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**Mini-Workshop: Recent Developments on Approximation  
Methods for Controlled Evolution Equations**

Organised by  
Birgit Jacob, Wuppertal  
Enrique Zuazua, Bilbao  
Hans Zwart, Enschede

1 November – 7 November 2015

ABSTRACT. This mini-workshop brought together mathematicians engaged in partial differential equations, functional analysis, numerical analysis and systems theory in order to address a number of current problems in the approximation of controlled evolution equations.

*Mathematics Subject Classification (2010):* 93C20, 93C25, 65Mxx.

**Introduction by the Organisers**

The mini-workshop *Recent Developments on Approximation Methods for Controlled Evolution Equations*, organised by *Birgit Jacob* (Wuppertal), *Enrique Zuazua* (Bilbao) and *Hans Zwart* (Twente) was held November 1st – 7th, 2015. This meeting was well attended with 16 participants with broad geographic representation.

Systems modelled by linear ordinary differential equations have long been studied and there exists a wide body of theory and design algorithms dealing with their control. The state describing such a system lies in a finite-dimensional vector space. This setting has its limitations, as many systems of interest, from the point of view of applications to industry and other disciplines, do not fall into this class. A more interesting generalisation is that to systems with an infinite-dimensional state space. This class includes delay systems, and systems modelled by functional differential equations and partial differential equations (PDEs), generally called *evolution equations*. This field finds applications in such diverse areas as aeronautics, mechanical and electrical engineering. Since they appear frequently

as models in these fields of applications, evolution equations with boundary control and boundary observation are of particular interest.

One of the key issues when addressing real applications is the effective control of those systems, which requires of significant effort from the point of view of mathematical analysis.

The talks were grouped into three main themes:

- Modeling and control of *real-live* problems
- Numerical analysis of PDE control
- Theoretical aspects of controller design and approximations for systems described by PDEs

In the first theme the following participants gave talks: *Rob Fey, Aitziber Ibañez, Jarmo Malinen, George Weiss*.

Furthermore, *Athanasios Antoulas, András Bátkai, Umberto Biccari, Nicolae Cîndae, Weiwei Hu, Orest Iftime, Kirsten Morris, Timo Reis* and *Hans Zwart* were the speakers of the second theme.

The last theme was covered by *Björn Augner, Birgit Jacob, Felix Schwenninger* and *Hans Zwart*. Although we have grouped them according to our themes, there was significant overlap between the approaches which stimulated many productive discussions.

The organizers and participants thank the Mathematisches Forschungsinstitut Oberwolfach for providing an inspiring setting for this mini-workshop, which allowed us to concentrate on the mathematics.

*Acknowledgement:* The MFO and the workshop organizers would like to thank the National Science Foundation for supporting the participation of junior researchers in the workshop by the grant DMS-1049268, “US Junior Oberwolfach Fellows”.

## Mini-Workshop: Recent Developments on Approximation Methods for Controlled Evolution Equations

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## Abstracts

### Model reduction of large-scale dynamical systems: An overview with applications to evolution equations

THANOS ANTOULAS

#### 1. MOR AND MOTIVATING EXAMPLES

*Model order reduction* (MOR) seeks to reduce the computational complexity and computational time in the simulation of large-scale dynamical systems by approximations of much lower dimension that produce nearly the same input/output response characteristics.

One version of the problem consists of a given physical system described by known sets of PDEs. These are then appropriately discretized to yield sets of ODEs (Ordinary Differential Equations). The number of the ODEs is reduced, which constitutes *model reduction*. Often the equations (PDEs or ODEs) are replaced by *data*, which can be measured or computed by means of DNS (Direct Numerical Simulation); in this case the reduced system is constructed from the data. Finally, the reduced system is used for simulation, design, or control.

Three motivating examples were mentioned, two based on equations and one on measurements. Example I: Viscous fingering in porous media. In this case the equations of the system are the PDEs (Partial Differential Equations) describing Darcy's law and advection diffusion equations. Example II: Thermal treatment of railroad rails. The equations here are convection diffusion PDEs with inhomogeneous Robin boundary conditions. Example III represents a microstrip device. Such devices consist of thin metallic conductors mounted on a flat dielectric substrate, itself mounted on a ground plate. Common applications are transmission lines, antennas and filters. Microstrip transmission lines are a common way to connect two devices. Such devices are described by means of measured *S-parameters* (Scattering parameter) matrices.

For an overview of MOR methods based on system equations, see [1, 6, 11]. In the sequel we will be concerned with a specific data-driven MOR method, namely that based on the Loewner framework. An overview of the Loewner framework is provided in [5].

#### 2. THE LOEWNER FRAMEWORK

The data-driven MOR framework that we propose is based on the Loewner matrix, which was introduced by Karel Löwner (later Charles Loewner) in the paper: Über monotone Matrixfunktionen, Math. Zeitschrift (1934).

Given a row array  $(\mu_j, \mathbf{v}_j)$ ,  $j = 1, \dots, q$ , and a column array  $(\lambda_i, \mathbf{w}_i)$ ,  $i = 1, \dots, k$ , the associated *Loewner matrix* whose  $(i, j)$ <sup>th</sup> entry is:  $\mathbb{L}_{i,j} = \frac{\mathbf{v}_i - \mathbf{w}_j}{\mu_i - \lambda_j}$ . If there is a known underlying function  $\mathbf{g}$ , then  $\mathbf{w}_i = \mathbf{g}(\lambda_i)$ , and  $\mathbf{v}_j = \mathbf{g}(\mu_j)$ .

**Main property.** Let  $\mathbb{L}$  be a  $q \times k$  Loewner matrix built from samples of a rational function  $\mathbf{g}(s)$ . If  $q, k \geq \deg \mathbf{g}$ , then  $\text{rank } \mathbb{L} = \deg \mathbf{g}$ , where the degree of rational function is the maximum between the degrees of the numerator and the denominator polynomials.

The exact or the numerical rank of  $\mathbb{L}$  is thus of central importance in this approach, depending on whether we wish to recover the original rational function or an approximant thereof.

**2.1. The Loewner framework and generalizations.** The Loewner framework has been worked out for linear systems with multiple inputs and outputs [10, 9, 5, 2]. It has been extended to parametric systems [4, 7, 8] and recently to nonlinear systems [3].

