

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Report No. 54/2016

DOI: 10.4171/OWR/2016/54

Large Scale Stochastic Dynamics

Organised by
Thierry Bodineau, Palaiseau
Fabio Toninelli, Villeurbanne
Bálint Tóth, Bristol/Budapest

13 November – 19 November 2016

ABSTRACT. The goal of this workshop was to explore the recent advances in the mathematical understanding of the macroscopic properties which emerge on large space-time scales from interacting microscopic particle systems. There were 53 participants, including 4 postdocs and graduate students, working in diverse intertwining areas of probability and statistical mechanics. During the meeting, 24 talks of 50 minutes were scheduled and an evening session was organised with 10 more short talks of 10 minutes, mostly by younger participants. These talks addressed the following topics : hydrodynamic limits and hydrodynamic fluctuations with a special emphasis on KPZ fluctuations, scaling limits in percolation and random walks, approach to equilibrium in reversible systems with a strong focus on kinetically constrained dynamics.

Mathematics Subject Classification (2010): 60-XX, 11Kxx, 62-XX, 90-XX, 91-XX, 92-XX, 93-XX, 94-XX.

Introduction by the Organisers

The workshop *Large scale stochastic dynamics* is the continuation of the highly successful series of Oberwolfach workshops with the same title previously organized by C. Landim, S. Olla and H. Spohn. This new edition, organised by T. Bodineau (Palaiseau), F. Toninelli (Lyon) and B. Toth (Bristol/Budapest), was well attended with over 50 participants with broad geographic representation.

The workshop was devoted to the wide mathematical problem of understanding emergent structures on large space-time scales in the evolution of physical systems. These are modelled by particle systems, namely high-dimensional Markov processes. In our choice of 24 talks, we tried to illuminate major recent advances

in the field and to expose and address at least some aspects of the works for each and every one of the participants. An evening session with short talks was the occasion to learn about the recent results of 10 participants and to trigger further discussions afterwards. A more detailed account of the long presentations is given below.

Hydrodynamic limits and fluctuating hydrodynamics

The aim of hydrodynamic limits is to explain mathematically the emergence of macroscopic transport phenomena observed in experiments, starting from microscopic dynamics of interacting particles.

S. Olla presented a recent work on deterministic dynamics with additional stochastic noise to model transport in mechanical systems and in particular superdiffusion in one-dimensional models. He showed that conserved quantities in these models obey a system of nonlinear partial differential equations.

M. Sasada provided a new perspective on the celebrated non-gradient method initiated by Varadhan to derive hydrodynamic limits. The main strategy is a systematic characterisation of closed forms by using ideas of cohomology.

M. Balazs showed how the behaviour of the so-called second class particle, for a wide class of asymmetric dynamics, can be related to the hydrodynamic limit. A key to this result is the proper initialisation of the second class particle.

C. Landim reviewed results on the metastability in the zero range process in the condensation regime. He also proposed a series of open problems on the derivation of the hydrodynamic behavior when condensation takes place. Further results on large deviations for reaction-diffusion systems were presented.

P. Ferrari presented a recent result on the box/ball model which is a deterministic cellular automata, all the randomness being encoded in the initial condition. He described the invariant measures and explained how the evolution can be followed in terms of records of a random walk.

The study of fluctuations around the deterministic macroscopic hydrodynamic limit is of great interest to understand the refined behavior of the microscopic dynamics.

M. Jara presented a new method to derive an equation describing non-equilibrium fluctuations. It is based on refined entropic estimates and it generalises the current approaches which are mainly limited to equilibrium fluctuations.

M. Simon gave a lecture on the different types of fluctuation scalings which occur when a mechanical system of infinitely many coupled oscillators is perturbed by different stochastic noises.

M. Gubinelli reviewed the strong and weak KPZ universality conjecture and he showed that, in the stationary regime, the notion of energy solutions of the KPZ equation uniquely characterizes solutions.

P. L. Ferrari surveyed the scaling limits arising in the asymmetric simple exclusion process depending on the different types of initial data. In particular, he presented a class of initial conditions for the TASEP for which the large-scale behavior

of fluctuations interpolates between the Baik-Rains distribution and the Tracy-Widom one.

T. Funaki explained how to renormalize systems of coupled KPZ equations which naturally arise when one considers anharmonic chains of oscillators with several conserved quantities.

Random walks and polymer models

J.D. Deuschel presented a new invariance principle for random walks in space- and time-dependent *balanced* random environment. The environment is assumed to be invariant and ergodic under space and time shifts. The result holds in the quenched sense, without assuming uniform ellipticity.

H. Spohn introduced models of directed polymers with complex random weights, which are motivated by wave transmission in disordered media. This opens the way to new challenging problems as the partition function is conjectured to obey some universal scaling, as is the case for real weights, even though it is a priori oscillatory.

P. Tarres presented a survey of the recent results connecting techniques of supersymmetric quantum field theory with the problem of recurrence/transience dichotomy for linearly edge-reinforced random walks. These techniques allow to prove the long standing conjectures about transience in 3 and more dimensions and weak reinforcement, respectively recurrence in any dimension and sufficiently strong reinforcement.

E. Bolthausen considered a *membrane model* where the interactions between heights are governed by the square of the Laplacian (rather than the Laplacian, as in the traditional interface models). It was proved that in 5 and more dimensions the pinned-down version, where the height at the origin is partially tied down to have value 0, the thermodynamic limit exists (as stationary random field) and its correlations decay exponentially with distance.

T. Seppalainen: In the first passage percolation problem it is known that there exists an asymptotic convex shape set of lattice points reached within passage time N , rescaled by the same N . (This follows from subadditivity.) In this talk a new variational approach to this problem was presented. A variational formula was formulated which characterizes the asymptotic shape.

F. Comets considered the random interlacement process in two dimensions. Due to the marginally recurrent nature of two-dimensional random walk, this process shows remarkable differences from the well-understood three (and more) dimensional random interacements. In particular, fine analysis of the fractal structure of the so-called late points (that is points on the two-dimensional torus which are not visited till late times) was presented.

Scaling in percolation

W. Werner presented *inter alia* recent results about percolation structures on continuous fractal domains defined as the complements of the two-dimensional Brownian loop coup of intensity $c < 1$. The construction is the scaling limit of the Edwards-Sokal coupling between Potts and Fortuin-Kasteleyn percolation models.

G. Kozma reviewed the Aizenmann-Grimmett argument on the perturbation of the critical point in percolation and explain its generalization to long range perturbations along random lines.

Approach to equilibrium in reversible systems.

Glauber dynamics are Markov chains that are reversible w.r.t. the Gibbs distribution of a statistical mechanics system. Understanding how quickly the process approaches the equilibrium distribution, and the occurrence of slowdown phenomena, gives insight on phase transitions, glassy or metastable behavior.

C. Toninelli gave an overview talk on kinetically constrained models (microscopic particle systems that model physical system undergoing a glassy or jamming transition) and presented sharp results on their relaxation time, showing links with bootstrap percolation.

O. Blondel discussed the behavior of the diffusion coefficient of a tagged particle moving in a kinetically constrained model, in the limit where vacancies are rare and the system is almost jammed. Results included notably the strict positivity of the coefficient for all particle densities.

A. Faggionato presented new results on so-called triangular and square plaquette models: these are two-dimensional spin systems with no equilibrium phase transition but a dramatic dynamical slowdown at low temperatures. In particular, she discussed the interplay between static lengthscales, glassy dynamics and the fractal space structure of excitations.

E. Lubetzky proved that the Glauber dynamics of the critical two-dimensional Potts model with $q \leq 4$ reaches equilibrium in time that is almost polynomial in the system size n , thereby significantly improving the previously known bounds that were of order $\exp(n)$.

C. Poquet discussed the Kuramoto mean field model of coupled rotators. In the limit of large number of rotators the model exhibits synchronization, that persists when the proper frequencies of the individual rotators are chosen random and i.i.d. The condensate moves with a speed related to the asymmetry of the disorder realization and, on large time scales, it has Brownian fluctuations.

H. Lacoin presented results on the cut-off phenomenon and sharp mixing time estimates for Markov chains describing a “card-shuffling” algorithm. The results have interesting implications on the mixing time for the Asymmetric Simple Exclusion process on a large but finite segment.

Summary. The workshop helped to update the participants on the state of the art and on the important pending open problems in the fields related to their domain of research. It triggered many interesting discussions and was the occasion to

initiate and pursue collaborations. The scientific presentations proved that this research field is still very active and is absorbing new ideas from other branches of mathematics and probability theory (conformal loop soups and the Gaussian Free Field, bootstrap percolation, cohomology, etc).

Acknowledgement: The MFO and the workshop organizers would like to thank the National Science Foundation for supporting the participation of junior researchers in the workshop by the grant DMS-1049268, “US Junior Oberwolfach Fellows”.

Workshop: Large Scale Stochastic Dynamics

Table of Contents

Wendelin Werner (joint with Jason Miller, Scott Sheffield, Titus Lupu) <i>Percolation through (some) fractal spaces</i>	3039
Tadahisa Funaki (joint with Masato Hoshino) <i>Coupled KPZ equations</i>	3040
Marielle Simon (joint with C. Bernardin, P. Gonçalves, M. Jara, M. Sasada) <i>Interpolation between standard and anomalous diffusion of energy</i>	3043
Patrik L. Ferrari (joint with S. Chhita and H. Spohn) <i>Limit distributions for KPZ growth models with spatially homogeneous random initial conditions</i>	3044
Jean-Dominique Deuschel (joint with N. Berger, X. Guo and A. Ramírez) <i>Quenched invariance principles for random walks in a balanced time-dependent environment</i>	3047
Gady Kozma (joint with Hugo Duminil-Copin, Marcelo Hilário, Gil Kalai, Ioan Manolescu, Vladas Sidoravicius and Vincent Tassion) <i>What can we learn from Brochettes?</i>	3048
Pierre Tarrès (joint with Franz Merkl, Silke Rolles) <i>Convergence of vertex-reinforced jump processes to an extension of the supersymmetric hyperbolic nonlinear sigma model</i>	3049
Erwin Bolthausen (joint with Alessandra Cipriani, Noemi Kurt) <i>Exponential decay of the correlations for the pinned membrane model</i> ..	3051
Timo Seppäläinen (joint with Nicos Georgiou, Arjun Krishnan, Firas Rassoul-Agha) <i>Variational formulas for percolation limit shapes</i>	3053
Francis Comets (joint with C. Gallesco, S. Popov and M. Vachkovskaia) <i>Cover time and cover process for the random walk on the 2-dimensional torus</i>	3054
Makiko Sasada (joint with Yukio Kametani) <i>Cohomological approach to the decomposition theorem for closed forms in the non-gradient method</i>	3055
Stefano Olla (joint with Tomasz Komorowski, Marielle Simon) <i>Temperature profiles in non-equilibrium stationary states</i>	3057
Márton Balázs (joint with Attila László Nagy) <i>How to initialize a second class particle ?</i>	3059

Otávio Menezes (joint with Milton Jara) <i>Invariance principle for a slowed random walk over symmetric exclusion</i>	3061
Balázs Ráth (joint with Omer Angel, Qingsan Zhu) <i>Branching Interlacements</i>	3061
Bálint Vető (joint with Patrik L. Ferrari) <i>Tilings of the Aztec diamond on a restricted domain and the hard-edge tacnode process</i>	3062
Bernard Derrida (joint with Zhan Shi) <i>Some recent works on the branching Brownian motion</i>	3063
Shirshendu Ganguly (joint with Fabio Martinelli) <i>Upper triangular matrix walk: cutoff for finitely many columns</i>	3064
Ellen Saada (joint with Lucie Fajfrová, Thierry Gobron) <i>Invariant measures of mass migration processes</i>	3065
Luca Avena (joint with Fabienne Castell, Alexandre Gaudillière and Clothilde Mélot) <i>RW kernels, spanning forests and multiscale analysis on graphs.</i>	3066
Benoît Laslier (joint with Fabio Toninelli) <i>How does ice melt at 0° C ?</i>	3067
Amol Aggarwal <i>Current Fluctuations for the Stationary ASEP</i>	3067
Oriane Blondel (joint with Cristina Toninelli) <i>Diffusions in Kinetically Constrained Models</i>	3069
Alessandra Faggionato (joint with P. Chleboun, F. Martinelli, C. Toninelli) <i>Static length scales and glassy dynamics in triangular and square plaquette models</i>	3070
Cristina Toninelli (joint with Fabio Martinelli) <i>Kinetically constrained models and bootstrap percolation: critical time and length scales</i>	3071
Christophe Poquet (joint with Lorenzo Bertini, Giambattista Giacomin, Eric Luçon) <i>Random long time dynamics of the stochastic Kuramoto model</i>	3074
Hubert Lacoin (joint with Cyril Labbé) <i>The Cutoff phenomenon for biased card shuffling and Adjacent transposition shuffle</i>	3075
Milton Jara (joint with Otávio Menezes) <i>Non-equilibrium fluctuations of interacting particle systems</i>	3076
Pablo A. Ferrari (joint with Chi Nguyen, Leonardo Rolla, Minmin Wang) <i>Invariant measures for the Box Ball System in \mathbb{Z}</i>	3078

Abstracts

Percolation through (some) fractal spaces

WENDELIN WERNER

(joint work with Jason Miller, Scott Sheffield, Titus Lupu)

The goal of this talk is to survey some recent and ongoing work about how to define and describe processes that can be viewed as “critical percolation” in some particular but fairly natural fractals. These percolative structures are relevant in the study of geometric structures embedded in a Gaussian Free Field, and in the two-dimensional case, in the study of scaling limits of critical lattice models.

Specifically, we focus on some fractal spaces obtained in d -dimensional space when $d \geq 3$, or in bounded planar domains by sampling a natural Poissonian cloud of Brownian loops of intensity c that we introduced a decade ago with Greg Lawler. Recall that in two dimensions, these loop-soups appear to be very closely related to SLE processes (more precisely, for each value of $c \leq 1$, joint work with Scott Sheffield showed that considering the clusters of loops in a loop-soup allows to construct collections of non-intersecting SLE_κ loops for some $\kappa = \kappa(c) \in (8/3, 4]$ – these so-called conformal loop ensembles are the conjectural scaling limit of critical Potts models and their fuzzy generalization to non-integer values for q for $q = q(\kappa) \in (1, 4]$), and that the properly renormalised occupation time measure of a loop-soup with intensity $c = 1$ (in higher dimension, one needs to look at a discretized version) is directly related with the square of the Gaussian Free Field (GFF) (by results of Yves Le Jan).

In the present talk, we describe aspects of the following two type of results:

- In joint work with Jason P. Miller (University of Cambridge) and Scott Sheffield (MIT) [1, 2], we construct directly in the continuum planar case the processes that can be viewed as critical percolation in the complement of the loop-soups for $c < 1$. The connected components of the complement of the loop-soup being conjectural scaling limit of Potts model clusters, our results provide the continuous analog of the Edwards-Sokal coupling between Potts and FK percolation models. We also explain what happens in the special critical case $c = 1$.
- In ongoing joint work [3] with Titus Lupu (ETH Zürich), we explain why the relation to the GFF leads to the existence of processes that can be interpreted as critical percolation in the space obtained by contracting all the loops in the $c = 1$ loop-soups. This leads to a procedure to connect Brownian loops in a loop-soup when $c = 1$ into structures that could be then viewed as excursion sets away from the origin by the GFF. We also discuss aspects of the dependence with respect to d .

REFERENCES

- [1] J.P. Miller, S. Sheffield, W. Werner, *CLE percolations*, arXiv:1602.03884.
 [2] J.P. Miller, S. Sheffield, W. Werner, *Non-simple SLE curves are not determined by their range*, arXiv:1609.04799.
 [3] T. Lupu, W. Werner, in preparation.

Coupled KPZ equations

TADAHISA FUNAKI

(joint work with Masato Hoshino)

We report the results obtained in [3]. Motivated by the nonlinear fluctuating hydrodynamics recently discussed by Spohn and others [1], [12], [13], we consider the following \mathbb{R}^d -valued coupled KPZ equation for $h(t, x) = (h^\alpha(t, x))_{\alpha=1}^d$ defined on the one dimensional torus $\mathbb{T} \equiv \mathbb{R}/\mathbb{Z} = [0, 1)$:

$$(1) \quad \partial_t h^\alpha = \frac{1}{2} \partial_x^2 h^\alpha + \frac{1}{2} \Gamma_{\beta\gamma}^\alpha \partial_x h^\beta \partial_x h^\gamma + \sigma_\beta^\alpha \xi^\beta, \quad x \in \mathbb{T},$$

for $1 \leq \alpha \leq d$. We use Einstein's convention, and $\xi(t, x) = (\xi^\alpha(t, x))_{\alpha=1}^d$ is an \mathbb{R}^d -valued space-time Gaussian white noise, which has the covariance structure

$$E[\xi^\alpha(t, x) \xi^\beta(s, y)] = \delta^{\alpha\beta} \delta(x - y) \delta(t - s).$$

We assume that the coupling constants $\Gamma_{\beta\gamma}^\alpha$ satisfy

$$(2) \quad \Gamma_{\beta\gamma}^\alpha = \Gamma_{\gamma\beta}^\alpha$$

and the diffusion matrix $\sigma = (\sigma_\beta^\alpha)_{1 \leq \alpha, \beta \leq d}$ is invertible.

The coupled KPZ equation (1) itself is ill-posed, so that we need to introduce its approximations; see [4] for a scalar-valued KPZ equation. A simple approximation of (1) is defined as follows. Let $\eta \in C_0^\infty(\mathbb{R})$ be a symmetric function satisfying $\int_{\mathbb{R}} \eta(x) dx = 1$. We set $\eta^\varepsilon(x) = \eta(x/\varepsilon)/\varepsilon$ for $\varepsilon > 0$ and consider the \mathbb{R}^d -valued KPZ approximating equation for $h = h^\varepsilon(t, x) \equiv (h^{\varepsilon, \alpha}(t, x))_{\alpha=1}^d$ with a smeared noise and a proper renormalization:

$$(3) \quad \partial_t h^{\varepsilon, \alpha} = \frac{1}{2} \partial_x^2 h^{\varepsilon, \alpha} + \frac{1}{2} \Gamma_{\beta\gamma}^\alpha (\partial_x h^{\varepsilon, \beta} \partial_x h^{\varepsilon, \gamma} - c^\varepsilon A^{\beta\gamma} - B^{\varepsilon, \beta\gamma}) + \sigma_\beta^\alpha \xi^\beta * \eta^\varepsilon,$$

for $1 \leq \alpha \leq d$, where $A^{\beta\gamma} = \sum_{\delta=1}^d \sigma_\delta^\beta \sigma_\delta^\gamma$, $c^\varepsilon = \frac{1}{\varepsilon} \|\eta\|_{L^2(\mathbb{R})}^2$ and $B^{\varepsilon, \beta\gamma}$ is another renormalization factor, which diverges as $O(-\log \varepsilon)$ as $\varepsilon \downarrow 0$ in general.

