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Mini-Workshop: Localising and Tilting in Categories

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ABSTRACT. The workshop brought together experts on localisation theory and tilting theory from different parts of mathematics with the aim of fully exploiting the power of some recent developments in so far rather independent contexts. The intensive exchange during the workshop is expected to lead to new and strengthened synergies and to new applications.

Mathematics Subject Classification (2010): primary 16xx, 18xx, secondary 13xx, 55xx, 79xx.

Introduction by the Organisers

The workshop *Localising and tilting in abelian and triangulated categories*, organised by Lidia Angeleri Hügel (Verona), Steffen Koenig (Stuttgart) and Changchang Xi (Beijing) was attended by 17 participants with a broad mathematical and geographical background. The aim was to bring together researchers from different branches of mathematics to discuss and compare recent results on localisation theory and tilting theory. In order to optimise interaction between different areas and techniques, the organisers gave the programme a precise structure and ordering to emphasize connections and point out applicability of the new results.

Historically, localisation theory has its origin in commutative algebra with the classical notions of a quotient field and of localisation of rings at prime ideals. Later on, these ring theoretic constructions were generalised to non-commutative rings and studied from a more categorical point of view, leading to a theory of localisation, first for abelian, and, more recently, for triangulated categories. Here the relevant notions are *t-structures* and *recollements*.

Localisation in abelian and triangulated categories plays an important role in many parts of mathematics. Commutative and non-commutative algebra, algebraic topology and homotopy theory, homological algebra and category theory have contributed in different and so far rather unrelated ways to better understanding localisations and to extending the scope of these methods. There are also close relations to microlocal analysis and to the study of \mathcal{D} -modules, which in turn relates to representation theory through Kazhdan-Lusztig conjectures/theorems.

Recent exciting developments link localisation theory with *tilting theory* - a fundamental branch of representation theory which allows to compare different categories of representations both in abstract and in combinatorial ways. In fact, tilting theory is undergoing a major change due to new and unexpected applications of localisation techniques. For example, localisation methods have turned out to be very useful in establishing *classification results* for tilting modules, both in commutative and in non-commutative situations.

The rapid progress in the theory during the last few years makes an exchange between researchers from different areas more necessary than ever. In fact, we believe that the impact of some of the recent developments has not been completely understood yet and can be strengthened much by combining and applying different techniques. A first step in this direction was done during the week in Oberwolfach.

A lot of attention has been devoted to the interplay between the topological and the algebraic perspective on localisation. New results in commutative algebra and algebraic geometry were intensively discussed also from the point of view of non-commutative ring theory. Recent developments in a more categorical algebraic setting were explored together with explicit and often combinatorial applications. The active and promising exchange during the workshop lays the foundation for new projects.

The positive outcome of the workshop is also due to the pleasant and well organised environment at the institute in Oberwolfach. In particular, the concept of bringing together such a small number of researchers in a mini-workshop creates an intense and almost private atmosphere, which turned out to be very productive. Finally, the active and helpful interaction between the different mini-workshops during the week should not remain unmentioned. Having parallel mini-workshops with a potential common interest proved to be a good idea.

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Abstracts

Morita theory for ring spectra

STEFAN SCHWEDE

In the two talks I tried to introduce and motivate homotopical algebra over structure ring spectra as a method to define interesting triangulated categories and produce triangulated equivalences. The stable homotopy category of algebraic topology is a tensor triangulated category, with symmetric monoidal product referred to as the *smash product*; the monoids are homotopy-associative ring spectra, and they represent multiplicative cohomology theories. While the notion of a homotopy-associative ring spectrum is useful for many things, it does not have a good enough module theory for our present purpose; for example, but the mapping cone of a homomorphism between such modules does not inherit a *natural* action of the ring spectrum, and the module category does not form a triangulated category.

So for a spectral analog of Morita theory we need a highly structured model for the category of spectra with a symmetric monoidal and homotopically well behaved smash product (before passing to the homotopy category). I advertised one particular model, the *symmetric spectra* as introduced by Hovey, Shipley and Smith [1], which I personally find the easiest to work with. The monoids are called *symmetric ring spectra*, and as far as their homotopy category is concerned, are equivalent to the older notion of *A_∞ -ring spectra*.

One main result of Morita theory for ring spectra is as follows. The category of modules over a symmetric ring spectrum R comes with a stable model structure (in the sense of Quillen), and the *derived category* $\mathbf{D}(R)$ is the localization at the class of stable equivalences. The derived category $\mathbf{D}(R)$ comes with a natural triangulated structure with coproducts, and the free module of rank 1 is a compact generator for $\mathbf{D}(R)$. If P is another compact generator of $\mathbf{D}(R)$, then ‘taking maps out of P ’ can be promoted to an exact functor

$$\mathrm{RHom}(P, -) : \mathbf{D}(R) \rightarrow \mathbf{D}(\mathrm{End}(P))$$

where $\mathrm{End}(P)$ is a certain symmetric ring spectrum, the *endomorphism ring spectrum* of P . This is the starting point for adapting various aspects of Keller’s ‘Morita theory’ for differential graded categories [2] to ring spectra. The results and many examples are summarized in the survey paper [3], and the generalization for a set of compact generators (as opposed to a single compact generator) is discussed in the original research paper [4] with Brooke Shipley.

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On Frobenius models for derived categories of Dynkin quivers

BERNHARD KELLER

(joint work with Sarah Scherotzke)

A *Frobenius model* for a triangulated category \mathcal{T} is a Frobenius category \mathcal{E} together with a triangle equivalence from the stable category of \mathcal{E} to \mathcal{T} . Inspired by the representation–theoretic approach (initiated in [1] and [3] and further developed in [2]) to Nakajima’s graded quiver varieties [4] we construct new Frobenius models for derived categories for path algebras of Dynkin quivers over a field. Remarkably, among these models, there is a ‘universal’ one. Moreover, already for the quiver A_2 , we obtain uncountably many new models. We construct these models as categories of finitely presented Gorenstein projective modules over the singular Nakajima category \mathcal{S}_C associated with a Dynkin quiver Q and a suitable set C of isoclasses of indecomposable objects of the derived category of Q . Then the universal model is associated with the set C of all isoclasses of indecomposables and each ‘minimal model’ is equivalent to the category of finite-dimensional modules over the repetitive algebra of an algebra B derived equivalent to the path algebra of Q . It is noteworthy that all ‘minimal models’ are abelian and form a unique derived equivalence class.

