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Mini-Workshop: Max Dehn: his Life, Work, and Influence

Organised by
David Peifer, Asheville
Volker Remmert, Wuppertal
David E. Rowe, Mainz
Marjorie Senechal, Northampton

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Abstract. This mini-workshop is part of a long-term project that aims to produce a book documenting Max Dehn's singular life and career. The meeting brought together scholars with various kinds of expertise, several of whom gave talks on topics for this book. During the week a number of new ideas were discussed and a plan developed for organizing the work. A proposal for the volume is now in preparation and will be submitted to one or more publishers during the summer of 2017.

Mathematics Subject Classification (2010): 01A55, 01A60, 01A70.

Introduction by the Organisers

This mini-workshop on Max Dehn was a multi-disciplinary event that brought together mathematicians and cultural historians to plan a book documenting Max Dehn's singular life and career. This long-term project requires the expertise and insights of a broad array of authors. The four organisers planned the mini-workshop during a one-week RIP meeting at MFO the year before.

Max Dehn's name is known to mathematicians today mostly as an adjective (Dehn surgery, Dehn invariants, etc). Beyond that he is also remembered as the first mathematician to solve one of Hilbert's famous problems (the third) as well as for pioneering work in the new field of combinatorial topology. A number of Dehn's contributions to foundations of geometry and topology were discussed at the meeting, partly drawing on drafts of chapters contributed by John Stillwell and Stefan Müller-Stach, who unfortunately were unable to attend. Cameron Gordon offered a brief talk on the problematic status of Dehn's Lemma, a topic that was discussed further by examining the letters Dehn and Helmuth Kneser exchanged during 1929, when both came to realize the serious difficulties that needed to be overcome to prove the lemma.

But Dehn was far more than just an eminent mathematician, and his influence extended well beyond research mathematics. He was also a remarkable scholar and teacher. He was the leader of the famous decade-long seminar on the history of mathematics at the University of Frankfurt. He was revered by his students, both in Germany before World War II and afterward in the United States, and he was remembered by many others as a man of unusually broad interests.

Indeed, in the last eight years of his life (1945 - 1952) Dehn taught mathematics, philosophy, Greek, and Italian at Black Mountain College (a unique if short-lived experiment in higher education in the mountains near Asheville, North Carolina). Black Mountain College focussed on the arts and crafts, and is celebrated today as a catalyst of 20th century art. Dehn's influence on those who taught and studied there will be another focus of our book. In addition to teaching elementary mathematics and projective geometry, and tutoring the few advanced mathematics students at BMC, Dehn taught a very popular course on Geometry for Artists and worked in close association with the painter Josef Albers and the weaver Anni Albers, with whom he overlapped there. His influence on both Albers was the theme of a talk by Brenda Danilowitz and Philip Ording. We were delighted to have an eye-witness from those years participate in this mini-workshop: Trueman MacHenry (York University, Toronto), who studied advanced mathematics with Dehn at BMC, offered his personal reflections on that period of his life.

To weave this all together, this project will go well beyond Dehn's accomplishments as a research mathematician and teacher by addressing his wider interests as a naturalist, artist, and thinker. We hope to convey the qualities that made him such an appealing figure to others.

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Max Dehn (1878-1952)

Volker R. Remmert

The aim of the talk was to give a rough overview of Dehn's biography.

1896-1900	Student in Freiburg and Göttingen
1900	PhD (Göttingen, D. Hilbert: *Die Legendreschen Sätze über die Winkelsumme im Dreieck*, in: *Math. Ann.* 53(1900), 404-439)
1900/01	Assistent in Karlsruhe (F. Schur)
1901	Habilitation (Münster, W. Killing: Hilbert's 3d Problem: *Über den Rauminhalt*, in: *Math. Ann.* 55(1901), 465-478)
1901-1911	Privatdozent in Münster
1911-1913	Extraordinarius in Kiel
1913-1921	Ordinarius TH Breslau
1915-1918	Service in WW I
1921-1935	Ordinarius in Frankfurt/Main (dismissed on the basis of the Nazi racial laws)
1938/1-4	Kent/England (boarding school)
1939	Emigration (Scandinavia)
1939/40	Visiting professor Trondheim/Norway (for Viggo Brun)
1940	USA (via Finland, Soviet Union, Japan)
1940/41	ass. Prof. of mathematics and philosophy, University of Idaho
1942/43	vis. Prof. of mathematics, Illinois Institute of Technology, Chicago
1943/44	Tutor, St. John's College, Annapolis, Maryland
1945-1952	Prof. of mathematics and philosophy, Black Mountain College

Lecturing at the University of Wisconsin, Madison 1946/47 (winter), 1948/49 (winter), and Notre Dame 1949 (summer)

1949	Dehn considers trip to India in 1950 (Levi)

Dehn had planned to return to Germany, albeit probably not on a permanent basis, after his retirement from Black Mountain College. In particular he intended to go to Frankfurt in September 1952 as visiting professor to take up the tradition of the famous *Mathematisch-historisches Seminar* (topic: *Mathematik in der Renaissance*). He also had been invited to Göttingen as visiting professor where he was to teach a course on the history of mathematics in February 1953 (topic: *Geschichtliche Darstellung der bedeutenden Resultate, Methoden und Begriffe in der Mathematik*).