**2.2. Summary.** In this talk we presented an approach to *data-driven model reduction*. Key tool is the *Loewner pencil* and tangential interpolation. Given input/output data, a singular high order model in descriptor form is constructed *without computation*; this constitutes a natural way to construct models and reduced models. Some features are:

- the SVD of  $\mathbb{L}$  provides a trade-off between accuracy and complexity;
- the approach does not force the inversion of  $\mathbf{E}$ ;
- the methodology can deal with many input/output ports as well as parameters and smooth nonlinearities.

The philosophy of this approach is: *collect data and extract the desired information*.

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### Boundary Stabilisation of Port-Hamiltonian Systems

BJÖRN AUGNER

(joint work with Birgit Jacob)

We are interested in stabilisation and control of beam equations. Quite a large class of them may be reformulated in the abstract *port-Hamiltonian* form [5]

$$\frac{\partial x}{\partial t}(t, \zeta) = \sum_{k=0}^N P_k \frac{\partial^k (\mathcal{H}x)}{\partial \zeta^k}(t, \zeta) =: (\mathfrak{A}x(t))(\zeta), \quad t \geq 0, \zeta \in (0, 1)$$

where the states are described by  $x(t, \zeta) \in \mathbb{K}^d$  ( $\mathbb{K} = \mathbb{R}$  or  $\mathbb{C}$ ). We assume that  $P_k = (-1)^{k+1} P_k^* \in \mathbb{K}^{d \times d}$  are symmetric and skew-symmetric matrices with  $P_N$  invertible, and  $P_0 \in L_\infty(0, 1; \mathbb{K}^{d \times d})$ . On the space  $X = L_2(0, 1; \mathbb{K}^d)$  with inner product  $\langle \cdot, \cdot \rangle_X = \langle \cdot, \mathcal{H} \cdot \rangle_{L_2}$  where  $\mathcal{H} \in L_\infty(0, 1; \mathbb{K}^{d \times d})$  is coercive as operator in  $\mathcal{L}(X)$  we define  $\mathfrak{A}$  on its maximal domain  $D(\mathfrak{A}) = \{x \in X : \mathcal{H}x \in H^N(0, 1; \mathbb{K}^d)\}$  and also boundary input and output maps  $\mathfrak{B}$  and  $\mathfrak{C}$ , respectively, which depend on the boundary trace only, i.e. the value of  $\mathcal{H}x$  and its derivatives at the boundary, and assume that the *port-Hamiltonian system*  $\mathfrak{S} = (\mathfrak{A}, \mathfrak{B}, \mathfrak{C})$  is *impedance passive*, i.e.

$$\Re \langle \mathfrak{A}x, x \rangle_X \leq \Re \langle \mathfrak{B}x, \mathfrak{C}x \rangle_{\mathbb{K}^{Nd}}.$$

We then consider the evolution equation in boundary control and observation form

$$\begin{aligned} \frac{d}{dt}x(t) &= \mathfrak{A}x(t), & x(0) &= x_0 \\ \mathfrak{B}x(t) &= u(t), & \mathfrak{C}x(t) &= y(t), \quad t \geq 0 \end{aligned}$$

where we couple input and output via a static  $m$ -dissipative map  $\phi : D(\phi) \subseteq \mathbb{K}^{Nd} \rightrightarrows \mathbb{K}^{Nd}$  and the feedback  $\mathfrak{B}x(t) \in \phi(\mathfrak{C}x(t))$  ( $t \geq 0$ ) or with an impedance passive control system, represented by an  $m$ -dissipative map  $N_c : D(N_c) \subseteq X_c \times \mathbb{K}^{Nd} \rightrightarrows X_c \times \mathbb{K}^{Nd}$  for  $X_c$  the state space of the controller, so that

$$\begin{pmatrix} \frac{d}{dt}x_c(t) \\ -y_c(t) \end{pmatrix} \in N_c \begin{pmatrix} x_c(t) \\ u_c(t) \end{pmatrix}, \quad t \geq 0.$$

As it turns out the resulting evolutionary equation is well-posed [4], [1] in the sense that one obtains  $m$ -dissipative operators  $A_\phi := \mathfrak{A}|_{D(A_\phi)}$  with  $D(A_\phi) = \{x \in D(\mathfrak{A}) : \mathfrak{B}x \in \phi(\mathfrak{C}x)\}$  or  $\mathcal{A}$  (defined on the product space  $X \times X_c$  in a suitable way), respectively. Then, provided that the resulting operators are dissipating enough energy, e.g.

$$\Re \langle \mathcal{A}(x, x_c), (x, x_c) \rangle \leq -\kappa |(\mathcal{H}x)(1)|^2$$

for systems with  $N = 1$ , when 0 is a uniformly exponentially stable equilibrium of the control system and the control system is, say, strictly input-passive, then 0

also is a globally uniformly stable equilibrium of the interconnected system [1], [2], [3]. Similar result also hold for higher order systems [1], [2], where for  $N = 2$  one needs a dissipation like  $|(\mathcal{H}x)(1)|^2 + |(\mathcal{H}x)'(1)|^2 + |(\mathcal{H}x)^{(j)}|^2$ , but less restrictive dissipation condition under more structural conditions, e.g. the Euler-Bernoulli beam structure. Finally we apply the abstract results to the stabilisation of a 1D-wave equation with static nonlinear boundary feedback.

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### Operator splitting for dissipative delay equations

ANDRÁS BÁTKAI

(joint work with Petra Csomós, Bálint Farkas)

The Lie–Trotter product formula and its generalizations to nonlinear semigroups play a central role in semigroup theory, as witnessed in the seminal monographs by Goldstein [9] or Engel and Nagel [8]. As it turns out, the Lie–Trotter product formula is widely used in numerical analysis under the name of operator splitting.

Our aim is to investigate the convergence of the Lie–Trotter product formula (or operator splitting) for nonlinear partial differential equations with delay.

Partial differential equations with delay play an important role in modeling physical, chemical, economical, etc. phenomena, since it is quite natural to assume that past occurrences effect the model. For further motivation see for example the monographs by Wu [13] or Bátkai and Piazzera [6].

There has been lots of work describing the asymptotic behavior and regularity of solutions, as well as in the numerical analysis of ordinary differential equations with delay, see for example the monograph by Bellen and Zennaro [7]. The numerical analysis of partial differential equations with delay, however, seems to be in its infancy. We aim to contribute to this topic by analyzing an operator splitting procedure for nonlinear partial differential equations with delay.

