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## Cryptography

Organised by  
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**ABSTRACT.** The Oberwolfach workshop Cryptography brought together scientists from cryptography with mathematicians specializing in the algorithmic problems underlying cryptographic security. The goal of the workshop was to stimulate interaction and collaboration that enables a holistic approach to designing cryptography from the mathematical foundations to practical applications. The workshop addressed fundamental research results leading to innovative cryptography for protecting security and privacy.

*Mathematics Subject Classification (2010):* 94A60.

### Introduction by the Organisers

The goal of the workshop Cryptography, organized by Johannes Buchmann (Darmstadt) and Shafi Goldwasser (Boston) was to stimulate interaction and collaboration between mathematicians and computer scientists that enables a holistic approach to designing cryptography from the theoretical foundations to practical applications. The topic of the workshop is highly relevant for both research and application. Cryptography has long been an integral building block of many cyber security solutions and has therefore been of critical importance to modern IT security. On the other hand technological progress presents multiple new scientific challenges. For instance the rise of cloud computing requires novel cryptographic approaches and constructions. Additionally future developments in quantum computing threaten current schemes, which shows the need for quantum resistant cryptography.

The talks given at the workshop covered important recent results in the areas relevant for the workshop. The talks on the mathematical foundations addressed both traditional and more recent algorithmic problems that serve as the security

basis of modern cryptography. A major topic of the workshop were obfuscation schemes. These primitives allow to hide the the source code of a given program against a computationally bounded adversary, while preserving the functionality of the program. It is possible to construct such schemes from multilinear maps or graded encoding schemes. The presented results showed that there is progress in the construction of obfuscation from multilinear maps, but for certain applications there are still powerful attacks on the underlying encoding schemes.

The talks on post quantum cryptography covered both new constructions and new cryptanalysis. In light of a recent call for proposals for post quantum schemes by NIST, new constructions for public key encryption and key exchange were presented from both lattices and codes. Furthermore there were talks on new attack both classical and quantum on certain schemes. The presentations on advanced cryptographic constructions included new constructions in attribute based encryption, updatable encryption and spooky encryption. These techniques allow for further functionalities, required to address the challenges of outsourced data. Multiple session dealt with the topic of secure multi-party computation, which enables multiple entities to jointly compute on data in a privacy preserving way. Novel constructions such as function secret sharing were very well presented during the talks of the workshop.

There were also several talks dealing with practical challenges, such as the possibility of trapdoored public parameters in widely used cryptographic libraries and privacy preserving operations on genomic data.

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## Workshop: Cryptography

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## Abstracts

### Rethinking Large-Scale Consensus Through Blockchain

RAFAEL PASS

We revisit classic questions in distributed computing in settings where players at any point can go offline.

We ask: Can we design consensus protocols under “sporadic participation” where at any given point only a subset of the players are online.

### Program Obfuscation: Outside the Black Box

OMER PANETH

Code is said to be obfuscated if it is intentionally difficult for humans to understand. Obfuscation is often used to conceal sensitive implementation details such as proprietary algorithms or licensing mechanisms.

A general-purpose obfuscator is a compiler that obfuscates arbitrary code (in some particular language) without altering the code’s functionality. Ideally, the obfuscated code would hide any information about the original code that cannot be obtained by simply executing it.

The potential applications of general-purpose obfuscators extend beyond software protection. For example, in computational complexity theory, obfuscation is used to establish the intractability of a range of computational problems. Obfuscation is also a powerful tool in cryptography, enabling a variety of advanced applications.

The possibility of general-purpose obfuscation was put into question by Barak et al. [1], who proved that such obfuscation cannot have ideal security. Nevertheless, they leave open the possibility of obfuscation with weaker security properties, which may be sufficient for many applications. Recently, Garg et al. [2] suggested a candidate construction for general-purpose obfuscation conjectured to satisfy these security properties.

We study the feasibility and applicability of different notions of secure obfuscation. In terms of applicability, we prove that finding a Nash equilibrium of a game is intractable, based on a weak notion of obfuscation known as indistinguishability obfuscation [4]. In terms of feasibility, we focus on a variant of the Garg et al. obfuscator that is based on a recent construction of cryptographic multilinear maps [3]. We reduce the security of the obfuscator to that of the underlying multilinear maps.

Our first reduction considers obfuscation and multilinear maps with ideal security [5]. We then study a useful strengthening of indistinguishability obfuscation known as virtual-grey-box obfuscation. We identify security properties of multilinear maps that are necessary and sufficient for this notion [6]. Finally, we explore the possibility of basing obfuscation on weaker primitives. We show that obfuscation is impossible even based on ideal random oracles [7].

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## Indistinguishability Obfuscation from DDH on 5-linear Maps and Locality-5 PRGs

HUIJA (RACHEL) LIN

We present a new construction of Indistinguishability Obfuscation (IO) from the following.

- Asymmetric L-linear map [1, 2] with subexponential Decisional Diffie-Hellman (DDH) assumption
- Locality-L polynomial-stretch pseudorandom generator (PRG) with subexponential security
- The subexponential hardness of learning with errors (LWE)

When plugging in a candidate PRG with locality 5 (e.g. [3]) we obtain a construction of IO from subexponential DDH on 5-linear maps and LWE. Previous IO constructions rely on multilinear maps or graded encoding schemes with higher degrees, more complex functionalities (e.g. graded encodings with complex label structures), and stronger assumptions (e.g. the joint-SXDH assumption).



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**Obfuscating Groups**

DENNIS HOFHEINZ

(joint work with Martin Albrecht, Pooya Farshim, Julia Hesse, Enrique Larraia, and Kenny Paterson)

We propose to use (indistinguishability) obfuscation to enhance the security and functionality of cryptographically useful groups. For instance, by attaching an encryption of the respective discrete logarithm to each group element, it is possible to efficiently implement a multilinear map over that group. The corresponding multilinear map is an obfuscated algorithm that knows the decryption trapdoor, and thus can extract and multiply the discrete logarithms of all involved group elements. Still, we can show that cryptographically useful computational assumptions hold in that group. Moreover, we show that similar constructions can be used to construct groups in which very strong computational assumptions (such as variants of the “Uber assumption” due to Boneh, Boyen, and Goh hold).

Our results imply new and abstract constructions of multilinear maps that allow, e.g., for multilinear variants of Groth-Sahai proof systems. However, while our constructions are abstract and modular, we use a number of strong building blocks: we use (subexponentially secure) indistinguishability obfuscation, fully homomorphic encryption, dual-mode zero-knowledge proof systems, and cyclic groups in which the Strong Diffie-Hellman assumption holds. (Of course, for our construction of groups in which Uber assumptions hold, we do not require Strong Diffie-Hellman groups.)

This talk surveys our results, and the main ideas from our constructions. In particular, this talk covers the results from [1, 2, 3].

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## From Search to Approximate-Decision, Locally, and while Preserving Exponential Hardness

BENNY APPLEBAUM

The Gap-ETH assumption (Dinur 2016; Manurangsi and Raghavendra 2016) asserts that it is exponentially-hard to distinguish between a satisfiable 3-CNF formula and one which is only  $(1 - \delta)$ -satisfiable. We show that this assumption follows from the exponential-hardness of solving *smooth* 3-CNF's. Here smoothness means that the number of satisfying assignments is not much smaller than the number of "almost-satisfying" assignments. We further show that the latter ("smooth-ETH") assumption follows from the exponential-hardness of solving constraint satisfaction problems over well-studied planted distributions, and, more generally, from the existence of an exponentially-hard locally-computable one-way function.

We also prove an analogous result in the cryptographic setting. Namely, we show that the existence of exponentially-hard locally-computable pseudorandom generator with linear stretch (EL-PRG) follows from the existence of an exponentially-hard locally-computable regular one-way functions.

None of the above assumptions (Gap-ETH and EL-PRG) was previously known to follow from the hardness of a search problem. Our results are based on a new construction of general (GL-type) hard-core functions which outputs linearly many hard-core bits, can be locally-computed, and uses only a linear amount of random bits.

## Cryptanalyses of Candidate Branching Program Obfuscators

SHAI HALEVI

(joint work with Yilei Chen and Craig Gentry)

We describe new cryptanalytic attacks on the candidate branching program obfuscator proposed by Garg, Gentry, Halevi, Raykova, Sahai and Waters (GGHRSW) [8] using the GGH13 graded encoding [7], and its variant using the GGH15 graded encoding as specified by Gentry, Gorbunov and Halevi [9]. All our attacks require very specific structure of the branching programs being obfuscated, which in particular must have some input-partitioning property. Common to all our attacks are techniques to extract information about the "multiplicative bundling" scalars that are used in the GGHRSW construction.

For GGHRSW over GGH13, we show how to recover the ideal generating the plaintext space when the branching program has input partitioning. Combined with the information that we extract about the "multiplicative bundling" scalars, we get a distinguishing attack by an extension of the annihilation attack of Miles, Sahai and Zhandry [10]. Alternatively, once we have the ideal we can solve the principle-ideal problem (PIP) in classical subexponential time or quantum polynomial time, hence obtaining a total break.

For the variant over GGH15, we show how to use the left-kernel technique of Coron, Lee, Lepoint and Tibouchi [5] to recover ratios of the bundling scalars. Once we have the ratios of the scalar products, we can use factoring and PIP solvers (in classical subexponential time or quantum polynomial time) [1, 4, 2, 3, 6] to find the scalars themselves, then run mixed-input attacks to break the obfuscation.

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**Nipped in the Bud: Graded Encoding Schemes that Did Not Make It**

ZVIKA BRAKERSKI

(joint work with Craig Gentry, Shai Halevi, Tancrede Lepoint, Amit Sahai,  
Mehdi Tibouchi)

Zeroizing attacks [4, 1, 2, 7] were shown to be a potent line of attacks against existing graded encoding candidates [4, 3, 5]. Although some applications do not seem to be affected by these attacks, they were used to break a number of applications (and many hardness assumptions on) these graded encoding candidates.

Roughly speaking, zeroizing attacks proceed by honestly computing many top-level encoding of zero, then using the prescribed zero-testing procedure to setup and solve a system of multilinear equations in the secret parameters of the scheme.