Special attention was paid to Dehn's views on the teaching of mathematics and the role the history of mathematics can play in it ($\rightarrow$ *Mathematisch-historisches Seminar*). An important source for this is the transcript of Dehn's talk *Probleme des mathematischen Hochschulunterrichts*, Bad Nauheim, Sept. 23, 1932 (*JBer DMV* 43(1934), Abt. 2, 71-78, p. 72). His approach to the teaching of mathematics is also reflected in his work as an editor in the editions of

- Moritz Pasch: *Vorlesungen über neuere Geometrie*, Berlin: Springer Verlag 1926 (Grundlehren 23), with an appendix of 80 pages by Dehn: *Die Grundlegung der Geometrie in historischer Entwicklung*,
- Arthur Schoenflies: *Einführung in die analytische Geometrie der Ebene und des Raumes*, Berlin: Springer Verlag 1931 (Grundlehren 21), revised by Dehn who also added appendices.

Dehn's PhD students

1912: Hugo Gieseking (1887-1915/WW I): *Analytische Untersuchungen über topologische Gruppen* (Killing/Münster)
1913: Jakob Nielsen: *Kurvennetze auf Flächen* (Kiel)
1913: Herbert Fuß: *Modulsysteme und höhere komplexe kommutative Zahlsysteme* (Kiel)
1923: Friedrich Wilhelm Schwan: *Extensive Größe, Raum und Zahl* (Frankfurt)
1923: Paul Kuhn: *Über die Gestalt der Integralkurven einer gewöhnlichen Differentialgleichung erster Ordnung in der Umgebung gewisser singulärer Punkte* (Frankfurt)
1926: Max Frommer (1904-1993): *Die Integralkurven einer gewöhnlichen Differentialgleichung 1. Ordnung in der Umgebung rationaler Unbestimmtheitsstellen* (Frankfurt)
1929: Wilhelm Magnus: *Über unendliche diskontinuierliche Gruppen mit einer definierenden Relation* (Frankfurt)
1929: Ott-Heinrich Keller: *Über die lückenlose Erfüllung des Raumes mit Würfeln* (Frankfurt)
1930: Ruth Moufang: *Zur Struktur der projektiven Geometrie der Ebene* (Frankfurt)
1936: Walter Wagner: *Über die Grundlagen der Geometrie und allgemeine Zahlsysteme* (Frankfurt)
1949: Joseph H. Engel: *Some contributions to the solution of the word problem for groups (Canonical Forms in Hypoabelian Groups)* (Madison)

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Max Dehn and Otto Blumenthal: two leading members of the Hilbert School

David E. Rowe

Max Dehn was one of Hilbert's earliest and most important students, so he has often been considered as a representative figure within the "Hilbert school." Clearly Dehn's early work on foundations of geometry was closely tied to Hilbert's work in that field, and his resolution of the third of Hilbert's 23 Paris problems made his name widely known in mathematical circles. After taking his doctorate in 1899, Dehn corresponded fairly regularly with Hilbert up until 1913, when he became a full professor at the Technische Hochschule in Breslau. After then they apparently no longer wrote each other at all, no doubt because their respective mathematical interests had grown very far apart. By 1910 Hilbert was largely focused on integral equations and related problems in mathematical physics, whereas Dehn's early works in topology and group theory appeared around this same time. As for Dehn's earlier career, we might want to re-examine just how deeply he was influenced by Hilbert's approach to geometry or mathematics in general; to what degree was his evolving style as a geometer and topologist influenced by his exposure to Hilbert's work as a student? Rather than pursuing this question, though, my talk took up a related issue, namely to assess Dehn's place in the Göttingen milieu he experienced at that time. For lack of direct sources relating to Dehn himself, I tried to illuminate his somewhat unusual place in Hilbert's school by looking at the case of another student, Otto Blumenthal.

Blumenthal and Dehn overlapped in Göttingen from 1897 to 1899, but afterward they probably had relatively few interactions. These mainly involved Mathematische Annalen, the journal in which Dehn published nearly all his more important mathematical papers. Blumenthal was its managing editor, and so knew these papers quite well, In an early letter from 1904 to Hilbert, Blumenthal refers to a course on foundations of geometry he was teaching in Marburg, noting that he hoped to get to present Dehn's work on the dissection of polyhedra, i.e. his solution of Hilbert's third Paris problem. Blumenthal's letters to Hilbert and his wife stand in sharp contrast to those written by Dehn to his former mentor. The former are filled with all kinds of news about mutual acquaintances and recent personal experiences, whereas Dehn's letters are almost exclusively restricted to mathematical concerns. Of course Blumenthal reported a great deal about mathematical matters as well, but Hilbert preferred that he begin with personal remarks, especially in the many letters that Blumenthal addressed to both him and his wife (throughout the years he addressed them as "Liebe gnädige Frau! lieber Herr Professor!"). Not surprisingly, Blumenthal's tone shifted markedly in letters written at the same time to his friend Karl Schwarzschild, a Dutzfreund who was three years older and, like him, grew up in Frankfurt am Main.

Hilbert certainly regarded Dehn as one of his brightest upcoming talents, but it might well be significant that Dehn chose to leave Göttingen almost immediately after completing his Habilitation there in 1900. After one year in Karlsruhe, he joined the faculty in Münster in 1901 as a Privatdozent. He remained in Münster for another ten years before assuming an associate professorship in Kiel. In the meantime, Blumenthal became a full professor at the TH Aachen already in 1905, just at the time he took over the position of managing editor of Mathematische Annalen. Professionally this proved to be a strong impediment for his professional career, a situation that became more and more obvious to the Annalen's most active board member, L.E.J. Brouwer.

