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Mathematical Instruments between Material Artifacts and Ideal Machines: Their Scientific and Social Role before 1950

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Abstract. Since 1950, mathematicians have become increasingly familiar with the digital computer in their professional practice. Previously, however, many other instruments, now mostly forgotten, were commonly used to compute numerical solutions, generate geometrical objects, investigate mathematical problems, derive new results, and apply mathematics in a variety of scientific contexts. The problem of characterizing the mathematical objects that can be constructed with a given set of instruments frequently prompted deep theoretical investigations, from the Euclidean geometry of constructions with straightedge and compass, to Shannon's theorem which, in 1941, stated that the functions constructible with a differential analyzer are exactly the solutions of algebraic differential equations. Beyond these mathematical considerations, instruments should also be viewed as social objects of a given time period and cultural tradition that can amalgamate the perspectives of the inventor, the maker, the user, and the collector; in this sense, mathematical instruments are an important part of the mathematical cultural heritage and are thus widely used in many science museums to demonstrate the cultural value of mathematics to the public. This workshop brought together mathematicians, historians, philosophers, collection curators, and scholars of education to address the various approaches to the history of mathematical instruments and compare the definition and role of these instruments over time, with the following fundamental questions in mind – What is mathematical in a mathematical instrument? What kind of mathematics is involved? What does it mean to embody mathematics in a material artefact, and how do non-mathematicians engage with this kind of embodied mathematics?

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Introduction by the Organisers

What is a mathematical instrument? The main aim of the workshop will be to question and make precise the meaning of this expression prior to 1950, that is, before the appearance of the digital computer. Instruments have been seen in very different ways in different mathematical cultures, which is one of the reasons we now have trouble characterizing the situation in a global and definite way. For example, at particular times, certain types of tables were seen as instruments and not distinguished in terminology from a brass, wooden or paper instrument performing the same function. Referring to two major ancient surveys, there are deep differences between the definitions and classifications found in Nicolas Bion's *Traité de la construction et des principaux usages des instruments de mathématiques* (1709), compared to those found in Friedrich Willers's *Mathematische Instrumente* (1926). The changing role and character of mathematical instruments is a sure reflection of the changing nature of mathematical disciplines generally, so there is a danger, in using too narrow a definition, of missing some important conclusions that might otherwise emerge. It is safer, therefore, to allow 'instrument' in the various epochs we will cover to mean what the contemporary mathematicians (again perhaps self-defined) took the term to mean. Fortunately, similar words are used in the language traditions we are likely to cover, and so terms such as 'organon', 'machine', 'device', 'apparatus', etc., are all valid.

In the earlier period there are many instruments that perform, what we would consider, mathematical operations, and so have necessarily been designed by people with mathematical skills, but whose professions and services may not immediately appear mathematical to us – surveying, navigation, cartography, artillery, etc. The 'mathematical' operations are not explicit – they (at least from our viewpoint, but perhaps not as seen in the period) are contained within the operation of the instrument. Astronomical instruments such as astrolabes, quadrants, Ptolemy's rulers, and equatoria, were the most common mathematical instruments for a long period during the Middle Ages. They exploit, to a large extent, geometrical properties in a clever way to transpose between coordinate systems or metrics. Dividing their scales and accurately adjusting them to parameters such as latitude or epoch would have required considerable mathematical skill. Another example is that of perspective machines, which represent an embodiment of a mathematical theory, even if this was not always perceived by the users of these machines. More generally, mechanical devices such as compasses, slide rulers, levers, balances, etc., were largely used by non-mathematicians (until today).

For the historian, one of interesting features of mathematical instruments is that they stand at the crossroad of the making and the use of material artefacts, and the development of abstract concepts, methods and theories. They are involved simultaneously in technology and mathematics (here we think of 'pure' mathematics as well as 'mixed' or 'applied' mathematics where these distinctions make sense). In the early days, the approach to mathematics was perhaps a more mixed one. From Nicomedes's conchoid tracer to Galileo's sector to Oughtred's circular slide rule, geometry sometimes experimented with using more than just compass and straightedge, and the calculators sometimes used graphical means to make approximations. Up to and including mechanical calculating machines and differential analyzers, all these tools permitted mathematicians and other scientists to experiment on numbers, curves, functions, and solutions of algebraic or differential equations in order to make conjectures and develop new theories. The rigorous and clever use of these devices was also closely linked to the creation of numerical and graphical methods of calculation that gave birth to numerical analysis as an autonomous discipline at the beginning of the 20th century. Far from actual calculation and measurement devices, mathematical instruments could be also thought of as 'ideal' machines: mechanical linkages described by Descartes, or tractional integraphs imagined by Leibniz and Vincenzo Riccati were not introduced at first to be manufactured and employed for actual use, but conceived for theoretical purposes, as a way of legitimizing the use of new curves and new types of constructions in geometry. Thus the situation appears complex: the workshop will have to examine, in depth, the creative and genetic role of instruments in the development of mathematics.

In studying mathematical instruments, the interaction between different group cultures and different mathematical traditions inevitably emerges, so our workshop will not neglect a more social approach. It starts from the observation that, at least in the ancient times, scholarly mathematics was mainly promoted by people with university training, who knew Latin (and sometimes Greek), who also had a rhetorical and methodological training in how to think and write; instrument makers did not generally have such an education. Nevertheless, the makers had access to a mathematical tradition, in part oral, in part written, that allowed them to often think of the same questions and problems as the scholarly people. On the subject of interaction between artefact and theory, this is not something necessarily associated with one mind, one inventor, one mathematician – it could also mean interaction between people from different groups, between those who materially and technically assemble instruments and those who offer theories on mathematical properties. Instruments are also what their users make them. In the history of technology, which sometimes comes close to that of mathematical instruments, one can write a narrative from the point of view of the makers, but also from the point of view of the users. This raises questions such as – What do these instruments stand for, and how are they designed, discussed, made, promoted, sold, used, explained, taught, represented, and advertised? Why were they bought, admired, published in so-called theatres, etc.? And also, of course, where were they collected, displayed, etc.? All these social interactions are what make mathematical instruments such unique research object. They are conceived, but also built (where? how? by whom?) and they are not used exclusively by mathematicians; that is to say that we have a large array of interested parties, and each needs to be considered and studied.

Many books and papers have already been published on the subject of mathematical instruments but, in our opinion, the majority of these studies have been conducted in too independent and specialized a way by mathematicians, philosophers of science, historians of mathematics, historians of computing, historians of astronomy, historians of technology, historians of engineering, museum conservators, private collectors, and researchers in mathematics education. The novelty of our workshop is that it gathers scholars from all these communities to work together. Regarding the organization of the meeting, we will try to balance general sessions devoted to defining, characterizing and classifying mathematical instruments at different times – taking into account the points of view of the different individuals involved – with specific sessions devoted to certain periods, certain geographical regions, certain areas of mathematics, certain professional milieus, certain types of instruments, or certain social aspects of their intervention. We hope that this will give rise to new insights into this broad-ranging subject and stimulate future, pertinent investigations.

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Workshop: Mathematical Instruments between Material Artifacts and Ideal Machines: Their Scientific and Social Role before 1950

Table of Contents

Jim Bennett
Mathematical instruments in Florence: A cataloguing project and its relevance for historians of the sixteenth century ... 3477

Volker Remmert
Iconography on early modern scientific instruments: Types and messages 3480

Peggy Aldrich Kidwell
Ideals embodied: Models of mathematical instruments in the United States, 1820-1950 ... 3482

Jeanne Peiffer
Sixteenth century perspective drawing instruments: Types, goals, status, locations, uses ... 3484

Maria G. Bartolini Bussi
Artifacts for perspective drawing: From painters' workshops to mathematics classrooms ... 3487

Michela Maschietto
Artifacts for geometrical transformation and drawing curves in the classrooms, workshops and exhibitions ... 3490

Samuel Gessner, Michael Korey
Equating the Sun: Variant mechanical realizations of solar theory on planetary automata of the Renaissance ... 3493

Martina Schiavon
Actors, places of exchange, and mathematical knowledge: Following the chronograph in the minutes of the Bureau des longitudes, France, 19th-20th centuries ... 3495

Antoni Malet
Measuring goods, or the social origins of the early modern "arithmetization" of ratio and proportionality ... 3499

Joachim Fischer
The early history of mechanical integration: The first five decades, 1814-1864 ... 3502

June Barrow-Green
"An exquisite machine": Olaus Henrici's harmonic analyser ... 3506

Marie-José Durand-Richard
Mathematical machines in tide analysis and prediction (1876-1950) 3510

Davide Crippa
Exactness in Leibniz's mathematics: Instruments to construct transcendental curves 3514

Dominique Tournès
Instruments for impossible problems: Around the work of Ljubomir Klerić (1844-1910) 3517

Pietro Milici
Tractional constructions as foundation of differential equations: Ancient open issues, new results, possible fallouts 3520

Seb Falk
Mathematical instruments in medieval monasteries 3523

Richard Kremer
Geometrical tools and mathematical practices: An exploration of BSB Cod.icon. 182 (1520s) 3525

Petra G. Schmidl
Comparing astrolabes 3529

Catherine Eagleton
What were portable astronomical instruments used for in late-medieval England, and how much were they actually carried around? 3533

Arianna Borrelli
Were early modern optical diagrams mathematical instruments? 3537

Renate Tobies
From academic to practical areas in Germany: The use and development of mathematical instruments in the first decades of the 20th century 3540

Loïc Petitgirard
From analogue to digital mathematical instruments: Examples of mathematical practices with machines in the 1950s and 1960s 3544

Marc Thomas
A brief history of the slide rule 3548

Thomas Morel
Subterranean geometry and its instruments: About practical geometry in the early modern period 3550

Nathalie Daval
The nomogram: An artifact changing during the First World War 3553

Abstracts

Mathematical instruments in Florence: A cataloguing project and its relevance for historians of the sixteenth century

Jim Bennett

I want to report on work located in traditional museum practice, in the area of collections and cataloguing. The resource I want to present is the collection – not the single instrument and still less written commentary on instruments in their period of use – and the activity I want to invoke is cataloguing.

For some years I have been working intermittently on a catalogue of mathematical instruments in the Museo Galileo in Florence. My initial brief from the museum was to catalogue the 'surveying instruments' and that title still conveys the core content, but it proved impossible to confine the scope of the catalogue to 'surveying' as we understand the discipline today. The instruments refused to respect that description, their functionality breaking through the boundaries we might expect would limit their design. It was often the case that no sooner had I selected an instrument, because it clearly fell into my assigned discipline, than I found that it was equally at home in other fields as well.

Surveying sits at the heart of practical geometry in the Renaissance and early-modern periods. The very name 'geometry' seems to point to origins in the measurement of land, while the impulse to extend geometrical practice into different areas of work, characteristic of instrumentation in the 16th century, was readily answered through the spread of techniques rooted in surveying. The very simplicity of geometrical survey facilitated applications in building, gunnery and fortification. Further, the trend to promote the underlying coherence of geometrical practice encouraged the natural tendency of instrument makers to multiply the functionality of their designs for commercial and reputational reasons. Space was readily found for surveying; even the noble astrolabe almost always includes the simple 'shadow square', of scarcely any relevance to astronomy but ostensibly useful in surveying.

The ubiquitous appearance of instrumental features pertinent to surveying, where proportionality is obviously relevant to mapmaking, makes it difficult to set the boundaries to a catalogue of 'surveying instruments' in a collection as rich as that in Florence. With the agreement of the museum, we settled on characterising the scope of my catalogue as 'surveying and related instruments', which covers quite a wide area of practical mathematics.

The goal of incorporating historic instruments into the evidence mustered by the historian of science has been a common aspiration in recent years, but integrating museum collections into mainstream research has proved a greater challenge. Here the collection in Florence has a considerable advantage. Museums will often acquire individual instruments, occasionally with some provenance, though much more often not, or they may acquire a collection assembled by an enthusiast. We might think of these usual museum collections as 'artificial'. In Florence most of the collection has survived since its accumulation by bodies and agencies that predate the museum by centuries, which gives much of it a coherent historical context and enhances its collective value as evidence. Its limitations are also readily understood: extravagance, both technical and decorative, is quickly obvious and points immediately to the princely context of the collection's creation.