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Ring epimorphisms and universal localisations

JORGE VITÓRIA

(joint work with Frederik Marks, Alice Pavarin)

Ring epimorphisms of a ring R are useful to study some subcategories of $R\text{-Mod}$ or of its derived category $\mathcal{D}(R)$. The restriction functor associated to a ring epimorphism f is fully faithful, and so is its derived functor when f is homological. In this talk we compare homological ring epimorphisms with universal localisations,

as defined by Schofield. If R is hereditary, these notions are known to coincide ([4]).

Theorem 1 (Marks-V.,[5]) *Let $f : R \rightarrow S$ be a ring epimorphism such that ${}_R S$ is a finitely presented R -module of projective dimension less or equal than 1. Then f is a homological ring epimorphism if and only if it is a universal localisation.*

Motivated by recent work on stratifications of derived module categories (e.g. [1]), we are interested in better understanding the Verdier quotient $\mathcal{D}(R)/\mathcal{D}(S)$.

Theorem 2 (Marks-V.,[5]) *Let $f : R \rightarrow S$ be a homological ring epimorphism such that ${}_R S$ is a finitely presented R -module of projective dimension less or equal than 1. If $\text{Hom}_R(\text{Coker } f, \text{Ker } f) = 0$, then f_* induces a recollement of $\mathcal{D}(R)$ by $\mathcal{D}(S)$ and $\mathcal{D}(\text{End}_{\mathcal{D}(R)}(K_f))$, where K_f is the cone of f in $\mathcal{D}(R)$. Moreover, if ${}_R S$ is projective, $\text{End}_{\mathcal{D}(R)}(K_f) \cong R/\tau_S(R)$, where $\tau_S(R)$ is the trace of S in R .*

Schofield's universal localisations were generalised in [3] to localisations with respect to sets of compact objects in $\mathcal{D}(R)$. These, however, do not always exist ([3]).

Theorem 3 (Marks-Pavarin-V.,[6]) *Let Σ be a set of compact objects in $\mathcal{D}(R)$. If $\mathcal{E}_\Sigma := \text{Ker}(\mathbb{R} \text{Hom}_R(\Sigma, -)) \cap R\text{-Mod}$ is a bireflective subcategory of $R\text{-Mod}$, then the generalised universal localisation $f : R \rightarrow R_\Sigma$ exists and $\text{Tor}_1^R(R_\Sigma, R_\Sigma) = 0$.*

Theorem 3 can be used to prove some known results: homological ring epimorphisms with a compact cone are generalised universal localisations ([2]) and, moreover, when the telescope conjecture holds for $\mathcal{D}(R)$, every homological ring epimorphism is a generalised universal localisation ([3]).

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Universal localisations and tilting modules for finite dimensional algebras

FREDERIK MARKS

In this talk we study universal localisations, in the sense of Schofield, for a finite dimensional \mathbb{K} -algebra A , where \mathbb{K} denotes an algebraically closed field. Motivated by a result in [3, Theorem 6.1.] for hereditary rings, we want to compare universal localisations of A with ring epimorphisms $f : A \rightarrow B$ fulfilling $\text{Tor}_1^A(B, B) = 0$. One possible way to approach this question is to discuss certain abelian subcategories of our initial module category, obtained via restriction induced by f . By studying these subcategories we get a complete answer in the following case.

Theorem 1 ([4]) *Let A be a Nakayama algebra and $f : A \rightarrow B$ be a ring epimorphism. Then f is a universal localisation if and only if $\text{Tor}_1^A(B, B) = 0$.*

As an application, we can provide a combinatorial classification of the homological ring epimorphisms for self-injective Nakayama algebras.

The idea of understanding universal localisations of A via “nice” subcategories of the category of all A -modules allows us to establish a comparison with tilting modules in the hereditary case. A key role is here played by a construction in [2], associating an abelian category to a given torsion class.

Theorem 2 ([4]) *Let A be a hereditary algebra. There is a bijection between*

- *the epiclasses of finite dimensional and monomorphic universal localisations of A*
- *the equivalence classes of finitely generated tilting A -modules*

where a universal localisation A_Σ corresponds to the tilting module $A_\Sigma \oplus A_\Sigma/A$. The inverse bijection is given by localising with respect to the set of non split-projective indecomposable direct summands of the tilting module in its torsion class.

A result of similar nature can be obtained for Nakayama algebras, using the notion of τ -tilting modules, which were recently introduced in [1].

Theorem 3 ([4]) *Let A be a Nakayama algebra. There is a bijection between*

- *the epiclasses of universal localisations of A*
- *the equivalence classes of support τ -tilting A -modules.*

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Localizations applied to tilting modules and algebraic K-theory I, II

HONGXING CHEN AND CHANGCHANG XI

This is a series of two talks. In the first part (presented by Xi), we use homological ring epimorphisms and generalized localizations to study infinitely generated tilting modules in terms of recollements of derived module categories. For infinitely generated 1-tilting modules, Happel's theorem on derived equivalences is extended to recollements of derived module categories. Namely, the derived category of the endomorphism ring of a good 1-tilting module admits a recollement of derived categories of the original ring and a new ring which is a universal localization (see [3]), and moreover, the new ring is zero if and only if the tilting module is finitely generated (see [2, 3]). This is applied to answer an open question in [1] on the Jordan-Hölder Theorem for stratifications of derived module categories by derived module categories. However, for infinitely generated tilting modules of higher projective dimension, that is, good n -tilting modules with $n \geq 2$, it is an open question whether such a recollement of derived module categories still exists, where the new ring is expected to be a generalized localization. In this talk, we first provide several equivalent characterizations of the existence of such kind of recollements, and then give the first counterexample to the open question for $n \geq 2$ (see [4]). This example is built from developments of general methods for judging when triangulated subcategories of derived module categories are equivalent to derived categories of rings via homological ring epimorphisms.

In the second part (given by Chen), we present properties of recollements of derived categories. Motivated by the discussions in [7, 8], we establish relationships for algebraic K-theory of rings involved in recollements. For a homological ring epimorphism from a ring R to another ring S , we show that if the left R -module S has a finite projective resolution by finitely generated projective modules, then the K-theory space of R decomposes as a product of the K-theory spaces of S and a differential graded ring \mathbb{T} (see [6]). If, in addition, \mathbb{T} has the non-zero cohomology only in degree zero, then we can replace \mathbb{T} in the above result by the 0-th cohomology ring $H^0(\mathbb{T})$. Moreover, given an arbitrary recollement of derived module categories of rings (with a natural condition), we describe the K-theory space of the ring in the middle as a product of the ones of the other two rings (see [5, 6]). As another consequence of our methods, we obtain a Mayer-Vietoris sequence (of infinite length) of algebraic K-groups for homological exact contexts, including homological Milnor squares (see [6]). This also reproves a result of Karoubi on K-groups of localizations.

