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Mini-Workshop: Feinstrukturtheorie und Innere Modelle

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Abstract. This workshop presented recent advances in fine structure and inner model theory. There were extended tutorials on hod mice and the Mouse Set Conjecture, suitable extender sequences and their fine structure, and the construction of true K below a Woodin cardinal in ZFC. The remaining talks involved precipitous ideals, stationary set reflection, failure of SCH in ZF, nonthreadable square sequences, reverse mathematics, forcing axioms, covering properties of canonical inner models, and "set theoretic geology."

Mathematics Subject Classification (2000): 03Exx.

Introduction by the Organisers

This was a successful workshop highlighting recent advances in inner model theory. There were 15 participants, 3 extended tutorials, and 8 shorter (1 to 2 hour) talks, as well as many small discussions.

The three extended tutorials were as follows:

(1) Grigor Sargsyan presented some of his work on the theory of hod mice (which are special kinds of hybrid mice; a hybrid mouse has a predicate for iteration strategies in addition to the usual predicate for the extender sequence). He also provided an outline of his proof of the Mouse Set Conjecture under a certain smallness assumption on the universe.

(2) Hugh Woodin discussed suitable extender sequences, for extenders which may witness supercompactness and beyond. He also discussed the corresponding fine structure theory of such sequences.

(3) John Steel presented the construction of the true core model under the assumption ZFC + "no inner model with a Woodin" (joint work with

Ronald Jensen). This solved a longstanding problem of how to run the construction without the additional assumption of a large cardinal in the universe.

The topics for the shorter talks were quite diverse. Moti Gitik discussed the strength of the existence of precipitous ideals without their normal counterparts (and introduced a nice game construction making use of Mitchell's Covering Lemma); Sean Cox presented lower bounds for stationary reflection at small cofinalities; and Peter Koepke presented an equiconsistency result for failure of SCH in choiceless models.

Martin Zeman presented a combinatorial result relating nonthreadable square sequences to nonreflecting stationary sets in extender models. Menachem Magidor spoke about the Proper Distributive Forcing Axiom and its relation to $\square_{\kappa,\omega}$ and $\square_{\kappa,\omega_1}$.

Gunter Fuchs discussed mantles and related classes (and whether these can be canonical models of ZF), and William Mitchell asked what possible generalizations of the Covering Lemma might hold for extender models with a Woodin cardinal. Itay Neeman gave an example of a necessary use of strong induction for a reversal, in reverse mathematics.

In addition to the talks, there were many small groups turning coffee into mathematics during the breaks.

Mini-Workshop: Feinstrukturtheorie und Innere Modelle

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Reflection of stationary sets at small cofinalities

Sean Cox

If κ is an ordinal of uncountable cofinality, $S \subseteq \kappa$ is stationary, and $\gamma < \kappa$ has uncountable cofinality, we say S *reflects at* γ if $S \cap \gamma$ is stationary in γ. Starting from a Mahlo cardinal, Harrington and Shelah in [1] obtained a model of "Every stationary subset of $\omega_2 \cap cof(\omega)$ reflects." In the other direction, if every stationary subset of $\omega_2 \cap cof(\omega)$ reflects, then Jensen's global square sequence for L can be used to show that ω_2 is Mahlo in L. Reflection for stationary subsets of a successor cardinal $\geq \omega_3$ are similarly equiconsistent with a Mahlo cardinal. However, requiring the reflection points to have small cofinality yields larger cardinals in K:

Let $\phi(\kappa)$ denote the statement "For every $\nu < \kappa^+$, there are stationarily many $\lambda < \kappa$ such that $o(\lambda) > h_\nu(\lambda)$," where h_ν denotes the ν-th canonical function on κ; note $\phi(\kappa)$ is slightly weaker than $o(\kappa) = \kappa^+$. Then

Theorem 1. *(C.) CON(ZFC + "every stationary subset of $\omega_3 \cap cof(\omega)$ reflects at some point in $cof(\omega_1)$") implies CON(ZFC + there is a κ such that $\phi(\kappa)$).*

The simultaneous variant of this reflection—i.e. "every pair of stationary subsets of $\omega_3 \cap cof(\omega)$ have a common reflection point in $cof(\omega_1)$—is at least as strong as $o(\kappa) = \kappa^+$.

