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Arbeitsgemeinschaft: Superrigidity

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Abstract. The purpose of the Arbeitsgemeinschaft was to review old and new phenomenas of rigidity in mathematics. The broad spectrum of such results was covered, such as Margulis-Zimmer superrigidity, cocyle and character rigidity.

Mathematics Subject Classification (2010): 20G05, 22E, 22A26, 28D05, 47C15.

Introduction by the Organisers

The Superrigidity Arbeitsgemeinschaft was attended by about 50 participants. Most of them were young researches or PhD students representing different branches of mathematics and with different backgrounds. Rigidity problems were studied since decades and (as it is visible in the program below) are still very attractive and dynamical areas of research. Rigidity is an interdisciplinary area of mathematics which combines elements of ergodic theory, Lie groups, actions of groups, von Neumann algebras and others. To deal with such a variety of material to cover the talks were divided into sections. Usually in each section first we discussed an appropriate classical rigidity results followed by recent ones. When it was possible, the focus was put on particular examples which illustrated the general theorems. The short description of each section with reference to the abstracts is given below.

It is a pleasure to thank the institute and the organizers for their effort for providing a pleasant and fruitful meeting.

B: Background talks. The purpose of this section was to review some classical material essential in the incoming sections. The starting point was the notion of amenable actions on measured spaces (à la Zimmer), which is an indispensable tool in most rigidity results (*B1*). Then we passed to the strong negation of amenability, that is Kazhdan property (T), (FH) and its consequences (*B2*). On the analytic side, we recalled some basic theory of von Neumann algebras (*B3*). To facilitate later talks on superrigidity of von Neumann algebras the notion of measured equivalence relation was introduced (*B4*).

C: Character superrigidity The introduction to characters and basics facts as correspondence between extremal characters and finite factors representation were given. Then we investigated character rigidity phenomenas of Thompson's group F (*C1*). The next examples we covered were recent results on characters rigidity of $PSL(n, \mathbb{Z})$ (by B. Bekka) and $PSL(2, k)$ for k infinite field (Peterson, Thom). The applications to freenes of ergodic actions and factor representations were discussed (*C2*). Following Creutz and Peterson we proved character rigidity for Λ an irreducible lattice in a product $G \times H$, where G is a non-compact simple Lie group with (T) and H is a totally disconnected non-discrete simple group with the Howe-Moore property (*C3 and C4*).

D: Deformation and rigidity techniques. Cocycle superrigidity. In this sections we dealt with Popa and Ioana Cocycle Superrigidity (*D1*) as well as basic applications to Orbit Equivalence rigidity (*D2*). The analytic counterpart was the rigidity phenomenon for crossed product von Neumann algebras associated with group actions (*D3 and D4*).

S: Margulis-Zimmer superrigidity for higher rank Lie groups and their lattice. Here we concentrated on the new approach to celebrated Margulis-Zimmer superrigidity due to U. Bader and A. Furman. After introducing the language of algebraic representations and birepresentations and proving crucial facts (*S1* and *S2*) the superrigidity theorem of Margulis for lattices in $SL_3(\mathbb{R})$ is demonstrated (*S3*). In the last lecture it was showed how classical Margulis normal subgroup theorem can be deduced from Factor Theorem (*S4*).

A: From Superrigidity to Arithmeticity of lattices It was showed how Margulis Arithmeticity theorem follows from Superrigidity theorem (*A1*). Subsequently we discussed Margulis Commensurator Superrigidity and how it implies arithmeticity criterion (*A2*).

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Arbeitsgemeinschaft: Superrigidity

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Abstracts

B1: Amenability of Groups and Group Actions

Carlos A. De la Cruz Mengual

This is a background talk on amenability of locally compact groups and of their actions. For groups, we present a characterization of amenability in terms of the existence of fixed points of their affine actions, as in [1], and discuss some examples. Then, we introduce the notion of amenable action: an action $G \curvearrowright (S, \mu)$ [where S is a Borel space and μ is a G-quasi-invariant measure on S] is amenable if for every compact metric G-space X, there exists a measurable map $S \longrightarrow \mathrm{Prob}\,(X)$ that is G-equivariant up to a null set. We conclude by showing that if Γ is a lattice of G and $P \leqslant G$ is closed and amenable, then the action $\Gamma \curvearrowright \frac{G}{P}$ is amenable; in particular, this holds if G is a semisimple Lie group and P is minimal parabolic. This fact in the latter situation is relevant when proving Margulis' superrigidity theorem.

References

[1] R. J. Zimmer, *Ergodic Theory and Semisimple Groups*, Monographs in Mathematics, vol. **81** (1984), Birkhäuser Verlag, Basel.