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Recollements of derived categories of dg algebras induced by partial tilting dg-modules

SILVANA BAZZONI

(joint work with Alice Pavarin)

The notion of a recollement of triangulated categories was introduced by Beilinson-Bernstein-Deligne [3] and it plays the role of an exact sequence of categories. As proved by Jørgensen in [5], a compact object of the derived category of a differential graded algebra (dg algebra) induces a recollement of derived categories of dg algebras. We consider the case of self-orthogonal compact dg modules, which we call partial tilting, obtaining a generalization of the Morita-type theorem proved by Rickard in [7]. Specializing to the case of partial tilting modules over a ring, we extend the results proved in [1] and [2] on triangle equivalences induced by infinitely generated tilting modules.

By results of Nicolas and Saorin in [6], the recollements of dg algebras are in bijection with equivalence classes of homological epimorphisms of dg algebras. For a finitely generated partial tilting module over a ring B , we give conditions under which the left hand term of the induced recollement is the derived category of a ring S linked to B via a homological ring epimorphism. If this happens, then the ring S plays the role of a “generalized universal localization” of B with respect to a projective resolution of the partial tilting module. We exhibit a class of examples of finitely generated partial tilting modules of projective dimension greater than one for which the above mentioned homological epimorphism exists.

This is in relation with the results proved by Chen and Xi in [4]. They show that this is always the case if the partial tilting module arises from a “good” infinitely generated tilting module of projective dimension one, but the case of projective dimension greater than one remains open. We note that our examples are not arising from good tilting modules, but our approach is more direct, since we fix a ring B and obtain recollements of the derived category of B , while starting with an infinitely generated good tilting module one obtains a recollement whose central term is the derived category of the endomorphism ring of the tilting module and this ring might be very large and difficult to handle.

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Bimodules and triangle equivalences

PEDRO NICOLÁS

(joint work with Manuel Saorín)

After works by Happel and Keller, it is well-known that a classical tilting module T over a ring A induces, via the total derived version of the Tensor-Hom adjunction, mutually quasi-inverse triangle equivalences between the derived categories $\mathcal{D}A$ and $\mathcal{D}B$, where $B = \text{End}_A(T)$. Thus, derived categories seem to be the right framework to deal with tilting modules.

Question: what can we say in the non-classical case?

The main result of [6] enlightens the situation:

Theorem. *Let A be a differential \mathbf{Z} -graded algebra. The following statements are equivalent:*

- 1) *The algebra A belongs to the smallest full subcategory of $\mathcal{D}A$ containing T and closed under shifts, extensions and direct summands.*
- 2) *The right total derived functor $\text{RHom}_A(T, ?) : \mathcal{D}A \rightarrow \mathcal{D}B$ is fully faithful and preserves compact objects.*

Notice that if $\text{RHom}_A(T, ?)$ is fully faithful, after [5, Proposition I.1.3], we know that it induces a triangle equivalence between $\mathcal{D}A$ and the quotient of $\mathcal{D}B$ by the kernel of the total left derived functor $? \otimes_B^{\mathbf{L}} T : \mathcal{D}B \rightarrow \mathcal{D}A$.

The previous theorem generalizes and complements results in [2] and [7], and it is related to work by Angeleri Hügel-Koenig-Liu [1] and Chen-Xi [3, 4].

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Bimodules and triangle equivalences

MANUEL SAORÍN

(joint work with Pedro Nicolás)

After the work of Happel [4], given ordinary associative unital algebras A and B , it is known that if ${}_B T_A$ is a classical tilting bimodule, then the functors $RHom_A(T, ?) : D(A) \rightarrow D(B)$ and $? \otimes_B^L T : D(B) \rightarrow D(A)$ define mutually quasi-inverse triangle-preserving equivalences, where $D(R)$ denotes the (unbounded) derived category of $Mod - R$, for any associative unital algebra R .

In the talk we presented applications of results on derived categories of dg categories from [5] to the case of ordinary algebras. The goal was to relax Happel's condition, giving necessary and sufficient conditions, in module-theoretical terms, for a bimodule ${}_B T_A$ in order for $? \otimes_B T$ or $RHom_A(T, ?)$ to be fully faithful (but not necessarily an equivalence).

We show that $? \otimes_B T : D(B) \rightarrow D(A)$ is fully faithful precisely when the following conditions hold:

- i) $B \cong End(T_A)$;
- ii) the canonical map $Hom_A(T, T)^{(\alpha)} \rightarrow Hom_A(T, T^{(\alpha)})$ is an isomorphism and $Ext_A^i(T, T^{(\alpha)}) = 0$ for all integers $i > 0$ and cardinals α ;
- iii) $RHom_A(T, ?) : D(A) \rightarrow D(B)$ preserves direct sums of objects in the localizing subcategory of $D(A)$ generated by T .

Since this last condition is not module-theoretical, it naturally arises the question of whether it is superfluous. Our impression is that it should not be the case, but it might be so if one assumes that T_A has finite projective dimension.

When one considers the functor $RHom_A(T, ?) : D(A) \rightarrow D(B)$, requiring that B is in its essential image is tantamount to requiring that $B \cong End(T_A)$ and $Ext_A^i(T, T) = 0$, for all $i > 0$. In this situation the functor is fully faithful and preserves compact objects if, and only if, there is an exact sequence $0 \rightarrow A \rightarrow T^0 \rightarrow T^1 \rightarrow \dots \rightarrow T^n \rightarrow 0$ in $Mod - A$, where the T^i are direct summands of finite direct sums of copies of T . Under the requirement that B is in the essential image, there are no known examples where $RHom_A(T, ?)$ is fully faithful but does not preserve compact objects. Hence we posed the question on the existence of such an example and linked it with a classical problem. Indeed if $RHom_A(T, ?)$ is fully faithful and B is in its essential image, then the ring homomorphism $A \rightarrow End({}_B T)^{op}$ is an isomorphism if, and only if, T_A is a Wakamatsu tilting

module. The posed question is equivalent in this case to a classical one, namely, whether a Wakamatsu tilting module has finite projective dimension.

As consequences of our results, we presented several connections of the fully faithfulness of $? \otimes_B^L T$ and $\mathrm{RHom}_A(T, ?)$ to the theory of (infinite dimensional) tilting modules, in particular to the good tilting modules introduced in [1], and to recollement situations recently studied, for instance, in [2], [3].