These attacks rely crucially on the linearity of the zero-testing procedure, and so some attempts were made recently to devise alternative zero-testing procedures that are non-linear. In one of these attempts [6], Gentry, Halevi and Lepoint recently described a variant of the GGH13 candidate scheme [4], in which the linear zero-testing procedure from [4] is replaced by a quadratic (or higher-degree) procedure.

We show that the Gentry-Halevi-Lepoint (GHL) variant remains susceptible to the same zeroizing attacks as the original GGH13. In particular, we show how to construct a native GGH13 zero-test parameter from the GHL quadratic zero-test parameter. In a nutshell, this is done by computing the “derivative” of the quadratic zero-test polynomial at a top-level encoding of zero, thus obtaining a linear zero-test polynomial that can be transformed into a native GGH13 zero-test parameter.

### 1. GGH WITH HIGH-DEGREE ZERO-TEST

The GGH13 graded encoding candidate [4] works over the quotient ring  $R_q = R/qR$  where  $R = \mathbb{Z}[x]/(x^n + 1)$  is the  $2n$ -th cyclotomic polynomial ring ( $n$  a power of two) and  $q$  is a large modulus.<sup>1</sup> The plaintext space is  $R_g = R/gR$  where  $g \in R$  is a small (secret) element. A level- $k$  encoding  $u$  of  $m \in R_g$  is such that  $u = [c/z^k]_q$  where  $c \in m + gR$  is small and  $z \leftarrow R_q$  is a random (secret) multiplicative mask. Encodings at the same levels can be added (and the encoded values get added modulo  $R_g$ ), and encodings can be multiplied as long as the sum of the levels remains smaller than the multi-linearity level  $\kappa$  (and the encoded values get multiplied modulo  $R_g$ ).

The GGH13 Zero-Testing. The zero-testing procedure of GGH13 consists in multiplying a level- $\kappa$  encoding  $u = [c/z^\kappa]_q$  by a public value  $p_{zt} = [h/g \cdot z^\kappa]_q$ , where  $h$  is a somewhat small secret value, so that

$$(1) \quad w = [u \cdot p_{zt}]_q = [h \cdot (c/g)]_q$$

has norm smaller than (say)  $q^{3/4}$  if and only if  $c \in gR$ , i.e. if and only if  $u$  is a top-level encoding of  $0 \in R_g$ . Now when  $c = gr$  over  $R$ , Eq. (1) holds over  $R$  and gives  $w = h \cdot r$  which is linear in  $r$ . This  $R$ -linearity can then be exploited in zeroizing attacks [4, 7].

Quadratic Zero-Testing. During the invited talk of CRYPTO 2015, Halevi described a tentative fix due to Gentry, Halevi and Lepoint (GHL) aiming at making the zero-testing procedure at least quadratic in the coefficients of the input (and therefore breaking the  $R$ -linearity of the zero-testing procedure at the core of the zeroizing attacks) [6]. For any encoding  $u = \sum_{i=0}^{n-1} u_i \cdot x^i \in R_q$ , denote  $\vec{u} = (u_0, \dots, u_{n-1}) \in \mathbb{Z}_q^n$  its vector of coefficients. The GHL zero-testing procedure is given by a quadratic polynomial  $p: \mathbb{Z}_q^n \rightarrow \mathbb{Z}_q$  such that

$$|p(\vec{u}) \bmod q| < q^{3/4} \iff u \text{ is a top-level encoding of } 0.$$

<sup>1</sup>Our attack extends to any cyclotomic polynomial ring  $R = \mathbb{Z}[x]/(\Phi(x))$  when  $\Phi$  has small enough coefficients. For ease of simplicity we restrict our description to  $\Phi(x) = x^n + 1$  for  $n$  a power of 2.

The key idea is to define  $p$  as  $p(\vec{u}) = \sum_{i,j} \alpha_{ij} \cdot \ell_i(\vec{u}) \cdot \ell_j(\vec{u})$  where the  $\alpha_{i,j}$ 's are small (say,  $\|\alpha_{ij}\|_\infty < q^{1/4}$ ) and the  $\ell_i$ 's are the linear equations corresponding to the multiplication by a native GGH13 zero-test parameter  $p_{zt}$  over  $R_q$ , i.e. such that  $w = [u \cdot p_{zt}]_q$  has coefficient-vector  $\vec{w} = (\ell_0(\vec{u}), \dots, \ell_{n-1}(\vec{u}))$  and (say)  $\|w\|_\infty < q^{1/4}$ . It is easy to generalize the GHL zero-testing procedure to a polynomial of higher degree  $d$  by considering monomials of the form  $\ell_{i_1}(\vec{u}) \cdots \ell_{i_d}(\vec{u})$ . Note, however, that describing the new zero-test polynomial takes  $\Theta(n^d)$  terms, hence for this zero-test procedure to be polynomial-time we need the degree  $d$  to be a constant.

## 2. CRYPTANALYSIS

The key idea of the attack will be to compute the “derivative” of the high-degree polynomial in a top-level encoding of 0, reducing its degree until we get back a linear polynomial.

**Definition.** Let  $p(x_0, \dots, x_{n-1}) \in \mathbb{Z}_q[x_0, \dots, x_{n-1}]$  be a polynomial. For all  $\vec{a} = (a_1, \dots, a_{n-1}) \in \mathbb{Z}_q^n$ , we define  $p'_a \in \mathbb{Z}_q[x_1, \dots, x_n]$  the derivative of  $p$  in  $\vec{a}$  as

$$p'_a(x_0, \dots, x_{n-1}) = p(x_0 + a_0, \dots, x_{n-1} + a_{n-1}) - p(x_0, \dots, x_{n-1}) \pmod q.$$

Note that if  $p(\vec{x})$  is of total degree  $t \geq 1$  in the  $x_i$ 's, then  $p'_a(\vec{x})$  is of total degree at most  $t - 1$ .

**Reducing the Degree.** Let  $p_d(\cdot)$  be the degree- $d$  zero-testing polynomial of GHL, so for every top-level encoding of zero  $x$  we have  $|p_d(\vec{x}) \pmod q| < q^{3/4}$  (say). Also let  $u \in R_q$  be some fixed top-level encoding of zero. For  $i = 1, 2, \dots, d - 1$  we compute  $p_{d-i}(\cdot)$  by deriving  $p_{d+1-i}(\cdot)$  at  $u$ , setting

$$p_{d-i}(\vec{x}) = p_{d+1-i}(\vec{x} + \vec{u}) - p_{d+1-i}(\vec{x}) \pmod q.$$

Clearly the total degree of each  $p_j$  is (at most)  $j$ , and in particular the last polynomial  $p_1(\vec{x})$  has degree (at most) 1.

Moreover, we can prove by induction on  $i$  that for every top-level encoding of zero  $v$  we have  $|p_{d-i}(\vec{v}) \pmod q| < 2^i \cdot q^{3/4}$ . This clearly holds for  $p_d$ , so now assume that it holds for  $p_{d+1-i}$  and we prove for  $p_{d-i}$ . Note that since both  $u, v$  are top-level encoding of zero then so is  $v + u$ , and therefore

$$\begin{aligned} |p_{d-i}(\vec{v})| &= |p_{d+1-i}(\vec{v} + \vec{u}) - p_{d+1-i}(\vec{v})| \\ &\leq |p_{d+1-i}(\vec{v} + \vec{u})| + |p_{d+1-i}(\vec{v})| < 2^{i-1}q^{3/4} + 2^{i-1}q^{3/4} = 2^i \cdot q^{3/4}. \end{aligned}$$

We conclude that for every top-level encoding of zero  $v$  we have  $|\rho + \sum_{i=1}^{n-1} \rho_i \cdot v_i| < 2^{d-1} \cdot q^{3/4}$  (and note that since  $d$  is a constant then  $2^{d-1} \cdot q^{3/4} \ll q$ ).

We note that the native GGH13 zero-test is linear whereas the polynomial  $p_1$  is above is affine, so recovering a native GGH13 zero-test parameter seem to require that we ignore the free term. Indeed, below we show that the free term  $\rho$  from above must be small, and therefore we can ignore it without affecting the zero-test result.

**Recovering a Native GGH13 Zero-Test Parameter.** Finally, we use the structure of the ring  $R_q$  to recover a native GGH13 zero-test parameter, i.e. a ring element  $r \in R_q$  such that  $\|r \cdot v \bmod q\| \ll q$  for every top-level encoding of zero  $v$ . Specifically we define  $r(X) = \rho_0 - \sum_{i=1}^{n-1} \rho_{n-i} \cdot X^i \in R_q$ , and we show that  $r$  is a native GGH13 zero-test parameter.

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### Improving Distributed Storage Systems Through Adaptive Social Secret Sharing

GIULIA TRAVERSO

(joint work with Denise Demirel, Sheikh M. Habib, Johannes Buchmann)

Due to the increase of digital data in the last years, it might be difficult for a user to have enough resources to store its data. A solution is to outsource these data to the Cloud and let one or multiple Cloud providers manage this storage through their storage servers. However, Cloud providers have to ensure certain guarantees such that for the user it is worth to pay for this service. More precisely, there are two protection aspects that Cloud providers must guarantee in the long-term. On the one hand, the data must remain confidential, i.e. no information should be leaked to a third party. On the other hand, the data must be retrieved efficiently any time it is needed. These two protection goals are clear when thinking of health records. Health records are sensitive data that if revealed can damage the patient (e.g. sexual diseases) but in case of an emergency they have to be retrieved fast and correctly (e.g. the blood group when transfusion is needed).

Confidentiality and retrievability are inherent to the behavior of the storage servers involved: when storage servers are untrustworthy, then confidentiality and retrievability might not be guaranteed any more. More precisely, honest but curious storage servers follow all the protocols correctly but are also prone to leak information to a third party or to collude. In this sense, they are untrustworthy with respect to confidentiality. Faulty storage servers do not respond or respond late when the data have to be retrieved, compromising the entire process. In this sense they are untrustworthy with respect to retrievability.