B2: Property (T), (FH) and cohomology

Stephan Tornier

In this talk we gave a definition of Property (T) for locally compact groups, examples and non-examples. We focused on the following material. Hereditary properties: Morphisms with dense image, extensions, lattices. Relation to amenability: A locally compact amenable groups has Property (T) if and only if it is compact. Consequences of Property (T): Unimodularity, compact Hausdorff abelianization, compact generation. Property (FH) for locally compact groups. Continuous affine isometric actions and continuous cohomology. Property (T) and reduced continuous cohomology. Dependance of Property (T) and (FH) on whether $\mathbb{R}$ and/or $\mathbb{C}$ are allowed as fields in the respective definition. Equivalence of Property (T) and (FH) for locally compact σ-compact groups. We gave a proof of (FH) $\Rightarrow$ (T).

References

[1] B. Bekka, P. de la Harpe, and A. Valette, *Kazhdans property (T)*, New Mathematical Monographs, vol. **11**, Cambridge University Press, Cambridge, 2008.

B3: Von Neumann algebras

Tim de Laat

In this talk, we recalled some basic theory on von Neumann algebras that was needed as background for the talks (C1)-(C4) and (D1)-(D4). Starting from the Bicommutant Theorem, we introduced the type decomposition, types of factors, conditional expectations, and normal faithful traces on finite von Neumann algebras. After that, we discussed amenability and property (T) for von Neumann algebras, which are operator algebraic analogues of these well-known properties for groups. We finished with the notion of Cartan subalgebras and Popa's intertwining-by-bimodules theorem. In the talk, the group von Neumann algebra served as a motivating example.

References

[1] N. Brown and N. Ozawa, *C^*-Algebras and Finite-Dimensional Approximations, Graduate Studies in Mathematics*, **88** (2000), American Mathematical Society, Providence, Rhode Island.

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[6] S. Popa, *Correspondences*, 1986. INCREST Preprint, Unpublished. www.math.ucla.edu/ popa/popa-correspondences.pdf

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[8] M. Takesaki, *Theory of Operator Algebras I*, Encyclopedia of Mathematical Sciences, Springer-Verlag (1979).

B4: Measured Equivalence Relations

Daniel Hoff

The primary goal of this talk was to give some of the background necessary to motivate and facilitate later talks on superrigidity in von Neumann algebras. The group measure space von Neumann algebra arising from a non-singular action of a countable group on a countably separated measure space was constructed and the type classification for factors arising in this way was given. Restricting to the case of standard probability spaces, Zimmer's Theorem on injective von Neumann algebras coming from amenable actions was stated. Then, in order to motivate the study of measured equivalence relations and Cartan subalgebras, Singer's Theorem was stated and discussed. Measured equivalence relations were then defined, and the talk concluded with brief remarks about their associated von Neumann algebras.

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[1] C. Houdayer, *An introduction to II_1 factors*, 2011. Lecture notes from the CIRM, Available at: wis.kuleuven.be/events/ihp2011/notes/vng2011-houdayer.pdf.
[2] R. V. Kadison and J. R. Ringrose, *Fundamentals of the theory of operator algebras.* Vol. **II**, Graduate Studies in Mathematics, vol. **16**, American Mathematical Society, Providence, RI, 1997. Advanced theory, Corrected reprint of the 1986 original
[3] C. C. Moore, *Ergodic theory and von Neumann algebras*, Operator algebras and applications, Part **2** (Kingston, Ont., 1980) (1982), pp. 179226.
[4] R. J. Zimmer, *Hyperfinite factors and amenable ergodic actions*, Invent. Math. **41** (1977), no. 1, 233.

C1: Character superrigidity: Thompson's group

Niels Meesschaert and Yash Lodha

In the first half of the talk, we give an introduction to characters on countable groups and prove that that there exists a one-to-one correspondence between extremal characters and equivalence classes of finite factor representations. In the second half of the talk, we give a short introduction to Thompson's group F and present Dudko and Medynets result ([1]) stating: every extremal character of F is either the regular character or arises as a homomorphism of the abelianisation F/F' to the unit circle.

References

[1] A. Dudko and K. Medynets, *Finite Factor Representations of Higman-Thompson groups.* Groups, Geometry, and Dynamics, to appear. `arXiv:1212.1230`.

C2: Character rigidity for special linear groups (after Peterson-Thom)

Chenxu Wen

A character on a group G is a positive definite function $\varphi : G \to \mathbb{C}$ which is invariant under conjugation and is normalized so that $\varphi(e) = 1$. Via the GNS-construction, characters are naturally related to the unitary representations of the group on finite von Neumann algebras.

