Mathematisches Forschungsinstitut Oberwolfach

Report No. 34/2009

DOI: 10.4171/OWR/2009/34

Mathematical Aspects of Hydrodynamics

Organised by
Gregory Seregin, Oxford - St. Petersburg
Vladimir Šverák, Minneapolis

July 19th – July 25th, 2009

Abstract. The workshop was devoted to discussions of recent developments and possible future directions of research in the field of mathematical hydrodynamics. Many of the leading experts in the theory of PDE's arising in fluid dynamics participated in this event. The topics included:

- Regularity, uniqueness and well-posedness problems for the Navier-Stokes equations
- Stability of Navier-Stokes solutions
- Open problems concerning the steady-state Navier-Stokes solutions
- Statistical approach to 2d hydrodynamics
- Inviscid limits of Navier-Stokes solutions
- Anomalous weak solutions of Euler's equation
- Finding physically reasonable classes of weak solutions of Euler's equations
- Local well-posedness of Euler's equations in optimal spaces
- Stability of solutions of Euler's equations
- Water waves
- Model equations
- Geometric approach to hydromechanical equations
- Selected compressible flow problems

Mathematics Subject Classification (2000): 35xx, 76xx.

Introduction by the Organisers

The workshop *Mathematical Aspects of Hydrodynamics*, organized by Gregory Seregin (Oxford - St. Petersburg) and Vladimir Šverák (Minneapolis) was held July 19th – July 25th, 2009. The meeting was well attended, more than 45 mathematicians participated. The program of the workshop consisted of 23 talks presented by leading researchers in Mathematical Fluid Mechanics coming from all around the world. The main topics covered by the workshop lectures, addressed 2D and 3D Euler and Navier-Stokes equations (stationary and non-stationary), Quasi-Geostrophic Equation, Hydrostatic Boussinesq equation and other model equations (Euler-α, Navier-Stokes-Voight etc.).

The lectures stimulated many interesting discussions and exchanges of ideas. Many participants appreciated the opportunity to learn more about a variety of approaches and points of views.

In addition to the scientific program, there were two events which might be worth mentioning. The first one was the traditional Wednesday afternoon hike to Oberwolfach-Kirche. The second event was an outstanding informal concert on Friday evening by Charles Doering (guitar) and László Székelyhidi (violin).

Many participants suggested that a conference devoted to mathematical aspects of fluid mechanics become a regular Oberwolfach workshop.

The unique atmosphere of the Institute was a significant factor in the success of the meeting. As always, the Oberwolfach staff contributed greatly by the perfect organizational work. The organizers would like to express once more their thanks to the Institute for the support and the flawless organization.

Workshop: Mathematical Aspects of Hydrodynamics

Table of Contents

Abstracts

New information provided by the shear flow for the 3d Euler equation

Claude Bardos
(joint work with Edriss S. Titi)

The "shear flow" is one of the most classical (or almost explicit) solutions of the 3d incompressible Euler equation. Analysis of this example leads to simple but may be important remarks on the Euler equation:

1. Existence of very unstable solutions (this was already observed by Di Perna and Lions).

2. Existence of initial data in $C^{0,\alpha}$ with solution in C^{0,α^2} this implies that in the Holder spaces the class $C^{0,\alpha}$ is not only sufficient but compulsory for well posedness.

3. All these pathological examples conserve the energy and this shows that relation between loss of regularity and energy decay for weak solution (the Onsager conjecture) is not completely true (regularity implies conservation of energy but conversely conservation of energy does not implies Holder regularity).

4. Existence of solution of the 3d Kelvin Helmholtz problem with (non analytic) singular interface. This statement is not true in 2d and comes from the fact that in 3d the linearized Kelvin Helmhlotz equation is no more elliptic.

References

[1] Claude Bardos, Edriss S. Titi, *Loss of smoothness and energy conserving rough weak solutions for the 3d Euler equations*, preprint arXiv:0906.2029.

Stokes and Navier-Stokes equations under Navier type Boundary conditions. Regularity and inviscid limit

Hugo Beirão da Veiga
(joint work with Francesca Crispo)

We consider the evolutionary Navier-Stokes equations with a Navier slip-type boundary condition, and study the convergence of the solutions, as the viscosity goes to zero, to the solution of the Euler equations under the zero-flux boundary condition. We obtain quite sharp results in the 2-D and 3-D cases. However, in the 3-D case, we need to assume that the boundary is flat. Convergence is proved in $L^\infty(0,T;W^{k,p}(\Omega))$, for arbitrarily large values of k and p.

References

[1] H. Beirão da Veiga, F. Crispo, *Sharp inviscid limit results under Navier type boundary conditions. An L^p theory*, Journal of Mathematical Fluid Mechanics (in press), DOI 10.1007/s00021-009-0295-4.

[2] H. Beirão da Veiga, F. Crispo, *Concerning the $W^{k,p}(\Omega)$- inviscid limit for 3-D flows under a slip boundary condition*, *Journal of Mathematical Fluid Mechanics* (in press), DOI 10.1007/s00021-009-0012-3.

