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Mini-Workshop: Modular Representations of Symmetric Groups and Related Objects

Organised by
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April 24th – April 30th, 2011

Abstract. The mini-workshop focussed on the modular representation theory of the symmetric group and other closely related objects, including Hecke algebras and Schur algebras. The topics and problems discussed include computations of support varieties, vertices and sources for natural choices of symmetric group modules such as simple modules, Specht modules, and Lie modules, results on Carter–Payne homomorphisms and irreducible Specht modules, connections of symmetric group cohomology with algebraic group cohomology and algebraic topology, and positions of natural symmetric group modules in the Auslander–Reiten quiver.

Mathematics Subject Classification (2000): 20C30 (primary); 20C08, 20G05, 20J06, 05E10 (secondary).

Introduction by the Organisers

The Mini-Workshop *Modular Representations of Symmetric Groups and Related Objects*, organized by Susanne Danz (University of Oxford) and David Hemmer (University at Buffalo, SUNY), and attended by 17 participants was held April 24th–April 30th, 2011.

The representation theory of the symmetric group $\mathfrak{S}_n$ has been a highly active field of research for the past century. Whilst the theory over fields of characteristic 0 is rather well understood, the picture changes drastically when working over a field F of prime characteristic p, where even a number of very basic questions cannot be answered satisfactorially. In particular, the simple modules as well as the *Specht modules*, which play an outstanding role in the representation theory of the symmetric groups and Hecke algebras, are yet far from being fully understood.

It is, for instance, still not known which Specht $F\mathfrak{S}_n$-modules are indecomposable in characteristic 2; a similar problem arises for Specht modules of Hecke algebras. Only very recently was the classification of simple Specht modules for $F\mathfrak{S}_n$ completed by Fayers, while the problem remains open for the corresponding Hecke algebra.

The goal of this mini-workshop was, thus, to bring together leading experts in the representation theory of the symmetric groups and related objects (such as Hecke algebras and Schur algebras) with a broad background, in order to discuss recent developments, and break the ground for new progress on long-standing problems in the field.

The meeting addressed, in particular, the following aspects:

- cohomology of symmetric groups, extensions and homomorphisms between $F\mathfrak{S}_n$-modules, in particular, Specht modules (Fayers, Hemmer, Lyle, Nakano);
- vertices and sources of natural classes of $F\mathfrak{S}_n$-modules (Danz, Külshammer, Wildon);
- Auslander–Reiten theory (Erdmann);
- computing decomposition numbers for symmetric groups theoretically (Tan) and algorithmically (Müller);
- block theory of centralizer algebras related to the symmetric groups (Ellers, Murray);
- classification results concerning quasi-hereditary covers of Hecke algebras (Ariki);
- the Külshammer–Olsson–Robinson Conjecture on generalized ℓ-blocks of symmetric groups (Hill);
- generalized hook lengths and hook formulas (Olsson);
- the Foulkes Conjecture (Paget);
- spin fake and generic degrees for symmetric groups (Wang).

Every participant contributed a talk of about 50 minutes, making the workshop a very lively and productive event, and yet leaving ample time for informal discussions. An additional problem session on Thursday afternoon completed the programme. A summary of the questions raised at the problem session can be found at the end of this report.

We are delighted to thank the director, the administration, and the staff of the MFO for their hospitality and support throughout the meeting.

Mini-Workshop: Modular Representations of Symmetric Groups and Related Objects

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Abstracts

Quasihereditary Covers of Hecke Algebras of Type B_2

Susumu Ariki

It is now well known that integrable modules over a Kac–Moody Lie algebra are categorified by certain finite-dimensional algebras. In the classical case of the basic module of the affine Lie algebra in type A, the algebras are block algebras of Hecke algebras of type A. In our Fock space theory, the integrable module is embedded into the level one Fock space, and the latter is categorified by using q-Schur algebras. Let us consider the integrable modules in level two. Then they are categorified by block algebras of Hecke algebras of type B. Embed the module into the corresponding level two Fock space. Then, by using the category $\mathcal{O}$ for the rational Cherednik algebra (in type B), Rouquier [4] showed that the Fock space is categorified by quasihereditary covers of the Hecke algebras again. He also showed that if the parameters of the Hecke algebra are not equal to -1, then this quasihereditary cover is 0-faithful. Recall that, assuming the conjectures (P1)-(P15) by Lusztig, Geck showed that Hecke algebras of type B have various cellular algebra structures. Then the quasihereditary structures are related with the cellular algebra structures: the Schur functor sends the standard modules to Geck's cell modules [3]. The picture is explained in [2].

Based on these examples, some people have constructed a general theory of 1-faithful covers and the other people study Hemmer–Nakano phenomenon etc. Hence, it seems that people expect the validity of

(i) the existence of 0-faithful (or even 1-faithful) covers and its uniqueness,
(ii) the Schur functor sending standard modules to cell modules,

for a large class of cellular algebras. In this talk, I have explained the classification of quasihereditary covers with 5 simple modules (5 is the number of bipartitions of size 2) for the Hecke algebras in type B_2 and how (i) and (ii) fail in the cases when one of the parameters is -1. There exist quasihereditary covers for each of the cellular algebra structures, none of which is 0-faithful, and the Schur functor sends standard modules to cell modules in most cases but the image may be zero, or a direct sum of simple modules of various multiplicities in some cases.

The idea of the classification is that as we know that block algebras in type B_2 are either Nakayama algebras or special biserial algebras of very simple type by [1], we may compute all the possible quasihereditary covers by brute force calculation. The computation was carried out by my student Kazumasa Harada.

References

[1] S. Ariki, *Hecke algebras of classical type and their representation type*, Proc. London Math. Soc. (3) **91** (2005), 355–413.
[2] S. Ariki, *Finite dimensional Hecke algebras*, Trends in Representation Theory of Algebras and Related Topics, EMS series of Congress reports **2** (2008), 1–48.
[3] M. Chlouveraki, I. Gordon and S. Griffeth, *Cell modules and canonical basic sets for Hecke algebras from Cherednik algebras*, arXiv:1104.4070.

[4] R. Rouquier, *q-Schur algebras and complex reflection groups I*, Moscow Math. J. **8** (2008), 119–158.

The Lie Module of the Symmetric Group — Some Computational Experiments

Susanne Danz
(joint work with Jürgen Müller)

The Lie module $\mathrm{Lie}_p(n)$ of the symmetric group $\mathfrak{S}_n$ over a field F of characteristic $p > 0$ appears naturally in questions concerning the free Lie algebra, in homology theory, and in algebraic topology. If p does not divide n then $\mathrm{Lie}_p(n)$ is always projective, but if p divides n then $\mathrm{Lie}_p(n)$ has a non-zero projective-free part $\mathrm{Lie}_p^{\mathrm{pf}}(n)$. So, what can one say about $\mathrm{Lie}_p^{\mathrm{pf}}(n)$ in the latter case? How 'big' is it compared with $\mathrm{Lie}_p(n)$? And 'how far' is it from being projective? There are some general results known when n is not a p-power, and there are partial results reducing the case of arbitrary n to the case where n is a p-power. For n being a p-power, however, almost no information is available in the literature, even when it comes to very small examples.

In my talk I presented some computational data, obtained in joint work with Jürgen Müller, concerning indecomposable direct summands of $\mathrm{Lie}_2^{\mathrm{pf}}(8)$ and $\mathrm{Lie}_3^{\mathrm{pf}}(9)$ as well their vertices and sources (for a definition of these notions see Burkhard Külshammer's report). Building on the computational data, the following general questions arise:

Question 1. Let $k \geq 0$, and let $n := p^k$.

(a) Is $\mathrm{Lie}_p^{\mathrm{pf}}(n)$ indecomposable?

(b) If so, does $\mathrm{Lie}_p^{\mathrm{pf}}(n)$ have vertex E_n — an elementary abelian group of order n, acting regularly on $\{1, \ldots, n\}$? Are, moreover, the sources of $\mathrm{Lie}_p^{\mathrm{pf}}(n)$ always *endo-permutation modules*? What are their isomorphism types?

(c) What happens when n is not a p-power?

Endo-permutation modules for p-groups have been introduced by Dade [3] as generalizations of permutation modules. Furthermore, for abelian p-groups P, a classification of the isomorphism classes of indecomposable endo-permutation FP-modules has also been obtained by Dade [4].

Our computational data show that for $\mathrm{Lie}_2^{\mathrm{pf}}(8)$ and $\mathrm{Lie}_3^{\mathrm{pf}}(9)$, Parts (a) and (b) in Question 1 admit a positive answer. In these cases, we have also determined the explicit isomorphism types of the sources of $\mathrm{Lie}_p^{\mathrm{pf}}(n)$ (in the sense of Dade's classification).

As for Question 1(c), in ongoing work with R. Bryant, K. Erdmann, and J. Müller, we have formulated a reduction theorem expressing vertices of indecomposable direct summands of $\mathrm{Lie}_p^{\mathrm{pf}}(n)$ in terms of vertices of indecomposable direct summands of $\mathrm{Lie}_p^{\mathrm{pf}}(p^k)$ $(k \geq 0)$. The proof of this theorem uses recent results of

Bryant–Erdmann [1], and of Bryant–Lim–Tan [2], as well as work of Külshammer [5] on vertices of indecomposable modules for wreath products.