The idea of operator splitting is to decompose the differential equation into simpler equations which can be solved in an effective way, and then represent the solution of the original equation using product formulae, like the Lie–Trotter one. For ordinary differential equations, the theory seems to be quite complete, as witnessed in Hairer et al. [10, Section II.4,5]. There has also been enormous progress



in the theoretical investigation of splitting procedures for infinite dimensional systems in recent years, see for example the monograph by Holden et al. [11]. See also the recent papers by Bátkai et al. [3, 4, 5], where nonautonomous equations and spatial approximations are also considered. Unfortunately, abstract results analyzing the order of convergence are rather incomplete, and, as it seems, can be applied to delay equations only with considerable difficulty.

The idea to apply splitting procedures to delay equations is the following. Consider, e.g., a delayed reaction–diffusion equation of the form

$$u'(t) = \Delta u(t) + g \left( u(t-1) + \int_{-1}^0 \eta(\sigma) \cdot u(t+\sigma) \, d\sigma \right).$$

with some initial and boundary conditions. The delay term appearing here represents the two main classes of possible delays in applications: point delays corresponding to dependence on a single event in the past, and distributed delays (given by an integral term with an integrable kernel  $\eta$ ) corresponding to dependence on a whole time period in the past. In our opinion, distributed delays are often more realistic in modeling, but we will not restrict ourselves to them here.

The nonlinearity  $g$  can be chosen from a wide range of functions depending on applications, for example a rational function in chemical reactions, or a tanh type function in neural networks, see for example the monograph by Wu [13].

Since the delay term is in a way a “scalar operator” (in the sense that it does not mix the spatial variables), it is natural to decompose the equation into two sub-problems: the heat equation

$$w'(t) = \Delta w(t),$$

and a scalar-valued delay equation

$$v'(t) = g \left( v(t-1) + \int_{-1}^0 \eta(\sigma) \cdot v(t+\sigma) \, d\sigma \right).$$

Both equations can be solved numerically in an effective way. Note that the second equation becomes here an ordinary differential equation with delay, hence the methods described in Bellen and Zennaro [7] can be applied.

Our aim is to put this example in an abstract theoretical perspective explaining the convergence and analyze the order of convergence in some special cases. The results obtained will be published in detail in [2].

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## Boundary controllability for a one-dimensional heat equation with two singular inverse-square potentials

UMBERTO BICCARI

Let  $T > 0$  and set  $Q := (0, 1) \times (0, T)$ . We prove boundary controllability for the following one-dimensional heat equation on the domain  $Q$

$$(1) \quad \begin{cases} u_t - u_{xx} - \frac{\mu_1}{x^2}u - \frac{\mu_2}{(1-x)^2}u = 0 & \text{in } Q \\ u(0, t) = f(t), u(1, t) = 0 & \text{in } (0, T) \\ u(x, 0) = u_0(x) & \text{in } (0, 1) \end{cases}$$

presenting two singular inverse-square potentials with singularities located on the boundary. In particular, the main result of this work will be the following Theorem, presented in [1].

**Theorem 1.** *Let  $\mu_1$  and  $\mu_2$  be two real numbers such that  $\mu_1, \mu_2 \leq 1/4$ . For any time  $T > 0$  and any initial datum  $u_0 \in L^2(0, 1)$ , there exists a control function  $f \in L^2(0, T)$  such that the solution of (1) satisfies  $u(x, T) = 0$ .*

The upper bound for  $\mu_1$  and  $\mu_2$  is required for the well-posedness of the problem, and is related to a multi-polar Hardy-Poincaré inequality.

As it is by now classical, for proving Theorem 1 we will apply the Hilbert Uniqueness Method; hence the controllability property will be equivalent to the observability of the adjoint system associated to (1), namely

$$(2) \quad \begin{cases} v_t + v_{xx} + \frac{\mu_1}{x^2}v + \frac{\mu_2}{(1-x)^2}v = 0 & \text{in } Q \\ v(0, t) = v(1, t) = 0 & \text{in } (0, T) \\ v(x, T) = v_T(x) & \text{in } (0, 1). \end{cases}$$

This observability inequality will involve the normal derivative of the solution of (2) in the point from which we aim to control. In particular, we will prove the following inequality

$$(3) \quad \int_0^1 v(x, 0)^2 dx \leq C_T \int_0^T \left[ x^{2\lambda_1} v_x^2 \right] \Big|_{x=0} dt.$$

where we introduce a weighted normal derivative in order to compensate the singularity at  $x = 0$ . The inequality above, in turn, will be obtained as a consequence of a Carleman estimate for the solution of (2).

This work extends previous results on the internal controllability for equations of the type of (1), presented in [2], [3] and [4], to the case of boundary controllability.

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**Numerical controllability of the wave equation using time-space finite elements**

NICOLAE CÎNDEA

(joint work with Carlos Castro, Arnaud Münch)

This talk aims to present some recent results concerning the approximation of controls of minimal  $L^2$ -norm for the wave equation using a finite elements discretization of the space-time domain [2]. More precisely, the following wave equation with distributed control is considered:

$$(1) \quad \begin{cases} y_{tt}(x, t) - \Delta y = v(x, t)\mathbf{1}_\omega(x), & (x, t) \in \Omega \times (0, T) \\ y(x, t) = 0, & (x, t) \in \partial\Omega \times (0, T) \\ y(x, 0) = y_0(x), \quad y_t(x, 0) = y_1(x), & x \in \Omega, \end{cases}$$

where  $\Omega$  is an open domain of  $\mathbb{R}^n$  and  $\omega \subset \Omega$  is an open and non-empty subset of  $\Omega$ . The null controllability in time  $T > 0$  consists, for every  $(y_0, y_1) \in H_0^1(\Omega) \times L^2(\Omega)$ , in finding a control  $v \in L^2(\omega \times (0, T))$  such that

$$y(x, T) = y_t(x, T) = 0, \quad \forall x \in \Omega.$$

In order to approach such a control, the first step of the proposed method is to write the optimality condition corresponding to the minimization of the functional  $J^*$ , which appears in Hilbert Uniqueness Method (HUM), as a mixed formulation. This is possible by introducing a Lagrange multiplier for tacking into account that the dual variable  $\varphi$  (the functional  $J^*$  is minimized with respect to  $\varphi$ ) verifies the wave equation. The obtained mixed formulation is well posed as a consequence of the exact observability of the adjoint system associated to (1).

In a second step, the mixed formulation is numerically approximated using the finite elements method. More exactly,  $C^1$  finite elements are employed for the approximation of the dual variable  $\varphi$  and  $C^0$  Lagrange finite elements for the approximation of the Lagrange multiplier  $\lambda$ .