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Derived bi-duality via homotopy limit

HIROYUKI MINAMOTO

We show that a derived bi-duality dg-module is quasi-isomorphic to the homotopy limit of a certain tautological functor. This is a simple observation, which seems to be true in a wider context. From the view point of derived Gabriel topology, this is a derived version of results of J. Lambek ([3, 4]) about localization and completion of ordinary rings. However the important point is that we can obtain a simple formula for the bi-dual modules only when we come to the derived world from the abelian world.

One of the main motivations to study derived bi-duality was a derived completion theorem ([1, 2, 6]) which asserts that the completion of a commutative ring satisfying some conditions is obtained as a derived bi-commutator. Our simple formula for derived bi-dual modules enables us to give a generalization and an intuitive proof, which tells us that the derived completion theorem is of categorical nature. (We can also prove Koszul duality for dg-algebras with Adams grading satisfying mild conditions, which is included in a joint work with A. Takahashi.) Secondly, we prove that every smashing localization of a dg-category is obtained as a derived bi-commutator of some pure injective module. This is a derived version of the classical results in localization theory of ordinary rings ([7]).

These applications show that our formula together with the viewpoint that a derived bi-commutator is a completion in some sense, provide us a fundamental understanding of a derived bi-dual module.

(This talk was based on the preprint [5].)

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Localization and t -structures in algebraic geometry and beyond

LEOVIGILDO ALONSO TARRÍO

(joint work with Ana Jeremías López, Joseph Lipman, María José Souto Salorio)

Let X be a scheme. Gabriel identified hereditary torsion theories on $\mathrm{Qco}(X)$, the category of quasi-coherent sheaves on X with stable by specialization subsets in X through the functors $\Gamma_Z :=$ “sections with support in $Z \subset X$ ” that correspond to the torsion radical.

Let now Z be a closed subset of X with associated quasi-coherent ideal \mathcal{I} and consider the completion functor $\Lambda_Z : \mathrm{Qco}(X) \rightarrow \mathrm{Qco}(X)$ given, for a quasi-coherent sheaf \mathcal{G} , by the inverse limit of the powers $\mathcal{G}/\mathcal{I}^n\mathcal{G}$. This functor is neither right-exact nor left-exact, but it can be derived on the left by means of quasi-coherent flat resolutions, obtaining the derived functor $\mathbf{L}\Lambda_Z$. The so-called Greenlees-May duality [1] reads:

$$\mathrm{Hom}_{\mathbb{D}}(\mathbf{R}\Gamma_Z\mathcal{F}^\bullet, \mathcal{G}^\bullet) \cong \mathrm{Hom}_{\mathbb{D}}(\mathcal{F}^\bullet, \mathbf{L}\Lambda_Z\mathcal{G}^\bullet)$$

with $\mathcal{F}^\bullet, \mathcal{G}^\bullet \in \mathbb{D} := \mathbb{D}(\mathrm{Qco}(X))$. This formula is related to a series of previous results in local cohomology due (among others) to Matlis, Grothendieck, Hartshorne, Greenlees, Peskine, Szpiro.

An unexpected connection with homotopy arose when considering *Bousfield localizations*: Let \mathbb{T} be a triangulated category and \mathcal{L} a localizing subcategory, i.e. a triangulated subcategory stable for coproducts. We say that \mathcal{L} defines a localization in \mathbb{T} if the canonical functor $\mathcal{L} \hookrightarrow \mathbb{T}$ possesses a right adjoint. The problem of the existence of localizations was posed by Adams in 1973 and solved in the affirmative by Bousfield in 1979 [6] for a localizing subcategory with generator on the stable homotopy category. In 2000 the theorem was extended to the (unbounded) derived category $\mathbb{D}(\mathcal{A})$ of a Grothendieck category \mathcal{A} in [2].

From this result, these consequences follow:

- (1) The derived version of Gabriel-Popescu embedding.
- (2) The category $\mathbb{D}(\mathcal{A})$ has “small hom sets”.
- (3) Every complex in $\mathbf{K}(\mathcal{A})$ has a homotopically injective resolution.
- (4) The category $\mathbb{D}(\mathcal{A})$ satisfies Brown representability.

Notice that $\mathbf{Qco}(X)$ is a Grothendieck category, so these results have immediate geometric applications.

For the functor RI_Z there is an associated localization functor RQ_Z . This last functor is a Bousfield localization and these classes (for Z stable by specialization) classify all localizations that come from $\mathbf{Qco}(X)$. The functor LA_Z is also a Bousfield localization but it does not come from a Gabriel localization. In the case of a noetherian ring R , all Bousfield localizations in $\mathbf{D}(R)$ have been classified by Neeman [8] putting them in bijection with the subsets of $\mathrm{Spec}(R)$. This classification can be extended to semi-separated noetherian schemes X and quasi-coherent sheaves if the localizing subcategory satisfies that it is a \otimes -ideal of $\mathbf{D}(\mathbf{Qco}(X))$. These are put in bijective correspondence with subsets of X , see [4].

It is a remarkable observation due to Keller and Vossieck [7] that t -structures in the sense of Beilinson, Bernstein and Deligne [5] are a generalization of Bousfield localizations. Indeed, let \mathcal{U} be a suspended subcategory (i.e. stable for triangles but only for positive suspensions) of a triangulated category \mathbb{T} , then the pair $(\mathcal{U}, \mathcal{U}^\perp[1])$ constitutes a t -structure if and only if the canonical functor $\mathcal{U} \hookrightarrow \mathbb{T}$ possesses a right adjoint [7].

Applying the ideas of Bousfield localization to this new context we prove that if $\mathcal{U} \subset \mathbf{D}(\mathcal{A})$ is stable for coproducts and possesses a set of generators then it makes part of a t -structure. Also, the same holds if $\mathcal{U} \subset \mathbb{T}$ with \mathbb{T} any triangulated category but \mathcal{U} compactly generated.

As applications:

- (1) We give a new proof of Rickard's result: for a couple of rings R, S , we have that $\mathbf{D}^b(R) \cong \mathbf{D}^b(S)$ if and only if there is a tilting object E^\bullet in $\mathbf{D}^b(R)$ such that $\mathrm{End}_{\mathbf{D}(R)}(E^\bullet) = S$
- (2) We also obtain Beilinson's description of the bounded derived category of coherent sheaves on the n -dimensional projective space as the derived category of finitely generated modules over a finite dimensional algebra over the base field.

Both results employ the theory of the realization functor that compares the bounded derived category of the heart of a t -structure with the original category.