Second approximation of (1) suitable for studying invariant measures is introduced as follows. Let $\eta_2(x) = \eta * \eta(x)$, $\eta_2^\varepsilon(x) = \eta_2(x/\varepsilon)/\varepsilon$ and consider the following \mathbb{R}^d -valued equation for $\tilde{h} = \tilde{h}^\varepsilon(t, x) \equiv (\tilde{h}^{\varepsilon, \alpha}(t, x))_{\alpha=1}^d$ with a smeared noise and a proper renormalization:

$$(4) \quad \partial_t \tilde{h}^{\varepsilon, \alpha} = \frac{1}{2} \partial_x^2 \tilde{h}^{\varepsilon, \alpha} + \frac{1}{2} \Gamma_{\beta\gamma}^\alpha (\partial_x \tilde{h}^{\varepsilon, \beta} \partial_x \tilde{h}^{\varepsilon, \gamma} - c^\varepsilon A^{\beta\gamma} - \tilde{B}^{\varepsilon, \beta\gamma}) * \eta_2^\varepsilon + \sigma_\beta^\alpha \xi^\beta * \eta^\varepsilon,$$

for $1 \leq \alpha \leq d$, where $\tilde{B}^{\varepsilon, \beta\gamma}$ is a renormalization factor, which diverges as $O(-\log \varepsilon)$ as $\varepsilon \downarrow 0$ in general.

If $\hat{\Gamma}$ determined from Γ as

$$(5) \quad \hat{\Gamma}_{\beta\gamma}^\alpha = \tau_{\alpha'}^\alpha \Gamma_{\beta'\gamma'}^{\alpha'} \sigma_{\beta'}^{\beta'} \sigma_{\gamma'}^{\gamma'}$$

satisfies the trilinear condition

$$(6) \quad \hat{\Gamma}_{\beta\gamma}^\alpha = \hat{\Gamma}_{\gamma\alpha}^\beta,$$

for all α, β, γ , then the distribution of the derivative of the d -dimensional periodic and smeared Brownian motion $(\partial_x(\sigma B * \eta^\varepsilon))_{x \in \mathbb{T}} = ((\partial_x \sigma_\beta^\alpha B^\beta * \eta^\varepsilon(x))_{\alpha=1}^d)_{x \in \mathbb{T}}$ multiplied by σ is infinitesimally invariant for the tilt process $u = \partial_x \tilde{h}$ of the solution \tilde{h} of (4) with $\tilde{B}^{\varepsilon, \beta\gamma} = 0$. This was shown in [2] when σ is an identity matrix I , but is easily extended to the general setting with σ .

When $d = 1$ and $\Gamma_{\beta\gamma}^\alpha = \sigma_\beta^\alpha = 1$ for simplicity, the approximating equations (3) with $B^{\varepsilon, \beta\gamma} = 0$ and (4) with $\tilde{B}^{\varepsilon, \beta\gamma} = 0$ have the forms:

$$(7) \quad \partial_t h = \frac{1}{2} \partial_x^2 h + \frac{1}{2} ((\partial_x h)^2 - c^\varepsilon) + \xi * \eta^\varepsilon,$$

and

$$(8) \quad \partial_t \tilde{h} = \frac{1}{2} \partial_x^2 \tilde{h} + \frac{1}{2} ((\partial_x \tilde{h})^2 - c^\varepsilon) * \eta_2^\varepsilon + \xi * \eta^\varepsilon,$$

respectively. It is shown that the solution of (7) converges as $\varepsilon \downarrow 0$ to the so-called Cole-Hopf solution $h_{\text{CH}}(t, x)$ of the KPZ equation [7], [8], while the solution of (8) converges to $h_{\text{CH}}(t, x) + \frac{1}{24}t$ under the equilibrium setting [4] and the non-equilibrium setting for a maximal solution [10]. The method of [4] is based on the Cole-Hopf transform, which is not available for our multi-component coupled equation in general.

The existence of the limits of the solutions of two types of approximating equations (3) and (4) as $\varepsilon \downarrow 0$ is established based on the paracontrolled calculus introduced by Gubinelli et al. [5], [6]. The difference between these two limits is studied and this extends the results for the scalar-valued KPZ equation mentioned above. For $\kappa \in \mathbb{R}$ and $r \in \mathbb{N}$, $(\mathcal{C}^\kappa)^r := \mathcal{B}_{\infty, \infty}^\kappa(\mathbb{T}; \mathbb{R}^r)$ denotes the \mathbb{R}^r -valued Besov space on \mathbb{T} .

Theorem 1.1. (1) Assume $h(0) \in \bigcup_{\delta > 0} (\mathcal{C}^\delta)^d$, then a unique solution h^ε of the KPZ approximating equation (3) exists up to the survival time $T_{\text{sur}}^\varepsilon \in (0, \infty]$ (i.e. $T_{\text{sur}}^\varepsilon = \infty$ or $\lim_{t \uparrow T_{\text{sur}}^\varepsilon} \|h^\varepsilon\|_{C([0, t], (\mathcal{C}^\delta)^d)} = \infty$). With a proper choice of $B^{\varepsilon, \beta\gamma}$, there exists $0 < T_{\text{sur}} \leq \liminf_{\varepsilon \downarrow 0} T_{\text{sur}}^\varepsilon$ and h^ε converges in probability as $\varepsilon \downarrow 0$ to some h in $C((0, T], (\mathcal{C}^{1/2-\delta})^d)$ for every $\delta > 0$ and $0 < T < T_{\text{sur}}$. This T_{sur} can be chosen maximal in the sense that $T_{\text{sur}} = \infty$ or $\lim_{T \uparrow T_{\text{sur}}} \|h\|_{C([0, T], (\mathcal{C}^\delta)^d)} = \infty$.

(2) A similar result holds for the solution \tilde{h}^ε of the KPZ approximating equation (4) with some limit \tilde{h} under a proper choice of $\tilde{B}^{\varepsilon, \beta\gamma}$. Moreover, under a well-adjusted choice of the renormalization factors $B^{\varepsilon, \beta\gamma}$ and $\tilde{B}^{\varepsilon, \beta\gamma}$, one can make $h = \tilde{h}$.

Theorem 1.2. We assume the trilinear condition (6). Then, both $B^{\varepsilon, \beta\gamma}$ and $\tilde{B}^{\varepsilon, \beta\gamma}$ behave as $O(1)$, so that the solutions of (3) with $B^{\varepsilon, \beta\gamma} = 0$ and (4) with $\tilde{B}^{\varepsilon, \beta\gamma} = 0$ converge as $\varepsilon \downarrow 0$. In the limit, we have

$$\tilde{h}^\alpha(t, x) = h^\alpha(t, x) + c^\alpha t, \quad 1 \leq \alpha \leq d,$$

where

$$c^\alpha = \frac{1}{24} \sum_{\gamma, \gamma'} \sigma_\beta^\alpha \hat{\Gamma}_{\alpha' \alpha''}^\beta \hat{\Gamma}_{\gamma \gamma'}^{\alpha'} \hat{\Gamma}_{\gamma \gamma'}^{\alpha''}.$$

Another result is on a global-in-time existence of the limit process h under the condition (6). Let μ_σ be the distribution on the space $(\mathcal{C}_0^{-1/2-\delta})^d := \{u \in (\mathcal{C}^{1/2-\delta})^d; \int_{\mathbb{T}} u = 0\}$, $\delta > 0$, of $(\partial_x \sigma B)_{x \in \mathbb{T}}$, which is the limit in law of that of $(\partial_x(\sigma B * \eta^\varepsilon))_{x \in \mathbb{T}}$ as $\varepsilon \downarrow 0$.

Theorem 1.3. *We assume the trilinear condition (6). Then there exists a subset $H \subset (\mathcal{C}_0^{-1/2-\delta})^d$ such that $\mu_\sigma(H) = 1$, and if $\partial_x h(0) \in H$, the convergence to the limiting process h as above holds on whole $[0, \infty)$ almost surely. Moreover, the spatial derivative $u = \partial_x h$ of the limit process h is a Markov process on $(\mathcal{C}_0^{-1/2-\delta})^d$ which admits μ_σ as an invariant measure.*

Proposition 5.4 of Hairer and Mattingly [9] (combined with Theorem 1.3) shows that the limit process h exists on $[0, \infty)$ almost surely for all initial values $h(0) \in (\mathcal{C}^{1/2-\delta})^d$. Kupiainen and Marozzi [11] have a similar result to our Theorem 1.1-(1) and a part of Theorem 1.2 due to a different approach.

REFERENCES

- [1] P.L. FERRARI, T. SASAMOTO AND H. SPOHN, *Coupled Kardar-Parisi-Zhang equations in one dimension*, J. Stat. Phys., **153** (2013), 377–399.
- [2] T. FUNAKI, *Infinitesimal invariance for the coupled KPZ equations*, Memoriam Marc Yor – Séminaire de Probabilités XLVII, Lect. Notes Math., **2137**, 37–47, Springer, 2015.
- [3] T. FUNAKI AND M. HOSHINO, *A coupled KPZ equation, its two types of approximations and existence of global solutions*, arXiv:1611.00498.
- [4] T. FUNAKI AND J. QUASTEL, *KPZ equation, its renormalization and invariant measures*, Stoch. PDE: Anal. Comp., **3** (2015), 159–220.
- [5] M. GUBINELLI, P. IMKELLER AND N. PERKOWSKI, *Paracontrolled distributions and singular PDEs*, Forum Math., Pi, **3** (2015) 75pp.
- [6] M. GUBINELLI AND N. PERKOWSKI, *KPZ reloaded*, arXiv:1508.03877.
- [7] M. HAIRER, *Solving the KPZ equation*, Ann. Math, **178** (2013), 559–664.
- [8] M. HAIRER, *A theory of regularity structures*, Invent. Math., **198** (2014), 269–504.
- [9] M. HAIRER AND J. MATTINGLY, *The strong Feller property for singular stochastic PDEs*, arXiv:1610.03415.
- [10] M. HOSHINO, *Paracontrolled calculus and Funaki-Quastel approximation for the KPZ equation*, arXiv:1605.02624.
- [11] A. KUPIAINEN AND M. MARCOZZI, *Renormalization of generalized KPZ equation*, arXiv:1604.08712
- [12] H. SPOHN, *Nonlinear fluctuating hydrodynamics for anharmonic chains*, J. Stat. Phys., **154** (2014), 1191–1227.
- [13] H. SPOHN AND G. STOLTZ, *Nonlinear fluctuating hydrodynamics in one dimension: the case of two conserved fields*, J. Stat. Phys., **160** (2015), 861–884.

Interpolation between standard and anomalous diffusion of energy

MARIELLE SIMON

(joint work with C. Bernardin, P. Gonçalves, M. Jara, M. Sasada)

Over the last few years, anomalous behaviors have been observed for one-dimensional chains of oscillators. The rigorous derivation of such behaviors from deterministic systems of Newtonian particles is very challenging, due to the existence of conservation laws, which impose very poor ergodic properties to the dynamical system. A possible way out of this lack of ergodicity is to introduce stochastic models, in such a way that the qualitative behaviour of the system is not modified. One starts with a chain of oscillators with a Hamiltonian dynamics, and then adds a stochastic which keeps the fundamental conservation laws (energy, momentum and stretch, usually).

One may first investigate the macroscopic evolution of the fluctuation field (around equilibrium), associated to the conserved quantities. For the unpinned harmonic chain where the velocities of particles can randomly change sign (and therefore the only conserved quantities of the dynamics are the energy and the stretch), it is known [5] that, under a diffusive space-time scaling, the energy profile evolves following a non-linear diffusive equation involving the stretch. In [1] and [4] it has been shown that in the case of one-dimensional harmonic oscillators with noise that preserves the momentum, the scaling limit of the energy fluctuations is ruled by the *fractional* heat equation.

This talk aims to understand the *regime transition* for the energy fluctuations, and to describe the results of [2,3]. Let us consider the same harmonic Hamiltonian dynamics, but now perturbed by two stochastic noises S_1 and S_2 : both perturbations conserve the energy, but only S_1 preserves the momentum. If $S_2 = 0$, the momentum is conserved, the energy transport is superdiffusive and described by a Lévy process governed by a fractional Laplacian. Otherwise, the volume conservation is destroyed, and the energy normally diffuses. What happens when S_2 vanishes with the size of the chain? In this case, we can show that the limit of the energy fluctuation field depends on the evanescent speed of the random perturbation, we recover the two very different regimes for the energy transport, and we prove the existence of a crossover between the normal diffusion regime and the fractional superdiffusion regime.

One future research direction would be to describe very precisely the form of the critical regime for the *anharmonic chain*. Since the study of anharmonic systems is still very challenging, one can start with the weakly anharmonic case. The expected scenario is that as the strength of the anharmonicity increases, a crossover from a fractional superdiffusion regime to a different superdiffusion regime should appear. This crossover regime is too subtle to be described by Spohn's theory, and one needs a more accurate analysis of the system to get the precise critical values for which it crosses the different behaviors.

REFERENCES

- [1] C. Bernardin, P. Gonçalves, M. Jara, *3/4-fractional superdiffusion in a system of harmonic oscillators perturbed by a conservative noise*, Arch. Rational Mech. Anal. **220** (2016), Issue 2, 505–542.
- [2] C. Bernardin, P. Gonçalves, M. Jara, M. Sasada and M. Simon, *From normal diffusion to super diffusion of energy in the evanescent flip noise limit*, Jour. Stat. Phys. **159** (2015), No. 6, 1327–1368.
- [3] C. Bernardin, P. Gonçalves, M. Jara, and M. Simon: *Interpolation process between standard diffusion and fractional diffusion*, Preprint hal-01348503 (2016).
- [4] M. Jara, T. Komorowski, S. Olla, *Limit theorems for additive functionals of a Markov chain*, Ann. Appl. Probab. **19** (2009), No. 6, 2270–2300.
- [5] T. Komorowski, S. Olla, and M. Simon: *Macroscopic evolution of mechanical and thermal energy in a harmonic chain with random flip of velocities*, Preprint hal-01358979 (2016).

**Limit distributions for KPZ growth models with spatially
homogeneous random initial conditions**

PATRIK L. FERRARI

(joint work with S. Chhita and H. Spohn)

For stochastic growth models in the Kardar-Parisi-Zhang (KPZ) universality class over a one-dimensional substrate the height fluctuations “always” broaden as $t^{1/3}$. On the other hand the full probability density function depends on the choice of the initial data. As well known, for a flat initial surface, $h(x, t = 0) = 0$, the large t fluctuations of $h(0, t)$ are distributed according to GOE Tracy-Widom distribution [5, 18, 22]. In contrast, if the height profile is macroscopically curved, then GOE has to be replaced by GUE [2, 3, 7, 13, 16, 20, 21].

If as a surface growth model we consider the one-dimensional KPZ equation,

$$\partial_t h = \frac{1}{2}(\partial_x h)^2 + \frac{1}{2}\partial_x^2 h + \xi$$

with $\xi(x, t)$ normalized space-time white noise, then the time-stationary initial data are

$$h(x, 0) = B(x)$$

with $B(x)$ a two-sided Brownian motion. As shown in [6] (for other KPZ models, see [1, 4, 12, 15]),

$$h(0, t) \simeq -\frac{1}{24}t + (t/2)^{1/3}\xi_{\text{BR}}$$

for large t and the random amplitude ξ_{BR} is Baik-Rains distributed [4]. Recently, Quastel and Remenik [19] identified a large domain of attraction for GOE Tracy-Widom distribution. Roughly speaking, for a macroscopically flat initial profile, if it satisfies $|h(x, 0) - h(0, 0)| \simeq |x|^{1/2}$ for large $|x|$ is the borderline below which the height fluctuations are GOE Tracy-Widom distributed.

We consider translation invariant random initial data, for which height differences typically grow as $|x|^{1/2}$. More precisely, for the totally asymmetric simple exclusion process, TASEP, with initial slopes $\eta_j(t = 0) = \eta_j \in \{0, 1\}$, we allow

initial conditions such that $\{\eta_j | j \in \mathbb{Z}\}$ is a stationary stochastic process satisfying the functional central limit theorem

$$\lim_{\ell \rightarrow \infty} \frac{1}{\sqrt{\ell}} \sum_{j=0}^{[\gamma x \ell]} (\eta_j - \langle \eta_0 \rangle) = \sigma B(x)$$

for some $\sigma \geq 0$. Here γ is a scaling constant set by the fact that $\sigma = 1$ corresponds to stationary initial condition. We show that for each σ there is a distinct distribution function $F^{(\sigma)}(s)$.

Denote by ρ the expected density of particles and j the expected (infinitesimal) current of particles. Then if $j'(\rho) = 0$ holds, the time correlations are relevant around the origin and the height fluctuations, as obtained from $\eta_j(t)$, are governed by $F^{(\sigma)}(s)$ in the large t limit. If $j'(\rho) \neq 0$, then correlations spread at a non-zero velocity and $F^{(\sigma)}(s)$ will be observed after properly centering (see e.g. [12] in the $\sigma = 1$ case).

For the TASEP we prove that the limiting distribution is determined through a variational formula,

$$F^{(\sigma)}(s) = \mathbb{P}\left(\sup_{x \in \mathbb{R}} \{\sqrt{2}\sigma B(x) + \mathcal{A}_2(x) - x^2\} \leq s\right),$$

where $\mathcal{A}_2(x)$ is the Airy process and is independent of the two-sided Brownian motion $B(x)$. The proof employs several non-trivial results obtained only recently, the most important ones being tightness [9] for the point-to-point process with ending points on horizontal lines, and the one-point slow-decorrelation [10]. Finally one also needs to know the convergence of the finite-dimensional distributions [8]. These ingredients can be used to obtain a functional slow-decorrelation result, see [11] for the discrete time counterpart. Interestingly, this latter result then implies tightness of the point-to-point process along generic lines, which is a result not covered by the elegant and soft arguments of [9].