We provide a solution [6] to this framework, called  $AS^3$ , which is a social secret sharing scheme based on adaptive hierarchical secret sharing. Secret sharing [3] is a cryptographic primitive that distributes a document within a storage system by generating shares of that document. Each share is such that it reveals no information about the document itself and is stored within a different storage server. For threshold secret sharing schemes, a subset of a certain size of shares is sufficient to retrieve the document. In this sense, secret sharing is the right primitive for distributed storage systems to be based on because by design it offers confidentiality and retrievability. Social secret sharing [2] is equipped with a trust function which tests the behavior of the storage servers and grants different reconstruction power according to their trustworthiness. That is, more informative shares are distributed for the better behaving storage servers and less informative shares are distributed to the worse behaving storage servers. Hierarchical secret sharing [4] is used in this context as the underlying scheme for social secret sharing: the most trustworthy storage servers are treated as the most powerful participants in a hierarchical structure. However, current solutions [2], [1] fail at providing confidentiality and retrievability of the document outsourced. The first reason is that hierarchical secret sharing schemes are not flexible and thus they cannot rearrange the threshold and generate new shares in accordance to the updated trust values. We overcome this drawback by proposing the first Birkhoff interpolation based hierarchical secret sharing scheme that is dynamic [5]. The second reason is that, previous to our work, the notion of trust was not well defined and, thus, the trustworthiness of the storage servers could not be properly rated since the two different behavior of confidentiality and retrievability were not distinguished yet. In addition, we overcome this second problem by defining a new trust function that tests the storage servers with respect to confidentiality and retrievability in a separate way, outputting two trust values for each storage servers. Thanks to our countermeasures,  $AS^3$  is the first construction of social secret sharing that actually works effectively in the framework of distributed storage systems. Rules and checks for confidentiality and retrievability are given such that each time the trust functions updates the trust values, the threshold of the secret sharing scheme can be adapted such that honest but curious storage servers can never retrieve the document by themselves and the absence of the faulty storage servers cannot prevent the remaining ones from retrieving the document.

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**Attribute Based Encryption and Information-Theoretic Crypto**

HOETECK WEE

Can we encrypt data while enabling fine grained access control? We survey how addressing this question led to new connections and questions in information-theoretic cryptography.

**Composition: The Key to Differential Privacy’s Success**

GUY N. ROTHBLUM

Differential privacy provides a rigorous and robust privacy guarantee to individuals in the context of statistical data analysis. It has sparked a revolution in the study of privacy-preserving data analysis, impacting fields from computer science to statistics to legal scholarship. A key factor underlying this success is robustness under composition: when multiple differentially private algorithms are run on the same individual’s data, privacy degrades smoothly and gradually.

It is hard to overstate the importance of robustness under composition. In reality, individuals’ data are involved in multiple datasets and analyses. Privacy that does not compose offers only questionable protection. No less important, composition makes Differential Privacy *programmable*: differentially private algorithms for small or common tasks can be used as subroutines in larger more complex algorithms, and inherit the subroutines’ privacy guarantees (up to some degradation).

Composition has been key to differential privacy’s success, and understanding how privacy degrades under composition is a core issue in the study of privacy-preserving data analysis. The differential privacy community has attempted to achieve a more perfect understanding of composition. Recent works have made



significant advances in this study, touching on issues in complexity theory, probability theory, and shedding new light on the behavior of differentially private algorithms.

This talk will survey the study of composition in differentially private data analysis, from basic definitions and guarantees and all the way to the most recent developments and open questions in this exciting frontier.

### Accessing Data While Preserving Privacy

KOBBI NISSIM

We study the privacy-efficiency tradeoff of secure remote database systems. Such systems allow storing data on an untrusted server and accessing it efficiently while maintaining the privacy of the data. While strong cryptographic tools (e.g. FHE, ORAM, SFE) can be used, implementations experiment with weaker primitives with the hope of striking a good privacy-efficiency balance.

Our approach is implementation independent. We provide abstract models that capture fundamental leakage channels of such systems and provide reconstruction attacks using these leakage channels. We also present a new model- PP- storage, that provably protects data from these and other attacks, and implement DP-storage using ORAM and tools from differential privacy.

### The LWE-based key exchange – A Simple Provably Secure Key Exchange Scheme Based on the Learning with Errors Problem

JINTAI DING

Key exchange protocol enables two users to exchange keys in untrusted channels without sharing secret materials in advance. The first and celebrated key exchange protocol is the Diffie-Hellman key exchange protocol [16] which is also a fundamental construction in public key cryptography. It is simple and elegant, and, after its invention, countless applications based on Diffie-Hellman key exchange protocol or the Diffie-Hellman problem were proposed.

Diffie and Hellman [16] also introduced the notion of public key encryption, and Rivest, Shamir and Adleman [27] gave the first concrete public key encryption scheme. Namely, the well-known RSA encryption. With public key encryption in hand, one can construct a key exchange protocol. Instantiating with the RSA algorithm, the construction produces a very efficient key exchange protocol. However, the encryption-type key exchange protocol may have an important side-effect in practice: This approach relies on the user's private key to protect all the session keys, anyone with access to a copy of the private key can also uncover the session keys and thus decrypt everything.

The Diffie-Hellman protocol offers an alternative algorithm to RSA for cryptographic key exchange. The Diffie-Hellman protocol generates more secure session keys that can't be recovered simply by knowing the user's private key, a protocol security feature called *forward security*. In order to decrypt all communication,

now the adversary can no longer compromise just the user's private key, but the adversary has to compromise the session keys belonging to every individual communication session. In other words, using the Diffie-Hellman protocol, even an adversary knows the session key of some particular session, he still can not learn anything about the session keys established before this particular session. Actually, SSL also uses the Diffie-Hellman protocol to support forward security.

The motivation of this report is to build simple Diffie-Hellman like key exchange protocols based on lattices. Lattice-based public key cryptography has become a promising potential alternative to public key cryptography based on traditional number theory assumptions. One building block of lattice-based cryptography, especially in encryption, is the learning with errors (LWE) problem. After the introduction of LWE problem by Regev [26], it has attracted a lot of attentions in theory and applications due to its usage in cryptographic constructions with good provable secure properties. In a nutshell, the (decisional) LWE problem is to distinguish polynomially many noisy inner-product samples of the form  $(\mathbf{a}, b \approx \langle \mathbf{a}, \mathbf{s} \rangle)$  from uniformly random ones, where  $\mathbf{a} \leftarrow \mathbb{Z}_q^n$  and  $\mathbf{s} \leftarrow \mathbb{Z}_q^n$  are uniformly random.<sup>1</sup> An attractive property of the LWE problem is that Regev [26] shows that to solve the average-case LWE problem is at least as hard as to (quantumly) solve some worst-case hard lattice problems. Many lattice-based primitives based on LWE have been discovered, such as public-key encryption [26, 18, LP11], (hierarchical) identity-based encryption [18, 1, 15], functional encryption [3, 2, 9, 19] and fully homomorphic encryption [12, 11, 10].

In the constructions mentioned above, a matrix form of the LWE problem is always used (i.e., need sufficient many samples). The drawback of that is it results in large (say quadratic) key size. To further improve the efficiency, Lyubashevsky, Peikert and Regev [24] introduced the ring learning with errors (RLWE) problem, which is to distinguish polynomially many noisy ring multiplications  $(a, b \approx a \cdot s)$  from uniform distribution, where “ $\cdot$ ” is the multiplicative operation over some ring. It's shown in [24] that to solve the RLWE problem is at least as hard as to solve some worst-case problems in *ideal* lattices, instead of general lattices.

What motivates the work in this report is to try to build a simple key exchange protocol using the basic idea of Diffie-Hellman protocol but based on the LWE and RLWE problem. There are already related works in [21, 22, 14, 17], but as far as we know there is not yet until very recently any provably secure key exchange protocols based on the LWE problem as a direct generalization of the Diffie-Hellman key exchange protocol, which is elegant in terms of its simplicity. Our work was finished in 2012 [20]. Recent works on LWE-based Key exchange protocols [25], [8], [5], [7] are all variants of our protocol with minor modifications but some with significant contributions in concrete implementations.

To achieve our goal, we use the normal form of LWE problem suggested in [6] and introduce a new randomized method to eliminate bias, which may be of independent interest.

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<sup>1</sup> $\mathbf{s}$  is secret and remains the same in all the samples.

The key idea behind our new construction can be viewed as a way to share a secret given by the value of the bilinear function of two vectors  $\mathbf{x}$  and  $\mathbf{y}$  in  $\mathbb{Z}_q^n$ , where  $q, n$  are some integers, via the bilinear form:

$$Q(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \mathbf{A} \mathbf{y} = (\mathbf{x}^T \mathbf{A}) \mathbf{y} = \mathbf{x}^T (\mathbf{A} \mathbf{y}),$$

where  $\mathbf{A}$  is an  $n \times n$  matrix in  $\mathbb{Z}_q$ . Surely in order to make the system provably secure, we need to introduce the small errors to achieve our goal. The main contribution of our work is to use this simple idea to build a simple and provably secure key exchange scheme. The idea of such an "noise" or "approximate" KE appeared long time ago like the work of Buchmann and Williams [13], and recently the work [4] using coding theory. A new fundamental contribution of ours, which is something very different from the DH KE is that we developed a new idea of "signal functions" as an additional tool needed to round the approximate value to a shared secret without affecting the security. Furthermore, we extend our construction further based on the RLWE problem. Our construction is a significant additional step in showing how versatile the LWE assumption can be.

Besides, we also give an interactive multiparty key exchange protocol. This protocol can be viewed as a generalization of our two party protocol. Although the provable security of the protocol seems plausible but we do not know how to do it, and we leave it as an open problem.

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## From Minicrypt to Obfustopia via Private-Key Functional Encryption

ILAN KOMARGODSKI

(joint work with Gil Segev)

Functional encryption [15, 7, 14] allows tremendous flexibility when accessing encrypted data: Such encryption schemes support restricted decryption keys that allow users to learn specific functions of the encrypted data without leaking any additional information. We focus on the most general setting where the functional encryption schemes support an unbounded number of functional keys in the public-key setting, and an unbounded number of functional keys and ciphertexts in the private-key setting. In the public-key setting, it has been shown that functional encryption is essentially equivalent to indistinguishability obfuscation [3, 11, 1, 2, 6, 16].