References

[1] R. M. Bryant, K. Erdmann, *Block components of the Lie module for the symmetric group*, preprint, 2011.

[2] R. M. Bryant, K. J. Lim, K. M. Tan, *Asymptotic behaviour of Lie powers and Lie modules*, preprint, 2010.

[3] E. C. Dade, *Endo-permutation modules over p-groups I*, Ann. Math. **107** (1978), 459–494.

[4] E. C. Dade, *Endo-permutation modules over p-groups II*, Ann. Math. **108** (1978), 317–346.

[5] B. Külshammer, *Some indecomposable modules and their vertices,* J. Pure and Applied Algebra **86** (1993), 65–73.

Representations, Blocks, and Centers of Centralizer Algebras

Harald Ellers

(joint work with John Murray)

If R is a commutative ring, G is a finite group, and H is a subgroup of G, the *centralizer algebra* is $RG^H = \{a \in RG \mid ah = ha \quad \forall h \in H\}$. The speaker has a long-running project with John Murray to try to understand representation theory of ${kS_n}^{S_l}$ where k is a field of finite characteristic p, S_n is the symmetric group on n letters, $l \leq n$, and S_l is the subgroup of S_n consisting of all permutations that fix every number larger than l.

More precisely, assume that (R, F, k) is a sufficiently large p-modular system, where R is a discrete valuation ring, F is its field of fractions of characteristic 0, and k is its residue field of characteristic p. We would like to be able to do the following.

(1) Find the simple ${FS_n}^{S_l}$-modules.
(2) Find the simple ${kS_n}^{S_l}$-modules.
(3) Find the blocks. (Find a version of Nakayama's conjecture.)
(4) Find the decomposition matrices (at least in small cases).
(5) Relate all the above to p-local information. This means we want a version of Brauer's First Main Theorem for blocks of ${kS_n}^{S_l}$ and a version of Alperin's weight conjecture for simple ${kS_n}^{S_l}$-modules.

The first of these has long been well known — the simple ${FS_n}^{S_l}$-modules are all of the form $\mathrm{Hom}_{S_l}(U, V\downarrow_{S_l})$, where U and V are simple FS_l-modules, and simple FS_n-modules respectively.

For $l \geq (n-3)$, the block idempotents of ${kS_n}^{S_l}$ are all of the form ef, where e is a block idempotent of kS_n and f is a block idempotent of kS_l. In these cases, we also have fairly complete information about decomposition matrices. We conjecture that blocks of ${kS_n}^{S_l}$ will arise this way in all cases. This would be a consequence of a stronger conjecture: we expect that $\mathrm{Z}({\mathbb{Z}S_n}^{S_l})$ is generated as a $\mathbb{Z}$-algebra by $\mathrm{Z}(\mathbb{Z}S_n)$ and $\mathrm{Z}(\mathbb{Z}S_l)$. (There are examples of groups G and subgroups

H for which kG^H has block idempotents not in the algebra generated by $\mathrm{Z}(kG)$ and $\mathrm{Z}(kH)$. An example was found by Susanne Danz.)

For $l = n-1$, we have satisfactory versions of Brauer's Theorem and Alperin's Conjecture.

A key tool is a surjective map $\mathcal{H}_{n-l}^k \to k{S_n}^{S_l} f/\langle \mathrm{J}(\mathrm{Z}(kS_l))f\rangle$, where $\mathcal{H}_{n-l}^k$ is the degenerate affine Hecke algebra, f is a block idempotent of kS_l, and $\langle \mathrm{J}(\mathrm{Z}(kS_l))f\rangle$ is the ideal generated by the Jacobson radical of the center of the block associated to f.

The focus of this talk is the case $l = n-1$. Murray's talk is about $l = n-2$ and $l = n-3$.

Auslander–Reiten Sequences for Symmetric Groups

Karin Erdmann

(joint work with Susanne Danz)

The Auslander–Reiten quiver $\Gamma(B)$ of a finite-dimensional algebra B is the directed graph with vertices labelled by the isomorphism classes $[M]$ of indecomposable B-modules M, and where the arrows are defined in terms of irreducible maps (for details we refer to [1]). This can be thought of as part of a presentation of the module category.

Assume B is symmetric. Then the stable Auslander–Reiten quiver $\Gamma_s(B)$ is obtained from $\Gamma(B)$ by removing the indecomposable projective modules. When B is a block of wild representation type of some group algebra FG then by [4] any component of $\Gamma_s(B)$ is isomorphic to $\mathbb{Z}A_\infty$, or is a tube (isomorphic to $\mathbb{Z}A_\infty/\langle\Omega^{2d}\rangle$). If M is indecomposable non-projective, then the quasi-length $ql(M)$ of M is defined to be the distance of $[M]$ to the end of the component. One expects that distinguished classes of modules have small quasi-length.

Now assume $G = \mathcal{S}_n$, and let B be a wild block of FG. By [2], all simple modules in B have quasi-length 1. Consider the quasi-length of Specht modules S^λ in B. T. Jost has proved that $ql(S^\lambda) = 1$ if S^λ has simple socle and simple top [5]. Extending his technology, we prove

Proposition [3] *Assume M is a B-module with simple socle or a simple top. Then $ql(M) \leq 3$. Furthermore, if $ql(M) > 1$ then the component of M contains a simple module D which is exceptional.*

Here we call a simple module D exceptional if there is a unique simple module S such that $\mathrm{Ext}^1(D,S) \neq 0$. We have further detailed structural information, and this allows us, using other known results, to prove the following:

Theorem [3] *Assume B has p-weight 2, or B has weight 3 and $p = 3$. Then all Specht modules in B have quasi-length 1.*

The following reduction result applies to many Specht modules when p is large (the precise definitions can be found in [3]).

Theorem. [3] *Assume B and $\bar{B}$ are wild blocks of symmetric groups which form a $[w : k]$-pair. Suppose S^λ in B corresponds to $S^{\bar{\lambda}}$ in $\bar{B}$ under the partial Scopes equivalence. If λ and $\bar{\lambda}$ are sufficiently far away from the exceptional part of the block then $ql(S^\lambda) = ql(S^{\bar{\lambda}})$.*

References

[1] D. J. Benson, Representations and cohomology. I. Basic representation theory of finite groups and associative algebras. Cambridge Studies in Advanced Mathematics, 30. Cambridge University Press, Cambridge, 1991.

[2] C. Bessenrodt, K. Uno, *Character relations and simple modules in the Auslander–Reiten graph of the symmetric and alternating groups and their covering groups*, Algebr. Represent. Theory **4** (2001), no. 5, 445–468.

[3] S. Danz, K. Erdmann, *Specht modules in the Auslander–Reiten quiver,* preprint 2010.

[4] K. Erdmann, *On Auslander-Reiten components for group algebras*, J. Pure Appl. Algebra **104** (1995), no. 2, 149–160.

[5] T. Jost, *On Specht modules in the Auslander-Reiten quiver*, J. Algebra **173** (1995), no. 2, 281–301.

Yet Another Talk on Irreducible Specht Modules

Matt Fayers

(joint work with Sinéad Lyle)

We report on some recent progress with the classification of irreducible Specht modules for the Iwahori–Hecke algebra $\mathcal{H}$ in type A. The only outstanding case of this problem is where the quantum parameter q equals -1. We assume here that the underlying characteristic is zero. In this case, the author and Mathas have a conjectured classification of irreducible Specht modules, based upon a strange combinatorially defined class of partitions which we call *FM-partitions*. Our result is that half of this conjecture is true: the conjecturally reducible Specht modules really are reducible.

Apart from the results in our earlier paper [1] and standard techniques using induction and restriction of Specht and simple modules, there are two main tools used to prove reducibility of Specht modules.

Fock space calculations. Let $\mathcal{F}$ denote the q-deformed Fock space for the quantum algebra $U_v(\widehat{\mathfrak{sl}}_2)$. The submodule V generated by the empty partition has a Kashiwara–Lusztig *canonical basis* $\{G(\mu)\}$ indexed by the set of 2-regular partitions. Ariki's Theorem says that when the transition coefficients between the canonical basis for V and the standard basis for $\mathcal{F}$ are specialised at $v = 1$, the resulting integers are precisely the decomposition numbers for $\mathcal{H}$. Since these (unspecialised) coefficients are known to be polynomials with non-negative integer coefficients, the assumption that a Specht module is irreducible has strong implications for the canonical basis. We exploit this in the following statement: if we can find bar-invariant elements X, Y of V in which the coefficients of the standard basis element λ are *different* powers of v, then S^λ must reducible. For certain partitions λ, we can explictly construct such X, Y by starting with the canonical basis element $G(\nu)$ for ν an 'alternating' partition contained in λ and with the same number of parts as λ. So we obtain reducibility of a large class of Specht modules.

Homomorphisms between Specht modules. There has been a lot of interest in recent years in the computation of the space of homomorphisms between two Specht modules. This has received a boost recently, with the proof by Lyle [3] of two lemmas first proved by Fayers and Martin [2] for the symmetric group, which facilitate computation of homomorphisms. If we have an $\mathcal{H}$-homomorphism $S^\mu \to S^\lambda$, then by using dominance considerations, we can often deduce that S^λ is reducible.