Finally, since the space-time domain is discretized with no differentiation between the temporal and the spatial variables, this method is very appropriate to tackle the controllability of the wave equation using controls supported in time-dependent domains. Several numerical simulations are discussed and confirm the theoretical analysis of the proposed method. A similar strategy can be employed for the wave equation with boundary control [1].

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### **Controlling nonlinear resonances in thin-walled structures: modeling, simulations, and experiments**

ROB H.B. FEY

Thin-walled structures are often used in engineering practice because in their (axial, in-plane) loading direction, they have a high stiffness-mass ratio and a high (static) loading capacity. However, in the transversal direction, they may be quite flexible. Although the structure will be generally designed in such a way that static buckling will not occur, dynamic periodic base excitation of the structure still may lead to dynamic buckling (i.e. resonances) of the structure. This may lead to malfunctioning, fatigue, damage, or even collapse of the structure. In this work, first, nonlinear resonances of an archetype thin-walled structure will be studied in the low frequency range, both numerically and experimentally. Subsequently, they will be attenuated using control. The structure consists of a vertical slender beam, which carries a top mass. The beam is loaded statically by the weight of the top mass, which is below the static buckling load, and dynamically

by a vertical periodic base excitation. Piezoelectric (PZT) patches are attached on both sides of the beam, which can locally exert a control moment. The transversal displacement and velocity halfway the length of the beam are measured by means of a laser vibrometer. First, a single mode model of the uncontrolled structure is derived. The transversal displacement field is approximated by an analytical shape function with a corresponding generalized degree of freedom. This shape function is fitted to the lowest eigenmode of the system, which is obtained using finite element analysis. The beam is assumed to be inextensible. This implies that the axial displacement field and the transversal displacement field are kinematically coupled. Furthermore, the beam has a small geometric imperfection. It is assumed that this imperfection can be approximated by the analytical shape function mentioned above as well. The model is geometrically nonlinear due to 3rd order Taylor series approximations of the inextensibility constraint and the beam curvature. After formulating expressions for the kinetic energy, the potential energy, and the non-conservative forces, the equation of motion of the system in terms of the generalized degree of freedom is derived using Lagrange's equation. Viscous damping is accounted for by a linear and a quadratic term. Now, branches of periodic solutions are calculated by solving two-point boundary value problems in combination with path following techniques using the base excitation frequency as the varying parameter. In the uncontrolled case, the model predicts a severe  $1/2$  subharmonic (parametric) resonance, a harmonic resonance, and several superharmonic resonances. The largest resonances show softening behavior. On the periodic solution branches, cyclic fold bifurcations and period doubling bifurcations are met. The steady-state responses are also measured using stepped frequency sweep-up and stepped frequency sweep-down experiments. The numerical and experimental responses match very well. Now, a (model-based) feedback linearizing controller in combination with a linear controller is implemented to suppress the severe transversal resonances using the PZT patches. The linear controller consists of a gain in combination with a lead filter. Experiments show that near dangerous base disturbance frequencies, the vibration levels are reduced up to a factor 300! It can be concluded that the implemented controller very successfully suppresses all resonances of the uncontrolled system.

### Observer-based Feedback Stabilization of a Thermal Fluid

WEIWEI HU

We discuss the problem of designing an observer-based feedback law which locally stabilizes a two dimensional thermal fluid modeled by the Boussinesq equations. The investigation of stability for a fluid flow in the free convection problem is important in the theory of hydrodynamical stability (see [11]). The challenge of stabilization of the Boussinesq equations arises from the stabilization of the Navier-Stokes equations and its coupling with the convection-diffusion equation for temperature. The controllability and stabilizability of the Navier-Stokes equations and Boussinesq equations have been widely discussed (see [4], [5], [6], [7], [10]). In

this talk, we consider a boundary feedback stabilization problem. In particular, we consider a finite number of controls acting on a portion of the boundary through Robin boundary condition and construct a linear Luenberger observer based on point observations of the linearized system. The sensor locations for the measurement are determined by the geometric structure of the feedback functional gains (see [1], [2], [3]). It can be shown that the nonlinear system coupled with the observer through the feedback law is locally exponentially stabilizable. Numerical results are provided to illustrate the idea and suggest areas for future research.

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### Guidance by repulsion model: analysis, some simulations and optimal control

AITZIBER IBÁÑEZ

(joint work with Ramón Escobedo, Enrique Zuazua)

The most usual strategy to find an effective way to solve a guidance problem is the “Follow me” strategy, which is based on the effect of the attraction that one agents (or particle) exerts on the others. The major novelty of our work [1] is that we use a repulsion force (i. e., “move away from me” strategy) to solve a guidance problem.

We can find several examples of how different guidance problems can be solved using the previously mentioned "Follow me" strategy [2], [3], [4], [5]. Repulsion forces, like the one used in our work, are mostly used for collision avoidance or interception, like in [6]. In a very recent work [7] they show how this kind repulsion force can be used to describe a defender-intruder interaction, where the defender exerts a repulsion force in the intruder to expel it away from a protected target.

In the recent work [1] we describe a "guidance by repulsion" phenomenon describing the behaviour of two agents, a driver and an evader. The first one follows the evader but cannot be arbitrarily close to it, and the evader tries to move away from the driver beyond a short distance. The driver can display a circumvention motion around the evader, in such a way that the trajectory of the evader is modified due to the repulsion that the driver exerts on the evader.

We show how the evader can be guided to any target on the plane or to follow a sufficiently smooth path by controlling just one parameter which modifies its behaviour, activating or deactivating the circumvention mode and selecting the clockwise/counterclockwise direction of the circumvention motion.

We propose different open loop strategies to drive the evader from any given point to another assuming that both switching the control and keeping it on (i. e. when  $\kappa \neq 0$ ) has a cost, and we end presenting a feedback law which prevents the excessive use of the circumvention mode with a significant reduction of the cost.

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#### Model reduction by moment matching for infinite dimensional systems

OREST V. IFTIME

(joint work with Tudor C. Ionescu)

Infinite-dimensional systems are a wide class of systems that stems from modelling of phenomena and processes. Approximation of infinite-dimensional systems involves finding a finite-dimensional model of the original system with an irrational transfer function, satisfying appropriate constraints. During the past decades,

considerable advances have been made in this direction. One popular approach is based on finite-element methods, resulting in large scale models which are difficult to analyse and control. Alternatively, model reduction procedures can be performed directly on the infinite-dimensional systems. One way to approximate an infinite-dimensional system is to perform modal truncation, i.e., keep a finite number of terms from the modal decomposition of the transfer function of the given system. The drawbacks of this method consist of the difficulty of computing the transfer function and of the fact that the zeros of the given system are lost, see e.g., [1]. Another method is based on the of the controllability and observability Gramians, where one can obtain reduced order models by balanced truncation, optimal or sub-optimal Hankel norm approximations [2, 4]. Balanced proper orthogonal decomposition and balanced truncation coincide when using a particular balanced realization for infinite-dimensional systems [3].