The collection is a resource for considering the character of mathematics in the 16th and 17th centuries, though 'resource' has become an overused word, with no very strong meaning or character. What the collection offers is a setting, a framework, a context for thinking and discussing, a tool for suggesting connections and testing characterisations. The collection as a whole plays its part, as does the study of individual instruments, especially a study shaped and disciplined by the uncompromising demands of cataloguing.

The collection was assembled under the patronage of the Medici Dukes of Tuscany in the 16th and 17th centuries [2]. For reasons of space I shall select only a few instruments from the time of Cosimo I and his immediate successors. We know not only the occasion of forming the collection, linked to Cosimo's military, political and administrative ambitions, but also a lot about its original setting, housed in the remarkable *Sala delle carte geografiche*, designed, but never completed in the *Guardaroba* of the *Palazzo Vecchio*, as an 'immersive' cosmography of the earth and the heavens [1].

Not many instruments in the collection were solely for surveying. Even so straightforward an example as an azimuth theodolite [3] is also a sundial and a protractor. Combined functionality is typical and, if we look across the collection, the most common combination with surveying is gunnery and the military arts in general. The basic idea of the azimuth theodolite is taken far in this direction by this instrument [4] by Baldassarre Lanci of Urbino, dated 1557, which may well have been one of the first instruments in the *Guardaroba*. The circle with a degree scale, grouped again into eight winds, needs two pairs of sights and these are placed on legs that are curved in the manner of external callipers. As well as functioning as an azimuth theodolite, it is a gunner's calliper, so requires an additional scale, since the theodolite measures angles, but the callipers distance. While the callipers are for measuring shot, this instrument has further connections to gunnery: set on the moveable arm is a quadrant that acts as a gunner's level, clinometer and sight and when closed it is adapted to sit on a large gun. It is worth noting the steel points beyond the ends of the curved legs, which thus act as a pair of compasses (today we would say 'dividers') as well as serving as sighting arms and callipers. The gunner's clinometer and sight folds down and can be removed to make the function as compasses less cumbersome.

Pursuing the idea of the collection as a context for discussion, in another instrument [5] we find a further extravagant elaboration of a pair of callipers, incorporating a quadrant in degrees, a clinometer scale read by a plumb-line hanging from the pivot, and a divided arc for measuring shot (as well as an equinoctial sundial), while the terminal points again invoke a pair of compasses.

Terminal points suggest a different discursive track, beginning with straight-legged compasses [6] instead of curved callipers. The flat-legged model [7] is typically Italian and very often has two pairs of scales, one for the division of the line (the distance between corresponding values on the two scales divides the distance between the compass points by that value) and one for polygons (the distance between equivalent values is the side of the regular polygon with this number of sides, inscribed in a circle whose diameter is the distance between the points of the compass). Both functions are relevant to surveying, the former for drawing plans to scale, the latter for laying out polygonal forts. The design of the hinge that makes the flat legs possible also makes possible a different way of registering the opening, as an angle, though not necessarily in degrees [8].

We saw a connecting arc in one of the calliper instruments, graduated for shot, so not in degrees, but the idea of a connecting arc can be used as a different register of the angle between the legs than an index on the hinge. This instrument [9] was made in Lucca in 1604. The flat legs have the familiar pairs of scales for dividing the line and for polygons, and we note the steel points, so we have a relationship with the standard flat-legged compasses. The most prominent new feature is the arc and it has a degree scale, giving the opening of the legs. To use that for surveying would require sights, which are found as accessories in the case, where there are also two clinometers, a quadrant and a gimbaled magnetic compass in a ring divided by degrees and marked with the winds.

There is another significant new feature here. On the recto face both legs have the same linear scale running along their lengths from the pivot. At first sight this might seem to relate to the sector. The cataloguer, however, has to try to make sense of everything and in the case there is one pin sight mounted on a sleeve that can embrace either arm, along which it can move or be fixed by a clamping screw. There were surely originally two of these sights (there are empty spaces in the fitted case), used for sighting across the arms for surveying by triangulation, setting one sight to the baseline (to scale) and sighting across the arms to form a triangle similar to that on the ground. The sector does become a development from this type of instrument but not just yet.

I conclude with another rather elaborate instrument [10], made in Venice by Antonio Bianchini. It has features we recognise: the opening of the legs given by a degree scale at the hinge, positions marked on the hinge at the openings for the internal angles of regular polygons, a plumb-line incorporated into the hinge for measuring altitudes with the same sights, and a shadow square. If we look underneath, there is a strip of brass riveted to the fixed leg, close to the inner edge, that looks as though it accommodated some sliding fitting no longer extant. A cataloguer has to worry about all unexplained features; in this case the strip is for the sliding mount for a third rule (now missing) for use in triangulation.

In many ways this is a cosmographical instrument and particularly suited to being part of the explicitly cosmographical space of the *Guardaroba*. The date on the instrument, 1564, is exactly right for the unfulfilled cosmographical ambitions of the *Guardaroba*. On the instrument the magnetic compass with the winds, the sundial, the full wind rose, the altitude function, the table of latitudes, and the diagram for finding the altitude of Polaris all have cosmographical significance. Yet the residual points at the termini of the legs, now almost decorative rather than functional, help place the instrument in relation to many others in the collection.

I hope these few examples begin to show that it is possible to have a discourse within a collection of instruments in a manner equivalent to analysing a text and that this can be based on traditional practices from museum culture – a knowledge of the collection derived from the discipline of the catalogue.

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Iconography on early modern scientific instruments: Types and messages

Volker Remmert

During the *Scientific Revolution* scientific instruments, such as astrolabes, air pumps, microscopes and telescopes became increasingly important for the study of nature. In the early modern period they had not yet reached the status of standardized and impersonal means to study nature. Rather they usually were unique items which, by their function as well as their design, could serve the mediation between scholars, social elites and beyond. In this context the iconography on the instruments played a crucial role. In fact a great number of early modern instruments are adorned with images that in themselves have no relevance for the use of the instruments, as for instance the depiction of Atlas and Hercules on an astrolabe by Praetorius (1568, Dresden) or the line of tradition in astronomy and geometry on Bürgi's astronomical clock (1591, Kassel) stretching from the church fathers to Copernicus. As of now such imagery on instruments and its contexts have only sporadically been analysed.

My project *Iconography on early modern scientific instruments* specifically analyses the imagery on the instruments. It aims for the first time at a systematic analysis of the multifaceted visual material on the instruments asking for its role in the various contexts of the adorned instruments (genesis, function, use) and its importance for setting up or supporting stories/histories of success and relevance within the emerging field of the sciences. The iconography points to quite a few significant topics as, for instance, statements of specific positions in theoretical debates (e.g. Copernican question), mediation and illustration of knowledge, in particular by picturing the usability of the instruments, or the role of instruments as patronage artefacts with specific iconographic programmes.

The analysis of the imagery is likewise highly relevant in order to understand the intellectual, cultural and artistic contexts shaping and determining the production of instruments in the early modern period. It opens a window on the investigation of collaborative processes during the conception, design and construction of instruments in the multi-layered field between instrument makers, artists, artisans, patrons and scholars.

In my talk I outlined a preliminary list of topics, that I expect to be reflected in the imagery on the instruments and gave specific examples for each category:

1. Illustration of application/applicability
2. Contemporary fashionable or religious designs/topics (or ornaments), heraldic signs
3. Illustrations pertaining to mathematical sciences (e.g. artes liberales, quadrivium, astronomers, personifications, etc.)
4. Legitimization strategies
   - 4.1. Myths and legends
   - 4.2. Invention/construction of tradition
   - 4.3. Self-fashioning
5. Debate on the world systems
6. Illustrations drawing on printed material

Naturally, often the iconography is not necessarily exclusive to one category alone, but as I was not pursuing the in-depth analysis of specific instruments in my presentation, it sufficed to concentrate on particular aspects. It is to be noted that instruments as patronage artefacts are a category in themselves and run through all the above categories.

Obviously, the project poses many problems upon which I have scarcely touched. One is very elementary: how to find the material relevant to the project? While databases and printed catalogues greatly help, a more systematic survey will involve quite some travelling. Also, there is the problem of how to find ways to understand and analyze the role of collaborative processes, for instance between designer and craftsmen, between prospective patrons and designers, etc. to get to grips with the more challenging aspects of the specific iconography.

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Ideals embodied: Models of mathematical instruments in the United States, 1820-1950

Peggy Aldrich Kidwell

Novel social, mathematical and technical ideals have long been embodied in objects. In the United States, these included prototypes for new methods of instruction, models submitted to the Patent Office, early production models of machines, and one-of-a-kind computers. Four devices illustrate this embodiment of mathematical ideals. The first is an early numeral frame of a form brought to the U.S. by the French educator William S. Phiquepal and used to teach young children in New Harmony, Indiana, in the 1820s. The second is an 1857 model of an adding machine submitted to the United States Patent Office by Thomas Hill, a minister in Waltham, Massachusetts. The third is an early production model of a calculating machine designed by engineer George B. Grant in the 1870s. Finally, the mid-twentieth century ASCC Mark I computer suggests innovations of mathematician and computing pioneer Grace Murray Hopper.

These objects illustrate transformations in American mathematics education. The numeral frame used in New Harmony was part of an introduction of arithmetic teaching into the general education of young children in the northern states and eventually throughout the country. Hill not only wrote textbooks for children and served as president of Oberlin College and then Harvard University, but encouraged the development of undergraduate education that included specialization in a major subject such as mathematics. Grant attended a special school established at Harvard to train engineers in science and mathematics as well as practical subjects. Finally, Hopper obtained a PhD. in mathematics from Yale University. She did so considerably after the beginning of graduate work in the U.S., and was part of a group of American mathematicians with sufficient expertise to shape the programming of early computers.

These objects also suggest the changing place of mathematical instruments in American society. The teaching abacus developed in response to the Pestalozzian ideals of several French and English authors. A Russian abacus was brought to France by the mathematician Poncelet, who encountered it as a prisoner at the time of the Napoleonic wars. The reformer Phiquepal, aided by the Scottish philanthropist William Maclure, brought it to Philadelphia and then to the Indiana community of New Harmony. Phiquepal would soon abandon teaching and return to Europe, but several American firms took up production of the abacus and other inexpensive teaching apparatus for the schools [3]. Thomas Hill's interest in mathematical instruments was equally transient, although adding machines embodying some ideas from his patent would become common commercial products in the late nineteenth century [2].

Calculating machines and objects relating to them played a much larger role in the life and career of Grant. As a student Grant was asked to solve a problem relating to earthworks and embankments. His solution required building what Babbage called a difference engine. Grant built a difference engine – it was exhibited at the 1876 Centennial Exposition, a World's Fair held in Philadelphia.

He also began to build small, cheaper calculating devices. Grant exhibited calculating machines both in 1876 (see Fig. 1) and at the 1893 Columbian Exposition in Chicago. He continued to improve them throughout his life – none proved a commercial success. To build these instruments, Grant needed precise gears. To produce them he founded several successful gear works. He also wrote what became a standard treatise on the theory of gears and took out patents relating to gear making. Standard gears would be central to the development of the calculating machine industry in the United States. Moreover, gears from a company founded by Grant would be incorporated in such other computing devices as the differential analyzer built by Vannevar Bush of MIT in the 1920s [4].

FIGURE 1. A Barrel-Type Calculating Machine by George B. Grant, Gift of Robert K. Otnes, Smithsonian Negative Number AHB2016q012589

Similarly, Hopper's involvement with the ASCC Mark I computer transformed both her career and mathematical instruments. After receiving her PhD., Hopper taught mathematics at Vassar College. When the U.S. entered the fighting in World War II, she left there, enlisted in the U.S. Navy, and was sent to Harvard as a programmer for the Mark I. Her longest running wartime project was the calculation, printing and publication of tables of values of Bessel functions. Relevant instructions and programming tapes survive in her papers [1].

Computers occupied Hopper for the rest of her life. After World War II, she worked as a civilian at Harvard and then took a position as a "mathematician" at the Eckert-Mauchly Computer Company, the first American manufacturer of commercial electronic computers. She is remembered for her work with compilers; her development of the programming language FLOW-MATIC, which was far closer to spoken English than its predecessors; and her championship of the later English-based business language COBOL. At the time of the Vietnam War, women once again were welcome in the U.S. Navy. Hopper reenlisted, devoted herself to military uses of information technology, and remained in the Navy until retirement at age seventy-nine.