When examining the various applications of functional encryption (see, for example, the survey by Boneh et al. [8]), it turns out that *private-key* functional encryption suffices in many interesting scenarios. However, although private-key functional encryption may seem significantly weaker than its public-key variant, constructions of private-key functional encryption schemes are currently known based only on public-key functional encryption.

We settle the problem of positioning private-key functional encryption within the hierarchy of cryptographic primitives by placing it in Obfustopia. First, given any *quasi-polynomially-secure* private-key functional encryption scheme, we construct a (quasi-polynomially-secure) indistinguishability obfuscator for circuits with inputs of poly-logarithmic length and sub-polynomial size.

**Theorem 1** (Informal). *Assuming a quasi-polynomially-secure private-key functional encryption scheme for polynomial-size circuits, there exists an indistinguishability obfuscator for the class of circuits of size  $2^{(\log \lambda)^\epsilon}$  with inputs of length  $(\log \lambda)^{1+\delta}$  bits, for some positive constants  $\epsilon$  and  $\delta$ .*

Underlying our obfuscator is a new transformation from single-input functional encryption to multi-input functional encryption in the private-key setting. The previously known such transformation of Brakerski et al. [9] required a sub-exponentially-secure single-input scheme, and obtained a multi-input scheme supporting only a slightly super-constant number of inputs. Our transformation both

relaxes the underlying assumption and supports more inputs: Given any quasi-polynomially-secure single-input scheme, we obtain a multi-input scheme supporting a poly-logarithmic number of inputs.

We demonstrate the wide applicability of our obfuscator by observing that it can be used to instantiate many natural applications of (full-fledged) indistinguishability obfuscation for polynomial-size circuits. We construct a public-key functional encryption scheme (based on [16]), and a hard-on-average distribution of instances of a PPAD-complete problem (based on [5]).

**Theorem 2** (Informal). *Assuming a quasi-polynomially-secure private-key functional encryption scheme for polynomial-size circuits, and a sub-exponentially-secure one-way function, there exists a public-key functional encryption scheme for the class of circuits of size  $2^{(\log \lambda)^\epsilon}$  with inputs of length  $(\log \lambda)^{1+\delta}$  bits, for some positive constants  $\epsilon$  and  $\delta$ .*

**Theorem 3** (Informal). *Assuming a quasi-polynomially-secure private-key functional encryption scheme for polynomial-size circuits, and a sub-exponentially-secure injective one-way function, there exists a hard-on-average distribution over instances of a PPAD-complete problem.*

Compared to the work of Bitansky et al. [4], Theorem 2 shows that private-key functional encryption implies not just public-key encryption but leads all the way to public-key functional encryption. Furthermore, in terms of underlying assumptions, whereas Bitansky et al. assume a sub-exponentially-secure private-key functional encryption scheme and a (nearly) exponentially-secure one-way function, we only assume a quasi-polynomially-secure private-key functional encryption scheme and a sub-exponentially-secure one-way function.

In addition, recall that average-case PPAD hardness was previously shown based on compact *public-key* functional encryption (or indistinguishability obfuscation) for polynomial-size circuits and one-way permutations [12]. We show average-case PPAD hardness based on quasi-polynomially-secure *private-key* functional encryption and sub-exponentially-secure injective one-way function. In fact, as shown by Hubáček and Yogev [13], our result (as well as [5, 12]) implies average-case hardness for CLS, a proper subclass of PPAD and PLS [10].

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### Average-Case Fine-Grained Hardness, and what to do with it

PRASHANT VASUDEVAN

(joint work with Marshall Ball, Alon Rosen, Manuel Sabin)

We present functions that are hard to compute on average for algorithms running in some fixed polynomial time, assuming widely-conjectured worst-case hardness of certain problems from the study of fine-grained complexity.

We discuss the relevance of such average-case hardness to cryptography and present, as an illustration, an outline of a proof-of-work protocol constructed based on the hardness and certain structural properties of our functions.

### The Journey from NP to TFNP Hardness

MONI NAOR

We discussed how to how that there are hard problems in the complexity class TFNP- Total Function NP. For this class hardness results based on one way permutations or collision resistant hash functions. On the other hand there are all sorts of barriers for showing reductions based on  $P \neq NP$ . We prove hardness based

on hard-on-the-average problems in NP (joint work with Eylon Yogev and Pavel Hubáček). On related work we considered the complexity of the Ramsey problem. It was known to be hard for a graph based algorithm. Based on CRH we showed hardness when one is given a program for computing a graph.

### Homomorphic Secret Sharing, Part I: Function Secret Sharing from One-Way Functions

YUVAL ISHAI

(joint work with Elette Boyle and Niv Gilboa)

Fully homomorphic encryption (FHE) is a powerful cryptographic tool that can be used to minimize the communication complexity of secure computation protocols. However, known FHE schemes rely on a relatively narrow set of assumptions and algebraic structures that are all related to lattices. Moreover, the efficiency of known FHE schemes still leaves much to be desired.

This two-part talk covers new techniques for succinct secure computation. The idea is to replace FHE by “homomorphic secret sharing” (HSS), which allows a compact evaluation of a function on a secret shared input, and construct HSS schemes for useful function classes in a way that gets around some of the limitations of known FHE schemes.

The first part covers constructions of Function Secret Sharing (FSS) schemes for simple function classes from one-way functions. FSS can be viewed as a dual version of HSS, where the roles of the function and input are reversed. More concretely, the goal of FSS is to split a function  $f$  into succinctly described  $f_1, \dots, f_m$ , such that  $f(x) = f_1(x) + \dots + f_m(x)$  for every input  $x$ , and every strict subset of the  $f_i$  computationally hides  $f$ . We present efficient constructions of FSS schemes for point functions and decision trees based on any pseudo-random generator, and survey applications of these constructions in the context of efficient secure access to remote data and secure multi-party computation. The material of this part of the talk is based on [1, 2].

A big open question in the area is that of characterizing the function classes for which efficient FSS schemes can be based on one-way functions, or, more generally, understand the necessary and sufficient cryptographic assumptions for useful instances of FSS. A more concrete open question is the efficiency of one-way function based constructions of FSS schemes for the class of point functions in the case of  $m \geq 3$  parties. The best construction from [1] achieves a near-quadratic improvement over the naive solution of secret sharing the truth-table.

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## Homomorphic Secret Sharing, Part II: Succinct and Round-Efficient Secure Computation from DDH

ELETTE BOYLE

(joint work with Niv Gilboa and Yuval Ishai)

Fully homomorphic encryption (FHE) is a powerful cryptographic tool that can be used to minimize the communication complexity of secure computation protocols. However, known FHE schemes rely on a relatively narrow set of assumptions and algebraic structures that are all related to lattices. Moreover, the efficiency of known FHE schemes still leaves much to be desired.

This two-part talk covers new techniques for succinct secure computation. The idea is to replace FHE by “homomorphic secret sharing” (HSS), which allows a compact evaluation of a function on a secret shared input, and construct HSS schemes for useful function classes in a way that gets around some of the limitations of known FHE schemes.

The second part of the talk presents a construction of a powerful HSS scheme based on discrete-log-type assumptions. More concretely, under the Decisional Diffie-Hellman (DDH) assumption, we construct a 2-out-of-2 secret sharing scheme that supports a compact evaluation of branching programs on the shares. In fact, the output of the homomorphic evaluation is *additively* shared between the parties. We survey different applications of this HSS scheme, including succinct secure computation for  $\text{NC}^1$ , two-round secure multiparty computation parties, and other DDH-based applications that previously required FHE. The material of this part of the talk is based on [2, 3].

This work gives rise to several interesting open questions. While HSS for all polynomial-time computable functions can be based on the Learning with Errors (LWE) assumption via special types of FHE [1, 4], obtaining a similar result under DDH or other cryptographic assumptions is open. Our DDH-based HSS scheme has an inverse-polynomial error probability, where the running time of the evaluation algorithm grows linearly with the inverse of the error probability. An interesting open question is to eliminate this error or improve the dependence of the running time on the error. Another open question is to obtain similar results for the case of 3 or more parties. Finally, our 2-round protocol for general secure multiparty computation has two limitations that we would like to eliminate: it requires a Public Key Infrastructure (PKI) setup, and it is only efficient when the number of parties is constant.

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### **Equivocating Yao: Constant-Round Adaptively Secure Multiparty Computation in the Plain Model**

MUTHURAMAKRISHNAN VENKITASUBRAMANIAM

Yao’s circuit garbling scheme is one of the basic building blocks of cryptographic protocol design. Originally designed to enable two-message, two-party secure computation, the scheme has been extended in many ways and has innumerable applications. Still a basic question has remained open throughout the years. Can the scheme be extended to guarantee security in face of an adversary that corrupts both parties, adaptively, as the computation proceeds.

We answer this question in the affirmative. We define a new type of encryption called functionally equivocal encryption (FEE) and show that when Yao’s scheme is implemented with an FEE as the underlying encryption mechanism, it becomes secure against adaptive adversaries. We then show how to implement FEE from any one-way function.

### **Techniques in Lattice-based Cryptography**

VINOD VAIKUNTANATHAN

Many recent constructions of advanced lattice-based primitives such as attribute-based encryption, fully homomorphic encryption and signatures, predicate encryption and constrained pseudorandom functions rely on a handful of techniques that play with the learning with errors (LWE) problem and lattice trapdoors. We will present them and show as many constructions as time permits.

### **Revisiting Non-Malleable Commitments**

RAFAIL OSTROVSKY

(joint work with Michele Ciampi, Luisa Siniscalchi, Ivan Visconti)

Commitment schemes are a fundamental primitive in Cryptography. These schemes are protocols between two players: sender and the receiver. There exist two phases, a commitment phase and a decommitment phase. In the commitment phase the sender, with a secret input  $m$ , interacts with the receiver. In the end of this interaction we say that a *commitment* of the message  $m$  has been computed. Moreover the receiver still does not know what  $m$  is (i.e.  $m$  is hidden) and at the same time the sender during the decommitment phase can subsequently opens this commitment only to  $m$ .