We study the problem of model reduction by moment matching for a class of infinite-dimensional systems, based on the unique solution of an operator Sylvester equation [5]. The solution yields a class of parametrized, finite-dimensional, reduced order models that match a set of prescribed moments of the given system. We show that, by properly choosing the free parameters, additional constraints are met, e.g., pole placement, preservation of zeros. One of the main advantages of this method is the preservation of a finite number of zeros of the given infinite-dimensional system, yielding better approximations (including the case of minimum phase systems).

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### **Optimal actuator and observation location for time-varying systems on a finite-time horizon**

BIRGIT JACOB

(joint work with Xueran Wu, Hendrik Elbern)

The choice of the location of controllers and observations is of great importance for designing control systems and improving the estimations in various practical problems. For time-varying systems in Hilbert spaces, the existence and convergence of the optimal location based on linear-quadratic control on a finite-time



horizon is studied. More precisely, for an initial-time  $t_0 \in [a, b]$  we consider the time-varying system described by

$$x_r(t) = T(t, t_0)x(t_0) + \int_{t_0}^t T(t, s)B_r(s)u(s)ds, \quad t_0 \leq t \leq b,$$

where  $T(t, s)$  is a mild evolution operator on a separable Hilbert space  $H$  and  $B_r$  is a strongly measurable essentially bounded function with  $B(s) \in L(U, H)$  a.e., where  $U$  is a finite-dimensional Hilbert space. Further we assume that  $B_r$  depends on a parameter  $r \in \Omega$ . The linear-quadratic optimal control problem is the question of finding for  $x_0 \in X$  a control  $u_0 \in L^2(t_0, b; U)$  which minimizes the cost functional

$$J_r(t_0, x_0, u) = \langle x(b), Gx(b) \rangle + \int_{t_0}^b \|C(s)x(s)\|^2 + \langle u(s), R(s)u(s) \rangle ds.$$

As the initial state  $x_0$  is not fixed we define the optimal location by

$$\hat{\ell}(t_0) = \inf_{r \in \Omega} \max_{\|x_0\|=1} \min_{u \in L^2(t_0, b; U)} J_r(x_0, u),$$

For time-invariant systems on an infinite time horizon the optimal location problem has been studied in [1, 2]. In this talk we aim to extend these results to time-varying system on a finite-time horizon and we tread the problem for stochastical systems as well. The optimal location of observations for improving the estimation of the state at the final time, based on Kalman filter, is considered as the dual problem to the LQ optimal problem of the control locations. Further, the existence and convergence of optimal locations of observations for improving the estimation at the initial time, based on Kalman smoother is discussed.

The obtained results are applied to a linear advection-diffusion model with a special extension of emission rates.

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#### Non-linear dynamic stabilising control of port-Hamiltonian systems

YANN LE GORREC

(joint work with Hector Raminez, Hans Zwart)

Port-Hamiltonian systems can be used to describe ordinary and partial differential equations (pde's) with control and observation. Since in standard port-Hamiltonian systems there is no internal damping, the stabilisation has to be done via the control. For a port-Hamiltonian system described by a pde, linear damping at the boundary of the spatial domain is well-studied, and there are easy testable conditions for exponential stability, see e.g. [2, Chapter 9] or [1, 3, 4].

However, these conditions limit the class of controllers. For instance, these controllers need to have direct feed through. If these controllers don't satisfy these conditions, exponential stability cannot be achieved. Furthermore, it is interesting to know if similar results can be obtained when applying non-linear control. Since any physical controller will have some non-linear behaviour, this question is also practically relevant. In what follows we consider non linear controllers inducing asymptotic stability. The system we consider is the port-Hamiltonian system described by

$$\frac{\partial x}{\partial t} = P_1 \frac{\partial}{\partial \zeta} (\mathcal{H}x), \quad t \geq 0, \zeta \in (0, 1)$$

with boundary condition (homogeneous and control)

$$W_{B,1} \begin{pmatrix} \mathcal{H}x(1) \\ \mathcal{H}x(0) \end{pmatrix} = 0, \quad W_{B,2} \begin{pmatrix} \mathcal{H}x(1) \\ \mathcal{H}x(0) \end{pmatrix} = u(t),$$

and boundary observation

$$W_C \begin{pmatrix} \mathcal{H}x(1) \\ \mathcal{H}x(0) \end{pmatrix} = y(t).$$

The energy is given by  $E(t) = \frac{1}{2} \int_0^1 x^T (\mathcal{H}x) d\zeta$ . It is assumed that for  $u \equiv 0$  the homogeneous systems is governed by a contraction semigroup, and for classical solutions the following balance equation holds

$$\dot{E}(t) = u^T(t)y(t).$$

The dynamic, non-linear controller is of the form

$$\dot{x}_c(t) = \begin{pmatrix} 0 & I \\ -I & -R \end{pmatrix} \begin{pmatrix} \dot{P}(x_{c,1}) \\ K_2 x_{c,2} \end{pmatrix} + \begin{pmatrix} 0 \\ I \end{pmatrix} u_c(t), \quad y_c(t) = K_2 x_{c,2}$$

where  $x_c = \begin{pmatrix} x_{c,1} \\ x_{c,2} \end{pmatrix} \in \mathbb{R}^{n_c}$ , and  $R, K_2$  are symmetric and (strictly) positive. The energy of this system equals  $E_c(t) = P(x_{c,1}) + \frac{1}{2} x_{c,2}^T K_2 x_{c,2}$ . Hence for mechanical systems,  $x_{c,1}$  would be positions and  $x_{c,2}$  the velocities. It can be shown that

$$\dot{E}_c(t) = -x_{c,2}^T(t) R x_{c,2}(t) + u_c^T(t) y_c(t).$$

The control system is connected to the port-Hamiltonian system via  $u = -y_c$  and  $u_c = y$ . By doing this it is easy to see that the total energy is bounded, and among others this shows that the solution of the closed loop system exists globally. Furthermore, for smooth initial conditions, the solution is classical. Abstractly, the connected system can be written as  $\dot{z}(t) = Az(t) + N(z(t))$ , in which  $N$  represents the non-linear part. For smooth initial conditions, we can show that  $\dot{z}(t)$  is uniformly bounded. By the above equation, and the fact that  $A$  has compact resolvent, it follows that  $\{z(t), t \geq 0\}$  is relatively compact. Hence by LaSalle's theorem the solution will converges to the largest invariant set contained in  $\dot{E}(t) \equiv 0$ . A simple observability condition now gives that the closed loop system is asymptotically stable.

