest that occasional level shifts in the intercept of the GARCH model can bias the estimation towards an integrated GARCH model.

Another line of research has focused on explaining non-stationary behavior of volatility by long-memory models, such as the Fractionally Integrated GARCH (FIGARCH) model by Baillie, Bollerslev & Mikkelsen (1996). The FIGARCH model is not the only way of handling the 'integrated GARCH effect' in return series. Baillie & Morana (2007) generalized the FIGARCH model by allowing a deterministically changing intercept. Hamilton & Susmel (1994) and Cai (1994) suggested a Markov-switching ARCH model for the purpose, and their model has later been generalized by others. One may also assume that the GARCH process contains sudden deterministic switches and try and detect them; see Berkes, Horváth & Kokoszka (2003) who propose a method of sequential switch or change-point detection.

Yet another way of dealing with high persistence would be to explicitly assume that the volatility process is smoothly non-stationary and model it accordingly. Dahlhaus & Subba Rao (2006) introduced a time-varying ARCH process for modelling non-stationary volatility. Their tvARCH model is asymptotically locally stationary at every point of observation but it is globally non-stationary because of time-varying parameters. Engle & Gonzalo Rangel (2005) assumed that the variance of the process of interest can be decomposed into two components, a stationary and a non-stationary one. The non-stationary component is described by using splines, and the stationary component follows a GARCH process. The parameters of the latter are estimated conditionally on the spline component. I study two non-stationary GARCH models for situations in which volatility appears to be non-stationary. The first one is an additive time-varying parameter model, in which a time-dependent component is added to the GARCH specification. In the second alternative, the variance is multiplicatively decomposed into the stationary and non-stationary component as in Engle and Gonzalo Rangel (2005). These two alternatives are quite flexible representations of volatility and can describe many types of non-stationary behavior. Model building plays a central role in this approach. The standard GARCH model is first tested against these time-varying alternatives. If the null hypothesis is rejected, the structure of the time-varying component of the model is determined using the data. This is done by testing a sequential of hypothesis testing. After parameter estimation, the final model is evaluated by misspecification tests following the ideas in Eitrheim & Teräsvirta (1996) and Lundbergh & Teräsvirta (2002).

Open problem: The asymptotic properties of the maximum likelihood estimators of these nonlinear (time-varying parameter) GARCH models. Existing results on nonlinear GARCH models can be found in Straumann & Mikosch (2006) and Meitz & Saikkonen (in press). The idea of a multiplicative time-varying GARCH model can be applied to estimating the autoregressive conditional duration (ACD) model with diurnal variation. Typically, this is done by first estimating the diurnal variation, for example by splines and then the ACD parameters conditionally on the spline-adjusted durations. It appears that the same idea can also be apply to estimating the parameters of autoregressive models of logarithmic realized variance; see Andersen, Bollerslev, Diebold & Labys (2003).

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The Essence of Roy's Safety-first Principle and Asset-liability Management

Hoi Ying Wong
(joint work with Mei Choi Chiu, Hoi Ying Wong and Duan Li)

Roy (1952) proposes the safety-first principle for portfolio selection that minimizes the probability of disaster subject to the mean constraint that the final wealth if greater than the pre-selected disaster level. This paper studies the dynamic safety-first principle in continuous time and applies it to asset and liability management. The safety-first problem is traditionally approximated either by considering an upper bound for the disaster probability, derived from the Chebycheff inequality, or by dropping out the mean constraint. We investigate the consequence of the latter approximation and show that it makes the safety-first principle become a target-driven principle. We then apply a martingale approach to solve the latter approximation for an investor who has uncontrollable and nonreplicable liabilities. Without dropping out the mean constraint, we show that the safety-first principle is ill-posed if the martingale approach is employed. However, if an investor gives up unreasonably high profits and sets an upper bound for the funding level, then the problem can be solved analytically. Setting an upper bound for the funding level is practiced in the UK pension law. This provides an application of this study.

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