When Ludwig Bieberbach left Frankfurt in 1920 to assume a chair in Berlin, both Blumenthal and Dehn were considered as potential candidates for the position. Blumenthal received extremely strong support from Brouwer, who wrote to his friend Arthur Schoenflies, a key figure in the negotiations. Correspondence between Schoenflies and Brouwer reveals that other candidates were actually preferred, in particular Georg Polya and Leon Lichtenstein, the editor of Mathematische Zeitschrift. The Frankfurt faculty in the end settled on a list headed by Wilhelm Wirtinger, with Dehn and Johann Radon both sharing the second slot. Schoenflies hoped to attain someone with ambition to build a school in Frankfurt. When Brouwer insisted that Blumenthal had shown talents in this direction in Aachen, Schoenflies assured him that he would be a strong candidate when his own position came open the following year. This, however, was not to be.

In all likelihood the Göttingen mathematicians Edmund Landau and Richard Courant played a decisive role in promoting the candidacy of Carl Ludwig Siegel, who was chosen to fill the vacancy in Frankfurt in 1922. The new trio there – Dehn, Siegel, and Hilbert's former assistant Ernst Hellinger – harmonized exceedingly well, creating a stimulating environment for teaching. Dehn emerged as the leader of the Frankfurt historical seminar, a unique institutional devoted to reading older texts in their original languages. Unlike Göttingen, with its highly competitive atmosphere and Courant's new yellow series for spreading older and newer research, Dehn and his colleagues in Frankfurt were content to cultivate their own local culture in relative isolation. If Göttingen symbolized the transition to a modern research community, Frankfurt can be seen as a center that consciously strove to develop a very different kind of culture, one in which the urgency to churn out new publications played almost no role at all.

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Dehn's Early Mathematics

Jeremy J. Gray, John McCleary

In the late 19th century mathematicians sought new foundations for geometry in light of the discovery of non-Euclidean geometries and the development of differential geometry which brought analytic methods, together with the anxiety about the foundations of analysis, to bear on geometric problems. Among the most influential efforts is the treatise of Pasch [9] that attempts to found geometry on projective means that are considered by the author more natural and scientific. In the 1890's Hilbert offered a series of lectures on the foundations of geometry [8], culminating in the publication of his *Grundlagen der Geometrie* in 1899. Dehn was a student in Göttingen from 1896 under Hilbert, and his earliest work concerns foundational issues in geometry.

It is well-known that Dehn's first published work [2], essentially his Ph.D thesis, is his discovery of non-Archimedean plane geometries in the spirit of Hilbert's *Grundlagen*. Gray described how Dehn adapted earlier work of Legendre on non-Euclidean geometry, and how he rewrote the methods Hilbert had used to describe a new plane geometry based on a novel concept of pseudo-congruence. Going further, Dehn introduced a new non-Archimedean field in which Hilbert-style coordinate methods could be used to introduce and develop two new geometries, which he called non-Legendrean and semi-Euclidean.

The Archimedean character of a geometry—whether it satisfies Hilbert's Axiom of Continuity—plays an important role in Dehn's next papers. These treat Dehn's solution to Hilbert's Third Problem [7] which asks if it is possible to present two tetrahedra on equal bases and in equal altitudes *which can in no way be split up into congruent tetrahedra, and which cannot be combined with congruent tetrahedra to form two polyhedra which themselves could be split up into congruent tetrahedra.* These relations between tetrahedra are currently termed *scissors congruent*—when two polyhedra are related by splitting the first into a finite collection of tetrahedra that can be reassembled to give the second—and *stable scissors congruent* when two polyhedra can be combined with a finite number of congruent tetrahedra to form two polyhedra which themselves are scissors congruent. The same properties can be defined for polygons in the plane.

McCleary described the theory of area for polygons in the plane developed by Euclid in Book I of *the Elements* which is based on scissors congruence and stable scissors congruence. It was Gauss, in letters to Gerling (pages 241 and 244 of [5]), who asked if the successes of Euclid's theory of area could be made to work for volumes of polyhedra in space. Hilbert's Third Problem expresses doubt that such a theory is possible.

In references to the *Annalen* paper [4], Dehn mentions two attempts on Hilbert's Third Problem that appeared before Hilbert's 1900 Paris lecture: the French geometer, Raoul Bricard [1] published a brief account in 1896, and the Italian geometer, Guiseppe Sforza [10] in 1897. McCleary presented these approaches as well as the inadequacies of their arguments. Their work indicated an approach to the Third Problem that is properly realized and developed in Dehn's papers [3] and [4]. The notion of a *Dehn invariant* of a polyhedron is introduced by Dehn as a geometric object, a subset of a unit radius cylinder. It was later given a more algebraic definition, and generalizations of Hilbert's Third Problem to other geometries and higher dimensions began to be considered in the decades after Dehn's papers appeared. In the 1950's, a resurgence of interest in the problems of scissors congruence was based on Dehn's work.

Dehn published other works related to these geometric ideas between 1900 and 1907 that were not discussed, but will be considered in the subsequent paper/chapter. Dehn's geometric work raises some questions for the Dehn project: how to handle earlier mathematical work; how to relate his work to Hilbert's, and what to make of the absence of explicit mention of group actions (the Kleininan view of geometry); what was the reception of Dehn's ideas? The subsequent discussion suggested possible answers to these questions.