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**Webster's equation in acoustic waveguides**

JARMO MALINEN

Long wavelength acoustics in lossy, curved tubular waveguides can be approximated by the generalised Webster's model, consisting of the PDE

$$\frac{1}{c^2 \Sigma(s)^2} \frac{\partial^2 \psi}{\partial t^2} = \frac{1}{A(s)} \frac{\partial}{\partial s} \left( A(s) \frac{\partial \psi}{\partial s} \right) - \frac{2\pi\alpha W(s)}{A(s)} \frac{\partial \psi}{\partial t} \quad \text{for } s \in [0, \ell], t \geq 0,$$

together with the appropriate boundary conditions at  $s = 0, \ell$ . Here  $A$  represents the spatially varying intersectional area of the waveguide, the function  $\Sigma$  is a correction due to curvature, and the function  $W$  in the dissipation term is due to different inner and outer curvature radii of the waveguide. The solution  $\psi = \psi(s, t)$  is the velocity potential, and the acoustic pressure as well as the (perturbation) velocity can be obtained from it by partial derivatives.

Two problems related to Webster's model are considered, and their solutions are presented in the special case  $\alpha = 0$  and  $\Sigma(s) = 1$ . Firstly, the control  $A(0)\phi_s(0, t) = i(t)$  and observation  $p(t) = \rho\phi_t(\ell, t)$  are used, and it is shown that the impedance transfer function from  $i$  to  $p$  does not have zeroes if  $A$  is real analytic in a neighbourhood of  $\ell$ .

Secondly, first order perturbations are derived for the eigenvalues  $\lambda^2$  of the resonance problem

$$\left(\frac{\lambda}{c}\right)^2 \Psi = \frac{1}{A(s)} \frac{\partial}{\partial s} \left( A(s) \frac{\partial \Psi}{\partial s} \right) \quad \text{for } s \in [0, \ell]$$

for small perturbations  $A = A_0 + \epsilon B$  of a nominal area function  $A_0$ . As an application, local spectral inversion problem is considered. Given (measurement) data about the some of the eigenvalues  $\lambda_j^2$  relating to a true area function  $A$  (of which we only have a guess, i.e.,  $A_0 \approx A$ ), how can be find a first order correction  $B$  to  $A_0$ ?

As a practical application of both the problems, modelling of speech acoustics of vowels is discussed.

## Using approximations in controller design for infinite-dimensional systems

KIRSTEN MORRIS

Many control systems are modelled by partial differential equations (PDE's). The state of such systems evolves on an infinite-dimensional space. There are essentially two approaches to controller design for systems modeled PDE's: direct and indirect. In direct controller design, the original model is used design the controller. The resulting controller will often be infinite-dimensional and in a subsequent step, order reduction is used to obtain a finite-dimensional controller of acceptable order. In indirect controller design, a finite-dimensional approximation of the system is obtained at the start and controller design is based on this finite-dimensional approximation.

The chief drawback of direct controller design is that a representation of the solution suitable for calculation is required. For many practical examples, this is not possible. For this reason, indirect controller design is generally used in practice. The hope is that the controller designed using the finite-dimensional approximation has the desired effect on the original system. That this method is not always successful was first documented in Balas [1], where the term *spillover* was introduced. Spillover refers to the phenomenon that a controller which stabilizes a reduced-order model may not stabilize the original model. Systems with infinitely many poles either on or asymptoting to the imaginary axis are notorious candidates for spillover effects. Conditions under which the indirect approach to controller design works have been obtained and are presented. The difference between approximation of parabolic and of hyperbolic equations is illustrated by several simple equations.

The performance of controlled partial differential equations depends on the location of controller hardware, the sensors and actuators. A mechatronic approach, where controller design is integrated with actuator location, can lead to better performance without increased controller cost. The best locations may be different from those chosen based on physical intuition. Furthermore, since it is often difficult to move hardware, and trial-and-error is laborious when there are multiple sensors and actuators, analysis is crucial. Approximations to the governing equations, often of very high order, are required and this complicates not only controller design but also optimization of the hardware locations. Care needs to be taken in formulating the joint optimization/ controller design problem in order to obtain correct results. Numerical issues will be briefly discussed.

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### Balanced truncation model reduction for infinite-dimensional linear systems

TIMO REIS

(joint work with Mark. R. Opmeer, Tilman Selig, Winnifried Wollner)

Balanced truncation is one of the most popular model reduction methods for finite-dimensional input-output-systems governed by ordinary differential equations, i.e.,

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t),\end{aligned}$$

where  $A \in \mathbb{R}^{n \times n}$ ,  $B \in \mathbb{R}^{n \times m}$ ,  $C \in \mathbb{R}^{p \times n}$ . This technique relies on the solution of the observability and controllability Lyapunov equations

$$\begin{aligned}AP + PA^\top + BB^\top &= 0, \\ A^\top P + PA + C^\top C &= 0\end{aligned}$$

for positive semi-definite matrices  $P, Q \in \mathbb{R}^{n \times n}$ .

We consider this method for infinite-dimensional systems, i.e.,  $A, B, C, P$  and  $Q$  are operators acting on Hilbert spaces. Motivated by the ADI method for the determination of Gramian matrices [1], we develop a numerical scheme for the finite-rank approximation of the Gramian operators associated to infinite-dimensional systems. This enables us to determine finite-dimensional approximations of infinite-dimensional systems. Error bounds in the  $\mathcal{H}_\infty$  norm are provided. Our results will be illustrated by means of a boundary-controlled heat equation and delay-differential equations.

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## Results around the Cayley Transform Problem

FELIX L. SCHWENNINGER  
(joint work with Hans Zwart)

This work deals with the infinite-dimensional analogue of the following question.  
*Are the powers of the matrix*

$$(1) \quad \text{Cay}(A) := (A + I)(A - I)^{-1}$$

(where  $I$  denotes the identity) uniformly bounded if it is assumed that  $A \in \mathbb{C}^{N \times N}$ ,  $N \in \mathbb{N}$ , has a uniformly bounded matrix exponential, i.e.,  $\sup_{t \geq 0} \|e^{tA}\| < \infty$  ?