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On t -structures associated to filtrations in schemes

ANA JEREMÍAS LÓPEZ

(joint work with Leovigildo Alonso Tarrío, Manuel Saorín)

The concept of a t -structure on a triangulated category was introduced by Beilinson, Bernstein, Deligne and Gabber in [5]; it arose as a way of expressing in terms of categories of cohomological coefficients the concept of intersection homology of Goresky-MacPherson. B. Keller and D. Vossieck showed that a t -structure on a triangulated category \mathcal{T} is determined by its aisle [7]. An aisle is a suspended subcategory $\mathcal{U} \subset \mathcal{T}$ whose inclusion has a right adjoint. So, constructing t -structures is a matter of representing functors. Let $\mathcal{T} = \mathbf{D}(\mathcal{A})$, the derived category of a Grothendieck category \mathcal{A} . In [3], it was proved that the smallest suspended subcategory of \mathcal{T} containing a given collection of objects \mathcal{M} is an aisle, the aisle generated by \mathcal{M} . We present our results towards classification of t -structures on $\mathbf{D}_{\text{qc}}(X)$, the derived category of $\mathcal{A} = \mathcal{A}_{\text{qc}}(X)$, the category of quasi-coherent sheaves over a noetherian semiseparated scheme X .

A theorem of Neeman classifies triangulated t -structures a.k.a. Bousfield localizations in the unbounded derived category of modules over a commutative noetherian ring [8]. Neeman's result extends to a semi-separated noetherian scheme X classifying, in terms of subsets of X , Bousfield localizations on $\mathbf{D}_{\text{qc}}(X)$ whose localizing classes are \otimes -ideals in the monoidal sense (*rigid* localizing classes) [4]. In the affine case all localizing classes are *rigid*. It became clear that the monoidal structure of the derived category $\mathbf{D}_{\text{qc}}(X)$ is essential to understand Bousfield localizations determined by subsets of the scheme X . The *tensor compatible* (smashing) Bousfield localizations correspond to stable by specialization subsets of X . The *Hom compatible* localizations can be described via a certain formal duality relation, shedding a different light on the results in [1].

It is not possible to achieve a classification for general t -structures on $\mathbf{D}_{\text{qc}}(X)$, because already for $\text{Spec}(\mathbb{Z})$ they form a proper class [9]. So we restrict to study t -structures on the subcategory $\mathbf{D}_{\text{c}}(X) \subset \mathbf{D}_{\text{qc}}(X)$ of complexes with coherent homology (that turns out to be skeletally small). Two questions arise naturally, namely, classify t -structures on $\mathbf{D}_{\text{qc}}(X)$ with *nice* generators, and distinguish those that restrict to t -structures on $\mathbf{D}_{\text{c}}(X)$.

A filtration by supports is a decreasing map $\phi: \mathbb{Z} \rightarrow \mathbf{P}(X)$ such that the subsets $\phi(i) \subset X$ are stable by specialization. In the affine case, the t -structures on $\mathbf{D}_{\text{qc}}(X)$ generated by complexes in $\mathbf{D}_{\text{c}}^{\text{b}}(X)$ (or equivalently, in $\mathbf{D}_{\text{c}}^{-}(X)$) are precisely the t -structures on $\mathbf{D}_{\text{qc}}(X)$ generated by compact objects, and they are classified in terms of filtration by supports of X ([2]). For a general noetherian semiseparated scheme X the aisles associated to filtrations by supports are characterized using,

not only the canonical t -structure on $\mathbf{D}_{\text{qc}}(X)$, but also its structure of a closed monoidal symmetric triangulated category to establish the orthogonality. The aisles associated to filtrations have a good behaviour with respect to localization on open subsets.

Next question is to characterize those filtrations that provide t -structures on $\mathbf{D}_{\text{c}}(X)$. We say that a filtration by supports $\phi: \mathbb{Z} \rightarrow \mathbf{P}(X)$ satisfies the *weak Cousin condition* (notation inspired by [6]) if the following holds: for every $j \in \mathbb{Z}$ and $x, y \in X$ points, with $y \in \bar{x}$ and x maximal under y , if $y \in \phi(j)$ then $x \in \phi(j-1)$. Let $\mathcal{U}_{\phi} \subset \mathbf{D}_{\text{qc}}(X)$ be the aisle determined by the filtration ϕ . If $\mathcal{U}_{\phi} \cap \mathbf{D}_{\text{c}}(X)$ is an aisle of $\mathbf{D}_{\text{c}}(X)$ then ϕ satisfies the weak Cousin condition. If moreover X has a pointwise dualizing complex, then the converse holds.

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Noncommutative localization from a topologist’s point of view

ANDREW RANICKI

The talk was based on the survey article [1], and described some of the applications of noncommutative localization (particularly the Cohn localization of rings) to the topology of non-simply-connected manifolds. I concentrated on the applications to high-dimensional knot theory, where noncommutative localization is used to distinguish between the homotopy and homology types of the knot complement, and so detect knotting.

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Thick subcategories generated by Serre subcategories

RYO TAKAHASHI

For the past five decades, a lot of classification theorems of subcategories of abelian categories and triangulated categories have been given in ring theory, representation theory, algebraic geometry and algebraic topology. For details, see, for instance, [1, 2, 3, 4, 5, 6, 7, 8, 10] and the references therein. Reconstruction of an object from its support in the spectrum of a suitable commutative ring plays a crucial role in the proofs of those theorems.

Let R be a commutative noetherian ring. We denote by $\text{mod } R$ the category of finitely generated R -modules and by $D_{\text{sg}}(R)$ the singularity category of R . Let $S(R)$ be the set of prime ideals \mathfrak{p} of R such that $R_{\mathfrak{p}}$ is not a field, and let $\text{Sing } R$ be the singular locus of R . The main result of this talk is the following.

(1) There is a one-to-one correspondence between:

- the specialization-closed subsets of $S(R)$,
- the resolving subcategories of $\text{mod } R$ generated by a Serre subcategory of $\text{mod } R$.

(2) There is a one-to-one correspondence between:

- the specialization-closed subsets of $\text{Sing } R$,
- the thick subcategories of $D_{\text{sg}}(R)$ generated by a Serre subcategory of $\text{mod } R$.

It follows from (1) that a resolving subcategory of $\text{mod } R$ is generated by a Serre subcategory of $\text{mod } R$ if and only if it is closed under tensor products and transposes. Also, it is seen by (2) that if R is a hypersurface, then every thick subcategory of $D_{\text{sg}}(R)$ is generated by some Serre subcategory of $\text{mod } R$.

The details of results given in this talk can be found in [9].