In our work we consider the intriguing question of constructing round-efficient schemes that remain secure even against man-in-the-middle (MiM) attacks: non-malleable (NM) commitments [DDN91]. The round complexity of NM commitments after 25 years of research remains a fascinating open question. The original construction of [DDN91] required a logarithmic number of rounds and the sole use of one-way functions (OWFs). Then, through a long sequence of very exciting positive results [Bar02, PR05, PW10, LP11, Goy11, GLOV12], the above open question has been in part solved obtaining a constant-round (even concurrent) NM commitment scheme by using any OWF in a black-box fashion. On the negative side, Pass proved that NM commitments require at least 3 rounds [Pas13] when security is proved through a black-box reduction to falsifiable (polynomial or subexponential time) hardness assumptions.

More recently forward steps to reduce the round complexity has been made in [GRRV14, GPR16] but only for the (simpler) one-one case (i.e., just one sender and one receiver). In particular, Goyal et al. [GRRV14] showed a *one-one* 4-round NM commitment scheme based on OWFs only. The initial version of [GRRV14] claims concurrent non-malleability. Later on we have found an error in their security proof of the one-one and of the one-many cases (that also applies to the construction of [BGR<sup>+</sup>15]). The claim on concurrent non-malleability has then been withdrawn in the recent eprint version of [GRRV14] where a variation of their original scheme is presented and proved one-one non-malleable. The more recent work of Goyal et al. [GPR16] shows a 3-round one-one NM commitment scheme based on the black-box use of any 1-to-1 OWF. This claim has however been withdrawn in [GPR15]. Two very recent results of Ciampi et al. [COSV16b, COSV16a] obtain in 3 round, respectively, a concurrent non-malleable commitment and one-one non-malleable commitment schemes. These constructions both rely on one-way permutations secure against subexponential-time adversaries.

**Our Results.** We show a 4-round concurrent non-malleable commitment scheme based on the sole existence of OWFs, therefore solving a problem explicitly left open by [GRRV14]. We achieve this result combining two following two notions. 1) We define a new security notion for argument systems on unique-witness instances w.r.t. MiM attacks that we refer to as simulation-witness-independence (SimWI). 2) We consider a weaker form of non-malleability, *weak non-malleability* (wNM), where the MiM is restricted to playing well-formed commitments in the right sessions when receiving well formed commitments from the left sessions.

In our work we construct a 4-round one-many SimWI argument of knowledge (AoK) for unique-witness instances by relying on OWFs only and we prove that a subprotocol of [GRRV14] is a 4-round statistically binding concurrent wNM commitment scheme from OWF. Finally putting these two gadgets together we obtaining our main result: a 4-round concurrent non-malleable commitment scheme. The recent state of the art is summarized in Table 1.

We leave open two important questions. 1) The existence of 3-round concurrent NM commitment from falsifiable assumption against polynomial-time adversaries.

Paper	Rounds	Assumption	Concurrency
Goyal,/ Lin and Pass STOC 2011	$\geq 6$	OWFs	Yes
Goyal et al., FOCS 2012	$\geq 6$	BB OWFs	Yes
Goyal et al., FOCS 2014	4	OWFs	No
Ciampi et al + ePrint 2016 Ciampi et al., CRYPTO 2016	3	subexp OWPs	Yes
Our result	4	OWFs	Yes
Goyal et al., STOC 2016	3	BB subexp OWPs	No

TABLE 1. Comparison with recent positive results.

2) The existence of 4-round fully adaptive-input one-many SimWI argument systems from OWFs.

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## Practical Key Exchange with Forward Secrecy from Coding Theory

NICOLAS SENDRIER

### 1. INTRODUCTION

Quasi-Cyclic Moderate Density Parity Check (QC-MDPC) codes allows the design [6] of McEliece-like encryption schemes [5]. Those variants have several advantages, among which

- (1) shorter public keys,
- (2) a good security reduction.

### 2. QC-MDPC-McELIECE

**2.1. Shorter Keys.** Using quasi-cyclic codes for the McEliece encryption scheme was first proposed by Gaborit [2]. Generator and parity check matrices of quasi-cyclic codes are block circulant and can thus be fully described by giving one or a few rows instead of the whole matrix. The quasi-cyclic codes that we consider

here are of length  $n = 2p$  with index 2. This means that any generator or parity check matrix can be written in the form  $(A \mid B)$  where  $A$  and  $B$  are  $p \times p$  circulant matrices (each row is the cyclic shift of the previous one) and is entirely described by its first row  $(a, b)$ , a vector of length  $2p$ .

Moreover, binary  $p \times p$  circulant matrices are isomorphic to  $\mathcal{R}_p = \mathbf{F}_2[x]/(x^p - 1)$  and thus most statements can be written in terms of polynomials instead of codes or matrices.

**2.2. QC-MDPC codes.** A binary MDPC code admits a sparse parity check matrix  $H$  whose rows have a (small) Hamming weight  $w$ . A QC-MDPC code of index 2 admits a sparse parity check matrix  $H = (H_0, H_1)$  and is fully described by the two sparse circulant blocks  $H_0$  and  $H_1$ , or equivalently by two sparse polynomial  $h_0$  and  $h_1$  in  $\mathcal{R}_p$ . To decode  $t$  errors in such a code, one has to solve the following problem ( $|\cdot|$  the Hamming weight)

*Problem 1* (QC-MDPC Decoding).

input:  $s, h_0, h_1 \in \mathcal{R}_p$  with  $|h_0| = |h_1| = w/2$

output:  $e_0, e_1 \in \mathcal{R}_p$  such that  $e_0 h_0 + e_1 h_1 = s$  and  $|e_0| + |e_1| \leq t$

This problem can be solved with Gallager's bit flipping algorithm for LDPC codes [3]. The algorithm succeeds with high probability as long as  $w \simeq \sqrt{2p}$  and  $t \simeq \sqrt{2p}$ . Some suitable values of  $p, w, t$  are given at the end of this abstract. In Figure 1 we describe the QC-MDPC-McEliece variant in terms of polynomials. The decryption can be achieved with Gallager's algorithm.

<p><b>Parameters:</b> block size <math>p</math>, row weight <math>w</math>, error weight <math>t</math>, <math>\mathcal{R}_p = \mathbf{F}_2[x]/(x^p - 1)</math>  <math>(p</math> a prime, <math>w</math> even, <math>w/2</math> odd, <math>w</math> and <math>t</math> are close to <math>\sqrt{2p}</math>)</p> <p><b>Key Generation:</b> pick <math>h_0</math> and <math>h_1</math> in <math>\mathcal{R}_p</math> both of Hamming weight <math>w/2</math>  public key: <math>g = h_1 h_0^{-1}</math>  private key: <math>h_0, h_1</math></p> <p><b>Encryption:</b> <math>\mathcal{R}_p \rightarrow \mathcal{R}_p \times \mathcal{R}_p</math>  <math>m \mapsto (mg + e_0, m + e_1)</math> with <math> e_0  +  e_1  = t</math></p> <p><b>Decryption:</b> given a ciphertext <math>(u_0, u_1)</math>  solve <math>u_0 h_0 + u_1 h_1 = e_0 h_0 + e_1 h_1</math> with <math> e_0  +  e_1  \leq t</math></p>
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FIGURE 1. QC-MDPC-McEliece Scheme

### 3. SECURITY REDUCTION

Following [7], the system is secure on average as long as two assumptions hold:

- (1) Decoding  $t$  error in a binary quasi-cyclic  $[2p, p]$  code is hard on average.
- (2) The public key is indistinguishable from a random block circulant matrix of same size.

Those assumptions relate to the following two problems

*Problem 2* (QC Generic Decoding).

input:  $s, h \in \mathcal{R}_p$ , an integer  $t$

output:  $e_0, e_1 \in \mathcal{R}_p$  such that  $e_0 + e_1h = s$  and  $|e_0| + |e_1| \leq t$

*Problem 3* (QC Codeword Weight).

input:  $h \in \mathcal{R}_p$ , an integer  $w$

question: is there  $h_0, h_1 \in \mathcal{R}_p \setminus \{0\}$  such that  $h_0 + h_1h = 0$  and  $|h_0| + |h_1| \leq w$ ?

The Problem 2 is the generic decoding of (index 2) quasi-cyclic codes. It is NP, but its completeness status is unknown. It is widely admitted that the problem is hard on average.

**Open problem:** extend Alekhovich’s hardness results on average case decoding [1] to the quasi-cyclic case.

The Problem 3 relates to the indistinguishability of QC-MDPC codes. It is worth noting that the two statements are very similar. More, all known algorithms that can solve Problem 3 can solve Problem 2 for essentially the same cost. Even more, in the non quasi-cyclic setting, decoding and finding a word of small weight in a linear code are equivalently hard.

**Open problem:** prove that Problems 2 and 3 are polynomially equivalent.

#### 4. KEY EXCHANGE FOR QC-MDPC CODES

A key agreement procedure easily derives from QC-MDPC-McEliece (see Figure 2). At the end of the protocol, Alice and Bob share the secret error pattern  $e_0, e_1$ . The protocol security reduces to the two above problems. In addition, since key generation is easy one can use ephemeral key pairs with two nice features: (1) forward secrecy (if one instance of the protocol is compromised it does not affect past executions), and (2) less vulnerability to side channel attacks (as [4] which exploits decoding failures).