This can be rephrased as the question of stability of the *Crank-Nicolson* scheme. In the infinite-dimensional situation,  $A$  is assumed to be the generator a bounded  $C_0$ -semigroup  $(e^{tA})_{t \geq 0}$  on a Banach space  $X$ , which implies that  $\text{Cay}(A)$  defines a bounded operator and thus, one can again study whether  $\sup_{n \in \mathbb{N}} \|\text{Cay}(A)^n\| < \infty$ .

In contrast to the elementary matrix case, the answer is ‘no’ for general Banach spaces. On the other hand, it is ‘yes’ if  $(e^{tA})_{t \geq 0}$  is bounded analytic. For Hilbert spaces  $X$ , the question still remains open, but in this case, there are ‘more positive answers’; e.g., contraction semigroups and uniformly continuous semigroups. See, e.g., [3] for a detailed overview and references of the mentioned results.

We show that, in order to answer the question for Hilbert spaces, it suffices to consider exponentially stable  $C_0$ -semigroups  $(e^{tA})_{t \geq 0}$  with  $\sup_{t \geq 0} \|e^t e^{tA}\| < \infty$ .

It is well known that this problem is strongly related to the *Inverse Generator Problem*, i.e., the question if  $A^{-1}$  generates a bounded  $C_0$ -semigroup, whenever  $A$  generates a bounded  $C_0$ -semigroup and  $A^{-1}$  exists as a densely defined operator [1, 2, 4]. We show that both problems are indeed equivalent and can be reduced to the special case of exponentially stable semigroups. The presented results were derived in [3].

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**Coupled passive systems and strong stabilization of the SCOLE  
system using tuned mass dampers**

GEORGE WEISS

(joint work with Xiaowei Zhao)

Coupled infinite-dimensional systems have attracted much interest in recent years. From the vast literature, we mention here only the thesis of Villegas [5], the books of Lasiecka [4] and Jacob and Zwart [3]. In [6] we have developed a theory for the regularity, controllability and observability of coupled linear systems consisting of an infinite-dimensional strictly proper subsystem  $\Sigma_d$  and a finite-dimensional subsystem  $\Sigma_f$  connected in feedback. In [6] we assume that the feedthrough matrix of  $\Sigma_f$  is invertible and certain (not too restrictive) algebraic conditions hold - this is relevant for the current work. In this work we study the stability of a feedback system as in [6] where the subsystems are impedance passive. We are interested in two main issues: (1) the strong stability of the operator semigroup associated with  $\Sigma_c$ , (2) the input-output stability (meaning transfer function in  $H^\infty$ ) of  $\Sigma^c$ . We show that these properties hold under relatively mild and verifiable assumptions, that include the approximate observability of both systems.

As an application we tackle a problem that is relevant for the vibration suppression of wind turbine towers. We demonstrate that a Tuned Mass Damper (TMD) can be used to stabilize the SCOLE (NASA Spacecraft Control Laboratory Experiment) model. The SCOLE system is a well known model for a flexible beam with one end clamped and the other end connected to a rigid body. Originally it has been developed to model a mast carrying an antenna on a satellite, see Littman and Markus [1, 2]. This framework is suitable also to model wind turbine towers, since these are typically clamped at the bottom while the upper end of the tower is linked to the heavy nacelle.

To approach the practical engineering problem of stabilizing a wind-turbine tower, we should use the natural energy state space for the SCOLE model, where only strong stability can be achieved (because the control and observation operators are bounded).

We design and analyze a TMD control system to reduce the vibration of the SCOLE model. A TMD consists of a large mass, connected to the top of the structure to be stabilized via springs and dampers. The idea is that the TMD is tuned to a particular structural frequency and thus will resonate and dissipates input energy via the dampers when the structure is excited at that particular frequency. In this paper we analyze this technique in one plane only, so that there is only spring and one damper, connected in parallel between the rigid body of the SCOLE system and the mass component of the TMD. This mass component can be either put on a trolley moving along a rail, or it can be hanged via cables.

The mathematical model of the SCOLE system with a trolley  $\Sigma_c$  is the following set of equations:

$$\begin{aligned} \rho(x)w_{tt}(x,t) + (EI(x)w_{xx}(x,t))_{xx} &= 0, & (x,t) \in (0,l) \times [0,\infty), \\ w(0,t) &= 0, \quad w_x(0,t) = 0, \end{aligned}$$

$$\begin{aligned}mw_{tt}(l, t) - (EIw_{xx})_x(l, t) &= F_e(t) + k_1(p(t) - w(l, t)) + d_1(p_t(t) - w_t(l, t)), \\Jw_{xtt}(l, t) + EI(l)w_{xx}(l, t) &= T_e(t), \\m_1p_{tt}(t) &= k_1(w(l, t) - p(t)) + d_1(w_t(l, t) - p_t(t)),\end{aligned}$$

where the subscripts  $t$  and  $x$  denote derivatives with respect to the time  $t$  and the position  $x$ . The first four equations are the non-uniform SCOLE model  $\Sigma_d$  while the last equation describes the trolley system  $\Sigma_f$ .  $\Sigma_d$  and  $\Sigma_f$  are interconnected through the transverse speed of the nacelle  $y = w_t(l, t)$  and the force

$$u = k_1(p(t) - w(l, t)) + d_1(p_t(t) - w_t(l, t))$$

generated by the trolley system.  $l$ ,  $\rho$  and  $EI$  denote the beam's height, mass density function and flexural rigidity while  $w$  denotes its transverse displacement.  $\rho, EI \in C^4[0, l]$  are assumed to be strictly positive functions. The parameters  $m > 0$  and  $J > 0$  are the mass and the moment of inertia of the rigid body.  $F_e$  and  $T_e$  are the force and torque inputs acting on the rigid body. The state of  $\Sigma_d$  at the time  $t$  is

$$z(t) = [z_1(t) \ z_2(t) \ z_3(t) \ z_4(t)] = [w(\cdot, t) \ w_t(\cdot, t) \ w_t(l, t) \ w_{xt}(l, t)].$$

Its natural energy state space is

$$X^d = \mathcal{H}_l^2(0, l) \times L^2[0, l] \times \mathbb{C}^2,$$

where  $\mathcal{H}_l^2(0, l) = \{h \in \mathcal{H}^2(0, l) \mid h(0) = 0, h_x(0) = 0\}$ . Here  $\mathcal{H}^n$  ( $n \in \mathbb{N}$ ) denote the usual Sobolev spaces. The natural norm on  $X^d$  is (twice the physical energy)

$$\|z(t)\|^2 = \int_0^l EI(x)|z_{1xx}(x, t)|^2 dx + \int_0^l \rho(x)|z_2(x, t)|^2 dx + m|z_3(t)|^2 + J|z_4(t)|^2.$$