References

[1] Leo Harrington and Saharon Shelah, Some exact equiconsistency results in set theory, Notre Dame J. Formal Logic, volume 26, no. 2, 1985, 178–188.

Set Theoretic Geology meets Inner Model Theory

Gunter Fuchs
(joint work with Ralf Schindler)

In joint work with Joel Hamkins and Jonas Reitz, the first author introduced new kinds of inner models the definability of which relies on the following key fact which was proved by Richard Laver (and, independently, by W.H. Woodin):

Theorem 1. *There is a first order formula in the language of set theory $\phi(x, y)$ such that for every set sized partial ordering $\mathbb{P}$, the following holds:*

$$\mathbb{P} \Vdash \check{\mathsf{V}} = \{x \mid \phi(x, a)\},$$

where $a = \mathcal{P}(\mathsf{card}(\mathbb{P})^+)$.

So the ground model is uniformly first order definable in all of its set forcing extensions, using a parameter. This makes it possible to turn the usual direction of movement from a model to its forcing extension around and look downwards from a model to all of its set ground models, i.e., to all of the models of which it is a set forcing extension. It is easy to see that the class I of all a such that the class $\{x|\phi(x,a)\}$ is a ground model is definable. This enables us to make the following definition:

Definition 2. *The Mantle* $\mathbb{M}$ *is the intersection of all ground models. I.e.,*

$$\mathbb{M} = \{x|\forall a \in I \quad \phi(x,a)\}.$$

We were not able to prove in general that the mantle is a model of ZF. A more robust model is given by the following

Definition 3. *The generic Mantle,* $g\mathbb{M}$*, is the intersection of all ground models of all forcing extensions. In other words, it is the intersection of all mantles of all forcing extensions. So*

$$g\mathbb{M} = \{x \mid \forall \mathbb{P} \quad \mathbb{P} \Vdash \check{x} \in \mathbb{M}\}.$$

The generic Mantle is a forcing invariant inner model (of ZF). Another model that shows up naturally in this context was introduced by me and is now called the generic HOD:

$$g\mathsf{HOD} = \bigcap_{\alpha<\infty} \mathsf{HOD}^{\mathsf{V}^{\mathsf{Col}(\omega,\alpha)}}$$

It is also invariant under set forcing, and it is a model of ZFC.

Due to the nature of this new approach of constructing inner models, which in a sense tries to undo forcing and dig downwards, we refer to it as set theoretic geology.

Many questions arise about these types of inner models, maybe the most natural one being whether they are canonical in any way. The joint work with Hamkins and Reitz basically shows that this is not the case. Among other things, we showed that every model of ZFC is the mantle and the generic mantle of another model of set theory, obtained by class forcing over the first model. Another natural type of question is whether the mantle of a canonical model is also canonical. The first inkling showing that it might be promising to analyze the set theoretic geology of canonical models was the following:

Theorem 4. *If the universe is constructible from a set, then the Mantle, the generic Mantle and the generic* HOD *coincide. In particular, they all are inner models of* ZFC*.*

In ongoing research with Ralf Schindler, we are trying to analyze the set-theoretic geology of $L[E]$ models which have inner models with Woodin cardinals. Having inner models with Woodin cardinals allows us to use genericity iterations, a tool which once again proves to be very useful.

The first case we analyzed is that V is constructible from a set and has an iterable inner model with a Woodin cardinal. In this case, we find an iterable inner model with a Woodin cardinal, M, such that the mantle of V is the intersection of all linear iterates of M arising by repeatedly hitting its least total measure.

The second case is that V is an $L[E]$ model with a Woodin cardinal δ and unboundedly many cutpoints, such that there is a fully iterable Q-structure for $J_\delta[E]$. In this case, we show that the Mantle is contained in the intersection of all linear iterates of $L[E]$ arising by repeatedly hitting its least total measure.