[3] Y. Xiao and Z. Xin, *On the vanishing viscosity limit for the 3-D Navier-Stokes equations with a slip boundary condition*, *Comm. Pure Appl. Math.*, **60** (2007), 1027-1055.

Hydrostatic Boussinesq equations and Optimal Transport Theory

Yann Brenier

(joint work with Mike Cullen)

We establish a connection between Optimal Transport Theory and classical Convection Theory for geophysical flows. Our starting point is the model designed few years ago by Angenent, Haker and Tannenbaum [AHT] to solve some Optimal Transport problems. This model can be seen as a generalization of the Darcy-Boussinesq equations, which is a degenerate version of the Navier-Stokes-Boussinesq (NSB) equations. In a unified framework, we relate different variants of the NSB equations (in particular what we call the generalized Hydrostatic-Boussinesq equations) to various models involving Optimal Transport (and the related Monge-Amp‘ere equation). This includes the 2D semi-geostrophic equations and some fully non-linear versions of the so-called high-field limit of the Vlasov-Poisson system [NPS] and of the Keller-Segel for Chemotaxis. Mathematically speaking, we establish some existence theorems for local smooth, global smooth or global weak solutions of the different models. We also justify that the inertia terms can be rigorously neglected under appropriate scaling assumptions in the Generalized Navier-Stokes-Boussinesq equations.

We show how a "stringy" generalization of the AHT model can be related to the magnetic relaxation model studied by Arnold and Moffatt to obtain stationary solutions of the Euler equations with prescribed topology.

We prove that smooth solutions of the semigeostrophic equations in the incompressible $x - z$ setting can be derived from the Navier-Stokes equations with the Boussinesq approximation.

References

[1] Y. Brenier, M. Cullen, *Rigorous derivation of the $x-z$ semigeostrophic equations*, to appear in CMS 2009.

[2] Y. Brenier, *Optimal Transport, Convection, Magnetic Relaxation and Generalized Boussinesq Equations*, to appear in JNLS 2009.

Inviscid limit for damped and driven incompressible Navier-Stokes equations in $\mathbb{R}^2$

Peter Constantin

We consider the zero viscosity limit of long time averages of solutions of damped and driven Navier-Stokes equations in $\mathbb{R}^2$. We prove that the rate of dissipation of enstrophy vanishes. Stationary statistical solutions of the damped and driven Navier-Stokes equations converge to renormalized stationary statistical solutions of the damped and driven Euler equations. These solutions obey the enstrophy balance.

References

[1] P. Constantin and F. Ramos, *Inviscid limit for damped and driven incompressible Navier-Stokes equations in R2*, submitted to Comm. Math. Phys. http://arxiv.org/PS cache/math/pdf/0611/0611782.pdf
[2] R. DiPerna, P-L. Lions, *Ordinary differential equations, transport theory and Sobolev spaces*, Invent. Math. 98 (1989) 511-547.
[3] C. Foias, *Statistical study of the Navier-Stokes equations I*, Rend. Sem. Mat. Univ. Padova 48 (1972), 219-348.
[4] C. Foias, *Statistical study of the Navier-Stokes equations II*, Rend. Sem. Mat. Univ. Padova 49 (1973), 9-123.

Transport and Dissipation in Turbulent Fluid Flows

Charles R. Doering

The relation between the shape of the force driving a turbulent ow and the upper bound on the dimensionless dissipation factor β is presented. We are interested in non-trivial (more than two wave numbers) forcing functions in a three dimensional domain periodic in all directions. A comparative analysis between results given by the optimization problem and the results of Direct Numerical Simulations is performed. We report that the bound on the dissipation factor in the case of infinite Reynolds numbers have the same qualitative behavior as for the dissipation factor at finite Reynolds number. As predicted by the analysis, the dissipation factor depends strongly on the force shape. However, the optimization problem does not predict accurately the quantitative behavior. We complete our study by analyzing the mean ow profile in relation to the Stokes ow profile and the optimal multiplier profile shape for different force-shapes. We observe that in our 3D-periodic domain, the mean velocity profile and the Stokes flow profile reproduce all the characteristic features of the force-shape. The optimal multiplier proves to be linked to the intensity of the wave numbers of the forcing function.

References

[1] B. Rolling, Y. Dubief, C.R. Doering, *Variation on the Kolmogorov Forcing: Asymptotic Dissipation Rate Driven by Harmonic Forcing*, preprint arXiv:0903.1897
[2] Thomas H. van den Berg, Charles R. Doering, Detlef Lohse, Daniel P. Lathrop, *Smooth and rough boundaries in turbulent Taylor-Couette flow*

[3] B. Gallet, C. Doering, E. Spiegel *Instability theory of swirling flows with suction*
[4] Doering, Charles R.; Spiegel, Edward A. *Energy dissipation in a shear layer with suction*, Physics of Fluids, Volume 12, Issue 8, pp. 1955-1968 (2000).
[5] C. Doering and P. Constantin, *Energy dissipation in shear driven turbulence*, Phys. Rev. Lett. 69, 1648 (1992).
[6] D. Lathrop, J. Feinberg, and H. Swinney, *Transition to shear-driven turbulence in Couette-Taylor flow*, Phys. Rev. A 46, 6390 (1992).
[7] O. Cadot, Y. Couder, A. Daerr, S. Douady, and A. Tsinober, *Energy injection in closed turbulent flows: Stirring through boundary layers versus inertial stirring*, Phys. Rev. E56, 427 (1997).
[8] P. Constantin and C. Doering, *Variational bounds on energy dissipation in incompressible flows: II. Channel flow*, Physica D 82, 221 (1995).