Work of Phiquepal, Hill, Grant, and Hopper demonstrates a continuing interplay between new ideas, new kinds of objects, and actual commercial products. Mathematical concerns ranged from arithmetic to Bessel functions. Such social forces as education, religion, immigration, industrialization, and warfare shaped the instruments proposed. Some ideas played only a minor role in long lives, others proved transformative. These visions of new instruments, new components, and new ways of using old instruments proved of lasting influence. In summary, careful study of novel instruments and those who designed them offers rich rewards for the historian of mathematics and computing.

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Sixteenth century perspective drawing instruments: Types, goals, status, locations, uses

Jeanne Peiffer

No systematic and comprehensive study of early modern perspective instruments exists, although most studies on perspective include apparatuses and deal with numerous aspects of the question (See for instance [7], [1]). Only a few of these drawing devices survive and are on display in museums. They are mainly known by their descriptions in perspective treatises, by drawings, engravings, or by copies built with modern materials according to the instructions found in the ancient treatises. In my paper, I have followed one line of inventions, a German tradition originating with Albrecht Dürer, pursued by his Nuremberg followers, and proceeding a century later via Italy to France.

In his *Underweysung der messung* [4], Albrecht Dürer included four famous engravings of perspective instruments. He relied on the Albertian model of a perspectival representation (as described in Alberti's *De pictura*, 1435), the image of an object being defined as the intersection of the visual pyramid having at its apex the eye point and at its base the object to be represented. Each of Dürer's four devices has been described: Dürer's grid (Portrait of a nude, 1538), his glass (Portrait of a seated man,1525), his window (The painter of the lute, 1525) and the machine attributed in 1538 to Jacob Kayser. The function of Dürer's window has been discussed; since its use requires some measure of skill and is rather time-consuming, it appears to be more a demonstrative device than a working tool. Some of Dürer's followers, such as Hans Lencker or Wentzel Jamnitzer, have invented their own perspective instruments, elaborating upon some of Dürer's ideas [6]. At the turn of the century, Paul Pfinzing [9] and, to a lesser extent, Johann Faulhaber [5], built a German tradition of perspective instruments starting with Dürer (although this is not quite true since Leonardo da Vinci had already designed a device similar to Dürer's window in the *Codex Atlanticus*). Pfinzing gave his own interpretation of Dürer's window, but also of the apparatuses built in Nuremberg by the goldsmith Wentzel Jamnitzer and by the musician Hans Haiden. He claims to have seen Jamnitzer's instrument installed in a room of his house and crafted beautiful, but somewhat strange, engravings representing these instruments.

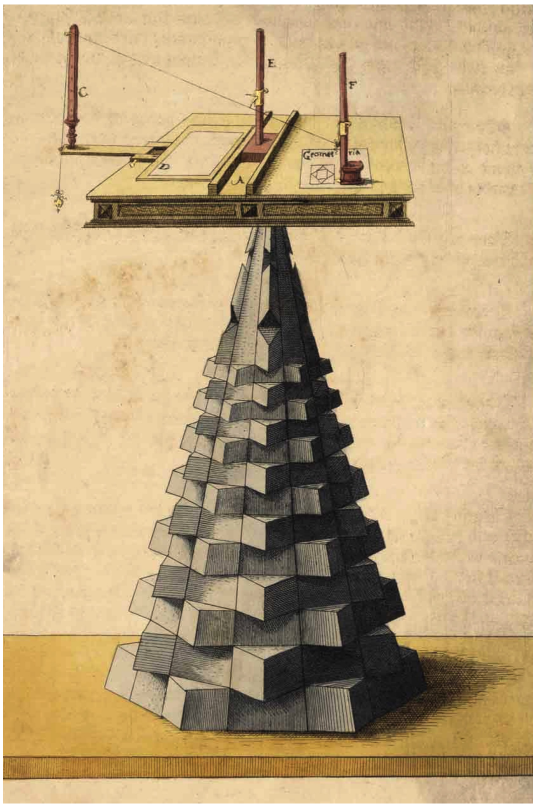

FIGURE 1. "Wentzel Jamnitzer Goldschmidt von Nürmberg / Anno 1568. der bringt deß Albrecht Dürers Perspectiv mit der Saiten wider an Tag / mit der verbesserung: An statt deß Rohms / und derselben darein gehefften Schnürlein oder Fäden / und auch an statt deß Steffts / richt er zwey lange Instrument / so man schieben und rucken kan / auff / damit er allein ohne hilff anderer Leuth arbeiten kan" ([9], fol. 9/10, `http://digital.bib-bvb.de/publish/viewer/43/162690.html`)

In the talk we have carefully described the changes, variants, and appropriations that occurred, not only in the conception of the instruments but also in their representations. Thus the human operators disappeared from Pfinzing's representation of Dürer's window and the lute was replaced by a plane. In Jamnitzer's version, as pictured by Pfinzing, the frame of the window has been replaced by a mobile pin. While Dürer's devices were placed in a domestic setting, the later ones built in the German tradition became portable, and parameters such as the distance from the eye, or that between the picture plane and the object, became variable.

As Filippo Camerota [3] has shown, Jamnitzer's instrument, a variant of Dürer's window, circulated via a woodcut by Jost Amman and came into the hands of the painter and friend of Galileo, Lodovico Cigoli, who copied it and designed his own, slightly different, versions of the instrument (in his ms "Prospettiva pratica", ca 1613, edited by Filippo Camerota, and in drawings kept in Berlin, London and Williamsburg). Cigoli's instrument came to be known in Europe via Jean-François Niceron's *Thaumaturgus opticus* [8], who had seen Cigoli's instrument in the collection of Louis Hesselin, a high-ranked official at the French court. According to Camerota [3], the instrument might have been offered to Hesselin by the grand-duke Ferdinando II dei Medici, to whom Cigoli's nephew had donated "Prospettiva pratica" (perhaps together with the instrument). The invention of perspective instruments had thus progressively become a goal of intellectual prestige aspired to by artists; from domestic use to display in prestigious collections.

Note that all perspective representations obtained by these devices were constructed pointwise. Some were meant to be used to draw a physical object, some only required a plan and an elevation of the object. The involvement of the latter two planes allows us to characterize these perspective devices as mathematical. The question of the uses of the perspective apparatuses still remains open and warrants the collection of further evidence.

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Artifacts for perspective drawing: From painters' workshops to mathematics classrooms

Maria G. Bartolini Bussi

1. Introduction. Aim of talk given at Oberwolfach was to present the collection of artefacts for perspective known as *Perspectiva Artificialis* (in short, perspectographs) at the University of Modena and Reggio Emilia (UNIMORE). This collection is a part of the larger collection of mathematical machines that includes also curve drawing devices, pantographs for geometric transformations and instruments for problem solving: the whole collection has been designed and built, drawing on historical sources, by the Associazione Macchine Matematiche, a no-profit association of school teachers who had used and are still using them for didactical purpose and popularization. The rationale for this demanding project of reconstruction, lasting some decades, was to allow students and visitors to handle them in order to appropriate mathematical and historical meanings. This report is linked to the one authored by Michela Maschietto.

2. A short history of the collection. The first public exhibition of the collection of mathematical machines at UNIMORE [3] was realized in 1992. At that time, the collection contained several dozens of curve drawing devices and pantographs for geometric transformations. Later the collection of perspectographs was started: now it contains more than 40 artefacts including painters' instruments for perspective drawings, anamorphoses, models of shadows and some dynamic representations of fundamental theorems (e. g. Stevin's theorem, De la Hire's theorem on homological transformation of a circle into a parabola). Among others, all the instruments designed by Dürer (1525) and by Barozzi & Danti (1682) have been reconstructed in actual size. Several public exhibitions with paper or digital catalogues have been realized. Also a small collection of instruments for blind people (*Geometry on the fingers*) exists, where the plexiglass planes used in many standard perspectographs are substituted by wire mesh planes (to allow tactile exploration crossing the plane with fingers), and rays are represented by thick threads which can be felt [7].

3. The function of perspectographs in the Western intellectual history. Perspectographs are paradigmatic examples of those instruments that are described in the introduction to the Oberwolfach workshop. In the following, some issues are outlined.

3.1. The genesis of the vanishing point. Perspectographs were first created for practical purposes and were used in painters' workshops with the aim of supporting the production of illusionistic images of the world. They were also used in the early practice of technical drawing for military art and architecture. They embody the mathematical model of projection from a point, representing the painter's eye (a single eye).

3.2. Perspective as a symbolic form. The development of perspective, even before its complete mathematisation, is strictly intertwined with a mathematical model of vision and with a theory of infinite and homogeneous space, which are different from the ancient ones and which are the basis for the modern attitude towards space. In this sense, perspective can be assumed, after Cassirer, as a 'symbolic form', or 'a concrete perceptible sign connected to a peculiar spiritual content and intimately identified with it', as defined by Panofsky ([5], p 50).

The very choice of central perspective (a unique Western choice) gave rise to a full vision of the world that characterized our own modernity. Panofsky demonstrated that even the mathematical perspective system of the Renaissance, the perspectiva artificialis of Brunelleschi and Alberti (considered by its inventors to be a universally valid method for depicting three-dimensional space), on closer inspection, does not conform with our visual reality, despite the five centuries (1400-1900, broadly speaking) during which it had been accepted as such.

3.3. Perspective and Desargues' metaphorical thinking. The function of perspective and of the instruments for perspective drawing had another important effect on the Western intellectual history: the genesis of projective geometry. Desargues' contribution to the foundation of projective geometry is often compared with Descartes' geometrical style. During the seventeenth century geometrical perception became separated, so to speak, into two relatively distinct forms of geometry, into two different geometrical styles. One of these is represented by the work of Descartes (1596-1650): the geometry of mechanical-metric activity. The straight line in Cartesian geometry corresponds to an axis of rotation or to the stiffness of a measuring rod. The other geometrical style is represented in the work of Desargues (1591-1661). The straight line of Desarguesian geometry is the ray of light or the line of sight. It is a geometry on which, among other things, perspective painting is based. In the collection of mathematical machines both styles are represented, by curve drawing devices for Cartesian geometry, and by perspectographs for Desarguesian geometry.

4. Perspectographs in formal and informal education. The collection of perspectographs *Perspectiva Artificialis* have the potential to start discussion about several issues, from the practical use of perspectographs to the origin of projective geometry and to the European intellectual history from the Renaissance period. The choice of the focus and the depth of discussion depend upon the audience's previous education.

Perspectographs may be used in formal education, within schools and universities, with more specific mathematical aims. Our research group in didactics of mathematics has realized several teaching experiments at different school levels. In the talk I present some results from our experiments in primary school ([1], [2], [4]). Fig. 1 shows a 3rd grader using a cardboard perspectograph to draw a chessboard (left) and the result (center) [6], and secondary school students exploring a Dürer's glass (right), in order to master the "discovery" or "invention" of the mathematical rules of perspective representation[1] [7].

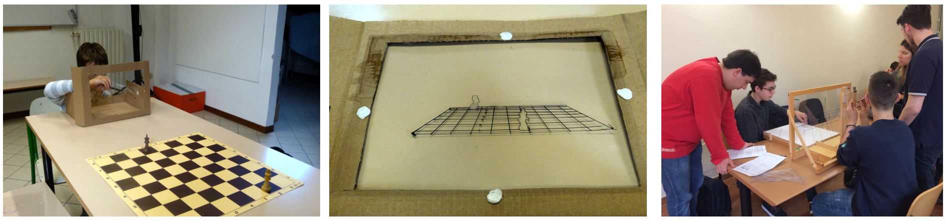

FIGURE 1. Drawing some basic points with the perspectograph (left) and tracing lines after opening it (center). Exploring perspective drawing with a laserpen (right)

Exhibitions[2] are cases of popularization of (or informal education about) mathematical heritage within the general history of western culture. The role played by perspective representation in the development of cultural identity in Europe make these artefacts good examples to be considered at the crossroads between different communities of practitioners, theoreticians and educators.

5. Acknowledgements. Several colleagues, teachers and students collaborated to the construction, enrichment, use and diffusion of the collection of perspectographs: Michela Maschietto, Marcello Pergola, Marco Turrini, Carla Zanoli, Annalisa Martinez, Simone Banchelli, Franca Ferri, Roberta Munarini, Margherita Rosi, Elisa Quartieri, Simona Vangelisti, Irene Ferrari.