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The derived category of a Leavitt path algebra

XIAO-WU CHEN

(joint work with Dong Yang)

This is a combination of extended abstracts of the two talks I presented at the Mini-Workshop, where the titles are ‘Leavitt path algebras as graded universal localizations’ and ‘Triangulated categories that are very close to module categories’, respectively. Both talks are concerned with the derived category of a Leavitt path algebra viewed as a differential graded algebra with zero differential. Such a derived category arises naturally from two quite differential setups; see [2].

One setup is the singularity category of a finite dimensional algebra with radical square zero. More precisely, let k be a field and Q be a finite quiver without sinks. Denote by kQ the path algebra and by J the two-sided ideal generated by arrows. Set $\Lambda = kQ/J^2$; it is a finite dimensional algebra with radical square zero. The *singularity category* $\mathbf{D}_{\text{sg}}(\Lambda)$ in the sense of Buchweitz and Orlov is by definition the Verdier quotient category of the bounded derived category of finite dimensional Λ -modules with respect to the subcategory of perfect complexes. Denote by $\mathbf{K}_{\text{ac}}(\Lambda\text{-Inj})$ the homotopy category of acyclic complexes of (possibly infinitely generated) injective Λ -modules; it is a compactly generated triangulated category, whose subcategory of compact objects is equivalent to $\mathbf{D}_{\text{sg}}(\Lambda)$. We aim to describe these two triangulated categories.

Recall that the path algebra kQ is \mathbb{Z} -graded by the length grading. We denote by $L(Q)$ the *Leavitt path algebra*, which is naturally \mathbb{Z} -graded such that the canonical map $\iota_Q: kQ \rightarrow L(Q)$ is a homomorphism of graded algebras. We mention that $L(Q) = \bigoplus_{n \in \mathbb{Z}} L(Q)^n$ is strongly graded with $L(Q)^0$ a von Neumann regular algebra.

Our first result claims a triangle equivalence between $\mathbf{K}_{\text{ac}}(\Lambda\text{-Inj})$ and $\mathbf{D}(L(Q)^{\text{op}})$, where $L(Q)^{\text{op}}$ is the opposite Leavitt path algebra. This equivalence restricts to a triangle equivalence on compact objects, in particular, we infer that $\mathbf{D}_{\text{sg}}(\Lambda)$ is triangle equivalent to the perfect derived category $\text{perf}(L(Q)^{\text{op}})$, which is further equivalent to the category $L(Q)^0\text{-proj}$ of finitely generated projective $L(Q)^0$ -modules; see [5, 1]. The main ingredient of the proof is the graded version of *universal localization* in sense of Cohn and Schofield, which applies to ι_Q . Another ingredient is Koszul duality, which claims a triangle equivalence between the homotopy category $\mathbf{K}(\Lambda\text{-Inj})$ and $\mathbf{D}(kQ^{\text{op}})$, where Q^{op} is the opposite quiver.

Another setup we consider is the comparison between triangulated categories and module categories along the line of [3, 4]. Our second result asserts that for a strongly graded ring $A = \bigoplus_{n \in \mathbb{Z}} A^n$ with A^0 left hereditary, there is a triangle equivalence between $\mathbf{D}(A)$ and $\Gamma\text{-GProj}$, where $\Gamma = A^0 \oplus A^{-1}$ denotes the trivial extension ring and $\Gamma\text{-GProj}$ denotes the *stable category of Gorenstein projective* Γ -modules; both triangulated categories differ from the graded A -module category by a two-sided ideal of square zero; see [3, 4]. This result applies to Leavitt path algebras, and relates the homotopy category $\mathbf{K}_{\text{ac}}(\Lambda\text{-Inj})$ to the stable category of Gorenstein projective modules over a certain trivial extension algebra.

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Resolving subcategories and t-structures over a commutative Noetherian ring

MANUEL SAORÍN

(joint work with Lidia Angeleri-Hügel)

Let R be a commutative noetherian ring and denote by $Mod - R$ (resp. $mod - R$) and $D(R)$ the category of all (resp. finitely generated) modules and the unbounded derived category of $Mod - R$, respectively.

Given an integer $n > 0$, recent results in [2] establish a bijective correspondence between resolving subcategories of $mod - R$ consisting of modules of projective dimension $\leq n$ and some finite decreasing sequences of specialization-closed subsets of $Spec(R)$. A resolving subcategory of $mod - R$ contains the projectives and is closed under taking direct summands, extensions and kernels of epimorphisms. On the other hand, by [1], there is a bijection between filtrations by supports of $Spec(R)$ and compactly generated t-structures in $D(R)$. By definition, such a filtration by supports is a decreasing map $\phi : \mathbb{Z} \rightarrow 2^{Spec(R)}$, where $\phi(i)$ is a specialization-closed subset of $Spec(R)$, for each $i \in \mathbb{Z}$. A natural question arises about a possible connection between (some) compactly generated t-structures of $D(R)$ and (some) resolving subcategories of $mod - R$.

In the talk we presented a recent result from [3] which gives a one-to-one correspondence between the following three sets:

- (1) Resolving subcategories of $mod - R$ consisting of modules of finite projective dimension;
- (2) Filtrations by supports ϕ such that $\phi(-1) = Spec(R)$ and, for each $i > 0$, $\phi(i)$ does not intersect the assassin of $E_i(R)$, the i -th term of the minimal injective resolution of R ;
- (3) Compactly generated t-structures of $D(R)$ containing the stalk complex $R[1]$ in their heart.

While the bijection between 2 and 3 is just a restriction of the bijection of [1], the hard part is the bijection between 1 and 3, which is based on the fact that if $(\mathcal{U}, \mathcal{U}^\perp[1])$ is a t-structure as in 3, then $\mathcal{Y} := \mathcal{U}^\perp \cap Mod - R$ is the right component

of a Tor-pair $(\mathcal{X}, \mathcal{Y})$ in $Mod-R$ (in the sense of [5]) such that $\mathcal{X} \cap mod-R$ consists of modules of finite projective dimension.

The result implies the existence of a maximum in the set of filtrations by supports in 2, which is the one corresponding to $\mathcal{P}^{<\infty}$, the resolving subcategory of all finitely generated modules of finite projective dimension. The explicit description provided of that maximal filtration by support allowed us to show a new angle of the (little) finitistic dimension of R . By definition, this is the supremum of the projective dimensions of all modules in $\mathcal{P}^{<\infty}$. For an integer $n > 0$, we show that $findim(R) \leq n$ if, and only if, $\bigoplus_{0 \leq i \leq n} E_i(R)$ is an injective cogenerator of $Mod-R$.