On a strength of no normal precipitous filter

Moti Gitik
(joint work with Liad Tal)

The notion of a precipitous filter was first introduced by T. Jech and K. Prikry in [4]:

Definition 1. *A filter F is* precipitous *if for every generic $G \subseteq F^+$, the ultrapower $Ult(V, G)$ is well-founded.*

They asked whether the existence of a precipitous filter over κ implies the existence of a normal precipitous filter over κ.

H-D.Donder and J-P.Levinski [1] introduced the following notion:

Definition 2. *A cardinal κ is called* ∞-semi-precipitous *iff there exists a forcing notion P such that the following is forced by the weakest condition:*
there exists an elementary embedding $j : V \to M$ with critical point κ and M transitive.

Clearly, if there is a precipitous filter over κ, then κ is ∞-semi-precipitous - just take P to be the forcing with the positive sets.

E. Schimmerling and B. Velickovic [8] proved that there is no precipitous ideal on $\aleph_1$ in $L[E]$ models up to at least a Woodin limit of Woodins. On the other hand $\aleph_1$ is always ∞-semi-precipitous in presence of a Woodin cardinal. So ∞-semi-precipitousness need not imply precipitousness at least in presence of large enough cardinals.

In the opposite direction the following was shown in [2](Thm. 3.11):

Theorem 3. *Assume that:*

(1) $\aleph_1$ is ∞-semi-precipitous.
(2) $2^{\aleph_1} = \aleph_2$.
(3) There is no inner model satisfying $(\exists\alpha\ o(\alpha) = \alpha^{++})$.
(4) $\aleph_3$ is not a limit of measurable cardinals in the core model.

Then there exists a normal precipitous filter on $\aleph_1$.

There is a huge gap between a Woodin cardinal and infinitely many measurable cardinals. The purpose of this paper is to improve Theorem 3 and to narrow the gap. Some methods developed here likely to be useful for other purposes as well.

Our aim is to prove the following theorem:

Theorem 4. *Assume that:*

(1) $\aleph_1$ *is* ∞*-semi-precipitous.*
(2) $2^{\aleph_0} = \aleph_1$ *and* $2^{\aleph}_1 = \aleph_2$.
(3) There is no inner model with a strong cardinal.
(4) In the core model, the set $\{\alpha < \aleph_3 \mid o(\alpha) \geq \alpha^+\}$ *is bounded in* $\aleph_3$.

Then there exists a normal precipitous filter on $\aleph_1$.

The main idea of the proof is to use specially chosen covering sets. For this purpose the following game is considered: Let $\mathcal{G}$ be a game (in V) with the following rules:

(R1) In step $2n$, player I chooses some function $g_n : \kappa \to [\lambda, \kappa^{++})$.
(R2) In step $2n+1$, Player II chooses a covering model X_n and an h^{X_n}-coherent system of indiscernibles $\mathcal{C}_n \subseteq \mathcal{C}^{X_n}$, such that $ran(g_n) \subseteq h^{X_n}[\mathcal{C}_n]$.
(R3) For every $n < m < \omega$, $\mathcal{C}_n \subseteq \mathcal{C}_m$, and for every $c \in \mathcal{C}_n$, $\alpha^{X_n}(c) = \alpha^{X_m}(c)$.

Player II wins if the game continues infinitely many steps. Otherwise (i.e, a step was reached in which player II cannot make a step such that rules (R2) and (R3) hold) player I wins.

It is shown that Player II has a winning strategy.

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On the consistency strength of the negation of the singular cardinals hypothesis without the axiom of choice

Peter Koepke

We present a fairly complete proof of the following theorem, which determines the consistency strength of a *surjective* failure of SCH.

Theorem 1. *For a fixed* $\alpha \geq 2$*, the following theories are equiconsistent:*

$$ZFC + \exists\kappa[\kappa \text{ is measurable}]$$

and

$$ZF + \neg AC + GCH \text{ holds below } \aleph_\omega + \text{There is a surjective } f : [\aleph_\omega]^\omega \to \aleph_{\omega+\alpha}.$$

The forcing direction starts from a ground model with a measurable cardinal κ and satisfying GCH. We define a *parallel Prikry forcing* which adjoins $\kappa^{+\alpha}$ Prikry sequences to a fixed measure on κ. The forcing interweaves the Prikry sequences in a regular ("parallel") fashion. Form a symmetric submodel N of the generic extension, generated by the system of equivalence classes modulo finite of the Prikry sequences. Initial cardinals are absolute between the ground model and N and no new bounded subsets of $\aleph_\omega$ are added. So GCH holds below κ. On the other hand, mapping countable subsets of κ to (the index of) their equivalence class modulo finite gives a surjection from $[\kappa]^\omega$ onto λ.