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Artifacts for geometrical transformation and drawing curves in the classrooms, workshops and exhibitions

Michela Maschietto

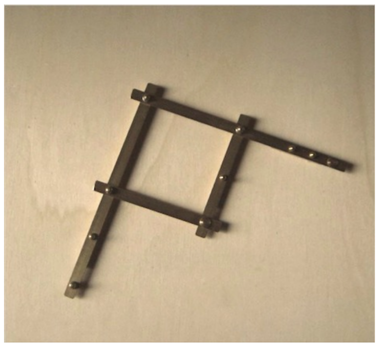
This contribution focuses on the use of some instruments, called "mathematical machines", in teaching and learning mathematics. In particular, we refer to some machines which have been built with a didactical aim by secondary school teachers on the basis of their descriptions in historical texts, ranging from classical Greek mathematics (linked to the theory of conic sections) to 20th century mathematics. These machines are currently collected in the rooms of the Laboratory of Mathematical Machines (MMLab, http://www.mmlab.unimore.it/) at the University of Modena and Reggio Emilia. The MMLab works for both mathematics education research and popularization of mathematics [1].

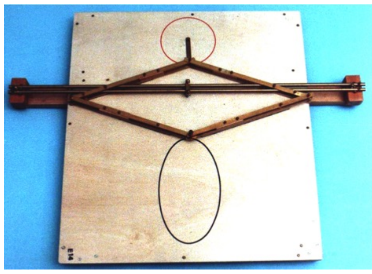
A mathematical machine (related to geometry) is an artefact designed and built for the following purpose: it aims at forcing a point, a line segment, or a plane figure to move or to be transformed according to a mathematical law that has been determined by the designer. A well-known mathematical machine is a pair of compasses. In the educational approach to the use of mathematical machines, the interest concerns not only what can be done with an instrument, but also how students using a machine can construct mathematical meanings embedded in the machine itself (and related to its structure and functioning) and justify its function. This paper mainly concerns pantographs for geometrical transformations and drawers for conic sections.

The history of instruments for geometry other than the compass and straight edge started with Descartes' work. In his *Géométrie* [3] Descartes studied curves that were mechanically obtained and worked to obtain their algebraic expression. This is the function of the hyperbola drawer. The instruments were above all considered as theoretical ones; Descartes thought about the curve as drawn by imaginary movements. However, Descartes described other instruments from a different perspective in other books. For instance, in the *Dioptrique* [3] the descriptions of the gardener's ellipse and the hyperbola drawer with tightened threads contain some practical elements for the user and their drawings show a hand where the pencil had to be placed to move the thread and draw the curve. In the same book, the author also described a machine based on the theory of conic sections by Apollonius for obtaining hyperbolic shapes for smoothing lenses. All these instruments have been constructed and used with secondary school and university students in several activities carried out by the MMLab.

In the development of the geometry of instruments ("geometria organica"), the interest passed from the conception of specific drawers to a more theoretical approach on drawers containing linkages. Our educational interest concerns linkages with one degree of freedom, corresponding to curve drawers, and linkages with two degrees of freedom, corresponding to pantographs. The latter ones are "local" instruments, in the sense that they determine a correspondence between limited plane regions, while geometric transformations are defined, globally, for all the points of the plane. An interesting example of pantograph was proposed by C. Scheiner in his *Pantographice, seu ars delineandi* [8]. This author constructed an instrument called "linear parallelogram"(see figure at the top right). This articulated figure is fixed in a plane by a point and can make homothetic transformations. As Scheiner showed on the title page of his book (see figure on the left), this pantograph works on the plane but it is also a component of a perspectograph for perspective drawings with the characteristic of drawing enlarged perspective images. Scheiner's idea of using some figures, in particular quadrilaterals, as components of other instruments was very fruitful. For instance, if two opposite vertices of an articulated rhombus are put into a groove and two pencils are inserted into the two free vertices, a mathematical machine for reflection is obtained. In our educational perspective, variations in the structure of a machine are important for fostering conjectures and arguments by the students. For instance, if two points are chosen at the same distance from a free vertex of the rhombus and they are put into the groove, does the new machine always make a reflection? The answer is quite intriguing: the two free vertices are corresponding points in an affine transformation [7]. This kind of instrument (see figure at the bottom right) was proposed by N. Delaunay for drawing an ellipse by a transformation of a circle [2].

Scheiner's contribution was also mentioned by G. Koenigs in his *Leçons de cinématique*: "La théorie des systèmes articulés ne date que de 1864. Sans doute on les a utilisés bien avant cette époque; il se peut même que quelque esprit amoureux de précision rétrospective découvre des systèmes articulés dans l'antiquité la plus reculée; nous apprendrions une fois de plus que tout siècle détient inconsciemment entre ses mains les découvertes des siècles futurs, et que l'histoire des choses devance très souvent celle des idées. Lorsque, en 1631, le P. Scheiner publia pour la première fois la description de son pantographe, il ne connut certainement pas l'idée générale dont son petit appareil n'était qu'une manifestation naissante; on peut même affirmer qu'il ne pouvait pas la connaître, car cette idée tient à la notion élevée de la *transformation* des figures, notion qui appartient à notre siècle et donne un caractère uniforme à tous les progrès qu'il a vus s'accomplir. Le mérite de Peaucellier, de Kempe, de Hart, de Lipkine est moins d'être parvenu à tracer avec des systèmes articulés telle ou telle courbe particulière, que d'avoir aperçu les moyens de réaliser avec ces systèmes de véritables transformations géométriques. Dans cette remarque réside ce qu'il y a de vraiment général dans la théorie des systèmes articulés" [4, p. 243].





Pantographs for geometrical transformations are exploited in Italian educational projects in school from 7th-grade students to 10th-grade students. Conic sections drawers are proposed in teaching experiments concerning a synthetic approach to conic sections in secondary school (11th-grade). All these projects are based on the methodology of a mathematics laboratory [5], in which students work in small groups with the mathematical machines, participate in mathematical discussions, but also solve some individual tasks. The teacher acts as a cultural mediator who constructs tasks involving the mathematical machines related to a chosen mathematical content (he/she uses a machine as tool of semiotic mediation) and manages collective discussions. From the perspective of mathematics education, the aim of the design research on conic section is to study if and how the mathematical machines can be used for defining the conic sections and for looking at their properties, from a unifying perspective of the curves. In this sense, the mathematical machines are involved in two didactical functionalities: introducing and defining a particular conic section, and fostering arguments and proving processes. The educational path consists of four parts (20 hours): 1) an introduction to linkages by the exploration of Van Schooten's compass; 2) the exploration of conic drawers with tightened threads (ellipse, parabola and hyperbola) for looking for the definition of these curves; 3) the exploration of conic drawers with crossed parallelograms (ellipse and hyperbola) for fostering arguments and proving processes; 4) a brief historical survey of conic sections: the definitions of conic sections by Menaechmus and Apollonius; the description of a perfect compass and Descartes' machine for hyperbolic lenses; Dandelin's Theorem and its proof.

This contribution ends with the presentation of the activities of the MMLab [6]. In the MMLab equipped rooms, we propose laboratory sessions to secondary school classes and to groups of university students. The topics are: conic sections and conic drawers, geometrical transformations, perspective, and the problem of the angle trisection. Each session takes approximately two hours and covers three steps (historical introduction, group work on mathematical machines, collective presentation of each group work). The MMLab also participates in cultural events in Modena, most importantly with the permanent exhibition on perspective, and to other exhibitions in other towns in Italy and abroad.

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Equating the Sun: Variant mechanical realizations of solar theory on planetary automata of the Renaissance

Samuel Gessner, Michael Korey

While the starry sky seems to rotate in unison around us once over the course of a day, we observers on Earth can make out several heavenly bodies that appear to have an additional movement against the background field of "fixed" stars. These are the two luminaries (the Sun and the Moon) and the five naked-eye planets (Mercury, Venus, Mars, Jupiter, and Saturn). The additional movements of these seven classical "planets," as they were often called, are proper to each – and wondrous. As seen from our vantage point on Earth, each seems to wander with varying speed relative to the stars, at times going faster or slower, or (with the exception of the luminaries) even occasionally going backwards. Their motions nevertheless show some regularity, something that Babylonian observers had already attempted to pin down in the second millennium BC. Eventually, planetary theory as developed by Claudius Ptolemy in Alexandria (2nd cent. CE) offered an impressively successful geometrical model, including realistic parameters and combining various uniform circular motions with appropriate offsets and eccentricities, to predict the observed positions of the planets [1].

The theory was received and refined by successive generations of mathematicians and astronomers over many centuries. For most of them the geometrical model served as a basis to compute tables and to compile guides to their use. As an alternative to these "digital" tables, at least since the 11th century, a class of specialized, analog mathematical instruments known as *equatoria* emerged as an approach for graphical computation, based on appropriately spaced, scaled, and turnable discs permitting results to be read off directly from their graduated scales. Some *equatoria* successfully united the representation of the geometrical model with a sense of the computations required to obtain values for the planetary coordinates. The development of spring-driven and hence compact (in comparison with weight-driven) clockwork mechanisms made considerable progress towards the end of the 15th century. This coincided with a fashion in princely circles to possess geared or clockwork-driven versions of these *equatoria*. Several noblemen interested in astrology and/or astronomy commissioned or acquired such planetary clocks, including Lorenzo Medici (by the maker Volpaia), the Habsburg Emperor Charles V (Torriani, Homelius), Cardinal Albrecht of Brandenburg (unknown maker), Cardinal Charles of Lorraine (Finé), Elector Ottheinrich of the Palatinate (Imser), Landgrave Wilhelm IV of Hessen-Kassel, and Elector August I of Saxony (the latter both by Baldewein) [2].

While Ptolemy's planetary theory served as a common reference, the devices that were actually built manifest distinct mechanical solutions in their quest to materially reproduce that theory. Remarkably, even when only considering Ptolemy's model for the Sun, the four surviving 16th-century planetary automata show three different means of realizing the solar anomaly, i.e., the Sun's non-uniform speed along the ecliptic in the course of a year. The oldest of the surviving machines (Paris) uses the uniform motion of an eccentric gear. Another (Vienna) incorporates an epicycle, positioned and turned at an angular speed so as to be geometrically equivalent to the first case, making use of the equivalence principle shown already by Apollonius in the 3rd cent. BC and proven in Ptolemy's treatise. On the other hand, the two machines made in Hessen (now in Kassel and Dresden) make use of a centred, circular gear with non-uniformly spaced teeth.

This difference in approach has not been clearly emphasized in the secondary literature, and the divergence does not seem to have been recognized as a historical problem needing explanation. With the support of the four institutions holding these magnificent machines, we have begun a careful on-site investigation, including measurement of the gearing, with the aim of determining the planetary parameters built into their mechanisms. We propose that the variety of approaches used attests to the differing expectations as to what such a mechanized mathematical model should fulfil. Simultaneously we suggest that this is an expression of the mathematical tools (e.g., geometrical reasoning vs. manipulation of tables and numbers) available to, or preferred by, the respective makers and commissioners of these clocks. In a certain sense, these variant mechanical approaches reflect a further (mechanical) contribution to the centuries-old reception and refinement of Ptolemaic theory.

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Actors, places of exchange, and mathematical knowledge: Following the chronograph in the minutes of the Bureau des longitudes, France, 19th-20th centuries

Martina Schiavon

Until 1854, standard methods of observing a stellar transit depended on coordinating the observed movement of the image across the wires of a micrometer and the audible sound of a standard clock: this was the "eye-and-ear method". Conveniently, the electric barrel-chronograph (or printed chronograph) only required a galvanic button to be pressed to self-register the observation on a paper-tape. The instrument introduced a new chronometric regime of vigilant surveillance by junior observers that, for accuracy, required knowledge of each one's "personal equation", a term used to indicate that person's typical reaction time, or the correction made for it [5]. Conceived around 1845 in the United States, the chronograph was first transferred to Great Britain, where it enjoyed unprecedented success; subsequently, it could be found it in almost all European astronomical observatories. However, a notable exception was France, where the chronograph wasn't used in observatories until the beginning of the 20th Century [4].