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Dehn's solution to the word problem for surface groups and its influence on geometric group theory

David Peifer

In his 1911 paper, *On Infinite Discontinuous Groups* [3], Dehn launched the study of groups given by finite presentations. He begins the paper by providing a general definition of (what are now known as) the Word, Conjugacy and Isomorphism Problems for finitely presented groups. Dehn then goes on to solve the word and conjugacy problems for the surface groups. The surface groups, the fundamental groups of orientable 2-manifolds, had been studied by Poincaré and others before Dehn. It was known that each surface group has a simple one relator presentation and acts by translations on a regular tesselation of the euclidean or hyperbolic plane. In his work to solve the word and conjugacy problem for the surface groups, Dehn was able to exploit this connection between the surface groups and geometry. A significant feature of the 1911 paper is Dehn's insight to use geometric ideas to solve the algebraic questions posed in the word and conjugacy problems.

A Cayley graph of a group (given by a presentation) has one vertex for each group element and directed edges labeled by group generators. There is an edge labeled by the generator a, from the vertex for group element g to h, whenever $h = ga$ in the group. By a careful analysis of the Cayley graph for a group (given by a presentation), Dehn was able to construct an interpretation of "distance" and "area" in the group. The distance between two group elements is the length of the shortest edge path in the Cayley graph between the vertices representing these group elements. This distance in now known as the word metric. In modern terms, we say that with this metric, the Cayley graph is a geodesic metric space.

Given a group by a finite presentation, Dehn's word problem asks, is there a finite algorithm that can determine if a finite product of generators and inverse generators (called a word) is equal to the identity in the group? Before Dehn's work, Dyck had shown that for any word equal to the identity, one can find a nice algebraic form (specifically, the word must be freely equivalent to a product of conjugates of the group relators). In his work, Dehn was able to find a planar diagram that geometrically represented Dyck's algebraic concept. This idea is now known as a Dehn diagram (also referred to as van Kampen diagram and small cancellation diagram).

In 1910, Dehn had given another proof of the solution to the word and conjugacy problem for surface groups. His earlier proof involved detailed analysis of the embedding of the Cayley graph in the hyperbolic plane. In his 1911 paper, Dehn provided a combinatorial proof that relies only on the group presentation and did not require any specific reference to hyperbolic geometry. Dehn had found the combinatorial consequences of the connection between the word problem and hyperbolic geometry. Dehn's solution to the word problem is now known as the Dehn Algorithm. In fact, Dehn had shown that for the surface groups his algorithm is very efficient. When given any word, Dehn's algorithm will determine if the word is equivalent to the identity, while requiring fewer processing steps than the length of the word. In modern terms we would say, Dehn had shown that the surface groups have a linear Dehn algorithm.

Over the years, this 1911 paper has had a continued influence on the study of infinite groups. The word and conjugacy problems have played a central role in combinatorial group theory. Dehn's Frankfurt student, Wilhelm Magnus, was a leader in this area, see [6]. Work in small cancellation theory, in the 1960's, is a direct generalization of Dehn's use of diagrams to solve the the word problem for surface groups, see for example [5]. Work by Cannon, in the early 1980's on

Kleinian groups, [1], and work by Gromov on hyperbolic groups, [4], were grand expansions of the Dehn's original solution to the word and conjugacy problems. In his almost 200 page paper, *On Hyperbolic Groups*, Gromov proves that his hyperbolic groups are exactly the groups that have a linear Dehn algorithm, thus generalizing Dehn's work on surface groups.

Dehn's use of diagrams to solve the conjugacy problem influenced the modern concept of a Dehn function (or isoperimetric function). The Dehn function for a group provides a measure of how complicated it is to solve word problem. It can also be interpreted – in a more geometric way – as a measure of how the length of the boundary of an enclosed region grows compared to the area. This talk examined how Dehn' s work has influenced these modern concepts from geometric group theory.

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Max Dehn's view on the philosophy of mathematics, and its relation to the history of mathematics

Moritz Epple

Is it possible to ascribe to Max Dehn a coherent position on the philosophy of mathematics, even if he did not directly intervene in the early 20th century debates on the foundations of mathematics?

In the first part of my contribution I discussed what views we can attribute to him in these foundational debates. Being close to David Hilbert in his early career, he learned about the latter's ideas on axiomatic foundations of geometry, and his joint paper with Poul Heegaard, published in 1907, sought to give a "modern" foundation of Analysis situs or topology [1]. To what extent can this contribution be considered as adhering to Hilbertian views on foundations? On the one hand, Dehn's attempt to give an axiomatic foundation of topology is very close in language and style to Hilbert's ealier *Grundlagen der Geometrie*. On the other hand, a closer look at the actual axioms provided by Dehn (who was responsible for the foundational aspects of the joint paper with Heegaard) reveals that he explicitly aimed at providing an intuitive interpretation, an "intuitive substratum" of topology, thus going beyond purely formal tasks for axiomatics.

Moreover, Dehn gave an essential role to combinatoric ideas in his contributions to geometric topology. To what extent can these be considered to be related to foundational views emphasizing the foundational role of combinatorics (even prior to logic) such as the ones advanced by Hans Hahn or in Kurt Reidemeister's essay "Exaktes Denken" in 1928 [2]? It emerges that while Dehn certainly was close to the ideas of both Hilbert's circle and the Vienna Circle, he never fully endorsed a formalist view on the foundations of mathematics. It is also interesting to note that he never engaged with a set theoretic, modernist approach to the foundations of topology such as the one developed in the wake of Felix Hausdorff's *Grundzüge der Mengenlehre* of 1914. On the contrary: Dehn was quite sceptical about the potential of set-theoretic foundations. Quite probably, Dehn's own mathematical ideas on geometric topology and in particular, the difficulty of providing non-intuitive proofs of crucial technical steps including those leading up to Dehn's notorious "lemma", defied such an interpretation of the foundations of mathematics. At the same time, I am not aware of any direct comments by Dehn on the more rigorously "intuitionist" ideas advanced by Brouwer and others.