L^p-estimates for stationary compressible fluids

Jens Frehse
(joint work with Mark Steinhauer, Wladimir Weigant)

We consider the Navier-Stokes equations for compressible isothermal flow in the steady two dimensional case and show the existence of a weak solution in the case of periodic and of mixed boundary conditions. Also we consider the three dimensional case and show the existence of a weak solution for homogeneous Dirichlet (no-slip) boundary conditions under the assumption that the adiabatic exponent satisfies some restriction. In particular we cover with our existence result the cases of a monoatomic gas and of air.

References

[1] J. Frehse, M. Steinhauer, W. Weigant *The Dirichlet Problem for Steady Viscous Compressible Flow in 3-D*, preprint University of Bonn, SFB 611, No. 347 (2007), http://www.iam.uni-bonn.de/sfb611/.
[2] J. Frehse, M. Steinhauer, W. Weigant, *The Dirichlet Problem for Viscous Compressible Isothermal Navier-Stokes Equations in Two-Dimensions*, preprint University of Bonn, SFB 611, No. 337 (2007), http://www.iam.uni-bonn.de/sfb611/.
[3] J. Frehse, M. Steinhauer, W. Weigant, *On Stationary Solutions for 2-D Viscous Compressible Isothermal Navier-Stokes Equations*, preprint University of Bonn, SFB 611, No. 330 (2007), http://www.iam.uni-bonn.de/sfb611/

Some large initial data to the Navier-Stokes equations giving rise to a global solution

Isabelle Gallagher
(joint work with Jean-Yves Chemin, Marius Paicu)

Classes of initial data to the three dimensional, incompressible Navier-Stokes equations were presented, generating a global smooth solution although the norm of the initial data (periodic or defined in the whole space) may be chosen arbitrarily large. Such initial data giving rise to global solutions have particular oscillatory properties or varies slowly in one direction or the norm blows up as the small parameter goes to zero. The proof uses the special structure of the nonlinear term of the equation.

References

[1] J.-Y. Chemin and I. Gallagher, *On the global wellposedness of the 3-D Navier-Stokes equations with large initial data*, Annales de l' Ecole Normale Superieure, 39, 2006, pages 679-698.

[2] J.-Y. Chemin and I. Gallagher, *Wellposedness and stability results for the Navier-Stokes equations in R3* to appear in Annales de l'Institut Henri Poincare, Analyse Non Lineaire.

[3] J.-Y. Chemin and I. Gallagher, *Large, global solutions to the Navier-Stokes equations, slowly varying in one direction*, to appear in Transactions of the Americal Mathematical Society.

[4] J.-Y. Chemin, I. Gallagher and M. Paicu *Global regularity for some classes of large solutions to the Navier-Stokes equations*, preprint arXiv:0807.1265.

Three-dimensional Stability of Burgers Vortices

Thierry Gallay
(joint work with Yasunori Maekawa)

Although the two dimensional stability of the Burgers vortex has been well studied by now, its stability with respect to three dimensional perturbations has been less understood since the linearized equation becomes much more complicated in this case. In Gallay-Wayne [1-2] it is mathematically proved that the Burgers vortex is locally stable for three dimensional perturbations at least for sufficiently small circulation numbers. New results concerning stability of Burgers Vortices in the symmetric and non-symmetric case are discussed. The talk is focused on this three dimensional stability problem in the case of arbitrary circulation numbers.

References

[1] Th. Gallay and C.E. Wayne. *Global stability of vortex solutions of the two-dimensional Navier-Stokes equation*, Comm. Math. Phys. 255 (2005), 97-129.

[2] Th. Gallay and C.E. Wayne, *Existence and stability of asymmetric Burgers vortices*, J. Math. Fluid Mech. 9 (2007), 243-261.

[3] Y. Maekawa, *Spectral properties of the linearization at the Burgers vortex in the high rotation limit*, J. Math. Fluid Mech., to appear.

[4] Y. Maekawa, *On the existence of Burgers vortices for high Reynolds numbers*, J. Math. Analysis and Applications 349 (2009), 181-200.

[5] Y. Maekawa, *Existence of asymmetric Burgers vortices and their asymptotic behavior at large circulations* Math. Models Methods Appl. Sci. 19 (2009), 669-705.

Strong L^p solutions to fluid-solid interaction problems

Matthias Hieber
(joint work with Matthias Geissert, Karoline Goetze)

In this talk we consider the movement of a rigid body in a Newtonian or Non-Newtonian fluid under the influence of gravitation. We show that in the first case the system consisting of the Navier-Stokes equations coupled with the balance laws for the momentum and the angular momentum admits a unique, local, strong solution in the Lp-setting. Note that the fluid-solid interface is a moving one and has to be found as part of the solution process. Our result will then be extended to the case of certain Non-Newtonian fluids. This is joint work with Matthias Geissert and Karoline Goetze.