As already proved in [17], $F^{(0)}(s) = F_{\text{GOE}}(2^{2/3}s)$, with F_{GOE} the GOE Tracy-Widom distribution. Our result indirectly implies that $F^{(1)}(s)$ equals the Baik-Rains distribution. The only other explicit solution corresponds formally to the limit $\sigma \rightarrow \infty$, which reads (after scaling s with $\sigma^{4/3}$)

$$\mathbb{P}\left(\sup_{x \in \mathbb{R}} \{B(x) - x^2\} \leq s\right).$$

An explicit representation is provided in [14]. Its probability density vanishes for $s < 0$ and decays as a stretched exponential with power $\frac{3}{2}$ for $s \rightarrow \infty$.

For all other values of σ we have to rely on Monte Carlo simulations, see Figure 1 for a plot of the densities of $F^{(\sigma)}$ for some values of σ .

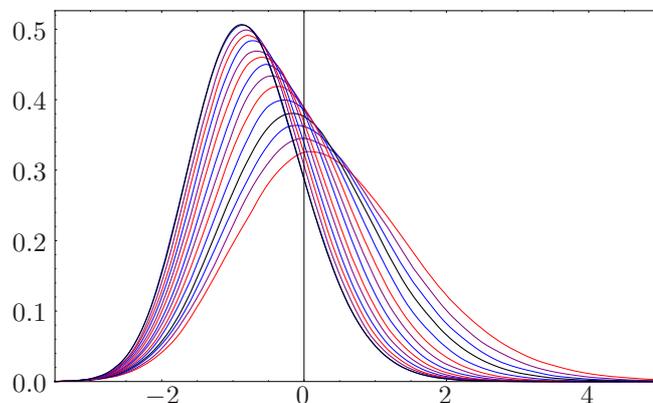


FIGURE 1. Probability densities of $F^{(\sigma)}(s)$ with $\sigma = \sqrt{\alpha/(1-\alpha)}$ from TASEP simulation until time $t_{\max} = 10^3$ and 10^6 runs. The different plots corresponds to the values $\alpha = 0, 0.05, 0.1, 0.15, 0.20, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.54, 0.58, 0.62$. The left-most black line is the exact rescaled GOE distribution ($\sigma = 0$), which overlaps with $\alpha = 0$ from the simulations. The black line in the middle is the stationary case ($\sigma = 1$).

REFERENCES

- [1] A. Aggarwal, *Current Fluctuations of the Stationary ASEP and Six-Vertex Model*, arXiv:1608.04726 (2016).
- [2] G. Amir, I. Corwin, and J. Quastel, *Probability distribution of the free energy of the continuum directed random polymer in 1+1 dimensions*, *Comm. Pure Appl. Math.* **64** (2011), 466–537.
- [3] J. Baik, P.A. Deift, and K. Johansson, *On the distribution of the length of the longest increasing subsequence of random permutations*, *J. Amer. Math. Soc.* **12** (1999), 1119–1178.
- [4] J. Baik and E.M. Rains, *Limiting distributions for a polynuclear growth model with external sources*, *J. Stat. Phys.* **100** (2000), 523–542.
- [5] J. Baik and E.M. Rains, *Symmetrized random permutations*, *Random Matrix Models and Their Applications*, vol. 40, Cambridge University Press, 2001, pp. 1–19.
- [6] A. Borodin, I. Corwin, P.L. Ferrari, and B. Vető, *Height fluctuations for the stationary KPZ equation*, *Math. Phys. Anal. Geom.* **18:20** (2015).
- [7] A. Borodin, I. Corwin, and V. Gorin, *Stochastic six-vertex model*, *Duke Math. J.* **165** (2016), 563–624.
- [8] A. Borodin and S. Péché, *Airy Kernel with Two Sets of Parameters in Directed Percolation and Random Matrix Theory*, *J. Stat. Phys.* **132** (2008), 275–290.
- [9] E. Cator and L. Pimentel, *On the local fluctuations of last-passage percolation models*, *Stoch. Proc. Appl.* **125** (2015), 879–903.
- [10] I. Corwin, P.L. Ferrari, and S. Péché, *Universality of slow decorrelation in KPZ models*, *Ann. Inst. H. Poincaré Probab. Statist.* **48** (2012), 134–150.
- [11] I. Corwin, Z. Liu, and D. Wang, *Fluctuations of TASEP and LPP with general initial data*, *Ann. Appl. Probab.* **26** (2016), 2030–2082.
- [12] P.L. Ferrari and H. Spohn, *Scaling limit for the space-time covariance of the stationary totally asymmetric simple exclusion process*, *Comm. Math. Phys.* **265** (2006), 1–44.

- [13] P.L. Ferrari and B. Vető, *Tracy-Widom asymptotics for q -TASEP*, Ann. Inst. H. Poincaré, Probab. Statist. **51** (2015), 1465–1485.
- [14] P. Groeneboom, *Brownian motion with a parabolic drift and Airy functions*, Probab. Theory Related Fields **81** (1989), 79–109.
- [15] T. Imamura and T. Sasamoto, *Polynuclear growth model with external source and random matrix model with deterministic source*, Phys. Rev. E **71** (2005), 041606.
- [16] K. Johansson, *Shape fluctuations and random matrices*, Comm. Math. Phys. **209** (2000), 437–476.
- [17] K. Johansson, *Discrete polynuclear growth and determinantal processes*, Comm. Math. Phys. **242** (2003), 277–329.
- [18] M. Prähofer and H. Spohn, *Universal distributions for growth processes in $1+1$ dimensions and random matrices*, Phys. Rev. Lett. **84** (2000), 4882–4885.
- [19] J. Quastel and D. Remenik, *How flat is flat in a random interface growth?*, arXiv:1606.09228 (2016).
- [20] T. Sasamoto and H. Spohn, *The crossover regime for the weakly asymmetric simple exclusion process*, J. Stat. Phys. **140** (2010), 209–231.
- [21] C.A. Tracy and H. Widom, *Level-spacing distributions and the Airy kernel*, Comm. Math. Phys. **159** (1994), 151–174.
- [22] C.A. Tracy and H. Widom, *On orthogonal and symplectic matrix ensembles*, Comm. Math. Phys. **177** (1996), 727–754.

Quenched invariance principles for random walks in a balanced time-dependent environment

JEAN-DOMINIQUE DEUSCHEL

(joint work with N. Berger, X. Guo and A. Ramírez)

We consider random walks in a space- and time- inhomogeneous balanced random ergodic environment which is not necessarily uniformly-elliptic. We will prove strong and weak versions of invariance principles under the quenched measure.

To be specific, we let \mathcal{M} be the set of probability measures on $\{e \in \mathbb{Z}^d : |e| \leq 1\}$, $d \geq 2$. A time-dependent **environment** is an element $\omega \in \Omega := \mathcal{M}^{\mathbb{Z}^d \times \mathbb{N}}$ with

$$\omega = \{\omega_n(x)\}_{(x,n) \in \mathbb{Z}^d \times \mathbb{N}} = \{\omega_n(x, e) : |e| \leq 1\}_{(x,n) \in \mathbb{Z}^d \times \mathbb{N}}.$$

We let \mathbb{P} be a probability measure on Ω which is *ergodic* with respect to the space-time shifts $\{\theta_{y,m}\}_{y \in \mathbb{Z}^d, m \geq 0}$ defined by

$$(\theta_{y,m}\omega)(x, n) = \omega(x + y, n + m).$$

For a given environment ω , the random walk $(X_n)_{n \geq 0}$ is a (possibly space- and time-inhomogeneous) Markov chain with law

$$P_\omega(X_{n+1} = x + e | X_n = e) = \omega_n(x, e).$$

We say that ω is *static* if $\omega_n(x) = \omega_m(x)$ for all m, n and x . When the environment (\mathbb{P} -almost surely) is static and satisfies the uniformly elliptic assumption $\min_{i=1, \dots, d} \omega(0, e_i) \geq \kappa$ for some positive constant κ , Lawler [4] proved the *quenched invariance principle* (QCLT). Namely, for \mathbb{P} -almost every ω , the law of the rescaled process

$$(X_{\lfloor tn \rfloor} / \sqrt{n})_{t \geq 0}$$

converges weakly to a Brownian motion with a *deterministic* nondegenerate diffusion matrix Σ . Later, the QCLT is proved by Guo and Zeitouni [3] for elliptic iid environment and any ergodic environment that satisfies the moment condition $E_{\mathbb{P}}[\prod_{i=1}^d \omega(0, e_i)^{-1}] < \infty$. Recently, it is shown by Berger and Deuschel [1] that the QCLT holds when the law P on static environment is i.i.d and *genuinely d -dimensional*. That is, $\mathbb{P}(\omega(0, e_i) > 0) > 0$ for all $i = 1, \dots, d$.

In our joint work, we will generalize all the aforementioned works to a possibly non-elliptic ergodic time-dependent balanced environment. Let $J_i = \inf\{n > 0 : X_i - X_0 = e_i\}$ for $i = 1, \dots, d$ and $J_0 = \inf\{n > 0 : X_n - X_0 = 0\}$. Our main result is the following theorem.

Theorem 1. Assume that $E_{\mathbb{P}}[\prod_{i=0}^d E_{\omega}[J_i]] < \infty$. Then \mathbb{P} -almost surely, the RWRE satisfies a CLT with a random nondegenerate diffusion matrix $\Sigma(\omega)$.

- (1) $\Sigma(\omega)$ can take only finitely many values.
- (2) When $d = 2$ and the environment is elliptic, then Σ is deterministic and the *QCLT* holds.
- (3) When $d \geq 3$, we have a counter-example that in a non-elliptic mixing random environment, the diffusion matrix is supported in a set of more than two values.

REFERENCES

- [1] N. Berger and J.-D. Deuschel, *A quenched invariance principle for non-elliptic random walk in i.i.d. balanced random environment*, Probab. Theory Relat. Fields **158** (2014), 91–126 .
- [2] J.-D. Deuschel, X. Guo and A. Ramírez, *Quenched invariance principle for random walk in time-dependent balanced random environment*, Ann. Inst. Henri Poincaré Probab. Stat. Accepted.
- [3] X. Guo and O. Zeitouni, *Quenched invariance principle for random walk in balanced random environment*, Probab. Theory Relat. Fields, **152** (2010), 207–230.
- [4] G. Lawler, *Weak convergence of a random walk in a random environment*, Comm. Math. Phys. **87** (1982), 81–87, .
- [5] G. Papanicolaou and S.R.S. Varadhan, *Diffusions with random coefficients*, Statistics and probability: essays in honor of C.R. Rao, pp. 547–552, North-Holland, Amsterdam, (1982).

What can we learn from Brochettes?

GADY KOZMA

(joint work with Hugo Duminil-Copin, Marcelo Hilário, Gil Kalai, Ioan Manolescu, Vladas Sidoravicius and Vincent Tassion)

We discussed two dimensional results from [2] as follows. Let $\delta > 0$ and “strengthen” each column of \mathbb{Z}^2 with probability δ , independently. Now let $\epsilon > 0$ be a second parameter and examine percolation on \mathbb{Z}^2 where every edge in a strengthened column is open with probability $p + \epsilon$ while all other edges are open with probability p , and all edges are independent. Then the result of [2] is that for every $\delta > 0$ and every $\epsilon > 0$, we have that the critical p for this process is strictly smaller than

$p_c(\mathbb{Z}^2)$. We sketched the proof, which combines the classic Aizenman-Grimmett argument [1], some near critical analysis, and finally the results of [4].

We then moved to discuss what would be needed for extending this result to 3 dimensions. The results uses a number of critical and near critical facts about percolation. Some of them known (in particular certain crucial lemmas from [3]) and others unpublished. In particular we mentioned the following yet unpublished result: the correlation length of percolation (in any dimension larger than 1) at $p_c + \epsilon$ is bounded above by $\exp(C/\epsilon^2)$.

REFERENCES

- [1] Michael Aizenman and Geoffrey Grimmett, *Strict monotonicity for critical points in percolation and ferromagnetic models*. J. Statist. Phys. **63:5-6** (1991), 817–835.
- [2] Hugo Duminil-Copin, Marcelo R. Hilário, Gady Kozma and Vladas Sidoravicius *Brochette percolation*. Preprint, available at: <https://arxiv.org/abs/1608.04963>.
- [3] Raphaël Cerf, *A lower bound on the two-arms exponent for critical percolation on the lattice*. Ann. Probab. **43:5** (2015), 2458–2480.
- [4] Harry Kesten, Vladas Sidoravicius and Maria Eulalia Vares, *Oriented percolation in a random environment*. Preprint, available at: <https://arxiv.org/abs/1207.3168>.

Convergence of vertex-reinforced jump processes to an extension of the supersymmetric hyperbolic nonlinear sigma model

PIERRE TARRÈS

(joint work with Franz Merkl, Silke Rolles)

Consider a finite undirected graph $G = (V, E)$ with edge weights $W = (W_{ij})_{i,j \in V}$. The vertex-reinforced jump process (VRJP) is a stochastic jump process $Y = (Y_t)_{t \geq 0}$ in continuous time with càdlàg paths, taking values in the vertex set V of G . The process starts in $Y_0 = i_0 \in V$; at time t , if $Y_t = i$, it has a jump rate to $j \sim i$ given by $W_{ij}L_j(t)$, where

$$(1) \quad L_j(t) = 1 + \int_0^t 1_{\{Y_s=j\}} ds, \quad i \in V$$

is the local time plus one at site j .

The VRJP was initially proposed by Werner and introduced by Davis and Volkov in [DV02, DV04] on trees. Further analysis on regular and Galton-Watson trees was conducted by Collecchio in [Col06, Col09] and by Basdevant and Singh in [BS12].

We first review recent progress on VRJP:

- explicit link with the edge-reinforced random walk (ERRW) by Tarrès [Tar11] and Sabot and Tarrès in [ST15]
- explicit correspondence, in [ST15], with a marginal of a supersymmetric hyperbolic sigma model introduced by Zirnbauer [Zir91]

- recurrence of VRJP on any graph of bounded degree for large reinforcement in [ST15], i.e. small conductances W_e , $e \in E$, using a result of Disertori and Spencer [DS10]; see Angel, Crawford, and Kozma in [ACK14] for an alternative proof not using the connection with a SuSy model
- transience of VRJP on \mathbb{Z}^d , $d \geq 3$ for small reinforcement in [ST15], using a result from Disertori, Spencer and Zirnbauer [DSZ10]
- representation through the Green function of a random Schrödinger operator by Sabot, Tarrès and Zeng [STZ15]
- link between a reversed version of the VRJP and the so-called Ray-Knight local time theorems by Sabot and Tarrès [ST16].

In [ST15], the marginals of the SuSy hyperbolic $H^{2|2}$ model arise in horospherical coordinates, as a function of the asymptotic proportions of local times at the vertices, but the other variables are not interpreted. On the other hand, the asymptotic analysis of the related edge-reinforced random walk by Keane and Rolles [KR00] (also Coppersmith and Diaconis [CD86]) naturally involves other variables than the asymptotic density, such as for instance the last exit tree of the walk.

In a joint work with Merkl and Rolles [MRT16] we prove that a similar asymptotic analysis can be carried out for the VRJP, and enables to interpret all the variables of an extension of the initial SuSy model by Zirnbauer [Zir91]. We show that in its tree version, the other variables in horospherical coordinates in that model arise on two different time scales as limits of the rescaled crossing numbers, rescaled fluctuations of local times, asymptotic local times on a logarithmic scale, endpoints of paths, and last exit trees.

REFERENCES

- [ACK14] O. Angel, N. Crawford, and G. Kozma. Localization for linearly edge reinforced random walks. *Duke Math. J.*, 163(5):889–921, 2014.
- [BS12] A.-L. Basdevant and A. Singh. Continuous-time vertex reinforced jump processes on Galton-Watson trees. *Ann. Appl. Probab.*, 22(4):1728–1743, 2012.
- [Col06] A. Collecchio. On the transience of processes defined on Galton-Watson trees. *Ann. Probab.*, 34(3):870–878, 2006.
- [Col09] A. Collecchio. Limit theorems for vertex-reinforced jump processes on regular trees. *Electron. J. Probab.*, 14:no. 66, 1936–1962, 2009.
- [CD86] D. Coppersmith and P. Diaconis. Random walks with reinforcement. *Unpublished manuscript*, 1986.
- [DS10] M. Disertori and T. Spencer. Anderson localization for a supersymmetric sigma model. *Comm. Math. Phys.*, 300(3):659–671, 2010.
- [DST15] M. Disertori, C. Sabot, and P. Tarrès. Transience of edge-reinforced random walk. *Comm. Math. Phys.*, 339(1):121–148, 2015.
- [DSZ10] M. Disertori, T. Spencer, and M.R. Zirnbauer. Quasi-diffusion in a 3D supersymmetric hyperbolic sigma model. *Comm. Math. Phys.*, 300(2):435–486, 2010.
- [DV02] B. Davis and S. Volkov. Continuous time vertex-reinforced jump processes. *Probab. Theory Related Fields*, 123(2):281–300, 2002.
- [DV04] B. Davis and S. Volkov. Vertex-reinforced jump processes on trees and finite graphs. *Probab. Theory Related Fields*, 128(1):42–62, 2004.

- [KR00] M.S. Keane and S.W.W. Rolles. Edge-reinforced random walk on finite graphs. In *Infinite dimensional stochastic analysis (Amsterdam, 1999)*, pages 217–234. R. Neth. Acad. Arts Sci., Amsterdam, 2000.
- [MRT16] F. Merkl, P. Tarrès, and S. W. W. Rolles. Convergence of vertex-reinforced jump processes to an extension of the supersymmetric hyperbolic nonlinear sigma model. Preprint arXiv:1612.05409, 2016.
- [ST15] C. Sabot and P. Tarrès. Edge-reinforced random walk, vertex-reinforced jump process and the supersymmetric hyperbolic sigma model. *JEMS*, 17(9):2353–2378, 2015.
- [ST16] C. Sabot and P. Tarrès. Inverting Ray-Knight identity. *Probab. Theory Related Fields*, 165(3-4):559–580, 2016.
- [STZ15] C. Sabot, P. Tarrès, and X. Zeng. The vertex reinforced jump process and a random Schrödinger operator on finite graphs. arXiv:1507.04660, to appear in *Annals of Probability*, 2015.
- [Tar11] P. Tarrès. Localization of reinforced random walks. *Preprint, available on <http://arxiv.org/abs/1103.5536>*, 2011.
- [Zir91] M.R. Zirnbauer. Fourier analysis on a hyperbolic supermanifold with constant curvature. *Comm. Math. Phys.*, 141(3):503–522, 1991.