In this report, my primary aim is to present some new and extraordinary archival sources, now free on-line, that I consider important in the history of mathematical (and precision) instruments: the minutes of the French Bureau des longitudes (1795-1932) [11]. Regarding the chronograph, the minutes help us assess how this instrument was utilized in France during the second half of 19th Century not by astronomers, but more by other interested parties such as naval and other military officers, and precision instrument makers. The minutes help us realise how crucial it is to consider the involvement of those professions with a "secondary role" in the development of mathematical instruments, something that is often forgotten. Looking beyond the definition of a chronograph as a "mathematical instrument", my other aim is to consider how "mathematical data" also became an "instrument" to increase precision and, more generally, assist in the administration of society.

The French Bureau des longitudes and its minutes: some keys elements. Created in 1795, the French Bureau des longitudes can be seen by its name as simply a facsimile of the British Board of Longitude (1714 to 1828). Indeed, in France, the Bureau worked as a "small academy", and remains in existence to this day [12]. Clearly, during its 223 years, the Bureau des longitudes has undergone many transformations whilst maintaining its reputation as a significant scientific and technological institution. The origin of the word "bureau", or office, in the title relates to the expertise and technical tasks entrusted to it by the revolutionary government, even if the plural in the word "longitudes" undoubtedly refers not to the mere solution of a technical problem (i.e. the definition of sea longitude with respect to a fixed meridian) but to all related scientific problems. It may also be noted that the Bureau was created during a historical period (the French Revolution) in which the Académie des science had been suppressed. We might hypothesise that the Bureau was an *attempt* to recreate a scientific academy, despite the special nature of its utility, because of its association with the Parisian and Military Observatories. Unlike the British board, the Bureau was conceived, and worked in practice, as a "small scientific assembly". Initially addressing only navigational problems, the Bureau came to embrace many branches of science and its applications, from astronomy, metrology, geodesy, and celestial mechanics, to earth sciences and, more recently, space science; the status of mathematics was, and still is, very high. Once named the depository of national instruments, the study and the development of precision instruments was considered very important inside the Bureau, and a place was thus reserved for an *artist*, or precision instrument maker; additionally, from 1854 onwards, posts were available for Navy and Artillery officers [8, 9].

The minutes (procès-verbaux) of the Bureau des longitudes form an extraordinary archive that allows the study of mathematical instruments relating to the period 1795 to 1932, not least in the context of international rivalry and competition. The principal authors of the minutes (typewritten from the early 20th Century) addressed to the tutelary minister of the Bureau (whose members were *fonctionnaires*) were the secretaries. The minutes detail scientific and administrative discussions, incoming and outgoing correspondence, and the election of new members. They also include the subjects to be examined in the Bureau's publications: the *Connaissance des temps* and the *Annuaire du Bureau des longitudes* from 1795; also the *Annales du Bureau des longitudes* from 1877 to 1949. For the period 1795-1932, the minutes comprise about 22,000 documents; these include unedited letters, scientific and technical papers received from, or written by, members, and preparatory studies submitted for the consideration of the Bureau prior to possible publication. The minutes also provide information on scientific expeditions organized by the Bureau in France and abroad. Their richness is evident from the words of Hervé Faye: "There would be a whole great chapter of science just in writing from our minutes [of the Bureau des longitudes] the summary of the discussions held on the physics of the globe, the projected or accomplished applications of optics, magnetism, electricity, thermometry, surveys" [3].

The minutes are now on-line on the website "Les procès-verbaux du Bureau des longitudes (1795-1932): un patrimoine numérisé" [11], where original manuscripts can be studied with their whole transcription. When searching for the word "chronograph", we find 118 occurrences. The first one is the minute dated 23rd October 1867, in which we read that the Bureau received the "description, with a figure, of a chronograph by M. Fleuriais" with a "small labeled package: Chronograph Records". The final instance appears on the 13th April 1932, the Bureau des longitudes agreeing to lend a chronograph belonging to the Paris Observatory to that of Nice.

Exploring the use of the chronograph as an instrument of mathematics. Let us consider where the word "chronograph" appears within the minutes of the Bureau des longitudes for the period 1867-1888 (the upper limit being the year in which Admiral Mouchez, the director of the Parisian observatory, introduced the chronograph in the meridian observatory service). We have already noted that in France, until the very end of the 19th Century, astronomers did not use the instrument [4]. However, the minutes reveal that it was largely employed in *other* professions such as hydrographical engineering and geodesy (more generally by the artillery). Navy officers used the printed chronograph to study, from the graphics traced on a paper-tape, the running of a chronometer; on the other hand, artillerists employed it in geodesic triangulation, when they needed to know the personal equation during the observation of light signals [6, chap. 1-2-3]. When exploring the minutes, the discussion regarding the paper-tape chronograph (or any other mathematical instrument such as the reiterative circle), shows us how astronomers or mathematicians generally worked together with Navy and Artillery officers *and* precision instrument makers so that, during the second half of the 19th Century at least, instrumental research and innovation came out of a strong collaboration between these professions. The success of this way of working on mathematical instruments is also apparent from the fact that the Bureau generally appointed military *officers* (and not really pure mathematicians) to collaborate, test and also control artist's technical work. Inside the Bureau des longitudes, theory and practice are inseparable and thus progress together. Great mathematicians, such as Henri Poincaré, for example, never worked alone and, as a member of the Bureau, he collaborated with officers and precision instrument makers [10].

Using the minutes of the Bureau des longitudes the paper band chronograph (and other mathematical instruments) may be explored as an *archival* document in the sense that it is possible to study the material, literary and even social technologies that inevitably were at play in instrumental research: from its concept through to its utilization, recording the discussions of each participant (and their status inside the Bureau), tests, adaptations, field experiments, re-adaptations and calibration, and also circulation, and transfer and rivalry with foreign instrumentation. Moreover, linking the chronograph with all its users and with the improvements in precision that characterized the second half of the 19th Century, we discover how the mathematical data produced with instruments were also essential to the development of the State. For instance, geodesy was the first step in surveying and then making precision maps that were essential for the Army, but also for the development of public transport (i.e. the rail network). Despite the current opinion that an instrument is no more than a "support for the theoretical development or the intuitive way" that would help scientific reasoning [2, p. 26], within the minutes we can study the co-development of instruments *and* mathematics, and see that the chronograph also served, in turn, to develop new branches of mathematics. Perhaps one of the best examples is the use of paper-tape registrations in the sound detection of enemy's guns during the First World War. Here the chronograph served to give a very accurate distinction between the ballistic and the muzzle waves of a gun. It was used to correct maps and to find new relationships in mathematics. Ground survey and the study of the nature of the propagation of sound benefitted. It helped to reason in terms of mathematical analysis, something that produced very important changes in mathematical research. Last but not least, the chronograph was related to ultra-fast cinematography in order to study and to develop ballistics ([1], [6, chap. 6-7], [7]).

In conclusion, the minutes of the Bureau des longitudes allow us to get a long term and even a "coherent historical approach" to instrumental studies. The discussions of the Bureau's members help us to understand the negotiations involved in instrumental research: the way in which the members convinced or dissuaded the French scientific community regards the employment of new mathematical instruments and, more generally, how they allowed historians to contextually reconstruct, what I call, *faires*. Without forgetting the practices, the *faires* focus our attention on the manner in which all the Bureau's employees – scientists, military officers, and instrument makers – *collectively* interacted in order to develop *science* (and mathematics in particular).

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Measuring goods, or the social origins of the early modern "arithmetization" of ratio and proportionality

Antoni Malet

As is well known, ratios were crucial in classical, medieval and early modern mathematics. Relations that today we express with functions and algebraic formulae were expressed by proportionality or equality of ratios. This was, for instance, Galileo's language in the *Discorsi*, in which he set about the mathematization of the science of motion. He used in particular Euclid's famous definition of proportionality in Book V of *Elements*, a definition that, until the 17th century, everybody agreed was obscure.

Today we understand ratios as quantitative relationships between magnitudes measured by their quotient. The shift to quotients is often called the "arithmetization" of ratios. To turn lengths, surfaces, times, etc. into numbers they must be *measured*. But magnitudes cannot be measured without a definition of ratio and proportionality since, for instance, the length of a straight line segment is the ***ratio*** between the whole segment and whatever segment has been chosen as unit. So we have come full circle: if you want to avoid Euclid's definition and define ratios by division, measure magnitudes, but if you want to measure magnitudes, define ratio first. When, in the 17th century, ratios were defined by division and the measures of magnitudes required the notion of ratio, the argument was circular.

It is a historical fact that the majority of mathematicians in the 16th and 17th centuries chose to ignore the circularity. When a few mathematicians showed that no cogent definition of proportionality was available outside of the *Elements*, their warnings were dismissed. This is puzzling because internal consistency is essential to mathematics.

Returning to Galileo, Euclid's equimultiple definition played a fundamental role in the *Discorsi*, in results written probably in the first decade of the 17th century and published in 1638. As Galileo's case shows that by the early 1600s, Euclid's definitions were basic tools of mathematics and of mathematization. However, by 1700, Euclid's definitions had disappeared. Natural philosophers certainly ignored them, with just a few mathematicians upholding them.

As outlined above, the theoretical problem of measuring is at the center of the argument to "arithmetise" ratios and so warrants further consideration. Measuring was a pervasive presence in early modern Europe. The best-known measuring instruments were ornate, elaborate and expensive. They suggest a princely setting or court context, and use by specialized practitioners in astronomy, the arts of war, navigation, and architecture. However, numerical measuring was a social practice at the very heart of daily life in early modern European societies, one that was particularly relevant for burghers of the towns. As a practical geometry book put it in 1585, “Mesurer [une longueur] est cognoistre combien la ligne droite d’entre les extremitez d’icelle longueur contient de mesures fameuses et vulgaires” [2, f. 1v]. Literally hundreds of early modern practical geometry books connected this idea of measure to particular methods of conducting practical measurements. Measuring was defined as an unproblematic practice grounded on an unproblematic theory. Measuring a magnitude amounted to counting how many times a unit (a “mesure fameuse et vulgaire”) was contained in it. For greater precision, one counted how many sub-units comprised what remained, and so forth. Since all of this is about measuring material magnitudes, and then paying for them, measuring in practice always ended up as a number. Most practical geometries took for granted that, in general, geometrical continuous magnitudes can be measured, hence they implicitly assumed the existence of an arithmetical continuum. In some authors we find this claim made explicit, as in P. A. Cattaldi: “scienze matematiche intendiamo quelle che considerano la quantità in astratto, ... [la Geometria] considera la [quantità] continua, che si conosce con la misura” [1, p. [1]]. It is relevant that the arithmetical continuum was introduced only in the 19th century. The early modern implicit assumption of the arithmetical continuum had no foundations within contemporary mathematics.

This paper argues that the changing mathematical/theoretical status of the “arithmetical” understanding of ratios in early modern mathematics – and connectedly the reinforced belief in the tacit idea of an arithmetical continuum – mirrors the changing social status of the practice of measuring in early modern societies. To explore the latter we focus on the sixteenth-century emergence of professions specializing in the measure of specific goods, thereby being part and parcel of the fabric of everyday life. Across Europe they engaged thousands of people with different degrees of technical expertise. As regulators of economic life (and more) in early modern societies, measuring professions were often embodiments of political authority. The following examples will make these points clearer. I will be focusing on 16th and 17th-century France, but everything suggests that what happened there is indicative of general trends in Western Europe.

The *Prévôt* of Paris, the main authority under the king for all town matters, had jurisdiction over bodies of officers whose task was to measure and gauge special kinds of goods, called “sworn measurers” and “sworn gaugers”. The word *jurés* (sworn) referred to both the fact that they had taken part in a public ceremony or signed a document to swear their commitment to strictly follow and enforce the legal regulations of the trade, and to identify them as officers whose number and remunerations were fixed by royal edict. The list includes, among others, the “sworn measurers” of grains and flour; sworn master builders; sworn measurers of charcoal; of salt; of garlic, onions and dry fruits; of lime; of wine and spirits; of wood (for fuel, not timber). This list is far from exhaustive, since many measuring professions were organized by, and reported to, the King or other authorities.

Sworn measurers of cloths (*auneurs de toiles*), measurers of wool cloths (*auneurs de draps*), and surveyors (*arpenteurs*) were, from 1575, a royal monopoly.