Two natural questions remain open: a) is it possible to extend the bijection in our main result in order to include all the resolving subcategories of $mod-R$?; b) what is the connection of our bijection with the bijection given in [4] between the resolving subcategories in 1 and the grade consistent functions on $Spec(R)$?

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Recollements of module categories

JORGE VITÓRIA

(joint work with Chrysostomos Psaroudakis)

Recollements of abelian categories are short exact sequences of abelian categories with nice properties. They are related to torsion-theoretical structures and categorical localisations. In this talk we discuss these relations, with particular focus on module categories. An example (which we call *typical*) of a recollement of $Mod-A$, for a ring A , is given by an idempotent element e of A :

$$\begin{array}{ccccc}
 & \xleftarrow{-\otimes_A A/AeA} & & \xleftarrow{-\otimes_{eAe} eA} & \\
 Mod-A/AeA & \xrightarrow{inc} & Mod-A & \xrightarrow{Hom_A(eA, -)} & Mod-eAe. \\
 & \xleftarrow{Hom_A(A/AeA, -)} & & \xleftarrow{Hom_{eAe}(Ae, -)} &
 \end{array}$$

It is well-known that recollements of a triangulated category are in bijection with its torsion, torsion-free (TTF) classes ([3],[4]). For recollements of abelian categories a similar result holds.

Proposition ([5]) *Equivalence classes of recollements of an abelian category \mathcal{A} are in bijection with bilocalising TTF-classes of \mathcal{A} .*

If \mathcal{A} has enough projectives and enough injectives, all TTF-classes are bilocalising (i.e., both localising and colocalising). There are, however, examples of TTF-classes in certain abelian categories which are not bilocalising.

Recollements of $\mathbf{Mod}\text{-}A$ are induced by an idempotent ideal I of A ([1],[5]). Although the left-hand term of the recollement will then necessarily be (equivalent to) a module category ($\mathbf{Mod}\text{-}A/I$) the same is not true for the right-hand side. An example of this can be obtained by considering a field \mathbb{K} and taking $A = \prod_{i=1}^{\infty} \mathbb{K}$ and $I = \bigoplus_{i=1}^{\infty} \mathbb{K}$ (a more general class of examples can be obtained by considering the rings in [6]). Still, the following result conjectured by Kuhn in [2] holds.

Theorem ([5]) *Any recollement of $\mathbf{Mod}\text{-}A$ such that all the terms are equivalent to module categories is equivalent to a typical recollement.*

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The Jordan-Hölder Theorem for derived categories

LIDIA ANGELERI HÜGEL

(joint work with Steffen Koenig, Qunhua Liu, Dong Yang)

Let A be a ring, and let $\mathcal{D}(A) = \mathcal{D}(\mathbf{Mod}A)$. The concept of a recollement is used to study stratifications of $\mathcal{D}(A)$ which may be regarded as analogues of composition series for groups or modules. Hereby A is said to be *derived simple* if $\mathcal{D}(A)$ does not occur as middle-term of a non-trivial recollement of derived module categories. A *stratification* is then given by a full rooted binary tree with root $\mathcal{D}(A)$ whose nodes are derived module categories with derived simple rings at the leaves.

A stratification thus breaks up $\mathcal{D}(A)$ into simple pieces employing recollements. This can reduce some problems to derived simple rings, by using for example that one can glue t-structures or compute homological invariants along recollements. There are several results in this direction, see work of Wiedemann, König, Happel, Keller, Liu-Vitória quoted e.g. in [5]. We also mention

Theorem [5] *If A, B, C are finite-dimensional algebras over a field with a recollement of $\mathcal{D}^-(\text{Mod}A)$ by $\mathcal{D}^-(\text{Mod}B)$ and $\mathcal{D}^-(\text{Mod}C)$, then there is an isomorphism of K -groups $K_*(A) \cong K_*(B) \oplus K_*(C)$.*

The analogy with composition series raises the question whether A satisfies

(JH) ‘*Derived*’ Jordan-Hölder theorem. $\mathcal{D}(A)$ has a finite stratification which is unique up to ordering and equivalence.

The following rings satisfy (JH):

- semi-simple rings,
- commutative noetherian rings,
- group algebras of finite groups,
- piecewise hereditary algebras.

For the first three cases, observe that any ring having a block decomposition with derived simple blocks satisfies (JH), with the stratification given by the derived categories of the blocks, and use that simple artinian rings, indecomposable commutative rings, and blocks of group algebras of finite groups are derived simple [2, 5, 7]. The fourth case is settled in [2, 3]. Here the stratification is given by the derived categories of the endomorphism rings of simple modules.

Finally, we exhibit two counterexamples. The first, taken from [6], is a hereditary non-artinian ring A for which $\mathcal{D}(A)$ has two stratifications, one of length 2 and one of length 3, without common simple factors. The second, taken from [5], is a finite dimensional algebra with two different stratifications of length 2.

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Comparing recollements - yoga on ladders

STEFFEN KOENIG

(joint work with Lidia Angeleri Hügel, Qunhua Liu, Dong Yang)

This talk continues that of Lidia Angeleri and reports on further results in [1]. For the sake of exposition we restrict the setup to finite dimensional algebras.

A recollement can be seen as a short exact sequence of triangulated categories, or some sort of semi-orthogonal decomposition, or a torsion-torsionfree (TTF) triple. We are interested in recollements where all three terms are derived module categories. Derived module categories exist on different levels; unbounded, bounded above or below, bounded, or as homotopy category of, say, finitely generated projective modules.

A recollement on any level always can be lifted to the unbounded level. A recollement on unbounded level does, however, not in general restrict to any of the smaller derived categories. We give precise criteria, for each choice of derived category, when restriction is possible.

These criteria are closely related with a concept of mutation for recollements: A *ladder* (or TTF n -tuple) is a diagram of derived categories and functors such that any three subsequent rows form a recollement. We give precise criteria when a given recollement can be extended ('mutated') upwards or downwards, that is, when appropriate adjoint functors exist. The height of the ladder is the number of recollements it contains, that is, the difference from the top to the bottom recollement, or infinite.