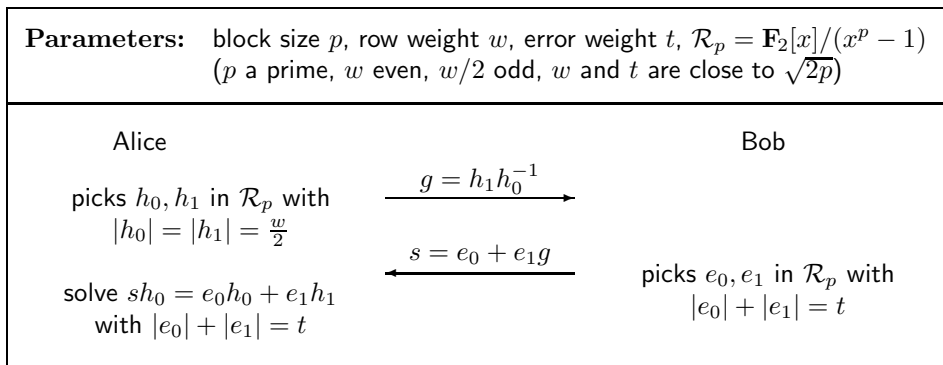


FIGURE 2. QC-MDPC Key Exchange Protocol (Sketch)

4.1. **Parameters.** Parameter selection requires some work. One has to make sure, through analysis and simulations that the designed number of errors can be corrected with high probability. The following table gives some possible values.

security	block size, $p$	$w$	$t$
80 bits	4801	90	84
128 bits	9857	142	134
256 bits	32771	274	264

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### Short Generators Without Quantum Computers: The Case of Multiquadratics

DANIEL J. BERNSTEIN, CHRISTINE VAN VREDENDAAL

Finding a short element  $g$  of a number field, given the ideal generated by  $g$  is a classic problem in computational algebraic number theory. Solving this problem recovers the private key in cryptosystems introduced by e.g. Buchmann-Maurer-Mller, Gentry, Smart-Vercauteren, Gentry-Halevi, and Garg-Gentry-Halevi. Work over the last ten years has shown that for some number fields this problem has a surprisingly low post-quantum security level. In this talk we will present an algorithm showing that for multiquadratic number fields this problem has a surprisingly low pre-quantum security level. Experimental results confirm the analysis.



## Interactive Coding with Nearly Optimal Round and Communication Blowup

Yael Tauman Kalai

(joint work with Klim Efremenko, Elad Haramaty)

The problem of constructing error-resilient interactive protocols was introduced in the seminal works of Schulman (FOCS 1992, STOC 1993). These works show how to convert any two-party interactive protocol into one that is resilient to constant-fraction of error, while blowing up the communication by only a constant factor. Since these seminal works, there have been many follow-up works which improve the error rate, the communication rate, and the computational efficiency.

All these works assume that in the underlying protocol in each round each party sends a single bit. This assumption is without loss of generality, since one can efficiently convert any protocol into one which sends one bit per round. However, this conversion may cause a substantial increase in *round* complexity, which is what we wish to minimize in this work. Moreover, all previous works assume that the communication complexity of the underlying protocol is *fixed* and a priori known, an assumption that we wish to remove.

In this work, we consider protocols whose messages may be of *arbitrary* lengths, and where the length of each message and the length of the protocol may be *adaptive*, and may depend on the private inputs of the parties and on previous communication. We show how to efficiently convert any such protocol into another protocol with comparable efficiency guarantees, that is resilient to adversarial error (for some fixed constant  $\epsilon > 0$ ), while blowing up both the *communication* complexity and the *round* complexity by at most a constant factor. As opposed to most previous work, our error model not only allows the adversary to toggle with the corrupted bits, but also allows the adversary to *insert* and *delete* bits. In addition, our transformation preserves the computational efficiency of the protocol. Finally, we try to minimize the blowup parameters, and give evidence that our parameters are nearly optimal.

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## Privacy Preserving Search of Similar Patients in Genomic Data

Tal Rabin

We described the results of the iDash Competition which we participated in. Showing the approximation algorithm that was optimized for the multiparty computation.

## Revisiting the NP-Hardness of Lattice Problems

SILAS RICHELSON

In this talk we state what is known about the NP-hardness of the two most well studied computational lattice problems: *shortest vector problem* (SVP) and the *closest vector problem* (CVP). Despite the syntactic similarities between SVP and CVP, currently hardness of approximation theorems for SVP are proven very differently than theorems for CVP. Proving hardness of CVP is easier and techniques used for CVP break down for SVP. This results in SVP having slightly worse approximation factors, randomized reductions and more complicated and difficult proofs. In this talk we will survey what is known and we present a framework for reductions to SVP which, if instantiated, would yield a hardness of approximation theorem for SVP analogous to what is currently known for CVP.

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## Cryptography with Updates

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(joint work with Prabhajan Ananth and Abhishek Jain)

The last decade has seen the advent of a vast array of advanced cryptographic primitives such as attribute-based encryption [1, 2], predicate encryption [3, 4, 5, 6], fully homomorphic encryption [7], fully homomorphic signatures [8, 9, 10], functional encryption [1, 11, 12, 13], constrained pseudorandom functions [14, 15, 16], witness encryption [17, 18], witness PRFs [19], indistinguishability obfuscation [20, 21], and many more. Most of these primitives can be viewed as “cryptographic circuit compilers” where a circuit  $C$  can be compiled into an encoding  $\langle C \rangle$  and an input  $x$  can be encoded as  $\langle x \rangle$  such that they can be evaluated together to compute  $C(x)$ . For example, in a functional encryption scheme, circuit compilation corresponds to the key generation process whereas input encoding corresponds

to encryption. Over the recent years, cryptographic circuit compilers have revolutionized cryptography by providing non-interactive means of computing over inputs/data.

A fundamental limitation of these circuit compilers is that they only support *static* compilation. That is, once a circuit is compiled, it can no longer be modified. In reality, however, compiled circuits may need to undergo several updates over a period of time. For example, consider an organization where each employee is issued a decryption key  $SK_P$  of an attribute-based encryption scheme where the predicate  $P$  corresponds to her access level determined by her employment status. However, if her employment status later changes, then we would want to update the predicate  $P$  associated with her decryption key. Known schemes, unfortunately, do not support this ability.

Motivated by the necessity of supporting updates in applications, in this work, we study and build *dynamic* circuit compilers. In a dynamic circuit compiler, it is possible to update a compiled circuit  $\langle C \rangle$  into another compiled circuit  $\langle C' \rangle$  by using an *encoded update string* whose size only depends on the “difference” between the plaintext circuits  $C$  and  $C'$ . For example, if the difference between  $C$  and  $C'$  is simply a single gate change, then this should be reflected in the size of the encoded update. Note that this rules out the trivial solution of simply releasing a new compiled circuit at the time of update.

**Background: Incremental Cryptography.** The study of cryptography with updates was initiated by Bellare, Goldreich and Goldwasser [22] under the umbrella of *incremental cryptography*. They studied the problem of incremental digital signatures, where given a signature of a message  $m$ , it should be possible to efficiently compute a signature of a related message  $m'$ , without having to recompute the signature of  $m'$  from scratch. Following their work, the study of incremental cryptography was extended to other basic cryptographic primitives such as encryption and hash functions [22, 23, 25, 24, 26, 27, 28], and more recently, indistinguishability obfuscation [29, 30].

**Our Goal.** In this work, we continue this line of research, and perform a systematic study of updatable cryptographic primitives. We take a unified approach towards adding updatability features to recently studied primitives such as attribute-based encryption, functional encryption and more generally, cryptographic circuit compilers. We, in fact, go further and also study updatability for classical protocols such as zero-knowledge proofs and secure multiparty computation.

To accomplish this goal, we introduce a new notion of *updatable randomized encodings* that extends the standard notion of randomized encoding [31] to incorporate updatability features. We show that updatable randomized encodings can be used to generically transform cryptographic primitives (discussed above) to their updatable counterparts.

**Updatable Randomized Encodings.** The notion of randomized encoding [31] allows one to encode a “complex” computation  $C(x)$  into a “simple” randomized function  $\text{Encode}(C, x; r)$  such that given its output  $\langle C(x) \rangle$ , it is possible to evaluate

a public `Decode` algorithm to recover the value  $C(x)$  without learning anything else about  $C$  and  $x$ . The typical measure of “simplicity” studied in the literature dictates that the parallel-time complexity of the `Encode` procedure be smaller than that of computing  $C(x)$ . Such randomized encodings are known to exist for general circuits based on only one-way functions [32] (also referred to as Yao’s garbled circuits [33], where the encoding complexity is in  $\mathbf{NC}^1$ ).

In this work, we study *updatable* randomized encodings (URE): given a randomized encoding  $\langle C(x) \rangle$  of  $C(x)$ , we want the ability to update it to an encoding  $\langle C'(x') \rangle$  of  $C'(x')$ , where  $C'$  and  $x'$  are derived from  $C$  and  $x$  by applying some update  $\mathbf{u}$ . We require that the update  $\mathbf{u}$  can be encoded as  $\langle \mathbf{u} \rangle$  which can then be used to transform  $\langle C(x) \rangle$  into  $\langle C'(x') \rangle$ , a randomized encoding of  $C'(x')$ .

The key efficiency requirement is that the running time of the `GenUpd` algorithm must be a fixed polynomial in the security parameter and the size of the update, and independent of the size of the circuit and input being updated. This, in particular, implies that the size of an update encoding  $\langle \mathbf{u} \rangle$  is also a fixed polynomial in the security parameter and the size of  $\mathbf{u}$ .

**Constructions.** In this work, we initiate the study of updatable randomized encodings. Our first result is a construction of multi-evaluation URE for general circuits that supports an unbounded polynomial number of sequential updates. The underlying assumption is a secret-key compact functional encryption scheme for general circuits that supports a single function key query. For the case of polynomially bounded updates, we can, in fact, relax our assumption to only *one-way functions*. We obtain this result by using a single-key compact secret-key FE scheme for an a priori *bounded* number of ciphertexts that is constructed from one-way functions [34, 35].

To the best of our knowledge, such an FE scheme has not been explicitly stated in the literature. However, it follows easily from prior work. Very roughly, a modified version of [34] FE scheme where the encryption and key generation algorithms are “flipped” yields a compact secret-key FE scheme with security for a single ciphertext based on one-way functions. We additionally construct updatable garbled circuits that supports an unbounded number of sequential updates from the family of bit-wise updates. We build such a scheme from worst-case lattice assumptions.