Exponential decay of the correlations for the pinned membrane model

ERWIN BOLTHAUSEN

(joint work with Alessandra Cipriani, Noemi Kurt)

The simplest example of a membrane model is the Gaussian measure on \mathbb{R}^D , $D \subset \subset \mathbb{Z}^d$ with Hamiltonian

$$H(\phi) := \sum_x (\Delta\phi)_x^2 = \langle \phi, \Delta^2 \phi \rangle,$$

and 0 boundary conditions. Here, Δ is the discrete Laplacian

$$\Delta f(x) := \frac{1}{2d} \sum_{y:|y-x|=1} [f(y) - f(x)].$$

The measure is described as

$$\mu_D(d\phi) := \frac{1}{Z_D} \exp[-H(\phi)] \prod_{x \in D} d\phi_x \prod_{x \notin D} \delta_0(\phi_x).$$

In physics literature, such measures have been introduced to model membranes, see for instance [2], [3]. The critical dimension is 4 : For $d \geq 5$, the field exists on $D = \mathbb{Z}^d$, by a thermodynamic limit, and has decay of correlations of order $|x - y|^{4-d}$. For $d = 4$, the variance of ϕ_0 for $D = D_N = \{-N, \dots, N\}^4$ is of order $\log N$. This case belongs to the class of logarithmically correlated models.

We investigate modified models with local pinning at the origin, i.e. measures of the form

$$\mu_D^\varepsilon(d\phi) := \frac{1}{Z_D^\varepsilon} \exp[-H(\phi)] \prod_{x \in D} (d\phi_x + \varepsilon \delta_0(d\phi_x)) \prod_{x \notin D} \delta_0(\phi_x)$$

where $\varepsilon > 0$ is the pinning parameter. There are other versions of local pinning which have a locally changed Hamiltonian. The above so-called δ -pinning is mathematically the most convenient one.

Our main result is the following

Theorem 1. Let $d \geq 5$, and let cov_N^ε denote the covariance under the measure $\mu_{D_N}^\varepsilon$. For any $\varepsilon > 0$, there exist $\eta(d, \varepsilon), C(d, \varepsilon) > 0$ such that

$$|\text{cov}_N^\varepsilon(\phi_x, \phi_y)| \leq C(\varepsilon, d) \exp[-\eta(d, \varepsilon)|x - y|]$$

uniformly in N , $x \neq y$, $x, y \in D_N$.

Method of proof: The basis is an expansion

$$\text{cov}_N^\varepsilon(\phi_x, \phi_y) = \sum_{A \subset D_N} \nu_N^\varepsilon(A) G_A(x, y)$$

where here $y \mapsto G_A(x, y)$ for $x \in D_N \setminus A$ satisfies

$$G_A(x, y) = 0, \quad y \in A \cup D_N^c,$$

$$\Delta_y^2 G_A(x, y) = \delta_{x, y},$$

and where

$$\nu_N^\varepsilon(A) := \frac{Z_{D_N \setminus A}^{\varepsilon=0}}{Z_{D_N}^\varepsilon} \varepsilon^{|A|}$$

The result then follows from properties of this measure, and a proof that $y \rightarrow G_A(x, y)$ is rapidly decaying when A is “sufficiently” dense. Information about the latter is obtained by adapting a method developed by Vladimir Mazya [4].

Open problems:

- The case $d = 4$ is open, but it is expected that there the model has a similar exponential decay of the correlations. The main difficulty is to derive appropriate properties of ν_N^ε .
- Completely open are properties under pinning for non-Gaussian models with Hamiltonians

$$H(\phi) := \sum_x V(\Delta\phi_x),$$

where V is not quadratic.

REFERENCES

- [1] E. Bolthausen, A. Cipriani, N. Kurt: *Exponential decay of covariances for the supercritical membrane model*. arXiv:1609.04258
- [2] S. Leibler: *Equilibrium statistical mechanics of fluctuating films and membranes*. In: *Statistical mechanics of membranes and surfaces* 45–103. World Scientific Singapore, 1989.
- [3] R. Lipowsky: *Generic interactions of flexible membranes*. *Handbook of biological physics* **1** : 521–602, 1995.
- [4] V. Maz'ya: *Lectures on isoperimetric and isocapacitary inequalities in the theory of Sobolev spaces*. *Contemporary Mathematics* **338** 307–340 (2003).

Variational formulas for percolation limit shapes

TIMO SEPPÄLÄINEN

(joint work with Nicos Georgiou, Arjun Krishnan, Firas Rassoul-Agha)

Consider standard first-passage percolation on the d -dimensional integer lattice \mathbb{Z}^d . Nonnegative independent and identically distributed edge weights $\{t(e)\}_{e \in \mathcal{E}_d}$ are given, indexed by the set of undirected nearest-neighbor edges between vertices of \mathbb{Z}^d :

$$\mathcal{E}_d = \{\{x, y\} : x, y \in \mathbb{Z}^d, |x - y| = 1\}.$$

The passage time from vertex x to vertex y is the minimal time along a path from x to y :

$$T_{x,y} = \inf_{\gamma} \sum_{e \in \gamma} t(e)$$

where the infimum is over paths $\gamma = \{x_0, x_1, \dots, x_n\}$ such that $x_0 = x$, $x_n = y$, and $|x_i - x_{i+1}| = 1$ for all $0 \leq i \leq n - 1$. The length of a path is arbitrary. We assume that $\mathbb{E}(t(e)^p) < \infty$ for some $p > d$, which is stronger than needed but it does guarantee that all that follows is proved. The limiting time constant is the law of large numbers limit

$$\mu(\xi) = \lim_{n \rightarrow \infty} n^{-1} T_{0, [n\xi]} \quad \text{for } \xi \in \mathbb{R}^d.$$

The limit exists almost surely. $[n\xi]$ is some choice lattice point close to $n\xi$.

The goal is to characterize the limit $\mu(\xi)$ with a variational formula. Assume now that the weights $\{t(e)\}$ are defined on a product probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with an action of the group $\{\theta_x\}_{x \in \mathbb{Z}^d}$ of translations. Let $\mathcal{R} = \{\pm e_1, \dots, \pm e_d\}$ be the set of admissible steps of paths. Introduce a potential function $V : \Omega \times \mathcal{R} \rightarrow \mathbb{R}$ which is now defined by $V(\omega, z) = t(\{0, z\})$ and satisfies $V(\theta_x \omega, z) = t(\{x, x + z\})$.

Define the following space of stochastic processes. Let \mathcal{K} denote the space of functions $B : \Omega \times \mathbb{Z}^d \times \mathbb{Z}^d \rightarrow \mathbb{R}$ with these properties:

$$\begin{aligned} \mathbb{E}[B(x, y)] &< \infty, \\ B(\omega, x, y) + B(\omega, y, z) &= B(\omega, x, z), \\ \text{and } B(\omega, x + u, y + u) &= B(\theta_u \omega, x, y). \end{aligned}$$

Given $B \in \mathcal{K}$, define (the negative of) its mean vector $h(B)$ by

$$h(B) \cdot x = -\mathbb{E}[B(0, x)] \quad \forall x \in \mathbb{Z}^d.$$

A variational characterization of μ is now given as follows:

$$\mu(\xi) = - \inf_{B \in \mathcal{K}_V} h(B) \cdot \xi$$

where the subspace \mathcal{K}_V of \mathcal{K} is defined as follows:

$$\mathcal{K}_V = \{B \in \mathcal{K} : \min_{z \in \mathcal{R}} [V(\omega, z) - B(\omega, 0, z)] \geq 0 \text{ } \mathbb{P}\text{-a.s.}\}.$$

This formula is in currently unpublished joint work with A. Krishnan and F. Rassoul-Agha. It is proved from a variational formula for a first-passage percolation problem with restricted path lengths. Let

$$G_{0,n,x} = \inf_{(x_k)_{k=0}^n} \sum_{k=0}^{n-1} V(\theta_{x_k} \omega, x_{k+1} - x_k)$$

where $(x_k)_{k=0}^n$ is an n -step path from $x_0 = 0$ to $x_n = x$. The limiting time constant for this problem is

$$g(\xi) = \lim_{n \rightarrow \infty} n^{-1} G_{0,n,[n\xi]}$$

for ξ in the convex hull of \mathcal{R} . The variational formula for $g(\xi)$, from article [1], is

$$g(\xi) = \sup_{B \in \mathcal{K}} \mathbb{P}\text{-ess inf}_{\omega} \min_{z \in \mathcal{R}} \{V(\omega, z) - B(\omega, 0, z) - h(B) \cdot \xi\}.$$

Given the formula above, the proof of the formula for $\mu(\xi)$ begins from the connection

$$T_{0,x} = \inf_{k \geq |x|_1} G_{0,k,x}.$$

REFERENCES

- [1] N. Georgiou, F. Rassoul-Agha and T. Seppäläinen, *Variational formulas and cocycle solutions for directed polymer and percolation models*, *Comm. Math. Phys.* **346** (2016), 741–779.

Cover time and cover process for the random walk on the 2-dimensional torus

FRANCIS COMETS

(joint work with C. Gallesco, S. Popov and M. Vachkovskaia)

Consider the simple random walk on the two-dimensional discrete torus $\mathbb{Z}_n^2 := \mathbb{Z}^d/n\mathbb{Z}^d$ with the starting point chosen uniformly at random. Let \mathfrak{T}_n be the *cover time* of the torus, i.e., the first moment when this random walk visits all sites of \mathbb{Z}_n^2 . Being the largest hitting times of torus points, \mathfrak{T}_n is the maximum of *dependent* random variables, it motivates much research efforts at the moment.

The cover time can be defined in arbitrary dimension d , but the dependence between different hitting times is weak for $d \geq 3$ because of the transience of the walk: asymptotics in this case are similar to the independent case. On the contrary, in dimension $d = 2$, hitting times are strongly correlated. It was shown in [4] that $\frac{\mathfrak{T}_n}{n^2 \ln^2 n} \rightarrow \frac{4}{\pi}$ in probability; later, this result was refined by the first correction to this limit. Then, some asymptotics of the cover time are much different from the independent case [1]. The reason is found in the structure of the set of *late points*, i.e., the set of points that are still unvisited up to a given time. This set is rather well understood in large dimension.

In dimension 3 or higher, A.-S. Sznitman introduced random interlacements in [5] to describe the trace of simple random walk the torus \mathbb{Z}_n^d . Random interlacements are a consistent approximation of this trace at times ranging from $O(n^d)$

to the typical cover time. Late points are approximately independently distributed, as a Bernoulli-Poisson process.

In two dimensions, by recurrence of the walk, the strict analogue of Sznitman's construction is trivial, and the set of late points has interesting fractal-like properties when the elapsed time is a fraction of the expected cover time.

In [2], we define two-dimensional random interacements at level α as a point process on \mathbb{Z}^2 : its complement \mathcal{V}^α is called the vacant set, and is characterized by:

$$\mathbb{P}[A \subset \mathcal{V}^\alpha] = \exp(-\pi\alpha \times \text{cap}(A)),$$

for all finite $A \subset \mathbb{Z}^2$ containing the origin. Here, $\text{cap}(A)$ denotes the (recurrent) capacity of a finite set A . We construct two-dimensional random interacements using simple random walk trajectories on \mathbb{Z}^2 conditioned on never hitting the origin. We prove that the law of the uncovered set around the origin at time $\frac{4\alpha}{\pi}n^2 \ln^2 n$ conditioned on the event that the origin is uncovered, is close to the law of two-dimensional random interacements at level α . This describes the structure late points in the neighborhood of a randomly chosen unvisited site.

The two-dimensional random interacements has interesting properties: invariance, long range dependence, ... Furthermore, it has a phase transition, of a different nature than percolation: the vacant set \mathcal{V}^α is a.s. infinite if $\alpha \geq 1$, and a.s. finite if $\alpha > 1$. The critical case $\alpha = 1$ requires a considerable effort [3].

REFERENCES

- [1] F. Comets, C. Galleco, S. Popov, M. Vachkovskaia (2013) *On large deviations for the cover time of two-dimensional torus*. Electr. J. Probab., **18**, article 96.
- [2] F. Comets, S. Popov, M. Vachkovskaia (2016) *Two-dimensional random interacements and late points for random walks*. Commun. Math. Phys. **343** (1), 129–164.
- [3] F. Comets, S. Popov *The vacant set of two-dimensional critical random interacement is infinite*. Preprint 2016.
- [4] A. Dembo, Y. Peres, J. Rosen, O. Zeitouni (2004) *Cover times for Brownian motion and random walks in two dimensions*. Ann. Math. (2) **160** (2), 433–464.
- [5] A.-S. Sznitman (2009) *Random walks on discrete cylinders and random interacements*. Probab. Theory Relat. Fields **145** (1-2), 143–174.

Cohomological approach to the decomposition theorem for closed forms in the non-gradient method

MAKIKO SASADA

(joint work with Yukio Kametani)

To prove the hydrodynamic limit for non-gradient models, applying the gradient replacement, introduced by Varadhan and Quastel in [7] and [4], is a standard and unique strategy so far. Its essential part is the so-called characterization of closed forms (cf. [1], [3]). This part requires a very complicated argument with a sharp spectral gap estimate. Even though the statement of the characterization theorem of closed forms is almost same for a wide class of models, we need to change the details of the proof depending on the specific model and it is not straightforward.

Also, to show the sharp spectral gap estimate for each model is usually a tough work.

In this talk, we aim to understand the common structure of the characterization of closed forms among different models. In particular, we reveal why the dimension and the explicit expression of the set of harmonic forms (or precisely, closed but not exact forms) do not depend on the details of the model, which we can guess from the previous works on the non-gradient models ([1], [2], [3], [5], [7]). For this purpose, we introduce a CW complex associated to the configuration space and reconsider the characterization of closed forms from algebraic and geometric points of view. We report new observations and results obtained by the study of this complex. They are the followings:

(i) The typical characterization theorem of closed forms required in the context of hydrodynamic limit is about the closed forms in $L^2(\nu)$ where ν is a probability measure on a configuration space. The theorem claims that any closed form (precisely any germ of closed form) is decomposed as a sum of an exact form and a harmonic form. Moreover, the space of harmonic forms is explicitly given. We study the closed forms which are local functions, and prove the similar decomposition of them by the exact forms and the harmonic forms. The space of harmonic forms are common for L^2 functions and local functions. The proof is very simple and able to apply very general models directly.

(ii) The statement and the proof of the characterization theorem for local functions do not relate to the probability measure ν nor the spectral gap estimate, so it turns out to be purely an algebraic problem.

(iii) From the characterization theorem for local functions, we know the dimension and the explicit expression of the set of harmonic forms. In fact, the dimension is exactly the first cohomology group of an abelian group acting on the configuration space.

(iv) Using the idea of the proof of the characterization theorem for local functions, for the case where the model has a good duality, we give an alternative proof of the characterization theorem for $L^2(\nu)$ functions where we do not use the spectral gap estimate. The example of the model having a nice duality is the lattice gas reversible under Bernoulli measures studied in [1].

(v) With our new observations, we can generalize these results of the characterization theorem of closed forms for the interacting particle systems in a crystal lattice instead of \mathbb{Z}^d . As mentioned in [6], the hydrodynamic limit for a non-gradient system in a crystal lattice is an important open problem. Our result gives a way to attack the problem.

REFERENCES

- [1] T. FUNAKI, K. UCHIYAMA AND H. T. YAU, *Hydrodynamic Limit for Lattice Gas Reversible under Bernoulli Measures, Nonlinear Stochastic PDEs*, Springer-Verlag New York **77** (1996),1–44.
- [2] F. HERNÁNDEZ, *Equilibrium Fluctuations for a Non Gradient Energy Conserving Stochastic Model, Commun. Math. Phys.*, **326** (2014), 687–726
- [3] C. KIPNIS AND C. LANDIM, *Scaling Limits of Interacting Particle Systems*, 1999, Springer.

- [4] J. QUASTEL, *Diffusion of color in the simple exclusion process*, *Comm. Pure Appl. Math.*, **45** (1992), 623–679.
- [5] M. SASADA, *Hydrodynamic limit for two-species exclusion processes*, *Stoch. Proc. Appl.* **120** (2010), 494–521.
- [6] R. TANAKA, *Hydrodynamic Limit for Weakly Asymmetric Simple Exclusion Processes in Crystal Lattices*, *Commun. Math. Phys.*, **315** (2012), 603–641.
- [7] S.R.S. VARADHAN, *Nonlinear diffusion limit for a system with nearest neighbor interactions II*, in *Asymptotic Problems in Probability Theory, Stochastic Models and Diffusions on Fractals*, Pitman Res. Notes Math. Ser., **283** (1994), 75–128.

Temperature profiles in non-equilibrium stationary states

STEFANO OLLA

(joint work with Tomasz Komorowski, Marielle Simon)

Systems that have more than one conserved quantity (i.e. energy plus momentum, density etc.), can exhibit quite interesting temperature profiles in non-equilibrium stationary states. For example in [1] it has been studied numerically a chain of coupled rotators, attached at the boundary to thermal Langevin thermostats and under a constant force on the last rotator, that keeps the dynamics in a stationary non-equilibrium state. Notably in these stationary states the temperature at the center of the system is considerably higher than at the boundary. We understand now that this effect is strictly related to the presence of more than one conserved quantity. We present here some analytical result on a more simple model where this phenomena can be proved rigorously and the corresponding temperature profile computed explicitly. The system is given by an unpinned chain of harmonic oscillators, whose dynamics include a force applied on the last particle, Langevin thermostats acting at the boundaries, and random flipping of the sign of the velocity of the particles. Energy and volume are conserved quantities, only changed by the border thermostat and force.