Using Lyle's machinery, the author was able to construct explicit homomorphisms to prove the reducibility of a certain class of Specht modules. This class is not particularly natural: it simply consists of the list of all Specht modules which we were unable to prove reducible by any other means.

References

[1] M. Fayers, S. Lyle, *Some reducible Specht modules for Iwahori–Hecke algebras of type A with $q = -1$'*, J. Algebra **321** (2009) 912–933.
[2] M. Fayers, S. Martin, *Homomorphisms between Specht modules*, Math. Z. **248** (2004) 395–421.
[3] S. Lyle, *On homomorphisms indexed by semistandard tableaux*, arXiv:1101.3192.

A Variety of Approaches to Specht Module Cohomology

David J. Hemmer

In this talk we discussed two different approaches to determining cohomology $\mathrm{H}^i(\Sigma_d, S^\lambda)$, where S^λ is a Specht module for the symmetric group Σ_d. Both approaches work only in odd characteristic.

The first uses Schur functor techniques to translate the problem (for small i) to the computation of $\mathrm{Ext}^i_B(\mathrm{H}^0(d), \lambda)$, where B is a Borel subgroup of $GL_d(k)$. Now using results of Doty on the structure of $\mathrm{H}^0(d)$ we can often prove vanishing and 'generic cohomology' type theorems. For example:

Theorem 1. [3] *Let p be odd and $\lambda \vdash d$. Then:*

$$\mathrm{H}^1(\Sigma_{pd}, S^{p\lambda}) \cong \mathrm{H}^1(\Sigma_{p^2d}, S^{p^2\lambda}).$$

We can also recover James' computation of $\mathrm{H}^0(\Sigma_d, S^\lambda)$ using this technique.

At this time we have no explicit realization of the isomorphism in Theorem 1 on the symmetric group side. In search of such a realization, we considered in [2] a combinatorial approach. The first step is to prove that (again for p odd), given a nonsplit short exact sequence:

$$0 \to S^\lambda \to U \to k$$

it must be the case that U embeds in the permutation module M^λ. Now the existence of such a U depends on finding a complementary vector $u \in M^\lambda$ so that the span $\langle S^\lambda, u\rangle$ gives the module U. Using James' famous kernel intersection theorem, this reduces to a combinatorial condition on u, written as a sum of tabloids. Using this we can make some computations, for example showing that $\mathrm{H}^1(\Sigma_{2p^a}, S^{(p^a,p^a)}) \neq 0$ and giving an explicit basis for a nonsplit extension, together with the symmetric group action.

We close by remarking that almost 30 years after James' computation of $\mathrm{H}^0(\Sigma_d, S^\lambda)$, we still do not have a description of $\mathrm{H}^1(\Sigma_d, S^\lambda)$, nor do we know if it can be more than one-dimensional.

References

[1] F. R. Cohen, D. J. Hemmer, D. K. Nakano, *Oh the cohomology of Young modules for the symmetric group*, Adv. Math. **224** (2010), 1419–1461.
[2] D. J. Hemmer, *A combinatorial approach to Specht module cohomology*, Algebra Colloq. to appear.
[3] D. J. Hemmer *Cohomology and generic cohomology of Specht modules for the symmetric group*, J. Algebra **322** (2009), 1498–1515.

Cartan Invariants of Symmetric Groups and Iwahori–Hecke Algebras

David Hill
(joint work with Christine Bessenrodt)

The theory of generalized blocks of symmetric groups was initiated by Külshammer, Olsson and Robinson in [8]. Using character-theoretic methods, they showed that many invariants of the usual block theory of symmetric groups over a field of characteristic p do not depend on p being a prime. This led the authors to define 'ℓ-blocks' of symmetric groups and a related ℓ-modular representation theory. They defined an appropriate analogue of the Cartan matrix associated to S_n for this theory and even conjectured that a certain set of numbers determined the invariant factors of this matrix [8, Conjecture 6.4]. In a related paper [3], Bessenrodt and Olsson conjectured a formula for the determinant of the Cartan matrix.

Using a new method developed in [9, 1], Brundan and Kleshchev [5] calculated an explicit formula for the determinant of the Cartan matrix of a block of the Iwahori–Hecke algebra, H_n, with parameter q a primitive ℓth root of unity. Donkin [6] showed that there is a direct link between ℓ-blocks of S_n and blocks of H_n. In particular, their respective Cartan matrices have the same determinant and invariant factors. Using this, together with the results of [5] and [3], Külshammer, Olsson and Robinson [8] verified the formula conjectured by Bessenrodt and Olsson [3] (see also the remarks at the end of [3]). It should also be noted that in [4], Bessenrodt, Olsson and Stanley obtained a more elementary proof of the formula for the determinant of the full Cartan matrix.

In [7], Hill investigated the invariant factors of the Cartan matrix associated to an individual block of H_n using the methods developed in [5]. When $\ell = p^r$ is a power of a prime satisfying $r \leq p$ these numbers were computed (see [7, Theorem 1.3]). Moreover, he conjectured that the same formula held for arbitrary r.

In [7] also another result was obtained. Namely, given the prime decomposition $\ell = p_1^{r_1} \cdots p_k^{r_k}$, the Cartan matrix of an ℓ-block of S_n is a product of Cartan matrices associated to $p_i^{r_i}$-blocks of S_n. Indeed, the invariant factors of the Cartan matrices of the $p_i^{r_i}$-blocks are nothing but the *elementary divisors* of the Cartan matrix of the associated ℓ-block. In particular, the invariant factors of the Cartan matrix associated to an ℓ-block of S_n can be recovered from the Cartan matrices associated to the $p_i^{r_i}$-blocks (see [7, Theorem 1.1, 1.2]).

In this talk, we explained recent joint work with C. Bessenrodt [2] in which we relate Hill's Conjecture to that of Külshammer, Olsson, and Robinson. In particular, we describe the precise relationship between [8, Conjecture 6.4], and the work [7], so that Hill's Conjecture is a refinement of the Küshammer–Olsson–Robinson Conjecture to blocks.

References

[1] S. Ariki, *On the decomposition numbers of the Hecke algebra of type $G(m,1,n)$*, J. Math. Kyoto Univ. **36** (1996), 789–808.

[2] C. Bessenrodt, D. Hill, *Cartan Invariants of Symmetric Grups and Iwahori–Hecke Algebras*, J. London Math. Soc. (2) **81** (2010) 113–128.

[3] C. Bessenrodt and J. B. Olsson, *A Note on Cartan Matrices for Symmetric Groups*, Arch. Math. **81** (2003), 497–504.

[4] C. Bessenrodt, J. Olsson, R. P. Stanley, *Properties of some character tables related to the symmetric groups*. J. Algebraic Combin. **21** (2005), no. 2, 163–177.

[5] J. Brundan, A. Kleshchev, *Cartan Determinants and Shapovalov forms*, Math. Ann. **324** (2002), 431–449.

[6] S. Donkin, *Representations of Hecke algebras and characters of symmetric groups*, in: Studies in Memory of Issai Schur, edited by A. Joseph, A. Melnikov, R. Rentschler, Progress in Mathematics **210** (2003), 49–67.

[7] D. Hill, *Elementary Divisors of the Shapovalov form on the basic representation of Kac–Moody algebras*, J. of Algebra, **319** (2008), 5208–5246.

[8] B. Külshammer, J.B. Olsson, and G.R. Robinson, *Generalized blocks for symmetric groups*, Invent. Math. **151** (2003), 513–552.

[9] A. Lascoux, B. Leclerc, and J.-Y. Thibon, *Hecke algebras at roots of unity and crystal bases of quantum affine algebras*, Comm. Math. Phys. **181** (1996), 205–263.

Vertices of Simple Modules for Symmetric Groups

Burkhard Külshammer

Let F be an algebraically closed field of characteristic $p > 0$, let G be a finite group, and let M be an indecomposable module over the group algebra FG. A *vertex* of M is a subgroup Q of G which is minimal with respect to the condition that the canonical homomorphism $FG \otimes_{FQ} M \longrightarrow M$ splits. In this case Q is a p-subgroup of G and unique up to conjugation. Moreover, there exists an indecomposable FQ-module V such that M is isomorphic to a direct summand of $FG \otimes_{FQ} V$. Then V is called a *source* of M; it is unique up to isomorphism and $N_G(Q)$-conjugation.

In the first part of my talk, I gave a survey on general properties of vertices and sources of indecomposable and, in particular, simple modules. Also, I stated Feit's finiteness conjecture (1980) on sources of simple modules and a related question by Puig (1994) on vertices of simple modules and defect groups of blocks.

In the second part of my talk, I concentrated on the special case where G is a finite symmetric group $\mathcal{S}_n$. In this case the simple FG-modules are parametrized by p-regular partitions λ of n. So far, there is no general conjecture on the structure of the vertices of the simple FG-modules D^λ. However, in recent years several special cases have been attacked successfully: hook partitions, two-part partitions, spin modules, completely splittable modules and blocks of small weight. I mentioned results by S. Danz, K. Erdmann, J. Müller, H. Wenzel, M. Wildon, R. Zimmermann and myself. Moreover, I stated several open problems and reported on computational aspects.