As an application we characterise derived simplicity on different levels and provide examples illustrating the subtleties arising: An algebra that is derived simple on unbounded levels, does not admit any recollement on any level and hence is derived simple on any level. The converse is unexpected:

Theorem. (a) *The algebra A is $D^-(\text{Mod})$ -simple (i.e. the category $D^-(\text{Mod}-A)$ does not admit a non-trivial recollement) if and only if $D(\text{Mod}-A)$ does not admit any ladder of height bigger than one.*

(b) *The algebra A is $K^b(\text{proj})$ -simple if and only if $D(\text{Mod}-A)$ does not admit any ladder of height bigger than two.*

(c) *The algebra A is $K^b(\text{proj})$ -simple if and only if it is $D^b(\text{Mod})$ -simple if and only if it is $D^b(\text{mod})$ -simple.*

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The universal localization of triangular matrix rings

ANDREW RANICKI

In the 1970's and 1980's Cohn, Bergman, Dicks and Schofield expressed the generalized free products and HNN extensions of rings in terms of the universal localization of triangular matrix rings: see Ranicki [1] and Sheiham [2]. The talk described the algebraic K - and L -theory of a stably flat (= homologically split) injective universal localization, in which the torsion groups are the Nil-groups of Farrell and the UNil-groups of Cappell, as well as the topological motivation from the codimension 1 splitting of homotopy equivalences of manifolds.

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Annihilation of Ext modules and generation of derived categories

RYO TAKAHASHI

(joint work with Srikanth Iyengar)

Uniform annihilation of Ext modules was originally a problem in number theory. In the 1980s it became a target in commutative algebra in relation to the Brauer–Thrall conjectures for maximal Cohen–Macaulay modules over a Cohen–Macaulay local ring [4, 7, 11]. It has been explored for more general rings so far [3, 10].

Strong finite generation of triangulated categories was introduced by Bondal and Van den Bergh [2] to investigate representability of functors. Rouquier [8, 9] defined the dimension of a triangulated category which refines strong finite generation, and connected it with the representation dimension of an artin algebra. The bounded derived category $D^b(R)$ of finitely generated modules over a commutative ring R was shown to be strongly finitely generated if R is either a ring essentially of finite type over a field [6, 9] or a complete local ring over a perfect field [1].

In this talk we analyse uniform annihilation of Ext modules and study strong finite generation of derived categories. Let Λ be a noetherian ring. We say that Λ has the *uniform annihilator property*, UAP for short, if there exist a nonzerodivisor x of the center of Λ and a positive integer n such that $x \operatorname{Ext}_{\Lambda}^n(M, N) = 0$ for all finitely generated Λ -modules M, N . It turns out that the UAP is widely satisfied; a reduced ring essentially of finite type over a field and an integral domain admitting a noncommutative resolution have the UAP. Also, the UAP decomposes the derived category $D^b(\Lambda)$ into two certain explicit subcategories. As an application, the theorem of Keller, Rouquier and Van den Bergh and the theorem of Aihara and Takahashi stated above are simultaneously recovered.

The details of results given in this talk are to be found in [5].

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Tilting modules arising from universal localisation for hereditary rings

FREDERIK MARKS

(joint work with Lidia Angeleri Hügel, Jorge Vitória)

Let A be a hereditary ring and T be a (possibly infinitely generated) tilting A -module. In this talk we discuss necessary and sufficient conditions for T to be equivalent to a tilting module of the form $A_\Sigma \oplus A_\Sigma/A$ for some monomorphic universal localisation $A \rightarrow A_\Sigma$, in the sense of Schofield. This question is motivated by recent work that uses ring epimorphisms and universal localisations to classify (infinitely generated) tilting modules (see [2] and [3]). For example, if A is a Dedekind domain, every tilting module arises from universal localisation ([2]). Moreover, if A is the Kronecker algebra, only the Lukas tilting module does not occur in this way ([3]).

The approach presented in this talk is based on a construction in [4], yielding a suitable abelian subcategory \mathcal{C} of the torsion class $Gen(T)$. It turns out that, in case T arises from universal localisation, \mathcal{C} describes precisely the full subcategory of all A_Σ -modules in $Mod(A)$. Furthermore, for an arbitrary tilting A -module T the category \mathcal{C} can always be used to define a universal localisation $A_{\mathcal{U}_T}$ at a set \mathcal{U}_T of finitely presented A -modules, which are $Ext^{\geq 0}$ -orthogonal to \mathcal{C} .

To use this construction it becomes necessary to impose some minimality conditions on the tilting module T . We call T minimal (respectively, strongly minimal), if the free A -module of rank one (respectively, every free A -module) admits a minimal left $Add(T)$ -approximation. Under this assumption we can assure that $A_{\mathcal{U}_T} \oplus A_{\mathcal{U}_T}/A$ is a tilting A -module and we are able to describe the projective

modules in \mathcal{C} . As a consequence, we get the following result.

Theorem ([1]) *Let A be a hereditary ring and T be a tilting A -module. The following are equivalent.*

- (1) T arises from universal localisation.
- (2) T is strongly minimal.
- (3) T is minimal and $\text{Gen}(T) = \{X \in \text{Mod}(A) \mid \text{Ext}_A^1(U, X) = 0 \forall U \in \mathcal{U}_T\}$.

As a corollary we get that, for example, all finitely generated tilting modules over any hereditary Artin algebra arise from universal localisation.

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Smashing subcategories of derived categories of rings of weak global dimension one

SILVANA BAZZONI

(joint work with Jan Stovicek)

The telescope conjecture arises in stable homotopy theory in works of Bousfield and Ravenel [1, 6] and it has been later formulated in the more general setting of compactly generated triangulated categories. It is a conjecture predicting a sort of finite type of smashing subcategories, where a full subcategory of a triangulated category with arbitrary coproducts is called smashing if it is the kernel of a Bousfield localization functor that preserves arbitrary coproducts. In its generality, the telescope conjecture has been disproved by Keller [2], even in the case of the derived category of rings. On the positive side, the telescope conjecture has been proved by Neeman for the derived category of noetherian commutative rings [4] and by Krause and Stovicek [3] for the derived category of hereditary rings.

The solution of the telescope conjecture goes parallel to the problem of characterizing smashing subcategories. Smashing subcategories of the derived category of differential graded algebras are strongly related to homological epimorphisms.

Nicolas and Saorin [5] proved that, given a differential graded algebra B , a full subcategory \mathcal{X} of the derived category of B is smashing if and only if there is a differential graded k -algebra C and a homological epimorphism $f: B \rightarrow C$ such that \mathcal{X} is the kernel of the functor $- \otimes_B^L C$.

We consider rings R with weak global dimension at most one. Making use of Kunneth formulas we prove that smashing subcategories of the derived category of R are in bijection with equivalence classes of homological ring epimorphisms originating in R . Specializing to the case of a valuation domain R we give a full description of the smashing subcategories of the derived category of R . As a consequence we prove that the telescope conjecture holds true for a commutative semi-hereditary ring if and only if every localization of R at a maximal ideal doesn't have non-zero idempotent ideals.

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