A variation on a theme of Caffarelli and Vasseur

Alexander Kiselev
(joint work with Fedor Nazarov)

The 2D surface quasi-geostrophic (SQG) equation has recently been a focus of significant research effort. This equation appears in geophysics, emerging under certain assumptions from Boussinesq approximation equations describing fluid in a three dimensional strongly rotating half-space. Part of the recent interest to the SQG equation may be due to the fact that it is the simplest-looking equation of fluid dynamics for which the question of global existence of smooth solutions is still open. Recently, there has been a progress in understanding SQG equation with critical dissipation. There are two very different proofs of global regularity in this case: a nonlocal modulus of continuity approach by Kiselev, Nazarov and Volberg and a method of Caffarelli and Vasseur, based on DiGiorgi-type iterative techniques. The KNV approach is shorter, while the CV approach is more general, showing Hölder regularization of solution to drift-diffusion equation with BMO norm estimate on the drift. The SQG equation result is then a particular consequence. In this talk, I will outline a third approach which allows to understand and recover some of Caffarelli-Vasseur results by more elementary means. This is a joint work with F. Nazarov.

References

[1] L. Caffarelli and A. Vasseur, *Drift diffusion equations with fractional diffusion and the quasi-geostrophic equation*, preprint arXiv:math/0608447, 25 pages.
[2] A. Kiselev, F. Nazarov and A. Volberg, *Global well-posedness for the critical 2D dissipative quasi-geostrophic equation*, Inventiones Math. 167 (2007) 445-453.
[3] A. Kiselev, F. Nazarov, *A variation on a theme of Caffarelli and Vasseur*, preprint arxiv:math/0908.0923, 9 pages.

Leray's inequality in 3D multi-connected domains

HIDEO KOZONO
(joint work with Taku Yanagisawa)

We consider the stationary Navier-Stokes equations in 3D bounded domains the boundary of which consists of several connected surfaces. It is known that if the given boundary data satisfies the "restricted" flux condition which means that the flux on each component of the boundary vanishes, then we have an existence theorem. However, it has been an open question whether or not the same existence result does hold under the "general" flux condition which implies that the total sum of each flux on the boundary component is zero. This problem was proposed by Leray in 1933 who had established the former existence theorem by means of the Leray inequality. Roughly speaking, we may regard the Leray inequality as the quadratic estimate of the nonlinear convection term in terms of the Dirichlet integral. In this talk, we will see that, in many cases, the Leray inequality necessarily yields the restricted flux condition.

REFERENCES

[1] H. Kozono, T. Yanagisawa, *Global Div-Curl lemma on bounded domains in R^3*, Journal of Functional Analysis Volume 256, Issue 11, 1 June 2009, Pages 3847-3859,
[2] H. Kozono, T. Yanagisawa, *Leray's problem on the stationary Navier-Stokes equations with inhomogeneous boundary data*, Mathematische Zeitschrift Volume 262, Number 1, 2009, pages 27-39.

Some Remarks on Euler Equation and Triebel-Lizorkin Spaces

PIERRE-GILLES LEMARIÉ-RIEUSSET

The solvability of the Cauchy problem for the non-stationary Euler equations in Triebel-Lizorkin Spaces is discussed.

REFERENCES

[1] D. Chae, *On the well-posedness of the Euler equations in the Triebel-Lizorkin spaces*, Comm. Pure Appl. Math 55 (2002), p. 654-678,
[2] Y. Zhou *Local well-posedness for the incompressible Euler equations in the critical Besov spaces* Annales de l'institut Fourier, 54 no. 3 (2004), p. 773-786

Behavior of solutions to Navier-Stokes equations in the scaling invariant spaces

Nataša Pavlović
(joint work with Jean Bourgain, Pierre Germain and Gigliola Staffilani)

In this talk we will discuss the Navier-Stokes equations in scaling invariant spaces. In particular, we will briefly recall regularity of so called "mild" solutions to the Navier-Stokes equations evolving from small initial data in a critical space in $\mathbb{R}^n$ (joint work with Pierre Germain and Gigliola Staffilani). Then we will describe a result on ill-posedness of the Navier-Stokes equations in the largest critical space in 3D (joint work with Jean Bourgain).

References

[1] Pierre Germain, Natasa Pavlovic, Gigliola Staffilani, *Regularity of solutions to the Navier-Stokes equations evolving from small data in* BMO^{-1}, preprint arxiv:math/0609781,
[2] Jean Bourgain, Natasa Pavlovic, *Ill-posedness of the Navier-Stokes equations in a critical space in 3D*, preprint arxiv:math/0807.0882.