An interesting case is provided by sworn cask gaugers (*jurés jaugeurs*). "Official gaugers" (*jaugeurs d'office*) for wine casks were established by a royal edict in 1550. From 1553 onwards, all commercial transactions involving casks had to be measured by the king's official gaugers. Under the great reformer, Henri IV, official gaugers were given the authority to "mark" (as inspected) all vessels, full or empty, in every wine merchant's shop. The gaugers were responsible for handing out to merchants and makers *échantillons* or *étalons*, standard models of units with the official mark. Cask makers (*tonneliers*) also fell under the authority of the sworn gaugers regarding the average capacity, the bulge or convexity in the barrel shape (*bouge*), and the distance between the circular bases and the staves' end (*jable*). If the gauger found casks whose gauge, *bouge* and *jable* were not reasonable, they would be confiscated and their tonnelier fined. To fulfill their obligations, gaugers had the right of visitation or inspection (see below). The *jurés mesureurs* were therefore the keepers of accuracy and fairness in trade, including in terms of legal responsibility.

Sworn master builders, established by royal edict in 1574, were in charge of mediating in conflicts between those who built (architects, master builders, carpenters) and those who ordered and paid for the building. They had the right of *visitation* to building sites; their main instrument (called the *toise*) was a rod on which one *toise* (6 Paris feet) and its subdivisions were marked. With it, measures were taken (*toiser*) of any disputed grounds. The standard for the *toise* was on public display, prominently fixed against the wall by the main staircase of the Grand Châtelet, fortress and seat of the Prévôt.

The measuring professions could resort to coercion through the right of *visitation*, which a contemporary source defined as: "the right the sworn officers of bodies and corporations have to visit the houses and workshops of the members of said corporations to inspect their standard weights and measures, their goods, and their works, so that in case of fraud, bad quality, or violation of regulations, to have them confiscated by police officers" [3, vol. 3, col. 652]. Judicial sentences provide evidence that measurers used (and abused) the right of visitation and the coercive force of the police.

All sorts of merchants (in bulk, retailers, or just intermediaries, *négociants*) were required to own and use measures that had been properly checked (*étalonnées*) by the corresponding *jurés mesureurs* (*Ordonnance*, March 1673). All the standards (*étalons*) were deposited and kept at the Hôtel de Ville; the more sensible ones (used to measure wine, flours and grains), had to be kept at the Chamber of Sworn Salt Measurers (*jurés mesureurs de sel*) (*Ordonnance*, December 1672). Since salt was a royal monopoly, the Sworn Salt Measurers' prominent role was probably related to the enormous significance salt taxation, the *gabelle*, had before the French Revolution. It was part of Sworn Salt Measurers' function to take care of the metallic *étalons* kept at the Hôtel de Ville and also to check the accuracy of the wooden measures used in marketplaces, cellars, and workshops, which had to be properly marked by the *jurés mesureurs de sel* as a warrant that they measured correctly. They had an annual right of visitation on retailers of grains, flours, and dry foods (*Ordonnance* 1672, ch. 25).

The *jurés mesureurs* became mediators between epistemic values, mathematical knowledge, economic interests, and political authority. The practice and conventions of measuring in the 16th and 17th centuries were not only economically consequential, but were backed by powerful political and symbolical arguments. Bylaws and ordonnances by royal and local authorities regulated measuring. The measurers themselves participated in the coercive apparatus of the state and town. The standards and tools for measuring were earmarked and kept in symbolically relevant buildings. Measuring, as explained above, made no sense within the mathematics of Euclid, Apollonius and Archimedes, where neither the measure of continuous magnitude nor the "arithmetical" understanding of ratio can be defined in general. However, when we consider the social and political organization sustaining the practice of measuring in early modern societies, we start to realize that theoretical considerations, even if they were concerned with mathematical consistency, might not be strong enough to stand their ground. It seems difficult to contemplate that the mathematical ideas underlying these practices can be ignored or taken lightly, be they theoretically inconsistent or otherwise.

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The early history of mechanical integration: The first five decades, 1814-1864

Joachim Fischer

Mechanical Integration in a narrow sense, i.e. meaning the theoretically exact determination of definite or indefinite integrals by purely mechanical means, saw the light slightly more than 200 years ago; to be more precise in 1814 [1]. Its history, however, has not yet been written, although the subject is almost closed by now: theories as well as the invention and production of instruments ended long ago; only one maker of purely mechanical planimeters remains to this day. The purpose of this talk is to look at the first half-century of Mechanical Integration, and, in particular, how the development from a one-of-a-kind device to mass production happened in the last decade, 1854 to 1864.

The field of application, where the need for such an instrument might have occurred first, is area measurement, be it in reality or on maps, drawings etc. In antiquity, several exact results were known for simple plane polygonal figures: rectangles, right triangles, parallelograms, trapezoids and general triangles; or, considering π as known and/or broadening the context to curved boundaries: circles, ellipses, parabolas, spirals etc. Covering a (completely irregularly bounded) area with a square grid, counting the squares and estimating parts of squares was an approximation method well known in the Middle Ages, or at least in the Early Modern Age. Another common approximation method was an overlay of stripes of the same width, accumulating the average lengths of the stripes and multiplying the result by the width. Only after the invention of differential and integral calculus by Newton and Leibniz in the 17th century did the utmost close connection between exact area calculation and (the computation of) definite integrals became sufficiently clear.

Nonetheless, it took almost one and a half centuries until a purely mechanical solution for the theoretically exact determination of the area of any even irregularly bounded figure was achieved. This is most astonishing since many protagonists in integral calculus, starting with Newton and Leibniz and continuing with the Bernoullis via Euler to Lagrange and others, used (mainly imagined) mechanical devices to illustrate and/or solve even complicated differential equations. However, a usable and mechanical solution for the most simple differential equation, $y' = f(x)$, obviously never came to any of these minds. When Johann Martin Hermann (1785-1841) conceived such a device for the first time in 1814, people who were informed of the invention were struck by its mechanical simplicity as well as by the fact that such a device actually existed. Although not published, Hermann's device is now undisputedly the first known instrument for theoretically exact integrating, able to measure precisely the area of any irregularly bounded plane figure.

The simple device uses only three mechanical elements: a cone, a wheel and a wedge, in the following self-explanatory configuration (see Fig. 1). Hermann explains this as a continuous multiplication and accumulation device (and we know that continuous multiplication and accumulation is nothing but integration). If the cone turns by dx, and if the touching point of the wheel with the cone is $f(x)$ away from the summit, then the wheel is turned by $f(x) \cdot dx$. As it turns, it continuously accumulates the differential turnings, and as the wedge can change the position of the touching point *during* turning, the multiplication factor $f(x)$ can also continuously change – and we have continuous multiplication with continuous accumulation. Therefore, a perfect (ideal) Hermann instrument can integrate and thus measure areas exactly!

The non-publication was obviously due to problems with Hermann's health; starting at least in 1822 he apparently had severe problems with high blood pressure, so was subsequently unable to work as a surveyor. However, the famous instrument makers Georg [von] Reichenbach (1771-1826) and Joseph Liebherr (1767-1840), as well as the mathematician, physicist and astronomer Johann Georg [von] Soldner (1776-1833), all knew of Hermann's invention; and at least Reichenbach and Soldner are reported to have said that they never believed such an invention possible. None of them, however, thought of making something out of it when Hermann was no longer able to do so. Of course it cannot be ruled out that

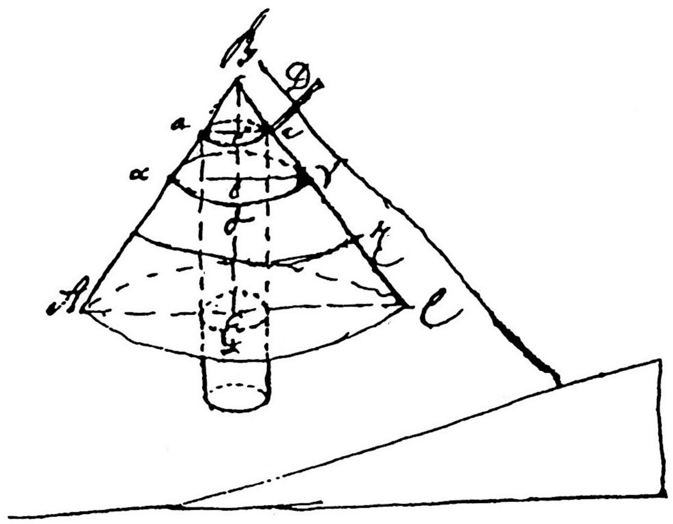

Figure 1. Hermann's planimeter: sketch from a manuscript, c1814/1815

Hermann annoyed these other men in some way and this may have been why the transformation into a fine working model was never finished. Hermann only had a rough prototype made in 1817, but which was still capable of giving an accuracy of 1/400; but this prototype was definitely scrapped in 1848. The number of planimeters extant in the decade 1814-1824 was just one.

In 1824, the situation for mechanical computation of areas changed. Tito Gonnella (1794-1867) had ideas for three new mechanisms or, at least in one case, what he must have considered a new mechanism: a) hyperbola + wheel, b) cone + wheel, and c) disc + wheel. While in 1825 [2] he published only the latter, it becomes clear from a subsequent, much more comprehensive, publication of 1841 [3], that his first idea was based on (a), a hyperbola + wheel configuration that was quickly discarded, probably for mechanical reasons. Then he went on to mechanism (b), very similar or – at least in theory – identical to that of Hermann (whom, of course, he did not know), and shortly before publication he saw that the aperture of the cone was nowhere used in the theory of the instrument and that the cone could therefore be flattened to a disc, resulting in (c). A prototype was finished and presented to scientists and others in 1824 and 1825; the Grand Duke of Tuscany ordered, in 1825, a "most exact" version to be made for his collection.

Originally, Gonnella had thought of Switzerland and her precision mechanics for manufacturing parts of this new machine. He sent drawings of both mechanisms (b) and (c), via an intermediary, to Swiss mechanics. It is not known, however, how much proliferation was made possible by this method of communication. Gonnella's ideas for mechanism (b) may have been compromised, which may have resulted in Johannes Oppikofer's (1782-1864) device, presented to a

Swiss learned society in the end of 1828. Whether this was a case of plagiarism or not is still open to discussion, but it is highly improbable that any new documents shedding more light on this will ever be found. On the other hand, Gonnella did not succeed with his plans and so in the middle of 1827 he continued to make the machine in Florence, probably in 1828; it was certainly finished by 1829, but using mechanism (c). Therefore, in the end of the 1820s, there existed Oppikofer machines with mechanism (b) and Gonnella machines with mechanism (c). But while Gonnella's publications were totally ignored in 1825 as well as in 1841, Oppikofer, with the help of the mechanic Heinrich Rudolf Ernst (1803-1863), succeeded in having a small number of planimeters built and, even more importantly, managed to spread the word regarding the possibility of exact mechanical integration to a still limited audience. The number of the planimeters added in the decade 1824-1834 was approximately 5.

Ernst began to produce an unknown but surely small number of Oppikofer type instruments (only one survives); and in 1839 he built at least 12 of these instruments with additional linear-logarithmic scales, a minor modification due to Léon Lalanne (1811-1892), of which type, again, only one survives. In 1839, Ernst again modified the original Oppikofer machine by making the cone only half the size of the original one, and changing the surface material from clock metal (bronze) to wood. He probably produced another small number of these modified instruments but more important were the anonymous publications – showing the machine in detail – in such widely-read journals as the *Bulletin de la Société d'Encouragement pour l'Industrie Nationale* and *Dingler's polytechnisches Journal*. The number of planimeters added in the whole decade 1834-1844 is probably, at most, 25.

Ignorance of Gonnella's most elegant solution (c), although published almost a quarter of a century earlier, led to its re-invention in 1848/49 by the Swiss Kaspar Wetli (1822-1889). Almost simultaneously, this re-invention of the disc and wheel mechanism was accompanied by a publication by Simon von Stampfer (1790-1864) in 1850. This publication appeared with minor changes in four different journals, and this, finally, was the breakthrough for exact mechanical integration: from 1850 onwards, knowledge of its possibility spread rapidly. And again almost simultaneously, three makers in three different locations/countries began manufacturing the instruments: Goldschmid in Zurich/Switzerland (from 1849); Starke in Vienna/Austria (also from 1849); and Ausfeld in Gotha/Thuringia (i.e. "Germany", from 1850 or 1851). The instruments were expensive, but there was demand amongst the public for them, and so the number of planimeters increased by approximately 150 in the decade 1844-1854.