More details can be found in [2] and in the papers in the list of references of [2]; more recent results are contained in [1], [3], [4].

References

[1] S. Danz, K. Erdmann, *The vertices of a class of Specht modules and simple modules for symmetric groups in characteristic* 2, to appear in Algebra Colloq.

[2] S. Danz, B. Külshammer, *Vertices of simple modules for symmetric groups: a survey*, in: Proceedings of the International Conference on Modules and Representation Theory, 'Babeş-Bolyai' University, Cluj-Napoca 2008, pp. 61–77.

[3] M. Kiyota, T. Okuyama, T. Wada, *The heights of irreducible Brauer characters in* 2*-blocks of the symmetric groups*, preprint.

[4] H. Wenzel, *Vertizes einfacher Moduln kleiner Dimensionen der symmetrischen Gruppen*, Diplomarbeit, Jena, 2011.

Homomorphisms Between Specht Modules

Sinéad Lyle

Let F be a field of characteristic $p \geq 0$ and let $q \in F^\times$. Choose $e \geq 2$ to be minimal such that $1 + q + \ldots + q^{e-1} = 0$, where we assume that such an e exists, and define $\mathcal{H} = \mathcal{H}_{F,q}(\mathfrak{S}_n)$ to be the corresponding Hecke algebra of the symmetric group $\mathfrak{S}_n$. For each partition $\lambda \vdash n$, we may define a module S^λ, known as a Specht module. In this talk, we discuss a way of computing the homomorphism space $\mathrm{Hom}_{\mathcal{H}}(S^\mu, S^\lambda)$ for λ and μ partitions of n. We then apply our technique to define a family of pairs of partitions where the homomorphism space is at least 2-dimensional. The first part of the work appears in [2] and the second in [3].

Recall that for each partition μ of n, we may define an element $m_\mu \in \mathcal{H}$, a 2-sided right ideal $\mathcal{H}^{\rhd\mu}$ and cyclic right $\mathcal{H}$-modules $M^\mu = m_\mu\mathcal{H}$ and $S^\mu = (\mathcal{H}^{\rhd\mu} + m_\mu)\mathcal{H}$. A homomorphism $\Theta : M^\mu \to S^\lambda$ therefore factors through S^μ if and only if $\Theta(m_\mu h) = 0$ for all $h \in \mathcal{H}$ such that $m_\mu h \in \mathcal{H}^{\rhd\mu}$. In fact, for $d \geq 1$ and $1 \leq t \leq \mu_{d+1}$, we may define an element $h_{d,t} \in \mathcal{H}$ such that $\Theta : M^\mu \to S^\lambda$ factors through S^μ if and only if $\Theta(m_\mu h_{d,t}) = 0$ for all d, t. Now for every semistandard λ-tableaux T of type μ, we may define a homomorphism $\Theta_{\mathsf{T}} : M^\mu \to S^\lambda$, where these maps form a basis of the homomorphism space if $e \neq 2$ and are linearly independent otherwise. We show that we may write $\Theta_{\mathsf{T}}(m_\mu h_{d,t})$ as the image of a linear combination of homomorphisms indexed by row-standard λ-tableaux of type $\nu(d,t)$ acting on $m_{\nu(d,t)}$. We then provide a result that allows us to rewrite a homomorphism Θ_{S} as a linear combination of homomorphisms indexed by other tableaux. It appears (although we do not have a proof) that these results are always sufficient to compute $\mathrm{Hom}_{\mathcal{H}}(S^\mu, S^\lambda)$. The potential problem is that our second lemma may not be sufficient to write a homomorphism Θ_{S} in terms of homomorphisms indexed by semistandard tableaux. However, in practice, the magic homomorphism calculator of Fayers has never encountered this problem.

The first family of examples of large-dimensional spaces between Specht modules appeared recently in the work of Dodge [1]. Dodge shows that for $p \geq 5$ and $k(k+1)/2+1 < p$, there exist partitions μ and λ such that $\dim(\mathrm{Hom}_{F\mathfrak{S}_n}(S^\mu, S^\lambda)) = k$; using the row and column removal theorems, there then exist $\alpha, \beta \vdash m$ such that $\dim(\mathrm{Hom}_{F\mathfrak{S}_m}(S^\mu, S^\lambda)) \geq l$ for any integer l. The following result, proved using the method above, generalizes this last result to Hecke algebras with arbitrary parameters $p \geq 0$ and $2 \leq e < \infty$.

Theorem 1. *For $a \geq b \geq c+1 \geq 4$, define partitions*

$$\mu = \mu(a,b,c,e) = (ae-3, be-3, ce-3, e-1, e-1),$$
$$\lambda = \lambda(a,b,c,e) = ((a+2)e-5, be-3, ce-3)),$$

of some integer n. Then $\dim(\mathrm{Hom}_{\mathcal{H}}(S^\mu, S^\lambda)) \geq 2$.

References

[1] C. Dodge, *Large dimensional homomorphism spaces between Specht modules for symmetric groups*, J. Pure App. Alg., to appear; arXiv:1103.0246.

[2] S. Lyle, *On homomorphisms indexed by semistandard tableaux*; arXiv:1101.3192.

[3] S. Lyle, *Large dimensional homomorphism spaces between Weyl modules and Specht modules*; arXiv:1103.5874.

Techniques for Finding Decomposition Numbers

Jürgen Müller

0. Introduction.

The techniques we report on have largely been introduced in the framework of the **Modular Atlas Project** [Wilson, Parker et al., ≥1984], aiming at the determination of the Brauer characters of the finite almost quasi-simple groups in the **Atlas of Finite Groups**. In particular, results are almost complete for 18 of the 26 sporadic groups, and are available electronically in [http://www.math.rwth-aachen.de/homes/MOC/].

The methods used can be divided into character-theoretic and module-theoretic ones. Notably, they are of general nature and, for example, applicable to the **symmetric groups** and their close relatives as well. More precisely, for $\mathcal{S}_n$ and $\mathcal{A}_n$ results are complete for $n \leq 18$ [Benson, 1987; M., 2000; Maas, M. 2011];

note that blocks of weight ≤ 4 for $\mathcal{S}_n$ are known generically [Fayers, 2007]. For the faithful blocks of the **Schur covers** $\widehat{\mathcal{S}}_n$ and $\widehat{\mathcal{A}}_n$ results are complete for $n \leq 17$, and $n = 18$ and $p = 3$ (at the day of writing) [Maas, 2011]; note that blocks of cyclic defect for these groups are known generically [M., 2003].

1. Computations with characters. [Hiss, Jansen, Lux, Parker, ∼1985]

Let G be a finite group, and let $[K, \mathcal{O}, k]$ be a splitting p-modular system. Then a $\mathbb{Z}$-basis BS $\subseteq \mathbb{Z}_{\geq 0}[\mathrm{IBr}]$ of the Grothendieck group $G_0(\text{mod-}kG)$ is called a **basic set** of Brauer characters, and similarly a $\mathbb{Z}$-basis PS $\subseteq \mathbb{Z}_{\geq 0}[\mathrm{IPr}]$ of the Grothendieck group $G_0(\text{proj-}kG)$ is called a **basic set** of projective characters. While a PS can always be found by induction from proper subgroups [Fong, 1963], it is only conjectured that there always is a BS contained in $\mathrm{Irr}(G)$; note that for $G = \mathcal{S}_n$ induction from $\mathcal{S}_{n-1}$ suffices to find a PS, and the ordinary characters parametrised by p-regular partitions provide a BS $\subseteq \mathrm{Irr}(\mathcal{S}_n)$.

The natural pairing $\mathbb{Z}[\mathrm{IBr}] \times \mathbb{Z}[\mathrm{IPr}] \to \mathbb{Z}$, where IBr and $\mathrm{IPr} = \mathrm{IBr}^*$ are a pair of mutually dual bases, leads to the notion of **Brauer atoms** BA := PS* and **projective atoms** PA := BS*. Now the strategy is to consider BS and PS as **approximations** to IBr and IPr; to find further Brauer characters and projective characters, for example by induction, tensoring, or using special techniques such as modular branching rules, the Jantzen–Schaper formula, or Scopes reduction for $G = \mathcal{S}_n$; to decompose them into the basic sets; and use positivity or negativity results to improve the basic sets or to show irreducibility.

2. Computations with modules: condensation.

Let A be a finite-dimensional F-algebra, where F is a field, and let $e \in A$ be an idempotent. Then the exact functor $C_e :? \otimes_A Ae \cong \mathrm{Hom}_A(eA, ?) : \text{mod-}A \to \text{mod-}eAe : V \mapsto Ve$ is called the associated **condensation functor** or **Schur functor** [Green, 1978; Auslander, 1974].

Letting Σ be the set of isomorphism classes of simple A-modules, C_e is an equivalence if and only if $\Sigma = \Sigma_e := \{S \in \Sigma; Se \neq \{0\}\}$ [Morita, 1958]. More generally, letting $\text{mod}_{\Sigma'}\text{-}A \subseteq \text{mod-}A$ be the full subcategory of A-modules with constituents in $\Sigma' \subseteq \Sigma$, and assuming that $\Sigma' \subseteq \Sigma_e \subseteq \Sigma$, then C_e restricts to a fully faithful functor $\text{mod}_{\Sigma'}\text{-}A \to \text{mod}_{\Sigma' e}\text{-}eAe$, and for any $M \in \text{mod}_{\Sigma'}\text{-}A$ induces an isomorphism between the submodule lattices of M and $C_e(M)$ [M., 1998].