2-D Navier-Stokes Equation and Intermediate Asymptotics

Mario Pulvirenti
(joint work with Emanuele Caglioti, Frederic Rousset)

We introduce a modified version of the two-dimensional Navier-Stokes equation, preserving energy and momentum of inertia, which is motivated by the occurrence of different dissipation time scales and related to the gradient flow structure of the 2-D Navier-Stokes equation. The hope is to understand intermediate asymptotics.

References

[1] E. Caglioti, M. Pulvirenti, F. Rousset, *2-D constrained Navier-Stokes equation and intermediate asymptotics*, preprint arxiv:math/0807.2197

Liquid crystals and Weil-Petersson geodesics

Tudor S. Ratiu
(joint work with François Gay-Balmaz, Jerrold E. Marsden)

The approach develops the theory of affine Euler-Poincaré and affine Lie-Poisson reductions and applies these processes to various examples of complex fluids, including Yang-Mills and Hall magnetohydrodynamics for fluids and superfluids, spin glasses, microfluids, and liquid crystals. As a consequence of the Lagrangian approach, the variational formulation of the equations is determined. On the Hamiltonian side, the associated Poisson brackets are obtained by reduction of a canonical cotangent bundle. A Kelvin-Noether circulation theorem is presented and is applied to these examples.

References

[1] Francois Gay-Balmaz, Tudor S. Ratiu, *The Geometric Structure of Complex Fluids*, preprint arxiv:math/0903.4294
[2] Anthony M. Bloch, Arieh Iserles, Jerrold E. Marsden, Tudor S. Ratiu, *A Class of Integrable Geodesic Flows on the Symplectic Group and the Symmetric Matrices*, preprint arxiv:math-ph/0512093

Well-posedness for the free-boundary 3-D compressible Euler equations in physical vacuum

Steve Shkoller
(joint work with Daniel Coutand, Hans Lindblad)

We prove a priori estimates for the three-dimensional compressible Euler equations with moving *physical* vacuum boundary, with an equation of state given by $p(\rho) = C_\gamma \rho^\gamma$ for $\gamma > 1$. The vacuum condition necessitates the vanishing of the pressure, and hence density, on the dynamic boundary, which creates a degenerate and characteristic hyperbolic *free-boundary* system to which standard methods of symmetrizable hyperbolic equations cannot be applied.

References

[1] Daniel Coutand, Hans Lindblad, Steve Shkoller, *A priori estimates for the free-boundary 3-D compressible Euler equations in physical vacuum*, preprint arxiv:math0906.0289.

What is a weak solution of the Euler Equation?

Alexander Shnirelman

I am going to discuss the construction of a more realistic 3-dim weak solution, whose kinetic energy monotonically decreases in time. This solution is also everywhere discontinuous and unbounded, while has some realistic features. The construction starts from a simple mechanical system having the property that the kinetic energy decreases, while there is no explicit friction; it requires Generalized Flows, introduced by Y. Brenier. At last, I am going to discuss the ways to the construction and theory of true, physically reasonable weak solutions of the Euler equations.

References

[1] Alexander I. Shnirelman, *Weak solutions of incompressible Euler equations with decreasing energy*, Seminaire Equations aux derivees partielles (Polytechnique) (1996-1997), Exp. No. 16, 9 p.
[2] Shnirelman, A. *Microglobal Analysis of the Euler Equations*, Journal of Mathematical Fluid Mechanics, Volume 7, Supplement 3, pp S387-S396
[3] Scheffer, V. *An inviscid flow with compact support in space-time*, J. Geom. Anal. 3, 4 (1993), 343-401.
[4] Shnirelman, A. *On the nonuniqueness of weak solution of the Euler equation*, Comm. Pure Appl. Math. 50, 12 (1997), 1261-1286.

[5] Shnirelman, A. *Weak solutions with decreasing energy of incompressible Euler equations*, Comm. Math. Phys. 210, 3 (2000), 541-603.

Recent results on strong and weak colutions of the Navier-Stokes Equations

Hermann Sohr
(joint work with Reinhard Farwig)

The first part of this talk yields the optimal initial value condition for the existence of a local (in time) unique strong solution of the Navier-Stokes equations in a smooth bounded domain. This condition is not only sufficient - there are several well-known sufficient conditions in this context - but also necessary, yielding therefore the largest possible class of such local strong solutions. A restricted result holds for completely general domains. In the second part, the well-known class of (global in time) Leray-Hopf weak solutions with zero boundary conditions and zero divergence will be extended to a larger class with corresponding nonzero conditions where boundary values and divergence are given satisfying certain compatibility conditions.

References

[1] R. Farwig, H. Sohr, W. Varnhorn, *On optimal initial value conditions for local strong solutions of the Navier-Stokes equations*, ANNALI DELL'UNIVERSITA' DI FERRARA (2009) 55:89-110

[2] R. Farwig, H. Kozono, H. Sohr, *Global weak solutions of the Navier-Stokes equations with nonzero boundary conditions*, preprint http://www3.mathematik.tu-darmstadt.de/fb/mathe/bibliothek/preprints.html

On a problem of magneto-hydrodynamics in multi-connected domains

Vsevolod A. Solonnikov

A proof of the solvability for the initial boundary value problem arising in magneto-hydrodynamics in multi-connected domains is presented.