Then came the big change: theory and instruments together stimulated the search for a more simple mechanical construction (once the possibility of theoretical exactness had been established). Among about ten different solutions, three more or less equivalent ones remained (but all were independent of each other): Jakob Amsler (1823-1912) in 1854, Albert Miller, Ritter von Hauenfels (1818-1897) in 1855 and Pavel Alexeevich Zarubin (1816-1886) also in 1855 came up with the idea of the Polar Planimeter – a name invented by Amsler to indicate that the instrument turns around a fixed point, i.e. the pole, of the instrument (see Fig. 2).

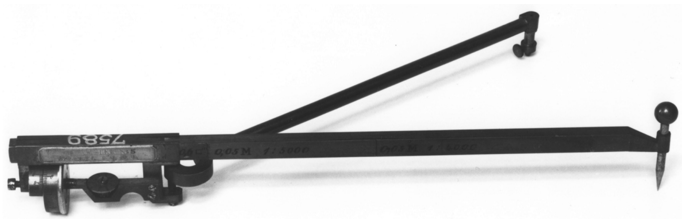

Figure 2. Amsler's polar planimeter

The instrument was simple, inexpensive and easy to use: three properties any object needs to become commercially successful. Of the three inventors, Amsler's enterprise (founded as early as the end of 1854) was the most prosperous one; the total number of previously produced planimeters was surpassed within only three years, and the number of planimeters manufactured in the decade 1854-1864 increased by an amazing 3500! Thus, by several small changes and one really big change, the final instrumental form of mechanically evaluating integrals was reached after five decades, and by 2001, one and a half centuries later, when production of mechanical planimeters ceased (with one exception that can be ignored in terms of numbers), the total number of mechanical integrating instruments, 95% or more of them planimeters, had reached an estimated 1.5 million!

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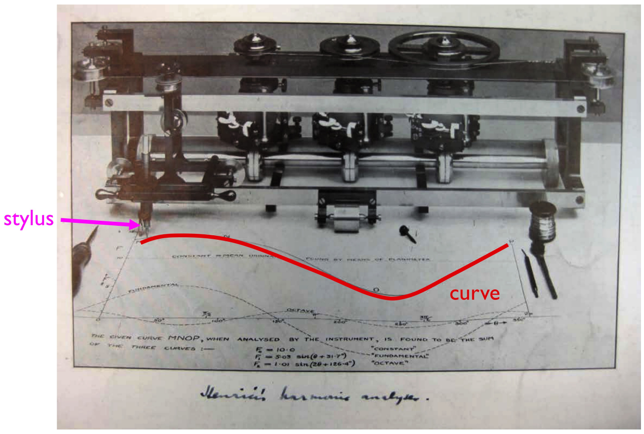
"An exquisite machine": Olaus Henrici's harmonic analyser

June Barrow-Green

In May 1894, Robert Ball, the Lowndean Professor of Astronomy and Geometry at Cambridge, described Olaus Henrici's new harmonic analyser as "an exquisite machine"[3], and he went on to expound on the machine's usefulness, especially in connection with the production of tide tables. Ball was reporting on a soirée held at the Royal Society at which Henrici's new machine was being exhibited. It was not Henrici's first harmonic analyser – he had had his first one constructed in 1889 – but it was a revised version of the new one he had designed in 1893 together with his assistant at the Central Technical College, Archibald Sharp, and which had been built by the renowned instrument maker Coradi of Zurich. Henrici's analyser was also displayed at the *conversazione* held in June 1896 at the Central Technical College[3] where a visitor reported that

> "[...] the smooth working of the latest form of Prof. Henrici's harmonic analyser, led the engineer to speculate on the time when all calculations, however complex, would be done by turning a handle, and when the brain would be left quite free to think and originate." [2]

As can be seen in the figure below, the analyser consists of multiple pulleys and glass spheres – rolling sphere integrators – connected to measuring dials. The image of a curve is placed under the device and the user moves a mechanical stylus along the curve's path tracing out the wave form. The resulting readings on the dials give the phase and amplitude of up to 10 Fourier coefficients.



Henrici had been led to the construction of his first harmonic analyser by the work of W. K. Clifford [7], his colleague at University College London (UCL), who in 1873 had provided "a beautiful graphical representation of Fourier's Series" [11, p. 113]. However, this 1889 machine did not work as well as Henrici hoped, not least because it gave only one Fourier coefficient at a time, and because the mechanism required to produce the simple harmonic motion introduced too much friction. He exhibited a revised version of this analyser at the German Mathematical Society's *Mathematical Models, Apparatus and Instruments Exhibition* held at the *Technische Hochschule* in Munich in 1893. Henrici was the lead organiser for the British exhibits, his German background combined with his interest in mathematical models and instruments making him a natural choice for the role.[4] He

[3]Olaus Henrici (1840-1918) was educated in Germany before making his career in London, first at University College, and then at the Central Technical College. See [4, p. 22].

[4]Britain provided more exhibits than any other country apart from Germany, with Henrici providing several geometrical models as well as his analyser. The other British organisers were Lord Kelvin and George Greenhill.

also wrote an article on harmonic analysers for the catalogue [9, p.125–136]. The exhibition had been due to take place in Nuremberg in 1892 but had had to be postponed to 1893 due to an outbreak of cholera but the exhibits had been sent in 1892 which explains why Henrici's new harmonic analyser, the one developed with Sharp, was not displayed.

Henrici was not the first to produce a harmonic analyser, credit for that goes to William Thomson (later Lord Kelvin) who in 1876 produced a rudimentary design for a machine which was fully realised in 1878. But Thomson's analyser, although capable of a high degree of accuracy, was large and difficult to manoeuvre. Henrici's intention, as described by him in the Munich catalogue [9, p. 134–135], was to produce a machine that was cheap, easier to handle and more portable than Thomson's, and which would be appropriate for applications where less accuracy was required. However, although the Henrici-Sharp analyser was certainly smaller than Thomson's, it wasn't cheap. In 1894, an analyser with five integrators was priced at £60 (c. £7,000 in 2017). The point was not lost on potential users. In 1920, Vannevar Bush observed that "The Coradi analyser is probably the most convenient machine, [...] very few of these instruments are in use, however, because of their cost." [5, p. 903]. In 1936 Bush again commented on the convenience of Henrici's machine but this time in the context of the relationship between the invention and the usage of such machines:

> "It is not much exaggerated to state that as many forms [of the harmonic analyser] have been invented as there are actual instruments in present use. Perhaps this is not undesirable, for it is certainly much more pleasant to invent a device of this nature than it is to operate the finished product. The writer pleads guilty to having invented several, none of which are in use. [...] The most convenient and precise is the Henrici-Coradi." [6, p. 659]

Bush's remark notwithstanding, some of Henrici's analysers were used, although it would seem not many. Felix Klein acquired one in 1894 [15], and in 1901 described it in his lecture course printed as *Anwendung der Differential- und Integralrechnung auf die Geometrie: eine Revision der Prinzipien* (1902) later edited and published as *Präzisions- und Approximationsmathematik* (1928).[5] References to the analyser appear in the literature elsewhere in Germany and in France but evidence for its actual use is sparse.

One person who did make good use of the analyser was the American physicist and astronomer Dayton Miller (1866-1941) who between 1908 and 1916 successfully used it for acoustic experiments [14]. Miller also encouraged his students to use it as evidenced by the following publications:

> Robert Sherwood Shankland, *The Dispersion of X-rays* (1933). Master's thesis. Included curve fitting with the Henrici analyser.
>
> Leslie Foldy, *The use of the Henrici Harmonic Analyzer to Obtain Frequency Spectra of Pulses* (1944). Report.

[5] For an English translation of the relevant pages, see [10, p. 79–82].

Miller corresponded with Gottlieb Coradi (1847-1929) about the analyser and in a letter to Miller, dated 30 November 1916, Coradi revealed that the analyser had not been the commercial success that he had hoped:

> "[...] it is really regrettable that this Analyzer of which you are proofing [sic] the high value in extent practical application is still nearly unknown. I am therefore greatly please[d] with your pamphlet books and photos and shall take the liberty to mention these when occasion occurs. The totality of work and idea of your researches is admirable and of most interesting results and I am very grateful to you as to one of the few of the possessors of a Henrici-Analyzer which have been kind enough to give me an idea of the work done with the apparatus. If this would have been more regularly the case, I would probably have had more practical success with the Analyzer when I would have been able to bring the different kinds of application to that general knowledge by means of notes in my catalogue and description." [8]

Another person who used the analyser for acoustic work was Carl Seashore who included a picture of it as the frontispiece to his book *Pyschology of Music* (1938) where it was described as "a symbol of the science of music".

Although Henrici's academic reputation, built while he was at UCL, was as a geometer, he had a background in engineering as well as in mathematics – he began his working life as an apprentice in an engineering works and later studied engineering under Ferdinand Redtenbacher (1809-1863), the founder of scientific mechanical engineering, at Karlsruhe. Even as a geometer he was an enthusiast for practical work, arranging for a workroom for his geometry students at UCL and producing many models of geometrical surfaces himself. With his move in 1884 to the Central Technical College and the setting up of his Mechanics Laboratory, he had the opportunity to become more focussed on applications, and in 1894, as well as producing a new version of his second analyser, he produced a report on planimeters for the British Association for the Advancement of Science [12]. Viewed from this perspective, Henrici's harmonic analyser – "Perhaps the most strikingly original piece of work done by Henrici" [13, p. xlvi] – can be seen as a natural synthesis of his mathematical and engineering talents, and despite the fact that it was not as widely used (or sold) as Henrici (or Coradi) had hoped, its ease of use and efficiency of design meant that it maintained a justifiably high status for over forty years.

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Mathematical machines in tide analysis and prediction (1876-1950)

Marie-José Durand-Richard

This talk presents three mechanical devices – the tide gauge, the harmonic analyser and the tide predictor – which were progressively introduced during the 19th century for recording, analyzing and predicting tides. It shows how different professional and academic cultures collaborated until the 20th century to establish a new control of ocean navigation, essentially with the colonial expansion of Great Britain and France.

I. Newton's and Laplace's theorization of tides. The phenomenon of tides has been observed for a very long time. Semi-diurnal tides, with two High and two Low Waters per day, were the best known on the Atlantic coasts, but no explanation was given other than the influence of the Moon. The first really efficient theorisation of tides was given by I. Newton (1643-1727). He applied the Three-Body Problem to the Water – assuming it covered the whole globe –, the Earth, and successively the Moon and the Sun. The main attraction is that of the Moon because of its proximity to the Earth. The Sun, because of its mass, also has a significant attraction. Newton's theory was correct in its basic principle, but it neglected the effect of the rotation of the Earth. So, some inequalities remained unexplainded, with theory also disagreeing with observation on some points.

The dynamical theory of P. S. de Laplace (1749-1827), was achieved in his impressive *Mécanique Céleste* (1799-1825). He formulated the differential equations describing the motion of the fluids attracted by the Sun and the Moon, but still with the simplifying hypothesis that water covered the complete surface of the Earth. Laplace identified three periodical species of tides – annual, diurnal and semi-diurnal – for each of the two attracting bodies, the nature of each being dependent upon the astronomical elements involved in its mathematical formulation. Laplace also gave a very general formulation of the elevation of a molecule at the surface of the sea above the equilibrium surface. But the resolution of the general case, when "the sea has a variable depth, surpasse[d] the forces of analysis", and observations were necessary to complete the theory of the ebb and flow of the sea for each relevant port.

So, the analysis of these "accidental circumstances" required better observational campaigns. In 1806, Laplace launched such a systematic campaign for Brest and a large number of French ports, which will last until 1835. In Great Britain, the most important campaigns were conducted in Liverpool (1774-1792) and in London (1808-1826). Tide scales were progressively set up in ports, but the reading of times and heights of High and Low Waters was difficult for untrained people because to determine these precisely required the level of the sea to be noted several times just before and just after maxima and minima, and mean values established.