To get the failure of SCH at $\aleph_\omega$ we collapse the elements of some Prikry sequence to $\aleph_n$s and make $\kappa = \aleph_\omega$.

For the converse, assume that there is no inner model with a measurable cardinal so that the Dodd-Jensen Covering Theorem for the core model K can be applied. By a standard covering argument a surjection $f : [\kappa]^\omega \to \aleph_{\omega+\alpha}$ where $\kappa = \aleph_\omega$ can be turned into a surjection $f' : \mathcal{P}(\aleph_2) \times \mathcal{P}^K(\kappa) \to \aleph_{\omega+\alpha}$. This contradicts the GCH assumption.

This research is part of a project with Arthur Apter who studied *injective* failures of SCH without the axiom of choice. Injective failures appear to have higher consistency strengths. A joint paper will appear in the JSL, a preprint is already available through our personal homepages.

Proper Distributive Forcing Axiom and Squares

Menachem Magidor

The Proper Forcing Axiom (PFA) is the statement that for every proper forcing notion P and a list of ω_1 dense subsets of P $\langle D_\alpha | \alpha \in \omega_1 \rangle$ there is a filter $G \subseteq P$ such that $G \cap D_\alpha \neq \emptyset$ for every $\alpha < \omega_1$. PFA is of course incompatible with CH in a very strong sense. An attempt to get a forcing axiom that is closer in character to CH is the following "Proper Distributive Forcing Axiom."

PDFA is the statement : If P is a proper σ distributive forcing notion and for every list of ω_1 dense subsets of P, there is a filter $G \subseteq P$ intersecting each of the given dense sets.

Like PFA, PDFA implies that $2^{\aleph_0} = \aleph_2$ but on the other hand it is consistent with several statements that are usually associated with CH like e.g. the bounding number $b = \aleph_1$. The main part of the talk was devoted to the proof of

Theorem 1. *Assuming the consistency of of a supercompact cardinal one can construct a model of PDFA in which $\square_{\kappa,\omega_1}$ holds for every κ. This is the best possible because PDFA implies that $\square_{\kappa,\omega}$ fails for every cardinal κ.*

A key tool in the proof of the previous Theorem is the following iteration theorem which may be useful in other contexts:

Definition 2. *Given a sequence of ω_1 reals $\langle a_\alpha | \alpha \in \omega_1 \rangle$ we say that a proper forcing notion P is nice with respect to the given sequence of reals if for large enough H_θ and every countable elementary substructure of H_θ N and every $p \in$*

$N \cap P$ if $a_{N\cap\omega_1}$ is Cohen generic over N then there is $q \leq p$ such that $q \Vdash$ '$G \cap N$ is generic over N' and $q \Vdash$ '$a_{N\cap\omega_1}$ is Cohen generic over $N[G \cap N]$.'

Theorem 3. *For every sequence of reals $\langle a_\alpha | \alpha \in \omega_1\rangle$ the countable support iteration of proper forcing notions which are nice with respect to the sequence , is nice with respect to the sequence.*

Remarks on covering

William Mitchell

I discussed several examples of situations in which it seemed possible that there might be a strengthened version of the covering lemma. The most interesting of these is in the case when there is a 1-iterable model with a Woodin cardinal, but no sharp for such a model. In this case the stationary tower forcing establishes that no model which is unchanged by forcing can be proved to have the weak covering property. I would like to know whether there is a possible version of the covering lemma in this case which would characterize the possible generic extensions of this model. As a test question, I asked the following: Suppose that $L[\mathcal{E}]$ is an extender model, and $L[\mathcal{E}][G]$ is an extension via the nonstationary tower forcing, yielding an embedding $L[\mathcal{E}] \to L[\mathcal{E}']$. Let M be the least mouse in $L[\mathcal{E}'] \setminus L[\mathcal{E}]$. Then is $L[\mathcal{E}][M] = L[\mathcal{E}][\mathcal{E}']$?