References

[1] O.A. Ladyzhenskaya, V.A. Solonnikov, *Solutions of some non-stationary problems of magneto-hydrodynamics for a viscous incompressible fluid*, Trudy Math. Inst. Steklov. 59 (1960) 115-173.

[2] O.A. Ladyzhenskaya, V.A. Solonnikov, *The linearization principle and invariant manifolds for problems of magneto-hydrodynamics*, J. Soviet Math. 8 (1977) 384-422.

Weak solutions of the Euler Equations: Non-uniqeness and dissipation

László Székelyhidi Jr.
(joint work with Camillo De Lellis)

In this paper we propose a new point of view on weak solutions of the Euler equations, describing the motion of an ideal incompressible fluid in $\mathbb{R}^n$ with $n \geq 2$. We give a reformulation of the Euler equations as a differential inclusion, and in this way we obtain transparent proofs of several celebrated results of V. Scheffer and A. Shnirelman concerning the non-uniqueness of weak solutions and the existence of energy–decreasing solutions. Our results are stronger because they work in any dimension and yield bounded velocity and pressure.

References

[1] Camillo De Lellis, Laszlo Szekelyhidi Jr, *The Euler equations as a differential inclusion*, preprint http://arxiv.org/abs/math/0702079

Inviscid Regularization of Hydrodynamics Equations: Global Regularity, Numerical Analysis and Statistical Behavior

Edriss S. Titi

Inviscid Regularization of Hydrodynamics Equations is described. Some results on solvability and regularity of the corresponding solutions and their numerical analysis and statistical behavior are discussed.

References

[1] C. Bardos, J. S. Linshiz, E. S. Titi, *Global regularity and convergence of a Birkhoff- Rott-α approximation of the dynamics of vortex sheets of the 2D Euler equations*, Comm. Pure Appl. Math., (to appear).
[2] Quansen Jiu, Dongjuan Niu, Edriss S. Titi, Zhouping Xin, *Axisymmetric Euler-α Equations without Swirl: Existence, Uniqueness, and Radon Measure Valued Solutions*, preprint arXiv:0907.2348
[3] A.-C. Bennis, R. Lewandowski, E. S. Titi, *Simulations de l'ecoulement turbulent marin ave un modele de deconvolution*, C. R. A. S., Paris (2009), (to appear).
[4] Y. Cao, E.M. Lunasin, and E.S. Titi, *Global well-posedness of the three dimensional viscous and inviscid simplified Bardina turbulence models*, Communications in Mathematical Sciences, 4 (2006), no.4, 823-84.
[5] B. Khouider, E.S. Titi, *An Inviscid regularization for the surface quasi-geostrophic equation*, Comm. Pure Appl. Math. (to appear). arXiv:math/0702067v1.
[6] V.K. Kalantarov, B. Levant and E.S. Titi, *Gevrey regularity of the global attractor of the 3D Navier-Stokes-Voight equations*, Journal of Nonlinear Science, 19 (2009), 133-152.
[7] B. Levant, F. Ramos and E.S. Titi, *On the statistical properties of the 3D incompressible Navier-Stokes-Voigt model*, Communications in Mathematical Sciences, (2009), (to appear).
[8] Y. Cao, Z.H. Musslimani and E.S. Titi, *Modulation theory for self-focusing in the nonlinear Schrodinger-Helmholtz equation, Numerical Functional Analysis and Optimization*, 30 (2009), 46-69.
[9] A. Cheskidov, D.D. Holm, E. Olson and E.S. Titi, *On a Leray-α Model of Turbulence*, Royal Society London, Proceedings, Series A, Mathematical, Physical and Engineering Sciences, 461 (2005), 629–649

Recent results in fluid mechanics

Alexis F. Vasseur

We will present, in this talk, new applications of De Giorgi's methods and blow-up techniques to fluid mechanics problems. Those techniques have been successfully applied to show full regularity of the solutions to the surface quasi-geostrophic equation in the critical case.

We will present, also, a new nonlinear family of spaces allowing to control higher derivatives of solutions to the 3D Navier-Stokes equation. Finally, we will present a regularity result for a reaction-diffusion system which has almost the same supercriticality than the 3D Navier-Stokes equation.

References

[1] A. Vasseur, *A new proof of partial regularity of solutions to Navier-Stokes equations*, NoDEA Nonlinear Differential Equations Appl., 14(5-6):753- 785, 2007.

[2] Alexis F. Vasseur *Higher derivatives estimate for the 3D Navier-Stokes equation*, preprint arXiv:0904.2422, 21 pages.

Is free surface potential Hydrodynamics integrable?

Vladimir E. Zakharov

If the spectral density of ocean wave is narrow in comparison with steepness, the modulation instability occurs. We have studied the nonlinear shape of this instability by performing massive numerical simulations of the Euler equations for ideal fluid with free surface. Our conclusion is the following - the modulational instability generates rogue waves.