Dehn's reluctance to fully engage with either of the dominant foundationalist views of the early 20th century was related to an approach that viewed mathematics as an essentially human activity, directing attention to both anthropological and historical considerations. The second part of my contribution therefore took a look at some of Dehn's essays on the cultural role of mathematics and on its historical development, including "Die geistige Eigenart des Mathematikers" (1928, [3]), "Das Mathematische im Menschen" (1932, [4]), "Raum, Zeit und Zahl bei Aristoteles, vom mathematischen Standpunkt aus" (1936, [5]), "Beziehungen zwischen der Philosophie und der Grundlegung der Mathematik im Altertum" (1937, [6]) and "Über Ornamentik" (1940, [7]). In these papers we find, on the one hand, a marked anthropological concern. Besides logical capabilities, according to Dehn all human beings share an aesthetic capability that he termed "mathematical emotion." Dehn considered this emotion, found already in the joy engendered by rhythms, ornaments, or architectural patterns, to be one of the most important motivations for producing mathematical knowledge on all its levels, from antiquity to present, from ordinary life and elementary education to research. On the other hand, and at the same time, the interplay of logical capability and mathematical emotion generates, in Dehn's view, a historical dynamics that can be followed through human history, and potentially in all cultures. Dehn's articles abound with examples from many periods and many parts of the world.

His papers on the interactions between philosophy and the foundations of mathematics in antiquity exemplify this general view. A closer look shows that Dehn's analysis of this interaction displays more affinity with Aristotelian views of the foundations of mathematics, emphasizing that mathematical knowledge is produced by the human soul from the world of sensory experience than with Plato's idealist views.

In the context of contemporary literature on the historical dynamics of mathematics, Dehn's views may be compared with authors such as Georg Cantor, Ernst Cassirer or Jean Cavaillès, who all believed in an intrinsic, mostly conceptual dynamics of mathematical knowledge. In contrast, and in harmony with his anthropological beliefs, Dehn thought that the dynamics of mathematical knowledge is more complex and in fact related to other domains of cultural, social, and political history as well. Such a view was indeed shared by other contemporary authors, including for instance his Frankfurt colleague Arthur Schoenflies, and Richard Courant in Göttingen.

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Max Dehn's Journey Through America

Marjorie Senechal

Max and Toni Dehn reached the United States on January 1, 1941, after an arduous three-month journey from Oslo through Siberia and Japan and across the Pacific Ocean. "In glorious sunshine we passed under the Golden Gate Bridge and sailed into the peaceful San Francisco Bay. I hope that sunshine and peace may be the meaning of this earthly constellation on New Year's Day and at the last moment of our long trip ([1])." But Max Dehn's journey through America, and through the landscape of American higher education, had just begun: in the course of the next several years, he would teach at widely scattered and wildly different institutions. His age (62), nativist hostiliity to refugees, especially German Jews; curricula reconfigured for the war effort; and faculty positions that would return to their former occupants at war's end together nudged, or lured, him from place to place. Friends and colleagues lamented that a mathematician of his stature did not readily find a permanent, and prestigious, home, but Dehn himself did not.

Max Dehn not only crossed the country geographically, he connected the dots in the landscape of American higher education (outside the circle of major private universities). His first stop was a (then) two-year state college in Pocatello, Idaho;

the next, a newly-formed hybrid engineering and design institute in Chicago, Illinois; then a small college devoted to “Great Books” in Annapolis, Maryland, and finally a still-young arts-centered ”experiment in community” in rural Black Mountain, North Carolina. Each of these institutions had its own special history and character, with strengths that appealed to him and weaknesses that did not. Black Mountain suited him well, providing scope for his many facets and interests: he taught ancient Greek, philosophy, elementary mathematics, and “geometry for artists” (some of whom he influenced profoundly), prepared two advanced mathematics students for graduate school, led community members young and old on nature walks in the mountains, and was a calming spirit in the tempestuous college governance. Nor was Dehn entirely isolated professionally: he stayed in touch with the wider mathematical community through occasional semesters at the University of Wisconsin, a leading public university, and a summer at the University of Notre Dame. He died at Black Mountain in 1952, a few weeks after he retired. (The college closed a few years later.)

This talk, which reflects a work in progress, is drawn largely from archives, including the Max Dehn papers at the Briscoe Center for American History at the University of Texas, Austin; the Black Mountain College collection at the Western Regional [State] Archives in Asheville, North Carolina,and the archives of the Illinois Institute of Technology and St. John’s College. It is also informed by an interview with Toni Dehn in the late 1970’s, which the interviewer made available for this workshop (it is not yet in the public domain) and by an unpublished memoir by the late Joseph H. Engel, Max Dehn’s Ph.D. student at the University of Wisconsin. I have also drawn on, and in some cases corrected or amended, published accounts of Max Dehn’s journey including [2],[3], [4], and [5]).