At the heart of this result is a new notion of *puncturable symmetric proxy re-encryption scheme* that extends the well-studied notion of proxy re-encryption [36]. For the case of polynomially bounded updates, we can relax our assumption to only *one-way functions*. We obtain this result by using a puncturable PRF scheme that can be based on one-way functions [37, 38].

**Applications.** We show how to use URE to transform any (key-policy) attribute-based encryption (ABE) scheme into *updatable ABE*. The same idea can be used in a generic way to build dynamic circuit compilers and obtain updatable functional encryption, updatable indistinguishability obfuscation, and so on. We describe two concrete applications, namely, *updatable non-interactive zero-knowledge proofs* (UNIZK) and *updatable multiparty computation* (UMPC). A notable feature

of these constructions is that they only require a URE scheme with *non-output-compact* updates and simulation-based security. Below, we briefly describe our main idea for constructing UNIZKs.

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### **The Hybrid Lattice-Reduction and Meet-in-the-Middle Attack: Improved Analysis and New Directions**

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(joint work with Florian Göpfert, Christine van Vredendaal)

Over the past decade, the hybrid lattice reduction and meet-in-the middle attack [14] (called the Hybrid Attack in the following) has been used to evaluate the security of many lattice-based cryptographic schemes such as the NTRU encryption scheme [13, 14, 11, 10, 12, 19], its recently proposed variant NTRU prime [2], a lightweight encryption scheme based on Ring-LWE with binary error [6, 5], and the signature schemes BLISS [7] and GLP [9, 7]. However, unfortunately none of the previous runtime analyses of the Hybrid Attack is entirely satisfactory: they are based on simplifying assumptions that may distort the security estimates. Such simplifying assumptions include setting probabilities equal to 1, which, for the parameter sets that have been analyzed in previous works, are in fact as small as  $2^{-80}$ . Many of these assumptions lead to underestimating the scheme’s security. However, some lead to security overestimates, and without further analysis, it is not clear which is the case. Therefore, the current security estimates against the Hybrid Attack are not reliable and the actual security levels of many lattice-based schemes are unclear.

In our work [21], we present a unified framework for the Hybrid Attack and give an improved runtime analysis of the attack that gets rid of incorrect simplifying

assumptions. In addition, we apply our analysis to evaluate the security against the Hybrid Attack for the NTRU, NTRU prime, and R-BinLWEEnc encryption schemes as well as for the BLISS and GLP signature schemes. Our results show that there exist in fact security over- and underestimates across the literature.

In our ongoing work [8], we present an improved quantum version of the Hybrid Attack, based on an idea sketched by Schanck [19], that replaces the meet-in-the-middle phase of the attack by a generalized version of Grover's quantum search algorithm for non-uniform distributions over the search space [4]. Therefore, our new Quantum Hybrid Attack can be applied to the Learning with Errors (LWE) problem [18, 16, 17] with arbitrary error distribution – a problem whose hardness is the foundation of many modern lattice-based cryptographic constructions. We provide a detailed analysis of the runtime complexity of our Quantum Hybrid Attack and apply it to the New Hope [1] and Frodo [3] key exchange schemes and the Lindner-Peikert [15] encryption scheme, which are based on LWE with discrete Gaussian (or Gaussian-like) error distributions. Our results show, that the Quantum Hybrid Attack outperforms all other Attacks covered by the LWE estimator with a restricted number of LWE samples [20] for most instances we analyzed, and is comparable for the remaining ones. Furthermore, we analyze the runtime of the Quantum Hybrid Attack on the R-BinLWEEnc [5] encryption scheme, which is based on LWE with binary error distribution, in order to showcase its improvement over the classical Hybrid Attack [6, 21].

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## Delegation with minimal time/space overhead

JUSTIN HOLMGREN

Assuming the existence of somewhat homomorphic encryption, we construct a (2-message)d elegation scheme with an improved asymptotic prover efficiency relative to the prior state of the art. Namely, if the underlying computation is a time- $T$ , space- $S$  computation, our prover runs in time  $T \cdot \text{poly}(\lambda)$  and space  $S + \text{poly}(\lambda)$ , where  $\lambda$  is a parameter determining the soundness of our protocol. The efficiency gap compared to prior work is especially pronounced when we restrict our attention to schemes where soundness is based on "standard" cryptographic assumptions. Our main technical contribution is showing that one can efficiently compute an arbitrary symbol of a classical PCP due to Babai, Fortnow, Levin, and Szegedy ([1]).

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## Beyond Hellman's Time-Space Trade-Offs

KRZYSZTOF PIETRZAK

(joint work with Hamza Abusalah, Joel Alwen, Bram Cohen, Danylo Khilko and Leonid Reyzin)

Proofs of space (PoS) were suggested in [DFKP15, RD16] as more ecological and economical proof systems to replace proofs of work, which are currently used in blockchain designs like Bitcoin. Existing PoS [DFKP15] are based on graph pebbling, much simpler and in several aspects more efficient schemes based on inverting random functions have been suggested, but they fail to give meaningful security guarantees due to existing time-memory trade-offs.

In particular, Hellman [Hel80] showed that any permutation over a domain of size  $N$  can be inverted in time  $T$  by an algorithm which is given  $S$  bits of auxiliary information, whenever  $N \approx S \cdot T$  (e.g.  $S = T \approx N^{1/2}$ ). For random functions a weaker attack  $N^2 \approx S^2 \cdot T$  (e.g.  $S = T \approx N^{2/3}$ ) exists.

We construct functions where for any constant  $k$  we can prove a lower bound of the form  $S^k \cdot T \in \Omega(N^k)$  (in particular,  $S = T \approx N^{k/(k+1)}$ ). Our construction does not contradict Hellman's attacks, which require that the function can be efficiently computed in forward direction. Our function cannot be efficiently evaluated, but its entire function table can still be computed in time quasilinear in  $N$ , which turns out to be sufficient to be used for PoS.

Our simplest construction is build from a random function  $g : [N] \times [N] \rightarrow [N]$  and a random permutation  $f : [N] \rightarrow [N]$  and is defined as  $h(x) = g(x, x')$  where  $f(x') = \pi(f(x))$ , where  $\pi$  can be any permutation on  $[N]$  without fixpoints. For this function we prove that any adversary, who gets  $S$  bits of auxiliary information,

makes at most  $T$  oracle queries, and inverts  $h$  on an  $\epsilon$  fraction of outputs must satisfy  $S^2 \cdot T \in \Omega(\epsilon^2 N^2)$ .

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**Spooky Encryption and its Applications**

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(joint work with Yevgeniy Dodis, Shai Halevi, Daniel Wichs)

Consider encrypting  $n$  inputs under  $n$  independent public keys. Given the ciphertexts  $\{c_i = \text{Enc}_{\text{pk}_i}(x_i)\}_i$ , Alice outputs ciphertexts  $c'_1, \dots, c'_n$  that decrypt to  $y_1, \dots, y_n$  respectively. What relationships between the  $x_i$ 's and  $y_i$ 's can Alice induce?

Motivated by applications to delegating computations, Dwork, Langberg, Naor, Nissim and Reingold [DLN<sup>+</sup>04] showed that a semantically secure scheme disallows *signaling* in this setting, meaning that  $y_i$  cannot depend on  $x_j$  for  $j \neq i$ . On the other hand if the scheme is homomorphic then any *local* (component-wise) relationship is achievable, meaning that each  $y_i$  can be an arbitrary function of  $x_i$ . However, there are also relationships which are neither signaling nor local. Dwork et al. asked if it is possible to have encryption schemes that support such “spooky” relationships. Answering this question is the focus of our work.

Our first result shows that, under the *learning with errors* (LWE) assumption, there exist encryption schemes supporting a large class of “spooky” relationships, which we call *additive function sharing* (AFS) spooky. In particular, for any polynomial-time function  $f$ , Alice can ensure that  $y_1, \dots, y_n$  are random subject to  $\sum_{i=1}^n y_i = f(x_1, \dots, x_n)$ . For this result, the public keys all depend on common public randomness. This scheme is based on a recent multi-key fully homomorphic encryption scheme proposed by Clear and McGoldrick [CM15], later simplified by Mukherjee and Wichs [MW16].

Our second result shows that, assuming sub-exponentially hard indistinguishability obfuscation (iO) (and additional more standard assumptions), we can remove the common randomness and choose the public keys completely independently. Furthermore, in the case of  $n = 2$  inputs, we get a scheme that supports an even larger class of spooky relationships.

We discuss several implications of AFS-spooky encryption. Firstly, it gives a strong counter-example to a method proposed by Aiello et al. [ABOR00] for building arguments for NP from homomorphic encryption. Secondly, it gives a simple

2-round multi-party computation protocol where, at the end of the first round, the parties can locally compute an additive secret sharing of the output. Lastly, it immediately yields a function secret sharing (FSS) scheme for all functions. Thus, in particular, we obtain a function secret sharing scheme for all functions based on LWE (prior to our work such a result was only known based on sub-exponential indistinguishability obfuscation [BGI15]).

We also define a notion of *spooky-free* encryption, which ensures that no spooky relationship is achievable. We show that any non-malleable encryption scheme is spooky-free. Furthermore, we can construct spooky-free *homomorphic* encryption schemes from SNARKs (i.e., succinct non-interactive arguments of knowledge).

We mention some open problems:

- (1) Construct a *spooky free* and yet *homomorphic* encryption scheme from standard assumptions (e.g., LWE) or even from indistinguishability obfuscation. This would imply, in particular, succinct non-interactive arguments for NP (c.f. [GW11]).
- (2) Remove the common random string from LWE based construction.
- (3) Construct an encryption scheme that supports *all* spooky operations (our first construction supports only additive function sharing operations, whereas our second construction supports general operations but only for 2 keys).

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### Average-Case Fine-Grained Hardness

ALON ROSEN

(joint work with Marshall Ball, Manuel Sabin, Prashant Nalini Vasudevan)

We present functions that are hard to compute on average for algorithms running in some fixed polynomial time, assuming widely-conjectured worst-case hardness of certain problems from the study of fine-grained complexity.

We discuss the relevance of such average-case hardness to cryptography and present, as an illustration, an outline of a proof-of-work protocol constructed based on the hardness and certain structural properties of our functions.