The configurations of the system are given by sequences $(\mathbf{q}, \mathbf{p}) := \{q_x, p_x\}_{x=0, \dots, n}$, where $p_x \in \mathbb{R}$ stands for the momentum of the oscillator at site x , and $q_x \in \mathbb{R}$ represents its position. Thus the configuration space is $\Omega_n := (\mathbb{R} \times \mathbb{R})^{n+1}$. The interaction between two particles x and $x+1$ is described by the quadratic potential energy $V(q_x - q_{x+1}) = \frac{1}{2}(q_x - q_{x+1})^2$ of a harmonic spring relying the particles. At the boundaries the system is connected to two Langevin heat baths at temperatures T_- and T_+ . Furthermore on the right boundary is acting a force (tension) $\bar{\tau}_+$, eventually slowly changing in time at a scale t/n^2 . Notice that the system is unpinned. Consequently the absolute positions q_x do not have precise meaning, and the dynamics depends only on interparticle elongations $r_x = q_x - q_{x-1}$, $x = 1, \dots, n$. The configurations are then described by

$$(1) \quad (\mathbf{r}, \mathbf{p}) = (r_1, \dots, r_n, p_0, \dots, p_n) \in \mathbb{R}^n \times \mathbb{R}^{n+1}.$$

The energy is defined by the Hamiltonian:

$$\mathcal{E}_n := \sum_{x=1}^n \left\{ \frac{p_x^2}{2} + \frac{r_x^2}{2} \right\} + \frac{p_0^2}{2}.$$

The equation of the dynamics are given by

$$(2) \quad \begin{cases} dr_x(t) = n^2 (p_x(t) - p_{x-1}(t)) dt \\ dp_x(t) = n^2 (r_{x+1}(t) - r_x(t)) dt - 2p_x(t^-) d\mathcal{N}_x(\gamma n^2 t), & x \in \{1, \dots, n-1\} \\ dp_0(t) = n^2 r_1(t) dt - 2p_0(t^-) d\mathcal{N}_0(\gamma n^2 t) - \tilde{\gamma} n^2 p_0 dt + n\sqrt{2\tilde{\gamma}T_-} dw_0 \\ dp_n(t) = -n^2 r_n(t) dt + n^2 \bar{r}_+(t) dt - 2p_n(t^-) d\mathcal{N}_n(\gamma n^2 t) - \tilde{\gamma} n^2 p_n dt + n\sqrt{2\tilde{\gamma}T_+} dw_n \end{cases}$$

where $w_0(t)$ and $w_n(t)$ are two independent standard Wiener processes.

It is useful to use the generator of the dynamics in order to compute time evolutions. This is given by

$$(3) \quad L_t = n^2 (A_t + \gamma S + \tilde{\gamma} \tilde{S})$$

where

$$(4) \quad A_t = \sum_{x=1}^n (p_x - p_{x-1}) \partial_{r_x} + \sum_{x=1}^{n-1} (r_{x+1} - r_x) \partial_{p_x} + r_1 \partial_{p_0} + (\bar{r}_+(t) - r_n) \partial_{p_n},$$

$$(5) \quad Sf(\mathbf{q}, \mathbf{p}) = \sum_{x=0}^n (f(\mathbf{q}, \mathbf{p}^x) - f(\mathbf{q}, \mathbf{p}))$$

where \mathbf{p}^x is the velocities configuration with the sign of p_x changed. The generator of the Langevin heat bath at the borders is given by

$$(6) \quad \tilde{S} = \sum_{x=0,1,n-1,n} (T_x \partial_{p_x}^2 - p_x \partial_{p_x}), \quad T_0 = T_1 = T_-, \quad T_{n-1} = T_n = T_+.$$

Let us denote by $\langle \cdot \rangle$ the average with respect to this process, including on the initial conditions. The hydrodynamic limit result is the following:

$$(7) \quad \begin{aligned} \langle r_{[nu]}(t) \rangle &\xrightarrow{n \rightarrow \infty} r(t, u) \\ \langle p_{[nu]}^2(t) \rangle &\xrightarrow{n \rightarrow \infty} e_{\text{th}}(t, u), \end{aligned}$$

where $r(t, u), e_{\text{th}}(t, u)$ are solution of the diffusive system:

$$(8) \quad \begin{aligned} \partial_t r(t, u) &= \frac{1}{2\gamma} \partial_{uu}^2 r(t, u) \\ \partial_t e_{\text{th}}(t, u) &= \frac{1}{4\gamma} \partial_{uu}^2 e_{\text{th}}(t, u) + \frac{1}{2\gamma} (\partial_u r(t, u))^2, \quad (t, u) \in \mathbb{R}_+ \times [0, 1] \end{aligned}$$

with boundary conditions

$$(9) \quad \begin{aligned} r(t, 0) &= 0, & r(t, 1) &= \bar{r}_+(t), \\ e_{\text{th}}(t, 0) &= T_-, & e_{\text{th}}(t, 1) &= T_+ \end{aligned}$$

with the initial condition

$$(10) \quad r(0, u) = r_0(u), \quad e_{\text{th}}(0, u) = \mathcal{T}_0(u).$$

Take $\bar{\tau}_+$ constant in time and $T_- = T_+ = T$. If $\bar{\tau}_+ \neq 0$, the stationary profiles in the equations (8) satisfy:

$$(11) \quad \begin{aligned} r_{\text{ss}}(u) &= \bar{\tau}_+ u \\ \partial_{uu}^2 e_{\text{th,ss}}(u) + 2\bar{\tau}_+^2 &= 0, \quad e_{\text{th,ss}}(0) = e_{\text{th,ss}}(1) = T. \end{aligned}$$

i.e.

$$(12) \quad e_{\text{th,ss}}(u) = \bar{\tau}_+^2 u(1-u) + T.$$

Notice that the chain heats up at the center, reaching the maximum temperature at $e_{\text{th,ss}}(1/2) = T + \frac{\bar{\tau}_+^2}{4}$. Notice that this is independent of the sign of $\bar{\tau}_+$ and that this is a quadratic effect.

REFERENCES

- [1] Iacobucci, A., Legoll, F., Olla, S., Stoltz, G., *Negative thermal conductivity of chains of rotors with mechanical forcing*, Phys. Rev. E, **84** (2011) 061108.

How to initialize a second class particle ?

MÁRTON BALÁZS

(joint work with Attila László Nagy)

We greatly generalize P. A. Ferrari and C. Kipnis' [3] results on the behavior of the second class particle in the rarefaction fan of the totally asymmetric simple exclusion process. Versions of their results are shown to hold through for practically any attractive particle system (including zero-range, misanthrope models, and many more) with established hydrodynamic behavior.

The second class particle in exclusion processes is a unique object in the sense that, being the integer difference between two 0-1 random variables, it must see occupation number 0 for one of the coupled processes while 1 for the other. Generalizing the notion to other models with more options for a site poses the new question of what the distribution of the coupled pair should be at the site of the second class particle, and sets the old task of providing a one-site coupling with the following properties:

- the first marginal of the coupled pair is the desired stationary marginal of one of the two densities for the rarefaction fan initial data;
- the second marginal of the coupled pair is the desired stationary marginal of the other of the two densities;
- the coupled pair either agree or have one difference between them (zero or one second class particles).

This is easily achieved by the standard monotone coupling of Bernoulli variables for exclusion, and is also doable for the discrete Gaussian example we have seen as a stationary distribution of the *bricklayer processes*. However, there is no such coupling probability distribution for other natural examples like the classical zero range process (with iid. Geometric marginals) or independent walkers (iid. Poisson marginals).

The main novelty is thus the introduction of a *signed* coupling measure as initial data, which nevertheless results in a proper probability initial distribution for the site of the second class particle. This is made possible by the observation that the signed coupling measure that arises from the above conditions always assigns non-negative weights on the off-diagonal state (when we have a second class particle), the unwanted negative mass can only occur at the diagonal entry where there is no second class particle present. To obtain a probability distribution for the second class particle one simply rescales the non-negative, off-diagonal part of the signed coupling measure.

This distribution proves to be canonical in many senses and makes the extension of [3] possible via a tricky argument that makes use of conditioning the pair of processes *before* coupling them. Combined with strong recent results in hydrodynamic limits, we are able to identify the ballistically and diffusively rescaled limit distribution of the second class particle position in a wide range of asymmetric and symmetric models, respectively. The asymmetric result is similar to that achieved in [3] while the symmetric case provides a new way of proving central limit theorems for the second class particle.

We also point out a model with non-concave, non-convex hydrodynamics [2, 4], where the rescaled second class particle distribution has both continuous and discrete counterparts. As a by-product of our methods we reveal a very interesting invariance property of the one-site marginal distribution of the process underneath the second class particle. Finally, we give a lower estimate on the probability of survival of a second class particle-antiparticle pair.

REFERENCES

- [1] M. Balázs and A.L. Nagy. How to initialize a second class particle? <http://arxiv.org/abs/1510.04870>, accepted for publication in the *Annals of Probability*, preprint, 25 pages, 2015.
- [2] M. Balázs, A.L. Nagy, B. Tóth, and I. Tóth. Coexistence of shocks and rarefaction fans: Complex phase diagram of a simple hyperbolic particle system. *Journal of Statistical Physics*, 165(1):115–125, 2016.
- [3] P.A. Ferrari and C. Kipnis. Second class particles in the rarefaction fan. *Ann. Inst. H. Poincaré Probab. Statist.*, 31(1):143–154, 1995.
- [4] Thierry G. and Ellen S. Couplings, attractiveness and hydrodynamics for conservative particle systems. *Ann. Inst. H. Poincaré Probab. Statist.*, 46(4):1132–1177, 11, 2010.

Invariance principle for a slowed random walk over symmetric exclusion

OTÁVIO MENEZES

(joint work with Milton Jara)

We establish an invariance principle for a random walk driven by a simple exclusion process in one dimension. The walk has a drift to the left (resp. right) when it sits on a particle (resp. hole). The environment starts from equilibrium and is speeded up with respect to the walker. After a suitable space-time rescaling, the random walk converges to a sum of a Brownian motion and a Gaussian process with stationary increments, independent of the Brownian motion. The proof is based on the martingale approximation method of Kipnis and Varadhan. The most important step in the proof is a bound on the relative entropy between the environment process and the Bernoulli product measure (which is not invariant for the environment process).

REFERENCES

- [1] Avena, L., Franco, T., Jara, M. and Völlering, F. (2015) Symmetric exclusion as a random environment: hydrodynamic limits *Ann. Inst. H. Poincaré Probab. Statist. Volume 51, Number 3 (2015), 901–916*.
- [2] Jara, M. Menezes, O. Central Limit Theorem for a Slowed Random Walk over Exclusion Process. In preparation.

Branching Interlacements

BALÁZS RÁTH

(joint work with Omer Angel, Qingsan Zhu)

We consider critical branching random walk (BRW) with geometric offspring distribution and uniform starting point on the d -dimensional torus of side length N , and condition the total number of offspring to be equal to $\lfloor uN^d \rfloor$. We look at the limit of the law of the trace of this BRW as N goes to infinity for some fixed value of u and $d \geq 5$, and find that it is a random subset of \mathbb{Z}^d which can be constructed as the trace of a Poisson point process on the space of infinite trees embedded in the lattice. Our construction relies on the notion of contour process of a plane tree. Inspired by similar results about Sznitman's random interlacements, one studies the connectivity properties of this random subset of the lattice (the branching interlacement at level u) and its complement. The talk is based on joint work in progress with Omer Angel and Qingsan Zhu.

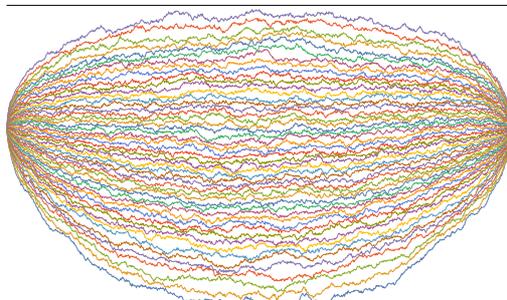


FIGURE 1. Illustration of $N = 50$ non-intersecting Brownian motions conditioned to stay below the black threshold.

Tilings of the Aztec diamond on a restricted domain and the hard-edge tacnode process

BÁLINT VETŐ

(joint work with Patrik L. Ferrari)

This work in progress is motivated by the tiling problem of the Aztec diamond. The Aztec diamond is a domain A_n in the two-dimensional integer lattice that consists of the union of squares of the form $[k, k + 1] \times [l, l + 1]$ which lie inside $\{|x| + |y| \leq n + 1\}$. One of all possible tilings of A_n by vertical or horizontal 2×1 domino is chosen uniformly at random. As $n \rightarrow \infty$, the upper part of the Aztec diamond A_n contains only horizontal domino in a special alignment which form the north polar region. The boundary of this region is the Arctic circle with limiting fluctuations given by the Airy_2 process on the $n^{1/3}$ scale, see [2].

To obtain a non-trivial interaction with the limiting Airy_2 process on the boundary of the north polar region, we consider a uniform tiling of the Aztec diamond A_n restricted to $y \leq R$ where the horizontal line $y = R$ is tangential to the Arctic circle. As a limit process, the Airy_2 process restricted to stay below a constant level is expected.

The same limit process was obtained in [1] as the $n \rightarrow \infty$ limit of n non-intersecting Brownian bridges conditioned to stay below a fixed threshold, see Figure 1. The scaling limit in this case appears if the limit shape of the region filled by the Brownian bridge paths is tangential to the threshold. In [1] the limiting distribution of the top Brownian bridge conditioned to stay below a function is described as well as the limiting correlation kernel of the system. It is a one-parameter family of processes which depends on the tuning of the threshold position on the natural fluctuation scale.

REFERENCES

- [1] P. L. Ferrari, B. Vető, *The hard-edge tacnode process for Brownian motion*, preprint, arXiv: 1608.00394
- [2] K. Johansson, *The arctic circle boundary and the Airy process*, Ann. Probab. **33** (2005), 1–30

Some recent works on the branching Brownian motion

BERNARD DERRIDA

(joint work with Zhan Shi)

Three recent results were presented during my short presentation:

- (1) It is well known that the most recent common ancestor of two particles, chosen at random among the rightmost particles of a branching Brownian motion (BBM), is either very close to the top of the tree or very close to its bottom. This is closely related to the broken symmetry of replicas in the mean field theory of directed polymers. In a recent work with Peter Mottishaw we have computed the leading finite size correction, i.e. the probability that the common ancestor is at any intermediate height on the tree [1]. Our result is universal as it remains unchanged for more general branching random walks.
- (2) With Zhan Shi [2], we have obtained some results on generalizations of the branching Brownian motion in presence of selection (L-BBM, N-BBM, branching random walk with coalescence). In the limit $L \rightarrow \infty$, $N \rightarrow \infty$ or of a very small coalescence rate, the large deviation function of the position of the rightmost particle exhibits a non-analytic dependence on the position.
- (3) With Zhan Shi [3], we have computed the large deviation function for negative large deviations of the position of the rightmost particle of a branching Brownian motion. This large deviation function is linear in a certain range, where the prefactor is a power law of time with an irrational exponent.

REFERENCES

- [1] Derrida, B., Mottishaw, P. (2016). On the genealogy of branching random walks and of directed polymers. EPL (Europhysics Letters), 115(4), 40005.
- [2] Derrida, B., Shi, Z. (2016). Large deviations for the branching Brownian motion in presence of selection or coalescence. Journal of Statistical Physics, 163(6), 1285–1311.
- [3] Derrida, B., Shi, Z. (2016). Large deviations for the rightmost position in the branching Brownian motion. preprint 2016 and work in preparation

Upper triangular matrix walk: cutoff for finitely many columns

SHIRSHENDU GANGULY

(joint work with Fabio Martinelli)

Random walks on the group $\mathcal{M}_n(q)$ of the $n \times n$ -upper triangular matrices with one along the diagonal and entries from the finite field \mathbb{F}_q , q a prime, have received quite a lot of attention, owing to the fact that they form basic examples of random walks on nilpotent groups. The random walk, sometimes called the *upper triangular walk*, in the case $q = 2$, is the Markov chain whose generic step consists in choosing uniformly at random a row among the first $(n - 1)$ -ones and adding to it the next row mod 2. It is easy to check that this chain is reversible w.r.t. the uniform measure on $\mathcal{M}_n(2)$. A natural variant, called the *lazy upper triangular walk*, entails to perform the above addition with probability $1/2$. We refer the interested reader to [9] for a nice review of the background on the literature related to this walk and other related variants.

In this work, we consider the continuous time version of the upper triangular walk where each row at rate one is updated by adding the row below it with probability $1/2$. Sharp bounds on the spectral gap were proven by Stong [6] implying, in particular, that the spectral gap $\lambda_2(n)$ is positive uniformly in n . Using an elegant argument, Peres and Sly [9] proved that the total variation mixing time $t_{\text{mix}}(n) = \Theta(n)$. From the above results it follows that $\lim_{n \rightarrow \infty} \lambda_2(n) \times t_{\text{mix}}(n) = +\infty$, a known necessary condition for the occurrence of the so called *mixing time cutoff* [2], i.e. a sharp transition in the total variation distance from equilibrium which drops from being close to one to being close to zero in a very small time window compared to the mixing time scale.

In [1], Y. Peres conjectured that, for many natural classes of reversible Markov chains, the above condition (sometimes referred to as the *product condition*) is also sufficient for the occurrence of cutoff, despite of the fact that, in full generality, this is known to be false (cf. [7, Chapter 18]). Thus it is a natural and interesting problem to decide whether the upper triangular matrix walk exhibits cutoff or not.

It has been observed before and was crucially used in [9], that the marginal process on a given column coincides with the East process [3–5] at density $1/2$, a well known constrained interacting particle system. The East chain is known to exhibit cutoff (cf. [8]), a result which, combined with the previous observation, suggests that the upper triangular walk in continuous time might do the same.

In this work we extend and complement the Peres-Sly result by proving that (i) the spectral gap of our chain is equal to the spectral gap of the East process on $n - 1$ vertices; (ii) the marginal chain on *finitely* many columns exhibits mixing time cutoff at the mixing time of the column with the largest index, among the chosen ones.

Whether the whole matrix has cutoff and mixes at the same location as the last column remains an intriguing open question! We also remark that, perhaps surprisingly, certain numerical evidence suggests that the mixing time of full chain is strictly larger (on a linear scale n) than the mixing time of one column.