References

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Max Dehn: His early life and his late life encounters with artists at Black Mountain College

Brenda Danilowitz, Philip Ording

Max Dehn was one of eight siblings born into a prosperous, secular, Jewish family in Hamburg in the last quarter of the nineteenth century. Little has been published about the Dehn family or Max Dehn’s early life and his schooling before his arrival in Freiburg as an eighteen year old university student in 1896. Through contact with Dehn's extended family in Germany and the USA, Danilowitz has begun to investigate the late 19th century history of Dehn and his family. In the first part of the talk she outlined some of this material including unpublished family photographs and her intention to fill out the story of Dehn and his family in this period.

Ording continued the talk by presenting a close reading of parts of Dehn's "Ueber Ornamentik" (1939), such as this concluding passage [2, pp. 152–153]:

> Aber unter all diesen Gestalten sind doch einige wenige Prinzipe zu erkennen, Grundtendenzen der menschlichen Seele, die auch mit Musik und Architektur und mit jeder Kunstäusserung untrennbar verbunden sind. Diese Prinzipe sind aber mathematischer Natur und aus vielen Ornamenten kann man mathematische Sätze herauslesen.
> (But among all these forms [of ornament], only a few principles must be acknowledged, basic tendencies of the human spirit which are inextricably bound up with music, architecture, and every artistic expression. These principles are mathematical in nature and mathematical propositions are discernible in many ornaments.)

Dehn arrives at these underlying principles of the art of ornamentation using "methods of applied mathematics" that he lays out in detail for his reader at the start of the essay. Taken as a whole, the article presents a unifying framework with which to apprehend works of art and suggests affinities between Max Dehn and the Bauhaus artists Josef and Anni Albers, whom he encountered in the US after taking up an appointment at Black Mountain College in the remote mountainous landscape of western North Carolina in 1945.

The college had been established in 1933 by a group of disgruntled academics who, led by John Andrew Rice, a classicist and follower of philosopher John Dewey, had resigned from Rollins College in Florida USA. Accompanied by a small coterie of devoted students, they were in search of greater academic freedom and especially freedom from bureaucratic and high-handed administrators. As an acolyte of Dewey whose theory of education was frequently broadly summarized as "learning by doing" Rice set out to make art education central to the general liberal arts curriculum then customary at American undergraduate colleges. These events, which coincided with the Nazis' closing of the by then internationally known Bauhaus in Germany, resulted in Bauhaus teacher Josef Albers and his wife Anni, both renowned teachers and artists, being invited to create the educational framework for Black Mountain College. After 1934, as events in Europe escalated towards war, and with the strong presence of the Alberses, several scholars from Germany, both friends of the Alberses and others who had heard of the college through other routes, including Harvard University and the Institute for Advanced Study at Princeton, began arriving. This story has been fully told elsewhere [1].

Max Dehn and his wife Toni were among the later arrivals. There were few mathematics students at the college and so Dehn turned his attention to teaching the Classics, Philosophy, and a course of his own devising, "Mathematics (or

Geometry) for Artists." A primary focus of this research will be Dehn's impact on artists at Black Mountain College: on the one hand on Josef and Anni Albers themselves, and on the other on his students, most notably Ruth Asawa, Dorothea Rockburne, and Lorna Blaine. In the last part of the talk, Danilowitz introduced works by these artists and posed the question of what methodologies could be employed to allow an understanding of how artists' works which appear to be related quite broadly to Dehn's intellectual concerns—for example Anni Albers's series of Knot drawings begun in 1947 or Josef Albers's Structural Constellations which allude to forms twisting in space—might be viewed through the prism of Dehn's mathematical and philosophical ideas and practice. What can we discover by analyzing students' notes and works from Dehn's classes? What did Dehn select from his mathematical vocabulary that he considered relevant for artists? Finally, did the educational philosophies of Dehn and Albers align, and if so, in what respects?

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[2] M. Dehn, *Ueber Ornamentik*, Norsk Matematisk Tidsskrift **21** (1939), 121–153.

Max Dehn at Black Mountain College

Trueman MacHenry

I have met two men in my life who make me think of Socrates: Max Dehn and [French Philosopher] Brice Parain — André Weil[1]

André Weil, one of the outstanding mathematicians of the last two centuries, was the the older brother of Simone Weil, the French philosopher and mystic The warm feeling he expressed toward Max Dehn was shared by the faculty and students at Black Mountain College. He was looked up to as a wise philosopher, and had the appearance of one also. Both he and his wife, Toni, were very active in the daily workings of the college. This included the duties of the faculty, who served as both the teaching staff and the governing council of the College, as well as community obligations, which included taking care of the cleanliness and repair of the grounds and the buildings, a chore usually reserved for Saturdays. The College was very informal – it has sometimes been accused of giving rise to the

[1] The quotation continues: "Both of them, like Socrates – as we picture him from the accounts of disciples – possessed a radiance which makes one naturally bow down before their memory: a quality both intellectual and moral, that is best conveyed by the word "wisdom"; for holiness is another thing altogether. In comparison with the wise man, the saint is perhaps just a specialist – a specialist in holiness; whereas the wise man has no specialty. This is not to say, far from it, that Dehn was not a mathematician of great talent; he left behind a body of work of very high quality. But for such a man, truth is all one, and mathematics is but one of the mirrors in which it is reflected – perhaps more purely than elsewhere. Dehn's all-embracing mind held a profound knowledge of Greek philosophy and mathematics."

hippy movement. But Dehn was often addressed as "Professor" – only occasionally as Max – and in the third person he was called 'Dehn,' as did his wife (as was usual in Germany). Toni Dehn was always addressed as 'Toni'.