A necessary use of strong induction for a reversal

Itay Neeman

My talk gave an example of a necessary use of strong induction for a reversal, in reverse mathematics.

Reverse mathematics deals with calibrating the strength of theorems of second order number theory (a.k.a. analysis). Strength is measured relative to a hierarchy of systems of axioms. It was realized early on that full induction is not needed in the base system, and so today the standard base system is RCA_0, consisting of the axioms of Peano arithmetic other than induction, Δ^0_1 comprehension, and induction limited to Σ^0_1 formulas.

There are of course theorems that reverse to system that include stronger induction, but it had seemed that for the *base* system in which the reversal is proved, Σ^0_1 induction suffices.

The origin for the work presented is my earlier work on the strength of Jullien's indecomposability theorem. The theorem states that if a scattered countable linear order is indecomposable, then it is either indecomposable to the left, or indecomposable to the right. The theorem was shown by Montalbán to be a theorem of hyperarithmetic analysis, and then, in the base system RCA_0 plus Σ^1_1 induction, I showed that its strength strictly between weak Σ^1_1 choice and Δ^1_1 comprehension.

It was expected at the time that, as usual, the use of Σ^1_1 induction in the reversal from the theorem to weak Σ^1_1 choice would prove unnecessary. This turns out not to be the case. In my talk I sketched a proof that the use is necessary. That is, in any system with weaker induction, the reversal fails. Put precisely:

Theorem 1. *In the system* $\mathsf{RCA}_0 + \Delta^1_1$ *induction, Jullien's indecomposability theorem does not imply weak* Σ^1_1 *choice.*

The proof of the theorem involves a combination of Steel forcing, and non-standard extensions of models of second order number theory.

The mouse set conjecture

Grigor Sargsyan

We will introduce the notion of a hod mouse and will establish their basic properties. A hod mice were first investigated by Woodin who studied the first ω levels of hod mice. In this sequence of talks, we will introduce hod mice below $AD_{\mathbb{R}}$ + "Θ is regular". A hod mouse is a *rigidly layered hybrid mouse.* A layered hybrid mouse is a hybrid mouse that is closed under the strategy its own initial segments. If $\mathcal{M}$ is a layered hybrid mouse then ν is a layer if after stage ν, $\mathcal{M}$ is being told how to iterate $\mathcal{M}|\nu$. A rigidly layered hybrid mouse is one in which all layers are cardinals. A hod mouse is rigidly layered hybrid mouse in which all layers are either Woodin cardinal or a limit of Woodin cardinals. We say $(\mathcal{P}, \Sigma)$ is a hod pair if $\mathcal{P}$ is a hod mouse and Σ is a strategy for $\mathcal{P}$ that has *hull condensation.* Given two hod pairs $(\mathcal{P}, \Sigma)$, let $\mathcal{M}_\infty(\mathcal{P}, \Sigma)$ be the direct limit of all Σ-iterates of $\mathcal{P}$. For the theory to have applications, one has to prove two things. 1 $\mathcal{M}_\infty(\mathcal{P}, \Sigma)$ is well defined and 2 $\mathcal{M}_\infty(\mathcal{P}, \Sigma) \subseteq \mathcal{H}$ (in fact, $= \mathcal{H}$). 1 is usually solved by showing that Σ has various Dodd-Jensen like properties and 2 is solved by comparison. The goal of the talks would be to illustrate how one proves comparison for hod pairs that are below $AD_{\mathbb{R}}$ + "Θ is regular". There are several applications of the theory of hod mice. For instance, one can use it to obtain an upper bound for the theory $AD_{\mathbb{R}}$ + "Θ is regular" which turns out to be weaker than Woodin limit of Woodins (contrary to existing expectations). By a result of Woodin, this implies that $MM(c)$ is weaker than a Woodin limit of Woodins.