REFERENCES

- [1] Peres, Y. American institute of mathematics (AIM) research workshop “sharp thresholds for mixing times” (Palo Alto, December 2004). Summary available at <http://www.aimath.org/WWN/mixingtimes>.
- [2] Aldous, David, and Persi Diaconis. “Shuffling cards and stopping times.” *The American Mathematical Monthly* 93.5 (1986): 333–348.
- [3] Cancrini, Nicoletta, et al. “Kinetically constrained spin models.” *Probability Theory and Related Fields* 140.3–4 (2008): 459–504.
- [4] Faggionato, Alessandra, et al. “The East model: recent results and new progresses.” arXiv preprint arXiv:1205.1607 (2012).
- [5] Aldous, David, and Persi Diaconis. “The asymmetric one-dimensional constrained Ising model: rigorous results.” *Journal of Statistical Physics* 107.5-6 (2002): 945–975.
- [6] Stong, Richard. “Random walks on the groups of upper triangular matrices.” *The Annals of Probability* (1995): 1939–1949.
- [7] Levin, David Asher, Yuval Peres, and Elizabeth Lee Wilmer. *Markov chains and mixing times*. American Mathematical Soc., 2009.
- [8] Ganguly, Shirshendu, Eyal Lubetzky, and Fabio Martinelli. “Cutoff for the East process.” *Communications in Mathematical Physics* 335.3 (2015): 1287–1322.
- [9] Peres, Yuval, and Allan Sly. “Mixing of the upper triangular matrix walk.” *Probability Theory and Related Fields* 156.3–4 (2013): 581–591.

Invariant measures of mass migration processes

ELLEN SAADA

(joint work with Lucie Fajfrová, Thierry Gobron)

We introduce in [5] the *mass migration process* (MMP), a conservative particle system on $\mathbb{N}^{\mathbb{Z}^d}$. It consists in simultaneous jumps of k particles ($k \geq 1$) between sites, with a jump rate depending only on the state of the system at the departure and arrival sites of the jump. On one hand it is a particular case of the dynamics studied in [7], and on the other hand it generalizes misanthropes, zero range and target processes [1, 3, 8], for which $k = 1$ always. In a mass migration zero range process (MM-ZRP) - resp. target (MM-TP) - the rates do not depend on the occupation number of the arrival site - resp. non-empty departure site - of the jump. The generalized zero range [9] (which may exhibit condensation) and the q -Hahn asymmetric zero range [2] (an exactly solvable model) are MM-ZRPs.

After the construction of MMP (done in the spirit of [1, 10]), our main focus is on its invariant measures, whose explicit knowledge is essential to study condensation (see [4]), or exactly solvable models. We derive necessary and sufficient conditions for the existence of translation invariant and invariant product probability measures. For asymmetric MM-ZRP and MM-TP, these conditions yield explicit solutions, and, if these processes are moreover attractive (we study attractiveness for MMP relying on [7], summarized in [12]), we obtain their extremal translation invariant, invariant probability measures. We study condensation for MMP, and its link with attractiveness (see also [11] on this link). Finally, we give the first proof of coexistence of attractiveness and condensation on a fixed finite volume (see [6]) on an example of MM-ZRP.

REFERENCES

- [1] Andjel, E.D.: Invariant Measures for the Zero Range Process. *Ann. Probab.* **10** (1982), no. 3, 525–547.
- [2] Barraquand, G., Corwin, I.: The q -Hahn asymmetric exclusion process. *Ann. Appl. Probab.* **26** (2016), no. 4, 2304–2356.
- [3] Coccozza, C. T.: Processus des misanthropes. *Z. Wahrsch. Verw. Gebiete* **70** (1985), no. 4, 509–523.
- [4] Evans, M. R. and Hanney, T.: Nonequilibrium Statistical Mechanics of the Zero-Range Process and Related Models. *J. Phys. A: Math. Gen.* **38** (2005), no. 19, R195–R240.
- [5] Fajfrovà, L., Gobron, T., Saada, E.: Invariant measures of Mass Migration Processes. *Electron. J. Probab.*, **21**, no. 60 (2016), 1–52.
- [6] Ferrari, P.A., Landim, C., Sisko, V.: Condensation for a fixed number of independent random variables. *J. Stat. Phys.* **128** (2007), no. 5, 1153–1158.
- [7] Gobron, T., Saada, E.: Couplings, attractiveness and hydrodynamics for conservative particle systems. *Ann. Inst. H. Poincaré Probab. Statist.* **46**, no. 4 (2010), 1132–1177.
- [8] Godrèche, C.: From urn models to zero-range processes: statics and dynamics. *Ageing and the glass transition*, 261–294, *Lect. Notes Phys.* **716**, Springer, Berlin, 2007.
- [9] Greenblatt, R.L., Lebowitz, J.L.: Product Measure Steady States of Generalized Zero Range Processes. *J. Phys. A: Math. Gen.* **39** (2006), no. 7, 1565–1573.
- [10] Liggett, T. M., Spitzer, F.: Ergodic theorems for coupled random walks and other systems with locally interacting components. *Z. Wahrsch. Verw. Gebiete* **56** (1981), no. 4, 443–468.
- [11] Rafferty T., Chleboun P., Großkinsky S.: Monotonicity and condensation in homogeneous stochastic particle systems. Preprint (2015). ArXiv:1505.02049
- [12] Saada, E.: Couplings and attractiveness for interacting particle systems. Oberwolfach Reports (OWR), Vol. 10, Issue 4, EMS Publishing House/MFO (2013), 3107–3109.

RW kernels, spanning forests and multiscale analysis on graphs.

LUCA AVENA

(joint work with Fabienne Castell, Alexandre Gaudillière and Clothilde Mélot)

We use ideas from large-scale stochastic dynamics to build a multiresolution scheme to analyse arbitrary functions on graphs. These types of problems emerge naturally in the context of signal processing. The goal is to obtain successive approximations at different scales of arbitrary functions on graphs which are used for signal classification, reconstruction and data compression. When the signal is defined on a graph having enough regularity structures, several methods (such as wavelets) are available in the literature and used in practice. When the regularity structure of the graph is lacking, very few methods are known. Our work [2, 3] aims at addressing this issue by using random spanning forests [1], loop-erased walks, determinantal structures, random walk kernels and intertwining of Markov chains.

REFERENCES

- [1] L. Avena and A. Gaudillière, *Random spanning forests, Markov matrix spectra and well distributed points*, arXiv:1310.1723v4 (2016), accepted for publication in Journal of Theoretical Probability.
- [2] L. Avena, F. Castell, A. Gaudillière and C. Mélot, *Approximate and exact solutions of intertwining equations through random spanning forests*, to appear (2016).

- [3] L. Avena, F. Castell, A. Gaudillière and C. Mélot, *Multiresolution analysis on weighted graphs through random forests and Markov intertwining*, to appear (2017).

How does ice melt at 0°C ?

BENOÎT LASLIER

(joint work with Fabio Toninelli)

Lozenge tilings can be seen as a random 2-dimensional surface embedded in \mathbb{R}^3 and can be therefore used as a model for an interface. In a work in progress, we consider a reversible dynamics on this model, corresponding physically to an interface at a point of phase coexistence, and prove that the surface follows a deterministic hydrodynamics limit at the diffusive time scale.

The dynamics was introduced by Luby, Randall and Sinclair to give a polynomial algorithm for generation of uniformly random lozenge tilings and we show that it is in some sense a gradient dynamics, allowing us to apply the H^{-1} method of Funaki and Spohn. The limit PDE is fully non-linear and has a non-trivial mobility coefficient but is strikingly explicit and was previously derived non-rigorously by the same authors in [1].

REFERENCES

- [1] B. Laslier, F. Toninelli, *Hydrodynamic limit equation for a lozenge tiling Glauber dynamics*, to appear in *Annales Henri Poincaré*

Current Fluctuations for the Stationary ASEP

AMOL AGGARWAL

The purpose of this short talk is to explain our recent result of [1], which accesses the scaling limit of the current of the stationary one-dimensional asymmetric simple exclusion process (ASEP).

Recall that the ASEP is a continuous-time interacting particle system on \mathbb{Z} . There is a one-parameter family of translation-invariant, stationary measures for this process, given by Bernoulli product measures, meaning that each site is occupied with some fixed probability $\rho \in (0, 1)$.

Denoting the tagged particles of the ASEP by $\dots < X_{-1}(t) < X_0(t) < X_1(t) < \dots$ at each time $t \geq 0$ (where we initialize $X_{-1}(0) \leq 0 < X_0(0)$), define for each $x \in \mathbb{R}$ the *current* $J_t(x)$ by the almost surely finite sum

$$J_t(x) = \sum_{i=-\infty}^{\infty} (\mathbf{1}_{X_i(0) \leq 0} \mathbf{1}_{X_i(t) > x} - \mathbf{1}_{X_i(0) > 0} \mathbf{1}_{X_i(t) \leq x}).$$

The following theorem provides the scaling limit for the current of the stationary ASEP along the characteristic line $x = (1 - 2\rho)t$.

Theorem 1.1 ([1, Theorem 1.4]). *Consider the ASEP with left jump rate L , right jump rate R , and stationary initial data with parameter ρ ; assume $\delta = R - L > 0$, and denote $\chi = \rho(1 - \rho)$. Then, for any $s \in \mathbb{R}$, we have that*

$$(1) \quad \lim_{T \rightarrow \infty} \mathbb{P} \left[J_{\delta^{-1}T}((1 - 2\rho)T) \geq \rho^2 T - \chi^{2/3} s T^{1/3} \right] = \Phi(s),$$

where $\Phi(s)$ denotes the Baik-Rains distribution, given by Definition 2 of [2].

Let us explain the context for the above theorem. Since the ASEP is a discretization of the *Kardar-Parisi-Zhang (KPZ) equation* $\partial_t \mathcal{H} = \partial_x^2 \mathcal{H} + (\partial_x \mathcal{H})^2 + \dot{\mathcal{W}}$ ($\dot{\mathcal{W}}$ is space-time white noise), it had long been believed that the scaling limit of the current of the stationary ASEP should converge to the scaling limit of the height fluctuations of the stationary KPZ equation.

The latter was recently analyzed in the work of Borodin-Corwin-Ferrari-Vető [4]; they showed that the height function of the stationary KPZ equation, after run for some large time T , is of order $T^{1/3}$ and scales to the Baik-Rains distribution $\Phi(s)$ above. Thus, it would be expected that the scaling limit of the current of the ASEP after run for some large time T should (along the characteristic line) also be of order $T^{1/3}$ and scale to $\Phi(s)$.

Special cases of this result were established by Ferrari-Spohn [5] and Balász-Seppäläinen [3], and the scaling limit of the ASEP with different types (step-Bernoulli) of initial data was later obtained by Tracy-Widom [6]; however, none of these methods seemed to directly apply to the stationary setting.

Theorem 1.1 above accesses the scaling limit for the current fluctuations of the stationary ASEP, thereby establishing the conjecture stated above.

A question left unresolved by this work is that of universality. For example, it remains unknown whether (1) holds for general stationary exclusion processes with non-nearest neighbor jumps.

REFERENCES

- [1] A. Aggarwal, Current Fluctuations of the Stationary ASEP and Six-Vertex Model, preprint, <https://arxiv.org/abs/1608.04726>.
- [2] J. Baik and E. M. Rains, Limiting Distribution for a Polynuclear Growth Model With External Sources, *J. Stat. Phys.* **100**, 523–541, 2000.
- [3] M. Balász and T. Seppäläinen, Order of Current Variance and Diffusivity in the Asymmetric Simple Exclusion Process, *Ann. Math.* **171**, 1237–1265, 2010.
- [4] A. Borodin, I. Corwin, P. Ferrari, and B. Vető, Height Fluctuations for the Stationary KPZ Equation, *Math. Phys. Anal. Geom.* **18**, Article 20, 2015.
- [5] P. L. Ferrari and H. Spohn, Scaling Limit for the Space-Time Covariance of the Stationary Totally Asymmetric Simple Exclusion Process, *Comm. Math. Phys.* **265**, 1–44, 2006.
- [6] C. A. Tracy and H. Widom, On ASEP With Step Bernoulli Initial Condition, *J. Stat. Phys.* **137**, 825–838, 2009.

Diffusions in Kinetically Constrained Models

ORIANE BLONDEL

(joint work with Cristina Toninelli)

KCM are interacting particle systems on \mathbb{Z}^d reversible with respect to a product Bernoulli measure with some parameter $1-q \in (0, 1)$. They evolve with either spin-flip or exchange dynamics, where updates are suppressed if a local constraint on the number of zeroes in the neighbourhood is not satisfied. We consider two classes of KCM: (1) non-cooperative models, the simplest example of which is the FA-1f model. The dynamics is spin-flip and updates are allowed at x provided at least one nearest-neighbour is empty. (2) KA models with parameter $j \in \{1, \dots, d\}$, where a particle at x is allowed to jump to an empty nearest neighbour y if x has at least j empty neighbours and y has at least $j-1$.

These models are known to be ergodic at any density q and the properties of their spectral gap investigated in the past [2, 3]. Here we want to study them “from the inside”. In spin-flip non-cooperative models such as FA-1f, we consider a probe particle, *i.e.* a simple random walk started at the origin and constrained to jump only between empty sites. In KA models, we follow the motion of a tagged particle.

In both these settings, due to reversibility and classical results, the probe or tagged particle satisfies an annealed invariance principle, with a diffusion coefficient $D(q)$ given by a variational formula [4] and depending on the density. The strength of this variational formula is that it allows to derive properties of the diffusion coefficient by comparison with an appropriate auxiliary dynamics.

For the probe particle in non-cooperative Kinetically Constrained Spin Models we find the correct (polynomial) asymptotic dependence of $D(q)$ as $q \rightarrow 0$ [1]. For the tagged particle in KA models, in collaboration with Cristina Toninelli, we announce a proof that $D(q) > 0$ for any $q \in (0, 1)$. In particular the proof fixes critical issues in the strategy sketched in [5].

An open problem is the derivation of the correct asymptotic order for $D(q)$, $q \rightarrow 0$ in the KA models (the bound we get should be off by an exponential). We also expect that in other KCM, there exists a regime where diffusion occurs (*i.e.* $D(q) > 0$) even though the KCM is not ergodic.

REFERENCES

- [1] O. Blondel, *Tracer diffusion in low temperature kinetically constrained models*, The Annals of Applied Probability, 2015, Vol. 25, No. 3, 1079–1107.
- [2] N. Cancrini, F. Martinelli, C. Roberto, C. Toninelli, *Kinetically constrained spin models*, Probab. Theory Related Fields 140 (2008), no. 3-4, 459–504.
- [3] N. Cancrini, F. Martinelli, C. Roberto, C. Toninelli, *Kinetically constrained lattice gases*, Comm. Math. Phys. 297 (2010), no. 2, 299–344.
- [4] H. Spohn, *Tracer diffusion in lattice gases*, J. Statist. Phys. 59 (1990), no. 5-6, 1227–1239.
- [5] C. Toninelli, G. Biroli, *Dynamical arrest, tracer diffusion and kinetically constrained lattice gases*, J. Statist. Phys. 117 (2004), no. 1-2, 27–54.

Static length scales and glassy dynamics in triangular and square plaquette models

ALESSANDRA FAGGIONATO

(joint work with P. Chleboun, F. Martinelli, C. Toninelli)

Plaquette models are finite range interacting spin systems with Glauber dynamics expected to exhibit a glassy behavior, despite the absence of dynamical constraints (present in kinetically constrained models) and disorder (present in spin glasses) [3, 5]. In particular, the triangular plaquette model on the triangular lattice and the square plaquette model on \mathbb{Z}^2 are supposed to have a dynamics similar to the East model and the FA1f model (which are kinetically constrained models). Plaquette models are an interesting object also in string theory [2] and in for cellular automata [6].

Since also the triangular lattice can be transformed into \mathbb{Z}^2 by a linear map, we can work directly on \mathbb{Z}^d . The plaquettes are given by the sets $P_* + z$, as z varies in \mathbb{Z}^d , where P_* is a fixed finite subset of \mathbb{Z}^d . Writing \mathcal{P} for the family of plaquettes, we fix uniformly bounded coupling constants $J(P)$, $P \in \mathcal{P}$. The resulting Hamiltonian is then given by $H(\sigma) = -\frac{1}{2} \sum_{P \in \mathcal{P}} J(P) \sigma_P$, where $\sigma \in \{-1, 1\}^{\mathbb{Z}^d}$ and $\sigma_P := \prod_{x \in P} \sigma_x$. The square plaquette model (SPM) corresponds to the case $d = 2$ and $P_* = \{0, 1\} \times \{0, 1\}$, while the triangular plaquette model (TPM) to the case $d = 2$ and $P_* = \{(0, 0), (0, 1), (1, 1)\}$.

Plaquette models with P_* of the form $B_1 \times B_2 \times \dots \times B_d$ (as for the SPM) are called *factorizable* and in [4] it has been proved the validity of the Dobrushin–Shlosman criterion [1] for any inverse temperature β , thus implying that for any β there is uniqueness of the infinite volume Gibbs measure μ_β , and this uniqueness is stable under small perturbations of the Hamiltonian. Their analysis is based on algebraic methods that do not apply to non factorizable systems as the TPM. We have provided a different method to derive the above results also for some non factorizable systems including the TPM. Restricting to the SPM and TPM we have also proved that the infinite volume Gibbs state μ_β is self-similar, in the sense that its marginal on any square sublattice (for SPM) or on any triangular sublattice of side a power of 2 (for TPM) can be identified with $\mu_{\beta'}$ for a suitable new inverse temperature β' . We have also proved that the critical correlation-decay lengthscale is given by $e^{\frac{\beta}{2}}$ and $e^{\frac{10-2}{15}\beta}$ for the SPM and the TPM, respectively.

We have also discussed the effect of boundary conditions in the Dobrushin–Shlosman sense in the TPM and SPM. To clarify, given finite subsets $V \subset \Lambda$ and a configuration τ , let us write $\mu_{\Lambda, V}^\tau$ for the marginal on V of the Gibbs measure on Λ with b.c. τ and inverse temperature β . Suppose that τ' is another boundary condition differing from τ at points $x \in X$ and denote by $B(x, \ell)$ the box with center x and radius ℓ , i.e. $B(x, \ell) = \{y : \|x - y\|_\infty \leq \ell\}$. Then, the distance in total variation of $\mu_{\Lambda, V}^\tau$ and $\mu_{\Lambda, V}^{\tau'}$ with $V = \Lambda \setminus \cup_{x \in X} B(x, \ell)$ is always $o(1)$ at low temperature if ℓ is larger e^β , while for $\ell \leq e^\beta$ one can exhibit examples where the total variation distance is not negligible. We shortly say that e^β is the critical lengthscale in the Dobrushin–Shlosman sense both in the TPM and SPM.

Another interesting lengthscale is the so-called *cavity length*, roughly the distance from the boundary at which the effect of the boundary condition on the bulk is negligible. Up to now, the analysis of this length scale remains an open problem.