The computational workhorse is **fixed point condensation**: Let $A = FG$ and $e = \frac{1}{|K|} \cdot \sum_{g \in K} g \in FG$, where $K \leq G$ such that $\mathrm{char}(F) \not| \, |K|$; then we have $C_e(M) \cong \mathrm{Fix}_K(M)$ as F-vector spaces. This has been implemented for permutation modules [Thackray, Parker, 1981], tensor products [Lux, Wiegelmann, 1994], induced modules [M., Rosenboom, 1997], and for direct condensation of permutation modules [Parker, Wilson, 1995; Lübeck, Neunhöffer, 1999; M., Neunhöffer, Wilson, 2003].

Further applications are, for example, **condensation with primitive idempotents** and submodule lattices [Lux, M., Ringe, 1995], **socle** and **radical**

series [Lux, Wiegelmann, 1996], **endomorphism rings** and direct sum decompositions [Szőke, 1998], and **condensation of Morita type** [Lux, 1997], ...

The Characters, Modules and Structure of a Centralizer Algebra in the Symmetric Group Algebra

John Murray
(joint work with Harald Ellers)

I discussed an ongoing project with Harald Ellers (Allegheny) to understand the centralizer algebras $RS_n^{S_l}$, where $l < n$. Fix a p-modular system (R, F, k). Each centralizer algebra RG^H has a basis C^+, where C ranges over the orbits of H on G. For $RS_n^{S_l}$ there is another basis which is analogous to a PBW basis for a Lie algebra, due to A. Olshanskii: $\sigma L_{l+1}^{m_1} \dots L_n^{m_n} C_\lambda^+$. Here $\sigma \in S_{n-l}$, the L_i's are Murphy elements, and C_λ is a conjugacy class of S_l. We hope to use this basis to prove that $Z(kS_n^{S_l})$ coincides with the polynomials in $L_2, \dots, L_n$ symmetric with respect to $S_l \times S_{n-l}$. Also this presentation shows that there is a surjective map $H_{n-l}(Z(RS_l)) \to RS_n^{S_l}$, where H_{n-l} is the degenerate affine Hecke algebra (daha) of degree $n - l$. In particular, modules for the centralizer algebra can be lifted to the daha. This allowed us to describe the simple modules, blocks and decomposition matrices in the cases $l = n-1, n-2, n-3$ (and probably $n - 4$). We illustrated the method with reference to $l = n - 2$. There are 3 'families' of simple daha modules, with one or two parameters. Each is defined for an arbitrary field, and is determined by its formal character. This makes the analysis of the modular decomposition of simple $RS_n^{S_l}$-modules almost trivial. In addition, we needed to apply some combinatorics on the abacus with p (not 2!) runners. Finally, we discussed the ordinary character theory of $FS_n^{S_l}$, using the fact that $FS_n^{S_l} \cong eFS_l \times S_n e$, where $e = \Delta(S_l)^+/|S_l|$. So the characters are indexed by pairs $\alpha \vdash n$, $\beta \vdash (n-l)$ such that $\beta \subseteq \alpha$. This has been exploited in the case $l = n-1$ by E. Strahov to produce a Murnaghan–Nakayama formula and an explicit isometry $\bigoplus_n \mathrm{Char}(FS_n^{S_{n-1}}) \to \Lambda[t]$, which is an isomorphism as Λ-modules. Here Λ is the ring of symmetric functions over F and t is an indeterminate over Λ. In particular, there are 'Schur' symmetric functions $S_{\alpha/\beta}$ involving t. We hope to generalize this to understand the characters of $FS_n^{S_l}$.

Cohomology of Symmetric Groups: Old and New Problems

Daniel K. Nakano

One of the most effective ways in studying the cohomology of symmetric groups has been the use of double centralizer theory. This involves symmetric group representations and general linear group representations. Let $n \geq d$ and V be the natural n-dimensional $G := GL_n(k)$ representation. The tensor space $V^{\otimes d}$ becomes a bimodule for the diagonal action of G and the action of the symmetric group Σ_d by place permutation.

One can now consider the functors

$$F(-) = \mathrm{Hom}_G(V^{\otimes d}, -) \text{ and } G(-) = \mathrm{Hom}_{\Sigma_d}(V^{\otimes d}, -).$$

The functor F is exact (and known as the Schur functor), and the functor G is left exact and admits higher right derived functors $R^\bullet G(-)$. A spectral sequence (cf. [2]) can be constructed by using these functors whose E_2-page involves cohomology for the Schur algebra $S(n, d)$. The spectral sequence abuts to the cohomology for the symmetric group. This approach has yielded a number of applications which involves stablity results [5] relating cohomology groups, and equivalences between Weyl and dual Specht filtrations [3].

There are still many open problems. Some of these include:

- Determining precise relationships between Ext^1 between simples for G and Σ_d.
- Determining whether self-extensions between simple modules of Σ_d vanish when the field has characteristic larger than 2.
- Calculating the first cohomology for Specht modules (cf. [4]).

An issue surrounding these questions involves the calculation of the higher right derived functors of G on specific modules. This was accomplished for the trivial module in recent work of Cohen, Hemmer and Nakano [1]. The calculation was used to precisely connect the determination of higher extensions between Young modules with the decomposition number theory of Schur algebras.

References

[1] F. R. Cohen, D. J. Hemmer, D. K. Nakano, *On the cohomology of Young modules for the symmetric group*, Adv. Math., **224**, (2010), 6551–6590.

[2] S. R. Doty, K. Erdmann, D. K. Nakano, *Extensions of modules over Schur algebras, symmetric groups and Hecke algebras*, Algebras and Representation Theory, **7**, (2004), 67–100.

[3] D. J. Hemmer, D. K. Nakano, *Specht filtrations for Hecke algebras of type A*, J. London Math. Soc., **69**, (2004), 623–638.

[4] D. J. Hemmer, D. K. Nakano, *On the cohomology of Specht modules*, J. Algebra, **306**, (2006), 191–200.

[5] A. S. Kleshchev, D. K. Nakano, *On comparing the cohomology of general linear and symmetric groups*, Pacific J. Math., **201**, (2001), 339–355.

Generalized Hook Lengths and Relative Hook Degree Formulas

Jørn B. Olsson

(joint work with Christine Bessenrodt and Jean-Baptiste Gramain)

In his work on unipotent degrees in reflection groups (J.Algebra, vol. 177, 1995) G. Malle used d-symbols as labels, defined hooks in d-symbols and associated lengths to the hooks. With these he was able to prove a 'hook formula' for the degrees.

In the paper 'Generalized hook lengths in symbols and partitions' we have introduced generalized hook length functions for d-symbols. Let $d \in \mathbb{N}$. A *d-symbol* S is a d-tuple of finite subsets $S = (X_0, X_1, \cdots, X_{d-1})$ of $\mathbb{N}_0$. A *hook* in S is a quadruple (a, b, i, j) where $i, j \in \{0, 1, \cdots, d-1\}, a \geq b \geq 0, a \in X_i, b \notin X_j$

and in addition if $a = b$ then $i > j$. Let $H(S)$ denote the set of hooks in S. We associate to S two other symbols $C(S)$ and $Q(S)$, the *core* and the (balanced) *quotient* symbol, such that $|H(S)| = |H(C(S))| + |H(Q((S))|$.

We consider a class of real-valued generalized hook length functions h on hooks (a, b, i, j) in d-symbols and give a decomposition of the multiset $\mathcal{H}(S)$ of all generalized hook lengths $h(z)$, where $z \in H(S)$ which is compatible with the multisets $\mathcal{H}(C(S))$ and $\mathcal{H}(Q(S))$ of the core and quotient of S. In fact, $\mathcal{H}(S) = \mathcal{H}(C(S)) \cup \overline{\mathcal{H}}(Q(S))$, where $\overline{\mathcal{H}}(Q(S))$ is obtained in a controlled way from $\mathcal{H}(Q(S))$ by adding multiples of d in conjunction with a possible sign change.

We give several applications of this. For instance, we show that the relative hook formula obtained by Malle and Navarro in the paper 'Blocks with equal height zero degrees' is in fact the well-known hook formula for the degree of the irreducible characters of the symmetric groups with the hooks suitably arranged and prove a generalization. If $\mathcal{H}(\lambda)$ is the multiset of hook lengths for a partition λ, and λ has d-core partition $\lambda_{(d)}$ then we have in particular $\mathcal{H}(\lambda_{(d)}) \subset \mathcal{H}(\lambda)$. Furthermore the remaining elements of $\mathcal{H}(\lambda)$ may be seen as *modified hook lengths* of a d-quotient partition for λ.