### Witness-Indistinguishable Verifiable Mix-nets

SALEET KLEIN

(joint work with Elette Boyle, Alon Rosen, and Gil Segev )

The goal of this work is to explore the possibility of constructing a simple mix-net that is secure against malicious verifiers and in addition is unconditionally sound. This would in particular mean that when applying the Fiat-Shamir transform to the proofs in the mix-net, anonymity would provably be guaranteed for *any choice* of a hash function. While soundness would still be heuristic, unconditional soundness of the protocols makes them less susceptible to theoretical doubts cast on the Fiat-Shamir transform in the case of certain computationally sound protocols [1].

Towards this end, we aim for a relaxed indistinguishability-based notion of anonymity, which is weaker than zero-knowledge and yet guarantees the privacy of voters in the system. We demonstrate how indistinguishability-based anonymity of an entire mix-net system can be attained, even if most of the underlying sub-protocols are merely WI. At the core of our analysis are new techniques for guaranteeing the existence of multiple witnesses in NP-verification relations upon which the soundness of mixnets is based.

We instantiate our ideas with a very simple and appealing Beneš-network based construction due to Abe [2, 3]. While this construction does not match the sub-linear verification efficiency of later mix-nets in the literature (verification time is quasi-linear in the number of voters), it does enjoy a number of desirable features, most notably high parallelizability. In addition, proving and verifying consists of invoking standard and widely used proofs of knowledge, making the mix-net easy to understand and implement.

Abe's mixnet was originally shown to be anonymous assuming honest verifiers, and specifically based on the honest verifier ZK property of the underlying proofs of knowledge. In the case of a *malicious* verifier, these sub-protocols are known only to be witness indistinguishable, alas this guarantees nothing in cases where there is a single witness. Moreover, in Abe's mixnet cases in which only one witness exists *cannot be ruled out*, and if indeed leakage on the single witness occurs in these situations we demonstrate that the system is *not anonymous*.

**Our Results.** We propose two different methods for modifying Abe’s original proposal so that it results in a verifiable mix-net anonymous against malicious verifiers and sound against computationally unbounded provers. Both methods require only minor changes to Abe’s original protocol:

**Lossy Abe mix-net:** This method is identical to Abe’s original proposal, with the only difference being that plain ElGamal encryption is replaced with an alternative, yet equally efficient, encryption scheme with the property that public-keys can be sampled using a “lossy” mode (this mode is only invoked in the analysis). When sampled with lossy public-keys encrypted ciphertexts do not carry any information about the plaintext.

**Injected Abe mix-net:** This method consists of running the original Abe mix-net with some additional dummy ciphertexts that are injected to the system for the purpose of proving D-WI without having to modify and/or assume anything about the encryption scheme in use (beyond it being re-randomizable). The analysis of this construction relies on combinatorial properties of the Beneš-network, and may turn out to be relevant elsewhere.

In both cases, we show that the entire transcript of the mixnet system satisfies the following natural anonymity property: *for any choice of votes and any two permutations on the votes, the corresponding views of an adversary are computationally indistinguishable.*

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#### NTRU Prime

TANJA LANGE

(joint work with Daniel J. Bernstein, Chitchanok Chuengsatiansup, Christine van Vredendaal)

Several ideal-lattice-based cryptosystems have been broken by recent attacks that exploit special structures of the rings used in those cryptosystems. The same structures are also used in the leading proposals for post-quantum lattice-based cryptography, including the classic NTRU cryptosystem and typical Ring-LWE-based cryptosystems.

This talk (1) proposes NTRU Prime, which tweaks NTRU to use rings without these structures; (2) proposes Streamlined NTRU Prime, a public-key cryptosystem optimized from an implementation perspective, subject to the standard design goal of IND-CCA2 security; (3) finds high-security post-quantum parameters for Streamlined NTRU Prime; and (4) optimizes a constant-time implementation of those parameters. The performance results are surprisingly competitive with the best previous speeds for lattice-based cryptography.

For more details see [1].

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### A kilobit hidden SNFS discrete logarithm computation

NADIA HENINGER

(joint work with Joshua Fried, Pierrick Gaudry, and Emmanuel Thomé)

We demonstrate that constructing and exploiting trapdoored primes for Diffie-Hellman and DSA is feasible for 1024-bit keys with modern academic computing resources.

The Number Field Sieve (NFS) was originally proposed as an integer factoring algorithm [3]. Gordon adapted the algorithm to compute discrete logarithms in prime fields [2]. For the past twenty years, the NFS has been routinely used in record computations. The NFS can handle an arbitrary prime  $p$  and compute discrete logarithms in  $\mathbb{F}_p^*$  in asymptotic time  $L_p(1/3, (64/9)^{1/3})^{1+o(1)}$ , using the usual  $L$ -notation, defined as  $L_p(e, c) = \exp(c(\log p)^e (\log \log p)^{1-e})$ .

Very early on in the development of NFS, it was observed that the algorithm was particularly efficient for inputs of a special form. Some composite integers are particularly amenable to being factored by NFS, and primes of a special form allow easier computation of discrete logarithms. This relatively rare set of inputs defines the Special Number Field Sieve (SNFS). It is straightforward to start with parameters that give a good running time for the NFS—more precisely, a pair of irreducible integer polynomials  $f$  and  $g$ , sharing a common root  $m$  modulo  $p$  and satisfying certain size and degree requirements—and derive an integer to be factored, or a prime modulus for a discrete logarithm. The complexity of SNFS is  $L_p(1/3, (32/9)^{1/3})^{1+o(1)}$ , much less than its general counterpart.

Gordon [1] suggested that one could craft primes so that SNFS polynomials exist, but may not be apparent to the casual observer. Heidi, a mischievous designer for a crypto standard, would select a pair of SNFS polynomials to her liking *first*, and publish only their resultant  $p$  (if it is prime) *afterwards*. The hidden trapdoor then consists in the pair of polynomials which Heidi used to generate  $p$ , and that she can use to considerably ease the computation of discrete logarithms in  $\mathbb{F}_p$ .

Let  $\mathbb{F}_p$  be a prime field, let  $\gamma \in \mathbb{F}_p^*$  be an element of prime order  $q \mid p - 1$ . We wish to solve discrete logarithms in  $\langle \gamma \rangle$ .

In Algorithm 1, we recall the method of Gordon to construct hidden SNFS parameters in a DSA setting, in which the subgroup order  $q$  is much smaller than  $p$ . The general idea is to start from the polynomial  $f$  and the prime  $q$ , then derive a polynomial  $g$  such that  $q$  divides the resultant of  $f$  and  $g$  minus 1, and only at the end check if this resultant is a prime  $p$ . This avoids the costly factoring of  $p - 1$  that would be needed to check whether there is a factor of appropriate size to play the role of  $q$ .

**Input** : The bit-sizes  $s_p$  and  $s_q$  for  $p$  and  $q$ ; the degree  $d$  of  $f$ .

**Output:** HSNFS parameters  $f, g, p, q$ .

Pick a random irreducible polynomial  $f$ , with  $\|f\| \approx 2^{s_q/2(d+1)}$ ;

Pick a random prime  $q$  of  $s_q$  bits;

Pick a random integer  $g_0 \approx 2^{s_p/d}/\|f\|$ ;

Consider the polynomial  $G_1(g_1) = \text{Res}_x(f(x), g_1x + g_0) - 1$  of degree  $d$  in  $g_1$ ;

Pick a root  $r$  of  $G_1$  modulo  $q$ ; if none exists go back to Step 1;

Add a random multiple of  $q$  to  $r$  to get an integer  $g_1$  of size  $\approx 2^{s_p/d}/\|f\|$ ;

Let  $p = |\text{Res}_x(f(x), g_1x + g_0)|$ ;

If  $p$  has not exactly  $s_p$  bits or if  $p$  is not prime or if  $q$  does not divide  $p - 1$ , then go back to Step 1;

Return  $f, g, p, q$ .

**Algorithm 1:** Gordon's hidden SNFS construction algorithm

Twenty-five years later, we reconsider the best-case scenario for Heidi: given a target size, what type of polynomial pair will give the fastest running time for a discrete logarithm computation? Back in 1992, when Gordon studied the question, the complexity analysis of the Number Field Sieve was not as well understood, and the available computing power was far less than today. At that time, the proposed parameter sizes for DSA were to generate a 160-bit prime subgroup modulo a 512-bit prime  $p$ , leading to difficulties satisfying the condition of Algorithm 1 unless a suboptimal degree  $d$  for polynomial  $f$  was chosen. Nowadays, popular DSA parameters are 1024-bit primes  $p$  with 160-bit subgroup order, leaving much room for the condition to hold, and it is possible to choose  $d = 6$ , which is optimal for our NFS implementation. Therefore, Gordon's algorithm can be run with optimal parameters, and detecting that  $p$  has this trapdoor seems out of reach.

We generated such a trapdoored 1024-bit prime. Our chosen prime  $p$  looks random, and  $p - 1$  has a 160-bit prime factor, in line with recommended parameters for the Digital Signature Algorithm. We then performed a special number field sieve discrete logarithm computation for this prime. To our knowledge, this is the first kilobit-sized discrete logarithm computation ever reported for prime fields. This computation took a little over two months of calendar time on an academic cluster using the open-source CADO-NFS software.



Twenty-five years ago, there was considerable controversy around the possibility of backdoored parameters for DSA. Our computations show that trapdoored primes are entirely feasible with current computing technology.

As can be expected from a trapdoor mechanism which we say is hard to detect, our research did not reveal any trapdoored prime in wide use. The only way for a user to defend against a hypothetical trapdoor of this kind is to require verifiably random primes.

Cryptosystems based on the hardness of discrete logarithms should have ceased to use 1024-bit primes entirely already a while ago. NIST recommended transitioning away from 1024-bit key sizes for DSA, RSA, and Diffie-Hellman in 2010 [4]. Unfortunately, such key sizes remain in wide use in practice. Our results are yet another reminder of the risk, and we show this dramatically in the case of primes which lack verifiable randomness.

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