As final part we have discussed some aspects of the Glauber dynamics. In particular, for the SPM we have proved that the infinite volume relaxation time (defined as the inverse of the spectral gap) is of Arrhenius type and we have explained some physical conjecture on the relaxation time of the TPM related to a hierarchical structure proposed in [5]. Further analysis of the dynamical aspects is a work in progress with A. Smith.

REFERENCES

- [1] R.L. Dobrushin, S.B Shlosman, *Constructive criterion for the uniqueness of Gibbs fields in Statistical physics and dynamical systems*, J. Fritz, A. Jaffe, D. Szasz (eds.), Basel, Birkhäuser (1985), 347–370.
- [2] D. Espriu, A. Prats, *Dynamics of the two-dimensional gonihedric spin model*, Phys. Rev. E **70** (2004) 046117.
- [3] J.P. Garrahan, *Glassiness through the emergence of effective dynamical constraints in interacting system*, J. Phys.: Condens. Matter **14** (2002) 1571–1579.
- [4] R. Fernández, J. Slawny, *Inequalities and Many Phase Transitions in Ferromagnetic Systems*, Commun. Math. Phys. **121** (1989) 91–120.
- [5] M.E.J. Newman, C. Moore, *Glassy dynamics and aging in an exactly solvable spin model*, Phys. Rev. E **60** (1999) 5068–5072.
- [6] S. Wolfram, *Statistical mechanics in cellular automata*, Rev. Mod. Phys. **55** (1983) 601–644.

Kinetically constrained models and bootstrap percolation: critical time and length scales

CRISTINA TONINELLI

(joint work with Fabio Martinelli)

In recent years, a great deal of progress has been made in understanding the behaviour of a class of monotone cellular automata, whose general definition has been given in [1]. Fix a finite collection of finite subsets of $\mathbb{Z}^d \setminus 0$, $\mathcal{U} = \{X_1, \dots, X_m\}$. \mathcal{U} is the *update family* of the process and each $X \in \mathcal{U}$ is an *update rule*. The \mathcal{U} -bootstrap percolation process on the d dimensional torus of linear size n , \mathbb{Z}_n^d , is then defined as follows. Given a set $A \subset \mathbb{Z}_n^d$ of initially infected sites, set $A_0 = A$, and define recursively for each $t \in \mathbb{N}$

$$A_{t+1} = A_t \cup \{x \in \mathbb{Z}_n^d : x + X_k \subset A_t \text{ for some } k \in (1, \dots, m)\}$$

In words, site x is infected at time $t + 1$ if the translate by x of at least one of the update rules is already entirely infected at time t , and infected sites remain infected forever. The set of sites that are eventually infected, $\cup_{t=0}^{\infty} A_t$, is called the \mathcal{U} -update closure of A and denoted by $[A]_{\mathcal{U}}$. This general class includes as specific examples the classical r -neighbour bootstrap percolation models (see [4] and references therein). In this case a site gets infected if at least r of its nearest

neighbours are infected, namely the update family is formed by all the r -subset of nearest neighbours of the origin.

The key issue is the global behavior starting from q -random initial conditions, namely when each site of \mathbb{Z}_n^d belongs independently with rate q to the initial set of infected sites. In particular one would like to know how large q should be in order that the closure typically covers the whole lattice. In [1–3] universality results for general \mathcal{U} -bootstrap percolation processes in dimension $d = 2$ have been established, yielding the behavior of the critical percolation threshold defined as

$$q_c(n; \mathcal{U}) = \inf\{q : \mathbb{P}_q([A]_{\mathcal{U}} = \mathbb{Z}_n^d) \geq 1/2\}$$

One can equivalently express these results in term of the critical length $L_c(q, \mathcal{U}) = \min\{n : q_c(n, \mathcal{U}) = q\}$. In turn, this length is naturally related to the infection time of the origin, $\tau(A, \mathcal{U}) := \min\{t \geq 0 : 0 \in A_t\}$. In particular, if A is q -random, and $\liminf_{n \rightarrow \infty} q_c(n, \mathcal{U}) = 0$, with high probability as $q \rightarrow 0$ it holds $\tau(A, \mathcal{U}) = L_c(q)$ [1].

Given a \mathcal{U} -bootstrap model one can consider the associated kinetically constrained model (KCM), a continuous time stochastic dynamics on \mathbb{Z}^d in which each vertex is resampled (independently) at rate one by tossing a q -coin if it could be infected in the next step by the \mathcal{U} -bootstrap model and it is not updated otherwise [6]. Since the constraint does not depend on the to-be-updated site, detailed balance holds w.r.t. the product measure μ which gives weight q to empty sites and $1 - q$ to occupied sites. Therefore μ is an invariant reversible measure for the process.

The basic issues concerning the long time behavior KCM are in general not trivial. In particular, due to the presence of constraints, there exist configurations which do not evolve under the dynamics, and relaxation to μ is not uniform on the initial configuration. Also, at variance with the cellular automata, these stochastic dynamics are not monotone: the presence of more zeros facilitates motion and can therefore also allow killing more zeros. Thus coupling and censoring arguments which have been developed for attractive dynamics (e.g. Glauber dynamics for Ising model) cannot be applied. The main interest of KCM is that for $q \rightarrow 0$ they reproduce some of the most striking features of the liquid/glass transition, a major and still largely open problem in condensed matter physics. In particular, they display an heterogeneous dynamics and anomalously long mixing times.

A key issue both from the physical and mathematical point of view is to determine the divergence of the time scales when $q \downarrow q_c = \liminf_{n \rightarrow \infty} q_c(n, \mathcal{U})$. A natural time scale is the mean of the random first time τ_0 at which the occupation variable at the origin is updated, $\mathbb{E}_\mu(\tau_0)$. Here the mean is over the process and the distribution μ for the initial configuration. Via a general argument based on the finite-speed of propagation, it is easy to establish (see [6]) that $\mathbb{E}_\mu(\tau_0)$ is lower bounded by the critical length of the corresponding cellular automata, L_c . Instead, there is not a direct connection with the cellular automata which allows to compute an upper bound on the time scales, and the best general upper bound is $\mathbb{E}_\mu(\tau_0) \leq \exp(cL_c^d)$ ([6]). Though this bound have been refined for special choices

of the constraints yielding in some cases the sharp behavior, the techniques are always ad hoc and valid only for very special choices of the constraints.

The main result that we present in this talk is a general toolbox which allows to obtain much tighter upper bound and hopefully identify the universality classes for the KCM critical scaling. In particular we apply our technique to the KCM with update rule corresponding to r -neighbour bootstrap percolation. This is a very popular KCM, known in physics literature as Friedrickson Andersen k -facilitated model. In this case the sharp scaling of L_c has been determined in a series of works (see [4] and references therein), leading to

$$L_c(q) = \exp_{(k-1)}\left(\frac{\lambda(d, k) + o(1)}{q^{1/(d-k+1)}}\right),$$

with $\lambda(d, k)$ an explicit constant and $\exp_{(r)}$ the r -times iterated exponential.

We prove that for $k = 2$ there exists $\alpha > 0$ such that

$$\mathbb{E}_\mu(\tau_0) = O\left(L_c(q)^{\log(1/q)^\alpha}\right)$$

and for $3 \leq k \leq d$ there exists $c > \lambda(d, k)$ such that

$$\mathbb{E}_\mu(\tau_0) \leq \exp_{(k-1)}(c/q^{1/(d-k+1)})$$

This, together with the lower bound $\mathbb{E}_\mu(\tau_0) \geq L_c$, establish a much tighter connection between $\mathbb{E}_\mu(\tau_0)$ and L_c than previous results. At this point one might think that $\mathbb{E}_\mu(\tau_0)$ is a function of the critical bootstrap length, perhaps scaling as power law. We explain that this is not the case and we discuss a class of the rules for which we prove that $L_c = 1/q^{\Theta(1)}$ and $\mathbb{E}_\mu(\tau_0) = q^{-\Theta(\log(1/q))} \gg L_c$.

REFERENCES

- [1] B.Bollobás, P.J. Smith and A.J. Uzzell, Monotone cellular automata in a random environment, *Combin. Probab. Computing*, 24 (2015), 687722
- [2] P. Balister, B.Bollobás, M.J. Przykucki and P.J. Smith, Subcritical neighbourhood family percolation models have non-trivial phase transitions, *Trans. Amer. Math. Soc.*, to appear
- [3] B.Bollobás, H. Duminil-Copin, R. Morris and P. Smith, Universality of two-dimensional critical cellular automata, preprint arXiv:1406.6680.
- [4] J. Balogh, B.Bollobás, and R. Morris The sharp threshold for bootstrap percolation in all dimensions, *Trans Am. Math. Soc.*, 364, 2667-2701, 2012.
- [5] F.Martinelli, C.Toninelli *in preparation*
- [6] N.Cancrini, F.Martinelli, C.Roberto, C.Toninelli Kinetically constrained spin models *Probability Theory and Related Fields* 140, p.459-504 (2008).

Random long time dynamics of the stochastic Kuramoto model

CHRISTOPHE POQUET

(joint work with Lorenzo Bertini, Giambattista Giacomin, Eric Luçon)

In this talk I have presented some recent progress in the study of the stochastic Kuramoto model [1], which is constituted of a population of N interacting rotators evolving according to the following system of stochastic differential equations:

$$d\varphi_j(t) = \delta\omega_j dt - \frac{K}{N} \sum_{i=1}^N \sin(\varphi_j(t) - \varphi_i(t)) dt + dB_j(t),$$

where $\varphi_j(t)$ is seen modulo 2π , $(B_j)_{1 \leq j \leq N}$ is a family of independent standard Brownian motions, and $(\omega)_{1 \leq i \leq N}$ is a family of i.i.d. random variables with distribution λ (called the disorder of the system).

On finite time intervals and in the limit of infinite population this model is described by a system of coupled PDEs [4, 7]: the empirical measure $\mu_{N,t} = \frac{1}{N} \sum_{j=1}^N \delta_{(\varphi_j(t), \omega_j)}$ converges to $p_t(\theta, \omega) d\theta \lambda(d\omega)$, where $p_t(\theta, \omega)$ satisfies

$$\partial_t p_t(\theta, \omega) = \frac{1}{2} \partial_\theta^2 p_t(\theta, \omega) - \partial_\theta \left[p_t(\theta, \omega) \left(\int_{\mathbb{R}} J(\theta - \theta') p(\theta', \omega') d\theta' \lambda(d\omega') + \omega \right) \right],$$

with $J(\theta) = -K \sin(\theta)$. Here $p_t(\theta, \omega)$ represents the limit distribution of the rotators having natural frequency ω .

When the interaction is strong enough and the disorder small enough, this system of PDEs admits a stable periodic solution of the form $p_t(\theta, \omega) = q(\theta - ct, \omega)$, where q is a synchronized profile [6]. In the case of symmetric disorder $c = 0$, the system admits a stable curve (in fact a circle) of stationary profiles.

For large but finite populations, the randomness brought by the thermal noise and the disorder is still present, and its effect on the system appear macroscopically on appropriate time scales. In the case of symmetric disorder, the finite-size fluctuations of the disorder appear at the time scale $\sqrt{N}t$: with probability converging to 1 the re-scaled empirical measure $\mu_{N, \sqrt{N}t}$ is close with high probability to $q(\theta - b_N t, \omega) d\theta \lambda(d\omega)$, where b_N depends on the fluctuation of the disorder [8]. At this time scale the finite-size fluctuations of the thermal noise play no role. On the other hand, in absence of disorder, these thermal fluctuations appear at the scale Nt : with probability converging to 1, $\mu_{N, Nt}$ is close to $q(\theta - \sigma W_N(t)) d\theta$, where $W_N(t)$ converges in distribution to a Brownian motion [2, 3].

These questions remain open for other graphs of interaction, in particular of Erdős-Rényi type: to my knowledge the existence of deterministic limit dynamics on bounded time intervals for edge density $p_N \ll \log N/N$ and interaction term appropriately re-scaled, and the study of the fluctuations for any edge density, have not been treated yet for Erdős-Rényi graphs of interaction [5].

REFERENCES

- [1] J.A Acebrón, L.L Bonilla, C.J Pérez Vicente, F. Ritort, R. Spigler, *The Kuramoto model: A simple paradigm for synchronization phenomena*, Reviews of modern physics **77** (2005), 137–185.
- [2] L. Bertini, G. Giacomin, C. Poquet, *Synchronization and random long time dynamics for mean-field plane rotators*, Probability Theory and Related Fields **160** (2014), 593–653.
- [3] R. Dahms, *Long time behavior of a spherical mean field model*, PhD thesis, Technische Universität Berlin, Fakultät II - Mathematik und Naturwissenschaften (2002).
- [4] P. Dai Pra, F. den Hollander, *McKean-Vlasov limit for interacting random processes in random media*, Journal of statistical physics **84** (1996), 735–772.
- [5] S. Delattre, G. Giacomin, E. Luon, *A Note on Dynamical Models on Random Graphs and Fokker-Planck Equations*, Journal of Statistical Physics **165** (2016), 785–798.
- [6] G. Giacomin, E. Luçon, C. Poquet, *Coherence stability and effect of random natural frequencies in populations of coupled oscillators*, Journal of Dynamics and Differential Equations **26** (2014), 333–367.
- [7] E. Luçon, *Quenched limits and fluctuations of the empirical measure for plane rotators in random media*, Electron. J. Probab **16** (2011), 792–829.
- [8] E Luçon, C. Poquet, *Long time dynamics and disorder-induced traveling waves in the stochastic Kuramoto model*, to appear in Annales de l'Institut Henri Poincaré (B) Probabilités et Statistiques

The Cutoff phenomenon for biased card shuffling and Adjacent transposition shuffle

HUBERT LACOIN

(joint work with Cyril Labbé)

We consider the biased card shuffling and the Asymmetric Simple Exclusion Process (ASEP) on the segment. We obtain the asymptotic of their mixing times, thus showing that these two continuous-time Markov chains display cutoff. Our analysis combines several ingredients including: a study of the hydrodynamic profile for ASEP, the use of monotonic eigenfunction, stochastic comparisons and concentration inequalities.

The ASEP can be defined as follows: k particles on a segment of length N jump independently with rate $p > 1/2$ to the right and $q = (1 - p)$ to the left. A restriction is added: each site can be occupied by at most one particle. The biased card shuffling which is a walk on the symmetric group which can be described as follows: for any n , the image of n and $n + 1$ are exchanged with rate p if the exchange puts them in increasing order and with rate q if not. The generator for this second chain is given by

$$\begin{aligned}
 \mathcal{L}_N f(\sigma) &:= \sum_{i=1}^{N-1} (p \mathbf{1}_{\{\sigma(i+1) < \sigma(i)\}} + q \mathbf{1}_{\{\sigma(i+1) > \sigma(i)\}}) [f(\sigma \circ \tau_i) - f(\sigma)] \\
 (1) \quad &= \sum_{i=1}^{N-1} p [f(\sigma^{i,+}) - f(\sigma)] + q [f(\sigma^{i,-}) - f(\sigma)].
 \end{aligned}$$

We let $\text{gap}_{N,k}$, gap_N , $T_{\text{mix}}^{N,k}(\varepsilon)$ and $T_{\text{mix}}^N(\varepsilon)$ the spectral gaps and total variation mixing times associated to these two chains. Our main result is the obtainment of sharp estimates for these mixing times which implies in particular that cutoff (abrupt convergence to equilibrium) holds.

For the biased card-shuffling:

Theorem 1. We have for every $p \in (1/2, 1]$ and $\varepsilon \in (0, 1)$

$$\lim_{N \rightarrow \infty} \frac{T_{\text{mix}}^N(\varepsilon)}{N} = \frac{2}{p-q}.$$

Moreover we have for every value of N and p

$$(2) \quad \text{gap}_N = (\sqrt{p} - \sqrt{q})^2 + 4\sqrt{pq} \sin\left(\frac{\pi}{2N}\right)^2.$$

And for the ASEP when the particle density tends to α :

Theorem 2. We have for every $p \in (1/2, 1]$, every $\alpha \in [0, 1]$ and every $\varepsilon \in (0, 1)$

$$\lim_{\substack{N \rightarrow \infty \\ k/N \rightarrow \alpha}} \frac{T_{\text{mix}}^{N,k}(\varepsilon)}{N} = \frac{(\sqrt{\alpha} + \sqrt{1-\alpha})^2}{p-q}.$$

Moreover for every N , every $k \in \{1, \dots, N-1\}$ and every p we have

$$(3) \quad \text{gap}_{N,k} = (\sqrt{p} - \sqrt{q})^2 + 4\sqrt{pq} \sin\left(\frac{\pi}{2N}\right)^2.$$

REFERENCES

- [1] H. Lacoïn and G. Labbé *Cutoff phenomenon for the asymmetric simple exclusion process and the biased card shuffling* (preprint).