In the trolley system  $\Sigma_f$ ,  $m_1 > 0$ ,  $k_1 > 0$  and  $d_1 > 0$  are the mass, spring constant and damping coefficient.  $p$  and  $p_t$  are the position and transverse velocity of the mass component of the trolley. The state of  $\Sigma_f$  is defined as

$$q(t) = [q_1(t) \ q_2(t)] = [p(t) - w(l, t) \ p_t(t)]$$

with state space  $X^f = \mathbb{C}^2$ , on which the norm is defined as  $\|q(t)\|^2 = k_1|q_1(t)|^2 + m_1|q_2(t)|^2$ . The state space of the SCOLE-trolley coupled system  $\Sigma_c$  is  $X_c$ , the product of the two state spaces of the subsystems.

Our results are that the SCOLE system with trolley  $\Sigma_c$  with input  $[F_e \ T_e]$ , state  $[z \ q]$  and output  $[u \ w_{xt}(l, t) \ y]$  is well-posed and regular on the energy state space  $X^c$  and that  $\Sigma_c$  is strongly stable on  $X^c$ .

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### Numerical approximations for port-Hamiltonian systems

HANS ZWART

An important subclass of port-Hamiltonian systems are those described by (linear) partial differential equations with control and observation at the boundary. A canonical example is the wave equation

$$\rho w_{tt} = \operatorname{div}(T \operatorname{grad}(w))$$

on an e.g. two dimensional spatial (bounded) domain  $\Omega$ . There are two ways in which this system can be presented as a port-Hamiltonian system, namely using abstract differential equations on Hilbert spaces or via differential forms. We begin with the abstract differential equation. For that we take the state

$$x = \begin{pmatrix} \rho w_t \\ T \operatorname{grad}(w) \end{pmatrix}$$

and write the partial differential equation as

$$\frac{\partial x}{\partial t} = \begin{pmatrix} 0 & \operatorname{div} \\ \operatorname{grad} & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\rho} & 0 \\ 0 & T \end{pmatrix} x = \mathcal{J} \mathcal{H} x.$$

The  $\mathcal{J}$  clearly gives the skew-symmetric structure associated to the conservation of energy for the wave. This means that the operator  $\mathcal{J} : e \mapsto f$  is skew symmetric with respect to the inner product in  $L^2(\Omega)$ .

The second representation of the wave equation uses differential forms. Thus  $x, \dot{x}$ , and  $\mathcal{H}x$  are seen as  $k$ -forms. The above mentioned  $\mathcal{J}$  becomes now the mapping

$$\mathcal{J}_{df} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} := \begin{pmatrix} 0 & d \\ d & 0 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$$

mapping the zero-form  $e_1$  and the 1-form  $e_2$  to the 2-form  $f_1$  and the 1-form  $f_2$ . The mapping  $\mathcal{H}$  is also replaced. The skew-symmetry of  $\mathcal{J}_{df}$  is now with respect to the wedge product.

There are now two research questions. The first is to understand the precise relation between these two ways of reformulation the wave equation. A start in solving this problem is given by Arnold et al in [1]. In this paper we can also see how the differential forms can be used in numerics. This is also the approach taken by van der Schaft et al, see e.g. [2]. What is missing is an error analysis.

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Reporter: Felix Schwenninger

## Participants

**Prof. Dr. Athanasios C. Antoulas**  
Department of Electrical and Computer  
Engineering  
Rice University, MS 380  
6100 South Main Street  
Houston, TX 77251-1892  
UNITED STATES

**Björn Augner**  
Fakultät für Mathematik und  
Naturwissenschaften  
Bergische Universität Wuppertal  
42097 Wuppertal  
GERMANY

**Prof. Dr. András Bátkai**  
Fakultät für Mathematik und  
Naturwissenschaften  
Bergische Universität Wuppertal  
Gaußstrasse 20  
42119 Wuppertal  
GERMANY

**Umberto Biccari**  
Basque Center for Applied Mathematics  
(BCAM)  
Alameda de Mazarredo No. 14  
48009 Bilbao  
SPAIN

**Nicolae Cindea**  
Laboratoire de Mathématiques  
UMR 6620 du CNRS  
Université Blaise Pascal - Clermont II  
63178 Aubière Cedex  
FRANCE

**Dr. Rob H. B. Fey**  
Department of Mechanical Engineering,  
Dynamics and Control  
Eindhoven University of Technology  
P.O. Box 513  
5600 MB Eindhoven  
NETHERLANDS

**Dr. Weiwei Hu**  
Department of Mathematics  
University of Minnesota  
206 Church Street S.E.  
127 Vincent Hall  
Minneapolis, MN 55455  
UNITED STATES

**Aitziber Ibanez**  
Basque Center for Applied Mathematics  
(BCAM)  
Alameda de Mazarredo No. 14  
48009 Bilbao  
SPAIN

**Dr. Orest Iftime**  
Mathematisch Instituut  
Rijksuniversiteit Groningen  
Postbus 800  
9700 AV Groningen  
NETHERLANDS

**Prof. Dr. Birgit Jacob**  
Fakultät für Mathematik und  
Naturwissenschaften  
Bergische Universität Wuppertal  
42097 Wuppertal  
GERMANY

**Prof. Dr. Jarmo Malinen**

Dept. of Mathematics & Systems  
Analysis  
Aalto University  
PO Box 11100  
00076 Aalto  
FINLAND

**Prof. Dr. Kirsten Anna Morris**

Department of Applied Mathematics  
University of Waterloo  
Waterloo ONT N2L 3G1  
CANADA

**Prof. Dr. Timo Reis**

Fachbereich Mathematik  
Universität Hamburg  
Bundesstrasse 55  
20146 Hamburg  
GERMANY

**Dr. Felix Schwenninger**

Fakultät für Mathematik und  
Naturwissenschaften  
Bergische Universität Wuppertal  
42097 Wuppertal  
GERMANY

**Prof. Dr. George Weiss**

Department of Electrical Engineering  
Systems  
Tel Aviv University  
Ramat Aviv 69978  
ISRAEL

**Prof. Dr. Hans J. Zwart**

Department of Applied Mathematics  
University of Twente  
P.O. Box 217  
7500 AE Enschede  
NETHERLANDS