Other things were not very informal as well. The degree program at Black Mountain was strongly patterned after European, and particularly German, programs of study. A student who planned to finish with a degree or a certificate from BMC first had to pass a qualifying examination. This was conducted by the entire faculty, the purpose being to assess whether a student had a reasonable grasp of general subject matter. Students who passed then selected an area of specialization. They were assigned a faculty advisor, who supervised a thesis or a project in the chosen field. On completion, the student was again examined before an assembly of the faculty, but this time the oral exam was conducted by an outside specialist. If all this was successful, the student was then awarded the degree or certificate.

Max Dehn came to Black Mountain in 1945 from St. John's, the "Great Books" College, four years before my arrival in 1949. Dehn died in the summer of 1952 just after I graduated. The College itself was founded in 1933 by John Rice and Theodore Dreier in 1933 as an experiment in education. Rice and Dreier, professors at Rollins College in Florida, were dissatisfied with the educational policies there, and so decided to leave Rollins in order to set up an institution that reflected their more liberal views. The title of the very fine book by Martin Duberman, *Black Mountain, An Exploration in Community*, reflects these intentions. Their plans included, among other things, a college with a strong emphasis on the arts, an idea that ultimately was fulfilled to such a degree that it became BMC's most famous aspect. In *The Arts at Black Mountain* by Mary Emma Harris, this point is explored and illustrated in great detail.

The hiring of Joseph and Anni Albers of Bauhaus fame was decisive in tilting BMC in the direction of the arts. Their presence acted like a magnet that attracted other European artists and scholars, a number of whom found their way to Black Mountain. While Dehn's mathematical friends were dismayed that he ended up an institution with no mathematical eminence at all, he harboured no such feelings. Not only the beautiful mountain setting but also the very unorthodox educational arrangements were, for him, both congenial and invigorating. He also enjoyed the range of interesting people at the College, plus the possibility of walks in the mountains looking for rare orchids and other botanical discoveries. It was a safe haven after a rather harrowing escape from the horrors of Nazi Germany. Dehn and Toni participated fully in all of the activities of the College, taking part, for example, in the Saturday work day devoted to cleaning and repairing the buildings and grounds. All members of the community were asked to do this as part of their commitment to BMC.

Dehn had deep interests in philosophy, both Eastern and Western. When I knew him, he devoted his afternoons to a nap and reading philosophy, usually sitting in his study in a straight back chair. His taste in Eastern philosophy centred on Lao Tse and Taoism, and his knowledge of classical Western philosophy was thorough, as would be expected of one with a European education during those years. At the time I was studying with him, he was also interested in the writings of the English philosopher Hume. But we also had interesting conversations about the ideas of Schopenhauer, Kant, and Hegel, philosophers I happened to be interested in at the time.

Both Dehn and Toni also had strong interests in poetry, especially the famous German poets. Among their favourites was Heinrich Heine. I studied German with Toni, and Heine was part of our reading regime. Dehn's languages included German, English, and French, along with classical Greek and Latin. John Rice was a classical scholar, and he left to the College a fine classical library with many works in Greek and Latin. With one or two exceptions, their principal reader was Max Dehn. One of the courses that Dehn taught at Black Mountain during my time there was classical Greek. Charles Olsen, the poet, and a faculty member of the College, was one of his Greek students. Olsen was rather displeased with Dehn's method of teaching languages. Dehn was of the opinion that one needed to know both grammar and vocabulary as part of learning a language. So his approach was based on a standard method for those who want to acquire a language after early childhood: one learned by memorizing. Olsen's idea was that language was something you came to by some kind of process of osmosis, perhaps in the same way that infants learn to speak; so there was a parting of the ways and Olsen left the course.

Dehn also taught a course called "Geometry for Artists", which involved a lot of perspective or projective geometry. These were well-attended classes, but often times they proved to be obscure for many of the students, who didn't have much previous academic training. Dehn taught more advanced courses in mathematics on an individual basis. During my time at BMC, he had two students who did a full-time degree course in mathematics, Peter Nemenyi and myself. Each of us had private meetings with Dehn, and these took place in his apartment study. I remember sitting with him side by side at his desk in front of a window that looked out onto the woods. Above me, to my right, was a shelf of books that included what was left of Dehn's mathematical library. On the shelf were the works of Riemann and Grassmann. And it was at this desk that I first learned about the ring of symmetric polynomials, a topic which has since been the focal point for my work and publications over the last many years.

These sessions almost always ended with a walk in the woods, which led to interesting and memorable talks. Sometimes Dehn would ask me about areas of philosophy I was interested in, and along the way he would point out a rare orchid hiding below the foliage. Botany, in general, and orchids, in particular, were matters about which he was an expert. These conversational walks were as valuable and interesting as our classes together, and my biweekly meetings with Dehn were always very pleasant and interesting. There was always a lot of conversation about the background of the mathematics I was studying with him, and he commented often about the people he had known as a student and teacher in Germany, for example, David Hilbert and Felix Klein. Dehn, along with Carl

Ludwig Siegel, had offered a well-known seminar in History of Mathematics when they were together at Frankfurt University. When he discussed mathematics, there was always a large portion of its history involved.

Dehn had a very gentle and tolerant disposition; so his wide and deep knowledge, which went far beyond mathematics, did not act as a barrier between him and his students. However, when accuracy and clarity were at stake, he could be very demanding. When I had completed the work on my thesis and the time came for a careful reading of it with him, he showed another side altogether. He went over the thesis with me in the apartment of Hazel Larsen, a well-known photographer at Black Mountain, who knew him as this very kind and tolerant man. Hazel was completely surprised how demanding and unrelenting he could be when truth and accuracy were at stake, and I too saw a side of Dehn that had not appeared before.