This result for partitions is applied in the paper 'On bar lengths in partitions' by Gramain and Olsson to obtain a similar result for the multiset of bar lengths in partitions with distinct parts (also called a bar partitions). This is possible for $d \geq 3$ odd. The starting point here for a given bar partition μ to consider its 'doubled' partition $D(\mu)$, as defined by I. Macdonald. The bars of μ is a subset of the hooks of $D(\mu)$. In a suitable setup the doubling process is compatible with the d-cores and d-quotients, allowing the use of the partition theorem.

Foulkes Modules for Symmetric Groups

Rowena E. Paget
(joint work with Mark Wildon)

For $m,\ n \in \mathbb{N}$, the symmetric group S_{mn} acts naturally on the collection of set partitions of a set of size mn into n sets each of size m; the resulting permutation module is the *Foulkes module* $H^{(m^n)}$. We denote its character by $\phi^{(m^n)}$. In 1950, H. O. Foulkes conjectured that if $m < n$, and χ^λ is an irreducible character of S_{mn}, then the multiplicity of χ^λ in $\phi^{(m^n)}$ is at least as great as the multiplicity of χ^λ in $\phi^{(n^m)}$, a conjecture which remains open.

Foulkes' Conjecture is concerned only with the ordinary character of $H^{(m^n)}$, yet these modules are also of interest in modular representation theory. For example, $H^{(2^n)}$ is known to possess a filtration by Specht modules and also a filtration by dual Specht modules. This was proved by the author using explicit homomorphisms from Specht modules to quotients of $H^{(2^n)}$ defined over any field.

We construct a new family of homomorphisms from Specht modules into Foulkes modules. A *set family of shape* (m^n) is a collection of n distinct m-subsets of $\mathbb{N}$. Such a set family $\mathcal{P}$ is *closed* if whenever $Y \in \mathcal{P}$ and $X \prec Y$ in the majorization order on m-subsets, then $X \in \mathcal{P}$. For a partition λ, we say that a closed set family

$\mathcal{P}$ has *type* λ if $\mathcal{P}$ has exactly λ'_i sets containing i for each i (where λ' denotes the conjugate partition).

Theorem 1. *Let m be odd. Let $\mathcal{P}$ be a closed set family of shape (m^n) and type λ. Then there is an injective $\mathbb{Z}S_{mn}$- homomorphism $f_{\mathcal{P}} : S^\lambda \to H^{(m^n)}$.*

We use these maps to describe the minimal partitions (in the dominance order) labelling irreducible constituents of Foulkes characters. A set family $\mathcal{P}$ of shape (m^n) and type λ is called *minimal* if there is no set family $\mathcal{Q}$ of shape (m^n) and type μ with $\mu \trianglelefteq \lambda$. A minimal set family is necessarily closed.

Theorem 2. *If m is even then the unique minimal constituent of $\phi^{(m^n)}$ is $\chi^{(m^n)}$, occurring with multiplicity one. If m is odd then χ^λ is a minimal constituent of $\phi^{(m^n)}$ if and only if there is a minimal set family of shape (m^n) and type λ, and, in this case, χ^λ occurs with multiplicity equal to the number of set families of shape (m^n) and type λ.*

We observe that the multiplicity of such a minimal constituent may exceed one. We also give a construction for a class of minimal set families.

Sign Sequences and Decomposition Numbers

Kai Meng Tan

The complete determination of the decomposition numbers of the symmetric group $\mathfrak{S}_n$ is a famous and longstanding open problem. In [3], Kleshchev described the decomposition numbers $d_{\lambda\mu}$, where the partition λ is obtained from μ by moving one node, in terms of latticed subsets of the sign sequence induced by λ and μ. He also described the branching coefficient $[D^\lambda{\downarrow_{\mathfrak{S}_{n-1}}} : D^\nu]$, where ν is obtained from λ by removing one node, in terms of normal nodes in λ.

In [1], Joseph Chuang, Hyohe Miyachi and the author obtained analogues of Kleshchev's formulas for the v-decomposition numbers $d_{\lambda\mu}(v)$ and the v-branching coefficients for the Fock space representation of the quantum affine algebra of $\mathfrak{sl}_e$.

In a recent joint work with Joseph Chuang and Wei Hao Teo [2, 4], we extended the results of [1] by describing the v-decomposition numbers $d_{\lambda\mu}(v)$, where λ is obtained from μ by moving any number of nodes subject to the condition that no two such nodes having adjacent e-residues, in terms of well-nested latticed paths of the sign sequence induced by λ and μ. Assuming only Kleshchev's branching coefficients $[D^\lambda{\downarrow_{\mathfrak{S}_{n-1}}} : D^\nu]$ where ν is obtained from λ by removing a normal node, we are able to show that when these v-decomposition numbers are evaluated at $v = 1$, we obtain the corresponding decomposition numbers. This in particular provides an alternative proof of Kleshchev's original decomposition numbers. We also obtained some other branching coefficients and their analogues in the Fock space in the process.

References

[1] J. Chuang, H. Miyachi, K. M. Tan, *Kleshchev's decomposition numbers and branching coefficients in the Fock space*, Trans. Amer. Math. Soc. **360** (2008), 1179–1191.
[2] J. Chuang, K. M. Tan, *Canonical bases of tensor products and decomposition numbers of symmetric groups*, in preparation.
[3] A. Kleshchev, *On decomposition numbers and branching coefficients of symmetric and special linear groups*, Proc. London Math. Soc. (3) **75** (1997), 497–558.
[4] K. M. Tan, W. H. Teo, *Sign sequences and decomposition numbers*, in preparation.

Spin Fake and Generic Degrees for Symmetric Groups

Weiqiang Wang
(joint work with Jinkui Wan)

1. Fake degrees and generic degrees of Weyl groups

Let W be a finite Weyl group and V be its reflection representation. When W is the symmetric group S_n, take $V = \mathbb{C}^n$. There is an induced action of W on the symmetric algebra S^*V. Denote by $(S^*V)_W$ the coinvariant algebra. According to Chevalley, $(S^*V)_W$ is a graded regular representation of W. The graded multiplicity of an irreducible W-module $\underline{\lambda}$ in $(S^*V)_W$,

$$d^\lambda(t) := \sum_{j\geq 0} t^j \dim \mathrm{Hom}_W(\underline{\lambda}, (S^jV)_W),$$

is called the fake degree of W. When $W = S_n$, we identify λ as a partition $\lambda = (\lambda_1, \lambda_2, \dots)$ of n. Let $n(\lambda) = \sum_{i\geq 1}(i-1)\lambda_i$ and h_{ij} be the hook lengths. Then

$$d^\lambda(t) = t^{n(\lambda)} \frac{(1-t)(1-t^2)\dots(1-t^n)}{\prod_{(i,j)\in\lambda}(1-t^{h_{ij}})}.$$

Denote by H_W the Hecke algebra of W of equal parameter v, with a basis $T_w, w \in W$. H_W is a symmetric algebra with a symmetrizing trace form τ given by $\tau(T_1) = 1$, and $\tau(T_w) = 0$ for $w \neq 1$. Over $\mathbb{C}(v)$, the irreducible H_W-modules are parametrized by $\underline{\lambda} \in \mathrm{Irr}(W)$. Denote by P_W the Poincare polynomial of W, by c_λ the Schur element and by $D^\lambda = D^\lambda(v) = P_W/c_\lambda$ the generic degree associated to λ (see [1]). The generic degrees are related to the fake degrees by Fourier–Lusztig transform for general W (see [2]). When $W = S_n$, it turns out that $D^\lambda(t) = d^\lambda(t)$, for all partitions λ.

2. Spin fake degrees for S_n

Our goal is to formulate and compute the spin fake degree and spin generic degree (associated to S_n). According to Schur 1911, the symmetric group S_n affords a double cover $\widetilde{S}_n$, with a distinguished central element z of order 2. All the algebras and modules in the remainder are understood to be $\mathbb{Z}_2$-graded. It is known that the representation theory of the spin symmetric group algebra $\mathbb{C}S_n^- = \mathbb{C}\widetilde{S}_n/\langle z+1\rangle$ is super-equivalent to its counterpart for Hecke–Clifford algebra $\mathfrak{H}_n^c :=$

$\mathcal{C}_n \rtimes \mathbb{C}S_n$. The simple $\mathfrak{H}_n^c$-modules $\underline{\xi}$ are parametrized by strict partitions ξ of n. Set $\delta(\xi)$ to be 0 or 1 when the length $\ell(\xi)$ is even or odd, respectively. Define the *spin fake degree* of $\underline{\xi}$ to be $d_-^{\xi}(t) := \sum_{j\geq 0} t^j \dim \mathrm{Hom}_{\mathfrak{H}_n^c}\left(\underline{\xi}, \mathcal{C}_n \otimes (S^j V)_{S_n}\right)$. The terminology is justified by our basic observation that $\mathcal{C}_n \otimes (S^* V)_{S_n}$ is a graded regular representation of $\mathfrak{H}_n^c$, and hence $d_-^{\xi}(1)$ is the degree of $\underline{\xi}$. Associated to a strict partition ξ, there is a notion of shifted diagram ξ^*, content c_{ij} and shifted hook length h_{ij}^* for each cell $(i,j) \in \xi^*$ (see [3]).