We started with the exact Euler equation for potential flow of infinitely deep incompressible fluid in two dimensions and performed canonical map of the domain filled by fluid onto the lower half-plane. Under this map, the hydrodynamic equations are transformed to the elegant Dyachenko equations, which are suitable for numerical solution by implementation of the standard spectral code. We used the code with adjusting time step and adjusting number of spectral modes. This number varied within the limits $10^4/10^6$. All experiments were done in the standard wave tank of the length 2π.

In the first group of experiments we studied the development of modulational instability of slightly perturbed Stokes wave. Both steepness and wave numbers of the initial wave could be essentially varied. The most impressive results are obtained for the Stokes with wave number $n = 100$ and $\mu \cong 0.1$. In this case the modulational instability is a slow process. On the initial stage we observed exponential growth of perturbation followed by formation of a kind of one-dimensional turbulence. After more than ten inverse growth rates of the instability (two or more periods of initial wave) the process ends up by formation of the freak wave of an amplitude exceeding initial average level of surface elevation in four-five times. Onset of the freak wave is a catastrophic event taking only a few wave periods.

In the second group of experiments we studied the formation of freak waves from envelope quasisolitons. The quasisolitons of small steepness, $\mu < 0.1$, propagate peacefully and could be fairly approximated by the Nonlinear Schrodinger equation (NLSE). Quasisolitons of a moderate amplitude, $0.1 < \mu < 0.14$, still propagate without changing their form, however, they are essentially asymmetric. The quasisolitons of high steepness, $\mu > 0.1$, are unstable. Their evolution leads to a formation of freak wave. The freak waves appear also in collisions of relatively small amplitude solitons.

In the third group of experiments we generated stochastic quasi-monochromatic waves from the white noise by including into equations of the narrow-band weak instability (so far in the range of wave numbers 30/40). At a certain level of nonlinearity, the growth of instability is arrested by a sporadic wave breaking. On this background we saw rare events of rogue wave formation.

References

[1] V.E. Zakharov, A.I. Dyachenko, A.O. Prokofiev, *Freak waves: Peculiarities of numerical simulations*, to appear in the book "Extreme Waves in the Ocean", Springer, eds. E.Pelinovsky and C. Harif.

[2] F. Dias, A. Dyachenko, V. Zakharov, *Theory of weakly damped free-surface flows: A new formulation based on potential flow solutions*, Phys. Lett. A (in press).

[3] V.E. Zakharov, A.I. Dyachenko, A.O. Prokofiev, *Freak waves as nonlinear stage of Stokes wave modulation instability*, Eur. J. of Mech. B/Fluids, 25(2006), 5, 677-692.

[4] A.I. Dyachenko, V.E. Zakharov, *Modulation instability of Stokes - freak wave*, JETP Letters, 81 (2005), 255.

Reporter: Alexander Mikhaylov

Participants

Prof. Dr. Claude Bardos
31, Avenue Trudaine
F-75009 Paris Cedex

Prof. Dr. Hugo Beirao da Veiga
Via Diotisalvi, 2
I-56126 Pisa

Prof. Dr. Yann Brenier
Laboratoire J.-A. Dieudonne
Universite de Nice
Sophia Antipolis
Parc Valrose
F-06108 Nice Cedex 2

Dr. Maria-Cristina Caputo
Department of Mathematics
The University of Texas at Austin
1 University Station C1200
Austin , TX 78712-1082
USA

Prof. Dr. Dongho Chae
Department of Mathematics
Sungkyunkwan University
Suwon 440-146
KOREA

Prof. Dr. Alexey Cheskidov
Department of Mathematics
The University of Chicago
5734 South University Avenue
Chicago , IL 60637-1514
USA

Prof. Dr. Peter Constantin
Department of Mathematics
The University of Chicago
5734 South University Avenue
Chicago , IL 60637-1514
USA

Prof. Dr. Camillo De Lellis
Institut für Mathematik
Universität Zürich
Winterthurerstr. 190
CH-8057 Zürich

Prof. Dr. Charles R. Doering
Department of Mathematics
University of Michigan
530 Church Street
Ann Arbor , MI 48109-1109
USA

Prof. Dr. Reinhard Farwig
Fachbereich Mathematik
TU Darmstadt
Schloßgartenstr. 7
64289 Darmstadt

Prof. Dr. Jens Frehse
Mathematisches Institut
Universität Bonn
Beringstr. 6
53115 Bonn

Prof. Dr. Susan Friedlander
Department of Mathematics
University of Southern California
DRB 1042 W 36th Place
Los Angeles , Ca 90089-1113
USA

Prof. Dr. Martin Fuchs
Fachrichtung - Mathematik
Universität des Saarlandes
Postfach 151150
66041 Saarbrücken

Prof. Dr. Giovanni Paolo Galdi
Department of Mechanical
Engineering
University of Pittsburgh
630 Benedum Hall
Pittsburgh PA 15261
USA

Dr. Isabelle Gallagher
U. F. R. de Mathematiques
Case 7012
Universite Paris 7
2, Place Jussieu
F-75251 Paris Cedex 05

Dr. Thierry Gallay
Institut Fourier
Universite de Grenoble I
BP 74
F-38402 Saint-Martin d'Heres -Cedex