K without the measurable

John R. Steel

(joint work with Ronald B. Jensen)

In this talk, we shall discuss

Theorem 1 (Jensen, Steel)**.** *There are* Σ_2 *formulae* $\psi_K(v)$ *and* $\psi_\Sigma(v))$ *such that, if there is no transitive proper class model satisfying* ZFC *plus "there is a Woodin cardinal", then*

(1) $K = \{v \mid \psi_K(v)\}$ *is a transitive proper class extender model satisfying* ZFC*,*

(2) $\{v \mid \psi_\Sigma(v)\}$ *is an iteration strategy for* K *for set-sized iteration trees, and moreover the unique such strategy,*
(3) *(Generic absoluteness)* $\psi_K^V = \psi_K^{V[g]}$, *and* $\psi_\Sigma^V = \psi_\Sigma^{V[g]} \cap V$, *whenever* g *is* V*-generic over a poset of set size,*
(4) *(Inductive definition)* $K|(\omega_1^V)$ *is* Σ_1 *definable over* $(J_{\omega_1}(\mathbb{R}))$,
(5) *(Weak covering) For any* K*-cardinal* $\kappa \geq \omega_2^V$, $cof(\kappa) \geq |\alpha|$; *thus* $\alpha^{+K} = \alpha^+$, *whenever* α *is a singular cardinal of* V *(Mitchell, Schimmerling* [1]*).*

It is easy to formulate this theorem without referring to proper classes, and so formulated, the theorem can be proved in ZFC. The theorem as stated can be proved in GB.

Items (1)-(4) say that K is absolutely definable and, through (1), that its internal properties can be determined in fine-structural detail. Notice that by combining (3) and (4) we get that for any uncountable cardinal μ, $K|\mu$ is Σ_1 definable over $L(H_\mu)$, uniformly in μ. This is the best one can do if $\mu = \omega_1$ (see [2, §6]).

The hypothesis that there is no proper class model with a Woodin cardinal in the theorem cannot be weakened, unless one simultaneously strengthens the remainder of the hypothesis, i.e., ZFC.

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The fine structure of suitable extender sequences

W. Hugh Woodin

We discuss the generalization of [2] and [3] to the case of suitable extender sequences, [4]. The approach is to use a combination of Jensen-indexing and Mitchell-Steel-indexing for the extenders. This leads to two types of active premice where in Jensen's approach there is only one type and where in the Mitchell-Steel approach there are three types.

Jensen in [1] introduced Jensen-indexing (defined as λ-indexing) as a modification of Mitchell-Steel-indexing in order to provide more uniformity to the fine-structure theory. In particular with Jensen-indexing the relevant structures are always amenable and the squashed premice of [2] disappear. However in the case of long extenders this does not seem to be possible without squashing and so the price paid for having a fine structure theory with long extenders is the resurrection of squashed premice.

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A fine structural non-threadable square sequence

Martin Zeman

A nonthreadable square sequence $\square(\lambda)$ where λ is regular is a sequence $\langle c_\alpha \mid \alpha \in \lim \cap \lambda \rangle$ such that each c_α is a closed unbounded subset of α, $c_{\bar\alpha} = c_\alpha \cap \bar\alpha$ whenever $\bar\alpha$ is a limit point of c_α and there is no closed unbounded subset C of λ that threads $\langle c_\alpha \rangle_\alpha$, i.e. that satisfies the requirement $C \cap \alpha = c_\alpha$ for all $\alpha \in \lim(C)$. We show that in Jensen-type extender model, a $\square(\kappa^+)$-sequence exists for every regular κ granting that there is a nonreflecting stationary subset of κ^+ concentrating on ordinals of cofinality strictly smaller than κ. It is likely that this construction can be refined to yield a coherent witness for non-reflection. It is known that $\square(\kappa^+)$ fails if κ is quasicompact, and quasicompact cardinals may exist in extender models. Hence some smallness assumption is necessary for a construction of a $\square(\kappa^+)$-sequence, in our case the smallness assumption is that on the existence of a nonreflecting stationary set. This work is joint with Kypriotakis.

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