Recently, Bekka proved in [1] that for $G = PSL(n, \mathbb{Z}), n \geq 3$, G has character rigidity. He also noticed that from this fact it follows that the only II_1-factor representation for these groups must be the left regular representation. More recently, Peterson and Thom [2], proved the character rigidity for $PSL(2, k)$, where k is a infinite field. They also got some applications for this result to the question of freeness of ergodic actions of those groups.

In the talk I will go through the main idea of the proof of the result of Peterson-Thom, and compare it with the Bekka's proof.

References

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D1: Deformation regidity methods for cocycles

Michael Cantrell

We prove Popa and Ioana Cocycle Superrigidity. Given a countable group Γ and a probability measure-preserving action $\Gamma \curvearrowright (X, \mu)$, we say $\Gamma \curvearrowright (X, \mu)$ is cocycle-superrigid if for every countable group Λ and every measurable cocycle $c : \Gamma \times X \to \Lambda$, c is measurably conjugate to a homomorphism, i.e. there exists a homomorphism $\rho : \Gamma \to \Lambda$ and a measurable map $\phi : X \to \Lambda$ such that for all $\gamma \in \Gamma$ and μ-a.e. $x \in X$, $c(\gamma, x) = \phi(\gamma \cdot x)\rho(\gamma)\phi(x)^{-1}$. A concrete version of Popa's Cocycle Superrigidity Theorem is that if Γ is a countable group with property T, then the Bernoulli action $\Gamma \curvearrowright ([0,1], lebesgue)^\Gamma$ is cocycle-superrigid. A concrete version of Ioana's Cocycle Superrigidity Theorem is that if Γ is a countable group with property T, then any profinite action of Γ is cocycle-superrigid, up to finite index.

References

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[4] S. Popa, *Cocycle and orbit equivalence superrigidity for malleable actions of wi-rigid groups*, Invent. Math. **170** (2007), no. 2, 243-295.

D2: Deformation rigidity methods for cocycles

Steven Deprez

Let $\Gamma \curvearrowright (X, \mu)$ be a free, ergodic, measure-preserving with cocycle superrigidity. Assume moreover that Γ does not have finite normal subgroups. We proved a theorem of Popa, that if such an action is orbit equivalent to some $\Lambda \curvearrowright (Y, \nu)$, then this orbit equivalence is actually a conjugation.

References

[1] A. Furman, *A survey of measured group theory, Geometry, rigidity, and group actions*, Chicago Lectures in Math., Univ. Chicago Press, Chicago IL, 2011 pp. 296-374.

[2] A. Furman, *On Popa's cocycle superrigidity theorem*, Int. Math. Res. not. IMRN **19** (2007).

S1: Algebraic representations of ergodic actions

Bruno Duchesne

Given an ergodic action of a locally compact T on a standard probability space (X, μ) and a homomorphism $S \to G$ where G is a real algebraic group, we explain how to represent algebraically the action $S \curvearrowright X$. It was the first talk in a series of three talks on the new approach of U. Bader and A. Furman on Margulis-Zimmer superrigidity.

1. Introduction

Margulis superrigidity theorem had a very strong impact because of its main consequence: the proof of arithmeticity of lattices in algebraic groups over local fields with higher rank. It still continue to inspire actual research. Recently U. Bader and A. Furman [2] published a new paper explaining a quite simple and systematic approach of superrigidity phenomenon in higher rank based on original ideas of Margulis and Zimmer [3, 4].

The general philosophy is based on the fact that ergodic actions in the algebraic world are poorer than general ones since they reduce to the homogeneous ones. The key tool to see this property is *smoothness*. This property allows the previous authors to construct the category of algebraic representations of ergodic actions and prove that this category has an initial object which is unique up to isomorphism. In particular, this uniqueness property will lead to the notion of *birepresentations* for commuting actions.

This report is based on a talk that was the first one among three talks on [2]. The two others were given by Thierry Stulemeijer and Jean Lécureux. The main goal was to emphasize ideas instead of technicalities. In particular, we didn't speak about cocyles nor algebraic groups over general fields with an absolute value. We concentrated on the case of a lattice Γ in $\mathrm{SL}_n(\mathbb{R})$ with $n \geq 3$ and proved that any homomorphism from Γ to some simple real algebraic group extend to $\mathrm{SL}_n(\mathbb{R})$.

2. Smoothness of algebraic actions

Let T be locally compact second countable group acting measurably on some standard probability space (Y, μ) preserving the measure class of μ.