Theorem 1 (Wan–Wang [4])**.** *Let ξ be a strict partition of n. Then,*

$$d_-^{\xi}(t) = 2^{-\frac{\ell(\xi)-\delta(\xi)}{2}} \frac{t^{n(\xi)} \prod_{(i,j)\in\xi^*}(1+t^{c_{ij}}) \prod_{r=1}^{n}(1-t^r)}{\prod_{(i,j)\in\xi^*}(1-t^{h_{ij}^*})}.$$

3. Spin generic degrees for S_n

Let $\mathcal{C}_n$ be the Clifford algebra generated by $c_1, \ldots, c_n$ with relations $c_i^2 = 1, c_i c_j = -c_j c_i$, for $i \neq j$. The Hecke–Clifford algebra $\mathcal{H}_n^c$ is a $\mathbb{C}(v)$-algebra generated by $T_1, \ldots, T_{n-1}$, $c_1, \ldots, c_n$, subject to the relation of Hecke algebra of S_n for $T_1, \ldots, T_{n-1}$, the Clifford algebra relation for $c_1, \ldots, c_n$, and the additional relation $T_i c_i = c_{i+1} T_i$, for $1 \leq i \leq n-1$. It follows that $T_i c_{i+1} = c_i T_i - (v-1)(c_i - c_{i+1})$. For definiteness, let us record that $(T_i - v)(T_i + 1) = 0$. For a partition $\mu = (\mu_1, \mu_2, \ldots, \mu_\ell)$ of n, let

$$\begin{aligned} T_{w_{\mu_j}} &= T_{\mu_1+\ldots\mu_{j-1}+1} \cdots T_{\mu_1+\ldots+\mu_j-1}, \quad 1 \leq j \leq \ell, \\ T_{w_\mu} &= T_{w_{\mu_1}} T_{w_{\mu_2}} \cdots T_{w_{\mu_\ell}}. \end{aligned}$$

Theorem 2. [5] *There exists a unique symmetrizing trace form $\daleth$ for $\mathcal{H}_n^c$ such that*

$$\daleth(T_{w_\mu}) = \left(\frac{v-1}{2}\right)^{n-\ell(\mu)}. \tag{1}$$

for all odd partitions μ of n.

There is a version of Theorem 2 for the spin Hecke algebra introduced in [6].

Associated to the trace form $\daleth$ and each strict partition ξ, we can define Schur elements c_ξ^- in a standard way, and then define the associated *spin generic degrees* $D_-^{\xi} = 2^n P_{S_n}/c_\xi^-$.

Theorem 3. [5] *The spin generic degrees for Hecke–Clifford algebra $\mathcal{H}_n^c$ coincide with the corresponding spin fake degrees, i.e., $D_-^{\xi}(t) = d_-^{\xi}(t)$, for all strict partitions ξ.*

The proof of Theorem 3 relies on a Frobenius type character formula for $\mathcal{H}_n^c$ which we develop.

References

[1] M. Geck and G. Pfeiffer, *Characters of Finite Coxeter Groups and Iwahori–Hecke Algebras*, London Math. Soc. Monographs, New Series **21**, Oxford University Press, New York 2000.

[2] G. Lusztig, *Characters of reductive groups over a finite field*, Ann. of Math Stud. **107**, Princeton University Press, 1984.

[3] I. G. Macdonald, *Symmetric functions and Hall polynomials*, Second edition, Clarendon Press, Oxford, 1995.

[4] J. Wan, W. Wang, *Spin invariant theory for the symmetric group*, J. Pure Appl. Alg. **215** (2011), 1569–1581.

[5] J. Wan, W. Wang, In preparation, 2011.

[6] W. Wang, *Spin Hecke algebras of finite and affine types*, Adv. in Math. **212** (2007), 723–748.

Vertices of Specht Modules

Mark Wildon

In a highly influential 1959 paper J. A. Green [2] defined the *vertex* of an indecomposable representation of a finite group. Vertices have since become a central object in modular representation theory, and feature in a number of important conjectures, including Alperin's Weight Conjecture. Despite this, comparatively little is known about the vertices of 'naturally occurring' modules, such as Specht modules for symmetric groups. Indeed, for many years, the only published work in this area was [5] on the vertices of the Specht modules $S^{(n-r,1^r)}$ in characteristic 2.

In my talk I discussed two results on vertices of Specht modules obtained using the Brauer correspondence for modules, as developed by M. Broué in [1].

Theorem 2 ([6, Theorem 2])**.** *The vertex of the Specht module $S^{(n-r,1^r)}$, defined over an arbitrary field of odd prime characteristic p not dividing n, is a Sylow p-subgroup of $S_{n-r-1} \times S_r$.*

Theorem 3 ([7, Theorem 1.1])**.** *Let λ be a partition and let t be a λ-tableau. Let $H(t)$ be the subgroup of the row-stabilising group of t which permutes, as blocks for its action, the entries of columns of equal length in t. If the Specht module S^λ, defined over a field of prime characteristic p, is indecomposable, then it has a vertex containing a Sylow p-subgroup of $H(t)$.*

Theorem 2 is of particular significance, because it leads to short proofs of a number of foundational results in the block theory of the symmetric group. For example, it implies that the defect group of a weight w block of a symmetric group in prime characteristic p is a Sylow p-subgroup of S_{wp}.

I ended my talk by surveying some interesting recent results obtained by K. J. Lim [4] using the idea of the cohomological complexity of a module. For reasons of space I will only state a special case of one of them here.

Theorem 4 (Lim [4, Theorem 3.2])**.** *Let F be a field of prime characteristic $p \geq 3$ and let μ be a partition. Suppose that the Specht module S^μ has an abelian vertex Q of p-rank m. If c is the complexity of S^μ and w is the weight of its block then*

$c = m = w$, and Q is an elementary abelian subgroup generated by w disjoint p-cycles in S_n.

This result is a step towards a classification of all indecomposable Specht modules with abelian vertex. The easier problem of classifying all Specht modules with cyclic vertex was solved in [6], where I showed that the Specht module S^λ, defined over a field of characteristic p, has a non-trivial cyclic vertex if and only if λ has p-weight 1.

References

[1] M. Broué, *On Scott modules and p-permutation modules: an approach through the Brauer homomorphism*. Proc. Amer. Math. Soc. **93**, no. 3 (1985), 401–408.

[2] J. A. Green, *On the indecomposable representations of a finite group*. Math. Zeitschrift **70** (1958/59), 430–445.

[3] D. Hemmer, *The complexity of certain specht modules for the symmetric group*. J. Algebr. Comb. **30** (2009), 421–427.

[4] K. J. Lim, *Specht modules with abelian vertices*. preprint (2011).

[5] G. M. Murphy, M. H. Peel, *Vertices of Specht modules*. J. Alg **86** (1984), 85–97.

[6] M. Wildon, *Two theorems on the vertices of Specht modules*. Arch. Math. (Basel) **81**, 5 (2003), 505–511.

[7] M. Wildon, *Vertices of Specht modules and blocks of the symmetric group*. J. Alg. **323** (2010), 2243–2256.

Open Problem Session

Some of the following open problems were collected during the session held Thursday afternoon at the workshop, others were submitted afterward or mentioned during talks. They are organized by presenter.

Dan Nakano

1. Let D_λ, D_μ be simple $k\Sigma_d$-modules. Express $\mathrm{Ext}^1_{k\Sigma_d}(D_\lambda, D_\mu)$ in terms of cohomology of modules for the general linear group GL_d. There are usually more tools available in algebraic groups to compute cohomology.

2. Kleshchev–Martin conjecture: For $p \geq 3$, is $\mathrm{Ext}^1_{k\Sigma_d}(D_\lambda, D_\lambda) = 0$?

3. Linnell conjecture: let G be a finite group, B_0 the principal block of kG and S a simple module in B_0. Can it be the case that $\mathrm{H}^i(G, S) = 0$ for all $i > 0$? For the case of restricted Lie algebras there is an easy example where this does happen, but no examples are known for finite groups. Can one verify it perhaps for symmetric groups?

4. The nucleus for $k\Sigma_d$: let

$$C = \{M \in B_0 \mid \mathrm{H}^j(G, M) = 0 \ \forall j > 0.\}.$$

The nucleus is defined as:

$$\mathrm{Nuc}(kG) = \bigcup_{M \in C} V_G(M).$$

Compute this nucleus for $G = \Sigma_d$.

John Murray

1. Conjecture (Külshammer et al.): Given $n \geq 1$ and $p = 2$, consider $\lambda = (n+1, n-1) \vdash 2n$. Then the Cartan invariant $C_{\lambda\lambda}$ is odd if and only if n is even.

Murray showed there is some odd Cartan invariant $C_{\mu\mu}$ in this block. In Mathas–James later for the Hecke algebra $H_n(\Sigma_d)$ it was odd in characteristic zero for $e = 2$ (using LLT). Maybe this is evidence. Remark: a group has a real element of 2-defect zero if and only if kG has an odd diagonal Cartan invariant. ($p = 2$).

2. Choose a prime p, non-negative integers $n \geq \ell$ (perhaps even fix $\ell = n-1$). Find an 'effective method' for enumerating all pairs of partitions $\alpha \vdash n, \beta \vdash \ell$, $\alpha \supset \beta$, and such that the diagrams of α, β contain given multisets of p-residues. This is related to the centralizer algebra $kS_n^{S_\ell}$. These pairs enumerate the ordinary representations in what are expected to be the p-blocks. By effective we mean something in the spirit of the p-abacus, you know how many there are and what they are.