Peter Nemenyi was an especially gifted student. His father was a professor of applied mathematics at George Washington University, and his half-brother was Bobby Fischer, the famous chess prodigy. Peter was raised by his father in a socialist community in Hungary before coming to the United States, and throughout his life he was committed to being of use to society. Peter's degree topic was Special Functions and his outside examiner was Emile Artin. He went on to Princeton University on a scholarship, where he got his Ph.D. under John Tukey. Afterward he became a very successful mathematician and he produced a number of results that now carry his name.

My thesis work was in Foundations of Geometry, an area in which Dehn earned his initial fame. Dehn had been a student of Hilbert, one of the most famous and productive mathematicians of the the last two or three hundred years. Hilbert is famous, among other things, for the twenty-three problems he posed as a challenge to the mathematicians of the twentieth century. He announced these at the Second ICM held in Paris in 1900. Reputations have been made ever since by those who came forth as solvers of these problems (Yandell 2002). At the age of only 22, Max Dehn was the very first such person. Already in 1900 he solved the third problem, which assured him a place among the leading names of his time. After that he made fundamental contributions in a number of areas of mathematics (Topology, Geometry, and Group Theory). Among his students were Ruth Moufang, Jacob Nielsen, and Wilhelm Magnus. The background reading for my thesis included Hilbert's famous work, *The Foundations of Geometry*, as well as Euclid's Elements. The only copy of the Elements available at Black Mountain belonged to Max Dehn. It was a beautiful edition from 1824, published in two volumes with facing pages in Greek and Latin. I had no Greek, but was able to read it in Latin. This book as well as two others were left to me after Dehn's death.[2]

My outside examiner was Alfred Brauer, who was then teaching at the University of North Carolina in Chapel Hill. Thanks to my work with Dehn, I was given a teaching fellowship at Chapel Hill, where I went to do my graduate work. There I taught a beginning course in algebra. After an obligatory stint in the U.S.

[2]The other two were *Manière universelle de Mr Desargues, pour pratiquer la perspective par petit-pied, comme le géométral*, Paris, 1648, and C. Jordan, *Cours d'Analse*, 2 vols., Paris , 1893.

Navy and later a Ph.D., I began my career as a university professor, now retired. My areas of interest in mathematics are geometry and algebra in which I have published a number of papers. Dehn did not have many students who specialized in mathematics at Black Mountain, but he was a strong influence on a number of students and faculty members at the College. Among these were Ruth Asawa and Dorothea Rockburne, both of whom became very well-known artists. There is an interesting remark by Mildred Harding, who recalls a conversation with Dehn during which he reflected that in his sixty years of teaching he “had 15 students of real mathematical talent.”

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Reporter: Antina Scholz

Participants

Brenda Danilowitz
The Josef and Anni Albers Foundation
88 Beacon Road
Bethany CT 06524
UNITED STATES

Prof. Dr. Moritz Epple
Historisches Seminar
-Wissenschaftsgeschichte-
Goethe-Universität Frankfurt
60629 Frankfurt am Main
GERMANY

Prof. Dr. Cameron M. Gordon
Mathematics Department
The University of Texas at Austin
1 University Station C 1200
Austin TX 78712-0257
UNITED STATES

Dr. Jeremy John Gray
Faculty of Mathematics and Computing
The Open University
Walton Hall
Milton Keynes MK7 6AA
UNITED KINGDOM

Dr. Cynthia Hog-Angeloni
Institut für Mathematik
Johannes-Gutenberg Universität Mainz
Staudingerweg 9
55128 Mainz
GERMANY

Prof. Dr. Jemma Lorenat
Pitzer College
1050 North Mills Avenue
Claremont, CA 91711
UNITED STATES

Prof. Dr. Trueman MacHenry
Dept. of Mathematics and Statistics
York University
4700 Keele Street
Toronto ON M3J 1P3
CANADA

Prof. Dr. John McCleary
Department of Mathematics
Vassar College
Box 69
Poughkeepsie NY 12604
UNITED STATES

StR i. R. Elisabeth Mühlhausen
Dammstrasse 2
37434 Krebeck
GERMANY

Dr. Philip Ording
Sarah Lawrence College
1 Mead Way
Bronxville NY 10708
UNITED STATES

Prof. Dr. David E. Peifer
Department of Mathematics
University of North Carolina at Asheville
Asheville NC 28804
UNITED STATES

Ilka Reinhard
Institut für Mathematik
Johannes-Gutenberg Universität Mainz
Staudingerweg 9
55128 Mainz
GERMANY

Prof. Dr. Volker Remmert
Wissenschafts- und Technikgeschichte
Historisches Seminar, Fachbereich A
Bergische Universität Wuppertal
Gaußstrasse 20
42119 Wuppertal
GERMANY

Antina Scholz
Wissenschafts- und Technikgeschichte
Historisches Seminar, Fachbereich A
Bergische Universität Wuppertal
Gaußstrasse 20
42119 Wuppertal
GERMANY

Prof. Dr. David E. Rowe
Institut für Mathematik
Johannes-Gutenberg Universität Mainz
Staudingerweg 9
55128 Mainz
GERMANY

Prof. Dr. Marjorie Senechal
Department of Mathematics
Smith College
Northampton, MA 01063-0001
UNITED STATES