Definition. *The action $T \curvearrowright Y$ is* smooth *if $T\backslash Y$ is countably separated as measurable space.*

The main point about smooth actions is the fact that ergodic smooth actions are actually transitive (the measure is supported on one orbit). This is a direct consequence of the definition. Let us give two examples. The first one is an irrational rotation of the circle. The orbits space is ugly, in particular not countably separated. The second one is the linear action of the diagonal group $\left\{ \begin{bmatrix} \lambda & 0 \\ 0 & \lambda^{-1} \end{bmatrix} \middle| \lambda \in \mathbb{R} \right\}$ of $\mathrm{SL}_2(\mathbb{R})$ on the plane $\mathbb{R}^2$. The action is perfectly smooth as one can convince himself with a drawing.

The reason for this action to be smooth is the fact it is algebraic. More precisely, a group acting algebraically on an algebraic variety has locally closed orbits (in the Haussdorff topology) [4, Theorem 3.1.3] and thanks to Effros-Glimm theorem [4, Theorem 2.1.14], this is equivalent to say that the action is smooth.

3. Algebraic representations

Assume that there is a continuous homomorphism $T \to G$ where G is some connected non compact simple Lie group without center.

Definition. *An* algebraic representation *of* $T \curvearrowright Y$ *is a measurable* T*-equivariant map* $Y \to V$ *where* V *is some algebraic variety on which* G *acts algebraically.*

Algebraic representations form a category for which morphisms are G-equivariant maps. One main point in [2] is to show there is an initial object in that category which is unique up to isomorphism. Moreover, using smoothness, it is shown this initial object V_0 is a *coset representation* that is there is an algebraic subgroup H such that $V_0 = G/H$.

A priori, this category can be reduced to the trivial representation, that is V_0 is a point. In case, the action $T \curvearrowright Y$ is ergodic and amenable then it is guaranteed there is a non-trivial algebraic representation. Let us sketch the argument. Fix P some minimal parabolic of G. One knows that G/P is a compact metrizable space and thus amenability implies there is a T-map $Y \to \mathrm{Prob}(G/P)$. Even if $\mathrm{Prob}(G/P)$ is not an algebraic variety, the action of G on $\mathrm{Prob}(G/P)$ is smooth and stabilizers of probability measures on G/P are algebraic [4, 1]. In particular from the map $Y \to \mathrm{Prob}(G/P)$ one get another map $Y \to G/H$ where H is an algebraic subgroup of G. This shows that the considered category is not reduced to the trivial algebraic representation.

References

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[2] U. Bader and A. Furman. *Algebraic Representations of Ergodic Actions and Super-Rigidity.* ArXiv e-prints, November 2013, 1311.3696.

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S2: bi-algebraic representations of bi-ergodic actions

Thierry Stulemeijer

Building on the previous talk about algebraic representation of ergodic actions, we define the slightly more general category of bi-algebraic representation of bi-actions.

We prove that under strong ergodic assumptions, this category still has an initial object. The proof follows essentially the same lines than the corresponding statement for algebraic representation.

Finally, we end the talk by showing the invariance of this initial object for commuting actions.

The talk covered section 9 of [1].

References

[1] U. Bader and A. Furman, *algebraic representations of ergodic actions and super-rigidity*, In preparation (2014)

S3: Higher rank Lattices Super-rigidity

Jean Lecureux

In this talk we combine the tools developed in the previous talks (algebraic representations and bi-representations) and give a complete proof of the superrigidity theorem of Margulis for lattices in $SL_3(R)$: let G be such a lattice, and H be a simple, non-compact, center-free Lie group. Then any morphism from G to H with Zariski dense image extends to a morphism from $SL_3(R)$ to H.

References

[1] U. Bader, A. Furman, *Algebraic Representations of Actions and Super-Rigidity.* arXiv:1331.3696

S4: Factor and normal subgroup theorems for lattices in products

Holger Kammeyer

In this talk we outlined how the classical Margulis normal subgroup theorem can be deduced from a Factor Theorem. This theorem measurably identifies spaces, which are acted on by a higher rank lattice, with a flag variety. Building on this, we presented the more recent normal subgroup theorem for lattices in products of just noncompact groups due to Bader–Shalom. The proof has a similar outline. We proved the key step, the intermediate factor theorem, and stressed that it crucially relies on the uniqueness of Poisson boundary maps.

References

[1] U. Bader, Y. Shalom, *Factor and normal subgroup theorems for lattices in products*, Invent. Math. **163** (2006) 2.

D3 & D4: Unique group-measure space decomposition

Remi Boutonnet and Peter Verraedt

We illustrate deformation-rigidity techniques for von Neumann algebras by proving the following theorem.