3. Brundan (2008): The centre of degenerate cyclotomic Hecke algebra $Z(H_n^f)$ is $F[x_1, \dots, x_n]^{S_n}$. Can his proof work to show that the centre $Z(kS_n^{S_l})$ is the set of polynomials in the Murphy elements that are symmetric under $S_l \times S_{n-1}$? Or any other proof?

4. Let $\lambda \vdash n$ be p-regular. Then $S^\lambda \downarrow_{\Sigma_{n-1}}$ has a filtration by Specht modules:

$$S^\lambda = S_1^\lambda \supset S_2^\lambda \supset \cdots \supset S_r^\lambda \supset S_{r+1}^\lambda = 0$$

where r is the number of removable nodes in λ, and $S_i^\lambda / S_{i+1}^\lambda \cong S^{\lambda_i}$ where λ_i is the partition of $n-1$ obtained by removing the ith removable node (counting bottom to top) from λ. Set:

$$J^\lambda := S^\lambda \cap S^{\lambda\perp}$$
$$J_i^\lambda := J^\lambda \cap S_i^\lambda.$$

Then

$$J^\lambda = J_1^\lambda \supseteq J_2^\lambda \supseteq \cdots \supseteq J_r^\lambda \supseteq J_{r+1}^\lambda = 0.$$

Is $J_i^\lambda \supsetneq J_{i+1}^\lambda$ if and only if i is a normal node of λ in the sense of Kleshchev? For details see http://www.maths.nuim.ie/documents/jmurrayactionLn.pdf.

Burkhard Külshammer

1. F a field, G a finite group, $H \leq G$. Define (Boltje–Danz–Külshammer)

$$T_i = FG \otimes_{FH} \cdots \otimes_{FH} FG$$

as an (FH, FH)-bimodule. Then $T_1 \mid T_2 \mid T_3 \cdots$. We know there is an n such that T_n and T_{n+1} have the same indecomposable summands up to multiplicity. The general question is what is the minimal n such that this happens. Our paper includes an answer for the symmetric group ($H = S_{n-1}$), where it does not depend on the field. What is the answer for the alternating group? The answer is known for characteristic 0, and bounds exist in prime characteristic. The problem comes from the theory of von Neumann algebras.

2. Find the vertices and sources of irreducible modules D^λ for the symmetric group. Only special cases are known. For example the vertices for the case where λ is a p-regular hook partition are known, except for $n = pw$, $\lambda = (n-p+1, 1^{p-1})$ where p is odd and $w \equiv 1 \pmod{p}$. The vertices are expected to be the Sylow p-subgroups of S_n.

3. Let $p = 2$, and let λ be a 2-regular partition of n. Let $V \neq F$ be the source of D^λ. Must V have even dimension?

Susanne Danz

1. Let $n = p^k$. Is the projective-free part of Lie(n) indecomposable? Does it have an endo-permutation module as a source? Is its vertex a regular elementary abelian subgroup?

2. Is $\mathrm{Lie}(n) \otimes (\mathrm{Lie}(n))^*$ a permutation module?

3. Suppose that $p = 2$. Does every indecomposable Specht $F\mathfrak{S}_n$-module that is not simple restrict indecomposably to $\mathfrak{A}_n$?

Weiqiang Wang

1. Let D^λ be a completely splittable S_n-module. Then it is known that the vertex of D^λ is just the defect group of the block. Let $\tilde{S_n}$ be the spin symmetric group. Then completely splittable representations have been classified (Wan, 2010 Journal of algebraic combinatorics) also independently by Hill et al. Is something similar true for vertices?

2. A problem in characteristic 0. Spin fake degree and spin generic degree are related to the spin symmetric group, and the spin Hecke algebra. Removing spin you add finite group of Lie type. Find a finite group of Lie type interpretation.

3. Let $k = \overline{k}$ be algebraically closed of characteristic $p > 0$. Let $k[x_1, \dots, x_n]_{S_n}$ denote the coinvariant algebra. What is the graded multiplicity of D^λ in this coinvariant algebra, or just in the socle of the coinvariant algebra? For the trivial module the multiplicity in the socle is known.

4. Let $G := GL_n(q)$, which acts naturally on $k[x_1, x_2, \dots, x_n]$. Consider $k[x_1, x_2, \dots, x_n]_G$, studied by topologists (Mitchell about 1985). This has the same composition factors as regular representation but is not isomorphic to the regular representation. In papers of Wan–Wang, two approaches are used, one is an invariant-theory approach. Can you answer the question as in Number 3 above? The answer is known 'around the Steinberg'. There is a conjecture about graded multiplicity in the socle 'around the trivial module'.

Karin Erdmann

1. Fact: Take M an FS_n-module, then there is a finite exact sequence

$$0 \to M \to Y_0 \to \cdots \to Y_d \to 0$$

where the Y_i are direct sums of Young modules.

Def: The *Young dimension* of M is the minimal such d. What is it? You get this resolution by taking an injective resolution for the Schur algebra and pushing it over. Is this the same of the injective dimension of $G(M)$? Even in blocks of weight 1 it is interesting. If you know this, perhaps one can obtain consequences about $\mathrm{Ext}(M, Y^\lambda)$.

2. Can a projective indecomposable module $P(D^\lambda)$ for the symmetric group have a 'waist', i.e., some layer in the radical filtration (other than the top or socle) that is simple?

3. Classify all λ such that $\mathrm{Ext}^1(D^\lambda, D^\mu) \neq 0$ for exactly one μ. We call these exceptional simple modules. These do appear in RoCK blocks, also in some blocks of weight 2.

4. Let $H_q(n)$ be the Hecke algebra of type A in prime characteristic. Given a module M, must there exist a periodic module W with $\mathrm{Ext}^1_H(W, M) = 0$?

Matthew Fayers

1. Let p be odd. Classify partitions λ such that λ is self-conjugate and S^λ has exactly two composition factors. (See Fayers' AMS paper). This would give you a solution to : 'Which characteristic zero irreducibles for A_n remain irreducible in characteristic p'. His paper has a conjectural answer.

David Hemmer

1. Say a partition μ is $p \times p$ if both μ and μ' are of the form $p\tau$. The Georgia VIGRE algebra group conjectures the complexity of a Specht module S^μ is maximal possible (the p-weight of the corresponding block) if and only if μ is *not* $p \times p$. We proved the easy direction, that the complexity is not maximal for $p \times p$. Prove the other direction. More generally, how much does the complexity drop in the $p \times p$ case? Work of Lim shows for the smallest case $\mu = (p^p)$ the complexity is $p - 1$.

2. Which D^λ can occur in the socles of non-projective Young modules? Which can occur in Specht module socles S^τ for τ not p-restricted. The answer to both is known in Rouquier blocks.

3. Cossey: Fix a block B and an abacus display for its p-core. Choose a runner. Give a bijection between p-regular partitions in the block and partitions for which the term in the p-quotient corresponding to this runner is empty. It is known the two sets have the same size (James–Kerber 6.2.2).

4. For which λ is the Ext algebra $\mathrm{Ext}^\bullet_{\Sigma_d}(Y^\lambda)$ a graded commutative algebra? Is it true for λ maximal in each block? Can one construct a natural action of this algebra on modules in the block?

5. Does the module $\mathrm{Lie}(n)$ have a Specht or dual Specht module filtration? How about the Foulkes module discussed by Wildon and Paget? (Answered for $\mathrm{Lie}(6), p = 3$ at the conference by J. Müller: has a dual Specht filtration, but no Specht filtration.)

6. Suppose $H \leq \Sigma_n$ is such that the permutation character on the cosets Σ_n/H is multiplicity free. Wildon has proved the permutation module in prime

characteristic has a Specht filtration, with a case-by-case analysis. Give a different proof not using the classification of such H.

Mark Wildon

Despite the importance of vertices to modern conjectures in representation theory, such as Alperin's Weight Conjecture, comparatively little is known about the vertices of 'naturally occurring' modules, such as Specht modules for recent groups. An ambitious target in this area would be solve the following problem:

Problem 1. *Classify all indecomposable Specht modules whose vertices are abelian.*

The easier problem of classifying all indecomposable Specht modules with cyclic vertex was solved by Wildon, who showed that the indecomposable Specht module S^λ, defined over a field of prime characteristic p, has a non-trivial cyclic vertex if and only if λ has p-weight 1.

Two recent results suggest techniques that may help to solve this problem. First of all Wildon used the Brauer Correspondence for modules, as developed by M. Broué to prove Theorem 3 in Wildon's report.

An immediate corollary is that if S^λ has an abelian vertex in characteristic p then $\lambda_i - \lambda_{i+1} < p^2$ for all i.

Lim used the idea of the complexity of a module to get information about vertices of Specht modules. His result is stated as Theorem 4 in Wildon's report.

By combining this theorem with an earlier result of Hemmer, Lim showed that any partition formed by $p \times p$-blocks has a non-abelian vertex. In particular, $S^{(p^p)}$ has a non-abelian vertex.

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