Prof. Dr. Matthias Hieber
Fachbereich Mathematik
TU Darmstadt
Schloßgartenstr. 7
64289 Darmstadt

Prof. Dr. Alexander Kiselev
Department of Mathematics
University of Wisconsin-Madison
480 Lincoln Drive
Madison , WI 53706-1388
USA

Dr. Gabriel Koch
Department of Mathematics
The University of Chicago
5734 South University Avenue
Chicago , IL 60637-1514
USA

Dr. Hideo Kozono
Mathematical Institute
Tohoku University
Sendai 980-8578
JAPAN

Prof. Dr. Pierre-Gilles Lemarie-Rieusset
Universite d'Evry Val d'Essone
Laboratoire d'Analyse et Probabilite
Departement de Mathematiques
Bd. Francois Mitterand
F-91025 Evry Cedex

Prof. Dr. Hans Lindblad
Department of Mathematics
University of California, San Diego
9500 Gilman Drive
La Jolla , CA 92093-0112
USA

Prof. Dr. Alex Mahalov
Dept. of Mathematics and Statistics
Arizona State University
Box 871804
Tempe , AZ 85287-1804
USA

Prof. Dr. Paolo Maremonti
Dipartimento di Matematica
Seconda Universita di Napoli
Via Vivaldi 43
I-81100 Caserta

Prof. Dr. Alexander Mikhaylov
Steklov Mathematical Institute
PDMI
Fontanka 27
St. Petersburg 191023
RUSSIA

Benson K. Muite
Mathematical Institute
Oxford University
24-29 St. Giles
GB-Oxford OX1 3LB

Prof. Dr. Natasa Pavlovic
Department of Mathematics
The University of Texas at Austin
1 University Station C1200
Austin , TX 78712-1082
USA

Dr. Eleonora Pinto de Moura
Mathematics Institute
University of Warwick
Gibbet Hill Road
GB-Coventry CV4 7AL

Prof. Dr. Mario Pulvirenti
Facolta di Scienze Matematiche
Universita di Roma
Piazza A. Moro 1
I-00185 Roma

Prof. Dr. Tudor S. Ratiu
Departement de Mathematiques
Ecole Polytechnique Federale
de Lausanne
CH-1015 Lausanne

Prof. Dr. Okihiro Sawada
Fachbereich Mathematik
TU Darmstadt
Schloßgartenstr. 7
64289 Darmstadt

Prof. Dr. Gregory A. Seregin
St. Hilda's College
Mathematical Institute
Cowley Place
GB-Oxford OX4 1DY

Prof. Dr. Timofey Shilkin
Steklov Mathematical Institute
PDMI
Fontanka 27
St. Petersburg 191023
RUSSIA

Prof. Dr. Steve Shkoller
Department of Mathematics
University of California, Davis
1, Shields Avenue
Davis , CA 95616-8633
USA

Prof. Dr. Alexander Shnirelman
Department of Mathematics
Concordia University
1455 De Maisonneuve Blvd. West
Montreal , QC H3G 1M8
CANADA

Dr. Roman Shvydkoy
Dept. of Mathematics, Statistics
and Computer Science, M/C 249
University of Illinois at Chicago
851 S. Morgan Street
Chicago , IL 60607-7045
USA

Prof. Dr. Hermann Sohr
FB 17: Mathematik/Informatik
Universität Paderborn
Warburger Str. 100
33098 Paderborn

Prof. Dr. Vsevolod A. Solonnikov
Steklov Mathematical Institute
PDMI
Fontanka 27
St. Petersburg 191023
RUSSIA

Prof. Dr. Michael Struwe
Departement Mathematik
ETH-Zentrum
Rämistr. 101
CH-8092 Zürich

Prof. Dr. Vladimir Sverak
School of Mathematics
University of Minnesota
127 Vincent Hall
206 Church Street S. E.
Minneapolis MN 55455-0436
USA

Dr. Laszlo Szekelyhidi
Hausdorff Center for Mathematics
Universität Bonn
Villa Maria
Endenicher Allee 62
53115 Bonn

Prof. Dr. Atusi Tani
Department of Mathematics
Faculty of Science and Technology
Keio University
3-14-1, Hiyoshi, Kohoku-ku
Yokohama 223-8522
JAPAN

Prof. Dr. Yasushi Taniuchi
Shinshu University
Faculty of Science
Dept. of Mathematical Sciences
Matsumoto 390-8621
JAPAN

Prof. Dr. Edriss S. Titi
Department of Mathematics
University of California, Irvine
Irvine , CA 92697-3875
USA

Prof. Dr. Alexis F. Vasseur
Department of Mathematics
The University of Texas at Austin
1 University Station C1200
Austin , TX 78712-1082
USA

Dr. Wojciech M. Zajaczkowski
Institute of Mathematics of the
Polish Academy of Sciences
P.O. Box 21
ul. Sniadeckich 8
00-956 Warszawa
POLAND

Prof. Dr. Vladimir E. Zakharov
P.N. Lebedev Physical Institute
Russian Academy of Science
Leninskiy prospekt 53
Moscow 117924
RUSSIA