Theorem ([1]). *Let Γ be a countable discrete group, and assume that*

- *there exists a nonamenable subgroup $H < \Gamma$ such that the pair (Γ, H) has relative property (T), and*
- *there exists a mixing orthogonal representation $\pi : \Gamma \to \mathcal{O}(H_{\mathbb{R}})$ and an unbounded 1-cocycle b for π.*

Let $\Gamma \curvearrowright (X, \mu)$ be a free ergodic p.m.p. action and put $M = L^\infty(X) \rtimes \Gamma$. Whenever $M = L^\infty(Y) \rtimes \Lambda$ for another free ergodic p.m.p. action $\Lambda \curvearrowright +(Y, \nu)$, there exists a unitary $u \in \mathcal{U}(M)$ such that $uL^\infty(Y)u^\star = L^\infty(X)$.

This result describes some rigidity fenomenon for crossed product von Neumann algebras associated with group actions. It allows to retrieve the orbit equivalence relation from the von Neumann algebra. We present the approach introduced by S. Vaes, [2].

References

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C3 & C4: Character rigidity for commensurators

F. Le Maître and S. Raum

In these two talks we gave a proof of the following theorem of Creutz-Peterson [1]: if Λ is an irreducible lattice in a product $G \times H$, where G is a non-compact simple Lie group with (T) and H is a totally disconnected non-discrete simple group with the Howe-Moore property, then every finite factor representation of Λ is either the regular representation or finite dimensional. As an example, one may consider $\Lambda = PSl_n(\mathbb{Z}[1/p])$ embedded diagonally into $PSl_n(\mathbb{R}) \times PSl_n(\mathbb{Q}_p)$. Such a result generalizes the Margulis Normal Subgroup Theorem for Λ, whose proof consists in playing property (T) against amenability. The first talk was devoted to the property (T) part, and used another result of Creutz-Peterson on the absence of nontrivial finite factor representations for H. It was shown that under the hypothesis that a finite factor representation π of Λ satisfies $\pi(\Gamma)''$ amenable, such a representation has to be finite dimensional. So the second talk focused on proving that whenever π is not regular, $\pi(\Gamma)''$ needs to be amenable. It relied on the Poisson boundary of $\pi(\Gamma)''$, which is an amenable von Neumann algebra and, using the contractivity of the action of G on its Poisson boundary, turns out to be isomorphic to $\pi(\Gamma)''$ in our setting.

References

[1] D. Creutz and J. Peterson, *Character rigidity for lattices and commensurators.* ArXiv e-prints, 2013.

A1: How Superrigidity implies Arithmeticity

Gregor Masbaum

Margulis' Arithmeticity Theorem states that irreducible lattices in semisimple groups of higher rank are arithmetic. The goal of the talk was to explain what is an arithmetic lattice and to deduce Margulis' Arithmeticity Theorem from Margulis' Superrigidity theorem.

References

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A2: Commensurator Superrigidty

Alin Galatan

The aim of the lecture is to sketch a proof of the fundamental theorem of Margulis concerning lattices in semisimple Lie groups, under the assumption that the commensurator of the lattice is dense in the ambient group. This way, we can prove arithmeticity of these lattices in semisimple Lie groups, not necesarily of higher rank (for example, $SL_2(\mathbb{R})$. We will actually prove the following:

Let S be a non-discrete locally compact, second countable group, and $\Gamma \subset S$ a lattice, Λ a dense subgroup in S such that $\Gamma \subset \Lambda \subset Commens_S(\Gamma)$. Then for any connected, simple, center-free, non-compact, real Lie group, and any $\rho : \Lambda \longrightarrow G$ a homomorphism with Zariski dense image, ρ extends to a continuous epimorphism $\hat{\rho} : S \longrightarrow G$.

The proof of this theorem will asume the existence and uniqueness of the boundary map as a black box. More exactly, we will use the following:

Given a lcsc group S, there exists a measurable S space (B, μ) with the following properties. Given any lattice $\Gamma \subset S$ and linear representation $\rho : \Gamma \longrightarrow G$ in a simple real Lie group, there exists a measurable Γ equivariant map $\phi : B \longrightarrow G/P$ where $P \subset G$ is a minimal parabolic subgroup. Moreover, such a map ϕ is unique.

Using this and the notion of quasi-projective transformations (pointwise limits of $PGL(d, R)$ acting say on a projective space) that were introduced by Furstenberg, we will prove the theorem, and say how the proof of arithmeticity for higher rank can be adapted for the commensurator case.

References

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