Non-equilibrium fluctuations of interacting particle systems

MILTON JARA

(joint work with Otávio Menezes)

Let us consider the following simple example of interacting particle systems. Let $n \in \mathbb{N}$ be a scaling parameter. Let $\Lambda_n = \mathbb{Z}^d/n\mathbb{Z}^d$ denote the discrete torus with n points. We think about Λ_n as a discrete approximation of the continuous torus $\mathbb{T}^d = \mathbb{R}^d/\mathbb{Z}^d$. Let $\Omega_n = \{0, 1\}^{\Lambda_n}$ be the state space of a continuous-time Markov chain that we will describe below. We denote by $\eta = (\eta_x)_{x \in \Lambda_n}$ the elements of Ω_n . For $x, y \in \Lambda_n$ and $\eta \in \Omega_n$, let $\eta^{x,y} \in \Omega_n$ be the configuration obtained from η by exchanging its occupation variables at x and y :

$$\eta_z^{x,y} = \begin{cases} \eta_y & ; z = x \\ \eta_x & ; z = y \\ \eta_z & ; z \neq x, y. \end{cases}$$

For $x \in \Lambda_n$ and $\eta \in \Omega_n$, let $\eta^x \in \Omega_n$ denote the configuration obtained from η by swapping its occupation variable at x :

$$\eta_z^x = \begin{cases} 1 - \eta_x & ; z = x \\ \eta_z & ; z \neq x. \end{cases}$$

We say that $x, y \in \Lambda_n$ are *neighbors* if $\sum_{i=1}^d |y_i - x_i| = 1$. We denote this by $x \sim y$. For $f : \Omega_n \rightarrow \mathbb{R}$, let $L_n f : \Omega_n \rightarrow \mathbb{R}$ be given by

$$L_n f(\eta) = n^2 \sum_{\substack{x, y \in \Lambda_n \\ x \sim y}} (f(\eta^{x,y}) - f(\eta)) + \sum_{x \in \Lambda_n} c_x(\eta)(f(\eta^x) - f(\eta)),$$

where $c_x(\eta) = c_x^+(\eta)(1 - \eta_x) + c_x^-(\eta)\eta_x$ and

$$c_x^+(\eta) = 1, \quad c_x^-(\eta) = 1 + \frac{b}{2d} \sum_{y \sim x} \eta_y.$$

We consider the family of Markov chains $\{\eta_t^n; t \geq 0\}_{n \in \mathbb{N}}$ generated by the operators L_n . We call this model the *reaction-diffusion* model with reaction rate c_x . It has been proved that for this model, the density of particles has a hydrodynamic limit given by solutions of the equation

$$\partial_t u = \Delta u + F(u),$$

where

$$F(\rho) = \int c_x(\eta)(1 - 2\eta_x) d\mu_\rho$$

and μ_ρ is the Bernoulli measure of density ρ . Notice that $F(\rho) = 0$ for $\rho = (1 + \sqrt{1 + b})^{-1}$. From now on, we fix this value of ρ and we take η_0^n with law μ_ρ . We denote by \mathbb{P}_n the law of this process and by \mathbb{E}_n the expectation with respect to \mathbb{P}_n . We define the *density fluctuation field* as the distribution-valued process $\{X_t^n; t \geq 0\}$ given by

$$X_t^n(f) = \frac{1}{n^{d/2}} \sum_{x \in \Lambda_n} (\eta_x^n(t) - \rho) f\left(\frac{x}{n}\right)$$

We prove that in $d = 1$, the density fluctuation field $\{X_t^n; t \geq 0\}$ converges in law to the solution of the stochastic PDE

$$\partial_t X = \Delta X + F'(\rho)X + \sqrt{2\rho(1 - \rho)} \nabla \dot{W}_t^1 + \sqrt{G(\rho)} \dot{W}_t^2,$$

where $G(\rho) = \int c_x d\mu_\rho$. In dimension $d = 2$, we can prove convergence of finite-dimensional distributions. The proof is based on a novel technique, which involves an *entropy estimate* that controls the distance of the law of the process $\eta^n(t)$ to the measure μ_ρ . More specifically, let g_t^n denote the density of the law of $\eta^n(t)$ with respect to μ_ρ :

$$\mathbb{E}_n[f(\eta^n(t))] = \int f(\eta) g_t^n(\eta) d\mu_\rho$$

for any $f : \Omega_n \rightarrow \mathbb{R}$. Let $H_n(t)$ denote the entropy of g_t^n :

$$H_n(t) = \int g_t^n \log g_t^n d\mu_\rho.$$

Then, we can prove that there exists a finite constant C , independent of t and n , such that

$$\frac{d}{dt}H_n(t) \leq \begin{cases} C & ; d = 1 \\ C \log n & ; d = 2 \\ Cn^{d-2} & ; d \geq 3. \end{cases}$$

This entropy bound allows to deal with the density fluctuation field adapting techniques from the case on which the measure μ_ρ is stationary with respect to the evolution of the process $\eta^n(t)$.

Our proofs of both the entropy estimates and the convergence of the density fluctuation field are very general and can be adapted for a large class of interacting particle systems with diffusive behaviour.

Invariant measures for the Box Ball System in \mathbb{Z}

PABLO A. FERRARI

(joint work with Chi Nguyen, Leonardo Rolla, Minmin Wang)

There is a *box* at each integer $x \in \mathbb{Z}$ which may contain one *ball* or be empty. Denote $\eta \in \{0, 1\}^{\mathbb{Z}}$ a ball configuration, with $\eta(x) := 1$ if there is a ball at x , else $\eta(x) := 0$. Take a configuration with a finite number of balls and let an empty *carrier* start from the left of the leftmost ball and visit the boxes in increasing order. At each box the carrier picks a ball if there is any and if the box is empty and the carrier has at least one ball, he deposits one ball in the box. Let $T\eta$ be the configuration obtained after the carrier visited all boxes. For instance:

$$\begin{array}{cccccccccccccccccccc} 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \eta \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & T\eta \\ 0 & 0 & 1 & 0 & 1 & 2 & 1 & 0 & 0 & 0 & 1 & 2 & 1 & 2 & 1 & 0 & 0 & 0 & 0 & \text{carrier} \end{array}$$

This cellular automaton called *Box-Ball-System (BBS)* was introduced by Takahashi and Satsuma [3], as a discrete system showing solitons, a phenomenon present in the Korteweg & de Vries (KdV) differential equation for $u(r, t) \in \mathbb{R}^+$, $r \in \mathbb{R}$, $t \in \mathbb{R}^+$ given by

$$(1) \quad \dot{u} = u''' + uu'$$

For the relation between BBS and KdV see Tokihiro et al [4], Takahashi and Matsukidaira [2] and Kato, Satoshi and Zuk [1].

We start the BBS with infinitely many balls. The set of configurations with density λ is defined by

$$\mathcal{X}_\lambda = \left\{ \eta \in \{0, 1\}^{\mathbb{Z}} : \lim_{y \rightarrow \pm\infty} \left| \frac{1}{y} \sum_{x=0}^y \eta(x) \right| = \lambda \right\}$$

We show that if $\lambda \in (0, \frac{1}{2})$ and $\eta \in \mathcal{X}_\lambda$, then $T\eta$ is well defined as limit of $T\eta_n$ for a sequence of finite ball configurations $\eta_n \nearrow \eta$ and $T\eta \in \mathcal{X}_\lambda$. Let $\mathcal{X} := \cup_{\lambda \in (0, \frac{1}{2})} \mathcal{X}_\lambda$.

at the *effective distance*. In case η has no k -podes, $M_k\eta$ contains only records. For instance,

..... 11101000 . 10 x . 10 ... 111000 . 10	η
..... 10 ... 10 x . 10 10	$M_1\eta$
..... x	$M_2\eta$
..... 111000 . x 111000	$M_3\eta$

The x indicates the position of the record at the origin.

Theorem 1. Let μ be a shift-stationary, mixing, invariant measure on \mathcal{X} with density $\lambda < 1/4$. Let η be a random ball configuration with distribution $\hat{\mu}$, the measure μ conditioned to have a record at the origin. Then

($M_k\eta : k \geq 1$) is a family of independent random configurations.

Let ρ_k be the expected number of k -podes between the record at the origin and the nearest record to the right of it, under $\hat{\mu}$. The proof of Theorem 1 is based in the following result.

Theorem 2. Let μ be a shift-stationary, mixing, invariant measure on \mathcal{X} and η with law $\hat{\mu}$. The k -component of $\hat{T}^t\eta$ is the k -component of a translation of η : there are functions $y(k, t, \eta)$ such that

$$M_k\hat{T}^t\eta = M_k\theta_{-y(k,t,\eta)}\eta$$

Assume $\sum_{m \geq 1} m^2\rho_m < \infty$. The translations have finite limits: if $\rho_k > 0$, then

$$\lim_{t \rightarrow \infty} \frac{y(k, t, \eta)}{t} =: v_k < \infty, \quad k \geq 1,$$

which satisfy the system of equations

$$(2) \quad v_k = k + \sum_{m > k} 2(m - k)(v_m - v_k)\rho_m, \quad k \geq 1.$$

For $m > k$, there exist constants $c(m, k)$ such that

$$c(m, k) \leq v_m - v_k \leq m - k.$$

If $\lambda < 1/4$ then $c(m, k) > 0$ for all $m > k$. Furthermore,

$$v_k \leq k + 2 \sum_{m \geq 1} m^2\rho_m.$$

Final remark. We show that the system starting with a mixing invariant measure may contain infinitely many rigid solitons described by the k -components of η , for $k \geq 1$. The k -components are independent but the soliton-soliton interactions produce temporary deformations of distances and modify average speeds as reflected by equations (2). The proof of independence in Theorem 1 requires $c(m, k) > 0$ which we prove in the regime $\lambda < 1/4$.

REFERENCES

- [1] KATO, T., TSUJIMOTO, S., AND ZUK, A. Spectral analysis of transition operators, automata groups and translation in BBS. *Communications in Mathematical Physics* (2016), 1–25.
- [2] TAKAHASHI, D., AND MATSUKIDAIRA, J. Box and ball system with a carrier and ultradiscrete modified kdv equation. *Journal of Physics A: Mathematical and General* 30, 21 (1997), L733.
- [3] TAKAHASHI, D., AND SATSUMA, J. A soliton cellular automaton. *Journal of The Physical Society of Japan* 59, 10 (1990), 3514–3519.
- [4] TOKIHIRO, T., TAKAHASHI, D., MATSUKIDAIRA, J., AND SATSUMA, J. From soliton equations to integrable cellular automata through a limiting procedure. *Physical Review Letters* 76, 18 (1996), 3247.

Participants

Amol Aggarwal

Department of Mathematics
Harvard University
Science Center
One Oxford Street
Cambridge MA 02138-2901
UNITED STATES

Dr. Luca Avena

Mathematisch Instituut
Universiteit Leiden
Postbus 9512
2300 RA Leiden
NETHERLANDS

Prof. Dr. Christophe Bahadoran

LM UMR 6620
Université Blaise Pascal
Clermont Ferrand II
24, Avenue des Landais
63177 Aubière Cedex
FRANCE

Dr. Márton Balázs

School of Mathematics
University of Bristol
University Walk
Bristol BS8 1TW
UNITED KINGDOM

Prof. Dr. Cédric Bernardin

Laboratoire J.-A. Dieudonné
UMR CNRS 7351
Université de Nice Sophia-Antipolis
Parc Valrose
06108 Nice Cedex 02
FRANCE

Dr. Oriane Blondel

Institut Camille Jordan
Université Claude Bernard Lyon I
43 blvd. du 11 novembre 1918
69622 Villeurbanne Cedex
FRANCE

Prof. Dr. Thierry Bodineau

Centre de Mathématiques Appliquées
École Polytechnique
Route de Saclay
91128 Palaiseau Cedex
FRANCE

Prof. Dr. Erwin Bolthausen

Institut für Mathematik
Universität Zürich
Winterthurerstrasse 190
8057 Zürich
SWITZERLAND

Prof. Dr. Pietro Caputo

Dipartimento di Matematica
Universita Roma III
Largo S. Leonardo Murialdo 1
00146 Roma
ITALY

**Prof. Dr. Ana Patrícia Carvalho
Goncalves**

Instituto Superior Técnico
Universidade de Lisboa
Avenida Rovisco Pais, 1
Lisboa 1049-001
PORTUGAL

Dr. Paul Chleboun

Mathematics Institute
University of Warwick
Gibbet Hill Road
Coventry CV4 7AL
UNITED KINGDOM

Prof. Dr. Francis M. Comets

U.F.R. de Mathématiques
Université Paris VII
Case 7012
75205 Paris Cedex 13
FRANCE

Prof. Dr. Bernard Derrida

Département de Physique, LP 5
École Normale Supérieure
24, rue Lhomond
75230 Paris Cedex 05
FRANCE

Prof. Dr. Jean Dominique**Deuschel**

Institut für Mathematik
Sekt. MA 7-4
Technische Universität Berlin
Straße des 17. Juni 136
10623 Berlin
GERMANY

Dr. Clément Erignoux

Instituto Nacional de Matemática
Pura e Aplicada
Jardim Botânico
Estrada Dona Castorina, 110
Rio de Janeiro, RJ 22460-320
BRAZIL

Prof. Dr. Alessandra Faggionato

Dipartimento di Matematica
Università di Roma "La Sapienza"
Istituto "Guido Castelnuovo"
Piazzale Aldo Moro, 2
00185 Roma
ITALY

Prof. Dr. Pablo A. Ferrari

Departamento de Matemática - FCEN
Universidad de Buenos Aires
Ciudad Universitaria
Pabellon 1
Buenos Aires C 1428 EGA
ARGENTINA

Prof. Dr. Patrik L. Ferrari

Institut für Angewandte Mathematik
Universität Bonn
Endenicher Allee 60
53115 Bonn
GERMANY

Prof. Dr. Tadahisa Funaki

Graduate School of Mathematical
Sciences
University of Tokyo
3-8-1 Komaba, Meguro-ku
Tokyo 153-8914
JAPAN

Dr. Shirshendu Ganguly

Department of Statistics
University of California, Berkeley
367 Evans Hall
Berkeley CA 94720-3860
UNITED STATES

Dr. Giambattista Giacomini

U.F.R. de Mathématiques
Université Paris Diderot
Bat. Sophie Germain
5, rue Thomas Mann
75205 Paris Cedex 13
FRANCE

Dr. Stefan Grosskinsky

Mathematics Institute
University of Warwick
Zeeman Building
Coventry CV4 7AL
UNITED KINGDOM

Prof. Dr. Massimiliano Gubinelli

Institut für Angewandte Mathematik
Universität Bonn
Endenicher Allee 60
53115 Bonn
GERMANY

Prof. Dr. Dmitri Ioffe

Faculty of Industrial Engineering
and Management, TECHNION
Israel Institute of Technology
Haifa 32000
ISRAEL

Dr. Milton Jara

Instituto de Matematica Pura e Aplicada
I M P A
Jardim Botânico
Estrada Dona Castorina, 110
Rio de Janeiro, RJ 22460-320
BRAZIL

Prof. Dr. Gady Kozma

Faculty of Mathematics and Computer
Science
The Weizmann Institute of Science
P.O. Box 26
Rehovot 76100
ISRAEL

Dr. Hubert Lacoïn

Instituto de Matematica Pura e Aplicada
I M P A
Jardim Botânico
Estrada Dona Castorina, 110
Rio de Janeiro, RJ 22460-320
BRAZIL

Prof. Dr. Claudio Landim

Instituto de Matematica Pura e Aplicada
I M P A
Jardim Botânico
Estrada Dona Castorina, 110
Rio de Janeiro, RJ 22460-320
BRAZIL

Dr. Benoit Laslier

UFR de Mathématiques
Université Paris Diderot (Paris VII)
Bâtiment Sophie Germain
8 Place Aurélie Nemours
75205 Paris Cedex 13
FRANCE

Dr. Eyal Lubetzky

Courant Institute of Mathematical
Sciences
New York University
251, Mercer Street
New York, NY 10012-1110
UNITED STATES

Dr. Fabio Martinelli

Dipartimento di Matematica
Universita Roma III
Largo S. Leonardo Murialdo, 1
00146 Roma
ITALY

Otávio Menezes

Instituto de Matematica Pura e Aplicada
I M P A
Jardim Botânico
Estrada Dona Castorina, 110
Rio de Janeiro, RJ 22460-320
BRAZIL

Dr. Jean-Christophe Mourrat

UMPA - ENS Lyon
46, Allée d'Italie
69007 Lyon Cedex
FRANCE

Peter Nejjar

Institute of Science and
Technology Austria (IST Austria)
Am Campus 1
3400 Klosterneuburg
AUSTRIA

Prof. Dr. Stefano Olla

CEREMADE
Université Paris Dauphine
Place du Marechal de Lattre de
Tassigny
75775 Paris Cedex 16
FRANCE

Dr. Christophe Poquet

Institut Camille Jordan
Université Lyon I
Bat. Braconnier
21, Avenue Claude Bernard
69622 Villeurbanne Cedex
FRANCE

Balázs Ráth

Institute of Mathematics
Budapest University of Technology
and Economics
1. Egy Jozsef U., Bldg. H
1111 Budapest
HUNGARY

Dr. Julien Reygner

CERMICS - ENPC
Cite Descartes, Champs-sur-Marne
6 et 8 avenue Blaise Pascal
77455 Marne-la-Vallée Cedex 2
FRANCE

Prof. Dr. Ellen Saada

C.N.R.S.
Laboratoire MAP 5
Université Paris Descartes
45, rue des Saints-Peres
75270 Paris Cedex 06
FRANCE

Dr. Makiko Sasada

Graduate School of Mathematical
Sciences
University of Tokyo
3-8-1 Komaba, Meguro-ku
Tokyo 153-8914
JAPAN

Prof. Dr. Timo Seppäläinen

Department of Mathematics
University of Wisconsin-Madison
480 Lincoln Drive
Madison, WI 53706-1388
UNITED STATES

Marielle Simon

INRIA Lille-Nord Europe
Parc Scientifique de la Haute Borne
Bat. A
40, Avenue Halley
59650 Villeneuve-d'Ascq
FRANCE

Dr. Sergio Simonella

Zentrum Mathematik
Technische Universität München
Boltzmannstrasse 3
85748 Garching bei München
GERMANY

Prof. Dr. Herbert Spohn

Zentrum Mathematik
Technische Universität München
Boltzmannstrasse 3
85748 Garching bei München
GERMANY

Prof. Dr. Pierre Tarrès

St. Hugh's College
St. Margaret's Road
Oxford OX2 6LE
UNITED KINGDOM

Prof. Dr. Cristina Toninelli

UFR de Mathématiques
Université Denis Diderot (Paris (VII))
5, rue Thomas Mann
75013 Paris
FRANCE

Dr. Fabio Toninelli

Institut Camille Jordan
Université Claude Bernard Lyon I
43, Bd. du 11 Novembre 1918
69622 Villeurbanne Cedex
FRANCE

Prof. Dr. Balint Toth

Department of Mathematics
University of Bristol
Bristol BS8 1TW
UNITED KINGDOM

Dr. Benedek Valko

Department of Mathematics
University of Wisconsin-Madison
480 Lincoln Drive
Madison, WI 53706-1388
UNITED STATES

Prof. Dr. Yvan Velenik

Département de Mathématiques
Université de Genève
Case Postale 64
2-4 rue du Lievre
1211 Genève 4
SWITZERLAND

Dr. Bálint Vető

Institute of Mathematics
Budapest University of Technology
and Economics
1. Egrý József U., Bldg. H
1111 Budapest
HUNGARY

Prof. Dr. Wendelin Werner

Departement Mathematik
ETH-Zentrum
Rämistrasse 101
8092 Zürich
SWITZERLAND

Prof. Dr. Ofer Zeitouni

Faculty of Mathematics
Weizmann Institute of Science
POB 26
Rehovot 76100
ISRAEL