However, long successions of averages could suppress contingent effects on tides. The best observational period, used both in Great Britain and in France, would cover a whole lunar cycle of 19 years, which corresponds to the return of the lunar node – the point where the lunar orbit intersects the ecliptic – in the same position.

II. The mechanization of recording, analyzing and predicting tides in Great Britain. In Great Britain, Laplace was soon considered a second Newton, extending his program by analytical methods, and one of the main goals of the contemporary, young, English algebraists and reformers was to supersede his methods for the solution of differential equations.

The British Association for the Advancement of Science, created in 1831, organised numerous committees for large projects whose economical and political impact had a national scope. It supported research on tide predicting for 90 years. J. W. Lubbock (1803-1865) was asked to analyse the best means of producing accurate tide tables. He designed "cotidal lines" on the charts, between points where the High Waters were simultaneous, and W. Whewell (1794-1866) also discussed the progression of tide waves all around the globe.

In 1831, the civil engineer H. Palmer designed a self-registering tide-gauge. To the floater, the scale and the pointer indicating the height of the tide, a rotating drum was added, with a pen drawing the curve directly onto a paper tape wound on the drum. With this device, new coastal measures were installed, with no less than 600 observatories on North Atlantic coasts.

In 1867, W. Thomson (1824-1907) – later Lord Kelvin – presided over a special committee of the British Association, bringing together astronomers, mathematicians, engineers, calculators, and officers of the Royal Navy, for the purpose of "promoting the extension, improvement, and harmonic analysis of Tidal Observations". As director of the Atlantic Telegraph Company, and head of the firm Kelvin and White Ltd in Glasgow, Thomson was directly concerned by the involvement of science and industry in public affairs. Reports of the committee used collections of data in ports from India, Mexico Gulf, California, Florid and Antarctica.

Decomposing the tide curve into a Fourier series first required the separation of the three species of waves, and other astronomical irregularities in the motion of the Sun and the Moon; only then could one pursue the harmonic decomposition of each one. Obtaining the coefficients of the decomposition required the result of integrating a product of two functions: that is what engineer J. Thomson (1822-1892) – Kelvin's brother – mechanised by combining several integrating disc-globe-cylinder systems, such that each of them provided a coefficient of the series.

After an experimental model, two harmonic analysers were built, the first in 1876 (11 integrators), the second in 1878 for the Meteorological Office (7 integrators). An operator had to follow the curve of the tide gauge, wound on a specific cylinder. The movement was transmitted by a fork to each integrator. On the 1876 model, the waves given by pairs of integrators were: the mean solar semi-diurnal S2, the mean lunar semi-diurnal M2, the luni-solar declinational diurnal K1, the lunar diurnal wave O1, and the solar diurnal P1. The index 1 corresponds to the diurnal tide, 2 to semi-diurnal tide. From a close relationship between astronomers and physicists, the following analysers will isolate more and more waves.

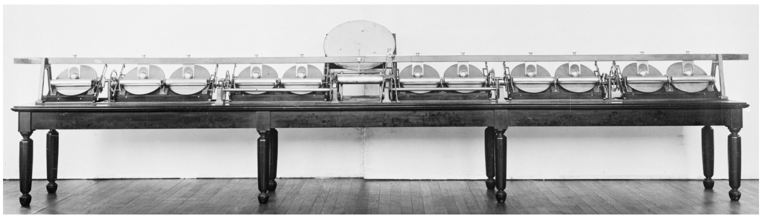

Figure 1. Harmonic Analyser (1876), Science Museum London SSPL. Inv. 187829

As the same time, W. Thomson devised the Tide Predictor. Its principle was simple: a flexible wire went under and over a succession of coplanar pulleys, the axis of each having a vertical harmonic movement corresponding to the various components. One end of the wire was fixed, a counterweight maintaining it taught at the other end, whose displacement indicated the double of the sum of the pulleys movements. A pen drew the trace of the resulting movement on a graphical paper wound on a rotating cylinder.

This first tide predictor (10 components) needed four hours to trace the predicted curve for one year, from observations made over only one month. During its construction, the calculator E. Roberts designed the TP No. 2 (24 components) for the Survey of India. Tide predictors with more and more components were produced by the firm Kelvin, Bottomley and Baird, and sold to France, Japan, Canada, Brazil and Argentina at the beginning of the 20th century. Their cost was about £5000, and it took two years to build them.

III. Tide predicting outside of Great Britain. The production of tide predictors quickly became a real industry. The harmonic method was well adapted

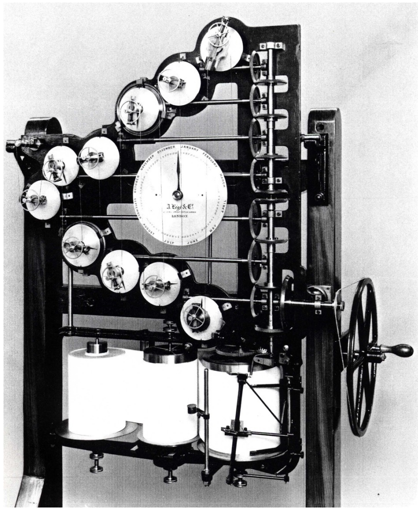

FIGURE 2. Tide Predictor (1876), Science Museum London SSPL. Inv. 1896-60

for distant ports of the British and French colonial empires, for which short-time observations were given by tide-gauges. For instance, the Tide Predictor No. 3 (16 components), built in 1881, was sold to the French Hydrographic Service in 1901, and used to obtain the tidal curves for the Cochinchine – later Indochine and Vietnam – seas. It was replaced by the new built TP No. 6 (26 components) in 1950, that worked until 1966.

Research into tides was not developed to any great extent in France. Nevertheless, the engineer hydrographer A. M. R. Chazallon (1802-1872) conceived a new tide gauge and set up a network of tide gauges, one of which was installed in Algiers in 1843. In the United States, the meteorologist W. Ferrel (1817-1891) established a review of existing tide predictors, with pictures (1912). His own were the TP No. 1 (1892, 19 components), and the TP No. 2 (1912, 37 components) which worked until 1966. A tide predictor was conceived on the same model in Germany in 1916, and another one (62 components) in 1938. After WW II, A. Dodson (1890-1968) conceived a TP (42 components), with copies for Argentina, India and Japan.

So, tide prediction by mechanical means supposed strong relationships between science, policy and industry, and was first realised in Great Britain because of the Industrial Revolution. It supposed close collaboration between engineers, physicists, mathematicians and astronomers, and opened the way for a global understanding of the tidal phenomenon all around the globe. It also contributed to the development of harmonic analysers in experimental physics.

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Exactness in Leibniz's mathematics: Instruments to construct transcendental curves

Davide Crippa

One of the explicit goals of Leibniz's early calculus was to overcome the Cartesian restriction on algebraic curves as the only legitimate objects of geometry, while keeping the same criterion of exactness based on constructions obtained via a unique and continuous motion ([2], [1], [3]). But on which methods of construction did Leibniz rely in order to trace non-algebraic curves?

Leibniz's mathematical corpus from 1673 to 1676 offers interesting but poorly known material relating to the construction of transcendental curves. In this talk, I shall examine several notes, letters and drafts from the period. In these documents, Leibniz considered several constructions of non-algebraic curves and discussed their possible geometrical nature. A common feature of these constructions is the use of a string (or thread) which can be wrapped around a curve and then extended to draw the desired curve. In more abstract terms, this construction is equivalent to accepting the possibility of rectifying any given arc. In the *Geometrie*, Descartes separates curves into two general classes: geometrical (algebraic) and mechanical (transcendental). To the latter class belong curves such as the quadratrix or the Archimedean spiral which, according to the classical definition, are constructible by synchronizing two independent motions: an impossible task in Descartes' Geometry.

Descartes then established several criteria to decide the nature of a curve on the basis of the method chosen for its description: either via a class of machines which involved systems of rulers (linkages) or strings, or via a pointwise construction, or via algebraic equations. Constructions using strings or threads are introduced in the *Geometrie* and their specific use to construct conic sections in the *Dioptrique*: the best-known example is perhaps the Gardener method for the construction of the ellipse. The possibility of using strings in a rectifying way is explicitly excluded as non-geometrical because it would allow the rectification of any curve, an operation not expressible with algebra.

By the second half of the 17th Century, a significant shift in the central and peripheral questions in geometry had occurred. In particular, the geometrical/mechanical divide was brought into question. Leibniz's early mathematical manuscripts reflect this shift in the mathematical landscape as they contain several studies of curves such as the cycloid, the curve described by the rolling of a circle on a straight line (whose physical applications became apparent in the construction of an isochronous pendulum), the involute of the circle, and the Archimedean spiral. These curves were recognized as mechanical on the basis of Descartes' criteria since all could be generated by a string used in a rectifying way (see Figs. 1, 2, 3).

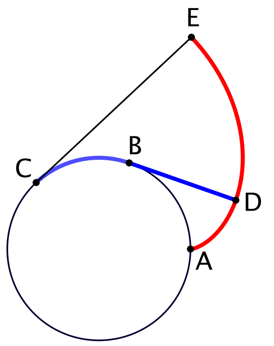

Figure 1. Involute of a circle. A string wrapped around ABC, unwrapped from D

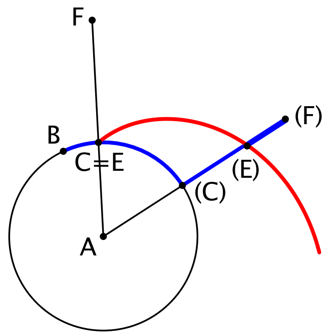

Figure 2. The curve of Bertet (Archimedean spiral). When the ruler rotates from AF to $A(F)$, the string fastened to BCF moves along CF such that $(C)(E) = C(C)$. The curve is traced by the moving end E with one continuous motion (*uno tractu*)

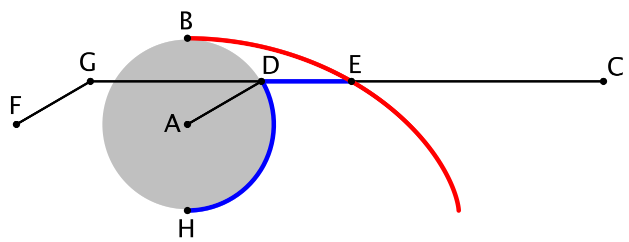

Figure 3. Cycloid. The string unwraps around the circle as the bar DC moves down

Leibniz discussed, in several manuscripts from 1674 and 1675, the status of the curves above with respect to the Cartesian criterion of exactness. According to him, both the cycloid and the involute should be given the status of geometrical curves ([5], p. 485). In other manuscripts of the same period, Leibniz referred to cycloids as geometrical curves. He was probably thinking of a way to generate the cycloid and other rolling curves which avoided two synchronized motions (as it happens in a rolling movement). It is indeed possible to construct a cycloid using one continuous motion if we employ a string that is bent into a curve or, in our terminology, a string used in a rectifying way (3). Leibniz toyed with the idea of extending the notion of geometrical exactness by permitting the use of strings in the rectifying mode. This would be an explicit violation of Cartesian exactness, but by conceding this possibility a construction of several transcendental curves using one continuous motion can be obtained. The manuscripts examined thus far indicate that Leibniz eventually discarded this way to extend the boundaries of Cartesian geometry by postulating a geometrical operation of rectification through the use of strings. We can suggest several reasons to explain the abandonment of this idea. Firstly, if one postulates an operation of rectification of any arc, then traditional classifications of problems and curves would lose their value. For instance, as Leibniz remarked in a manuscript of 1676, it would be sufficient to concede the possibility of rectifications in order to immediately solve difficult problems such as the quadrature of the circle, and it would be, therefore, a waste of time and energy to construct higher curves such as the cycloid or the Archimedean spiral for this task ([6], p. 146).

Furthermore, the generation of curves through strings used in the rectifying way, despite being able to produce several curves by one continuous motion, differs in a fundamental way from Cartesian machines. In fact, Leibniz observed that curves constructed by strings required "material" curves, namely curves previously constructed in the plane (like the circle, in the case of the cycloid), around which a string can be bent and unwrapped. The case of Cartesian machines is completely different, since in order to construct any algebraic curve it is sufficient to provide a system of linkages which ensure a unique and continuous motion, but no other curve needs to be assumed as previously constructed. Eventually, the discovery of tractional motion enabled Leibniz to overcome the difficulties related to string-constructions because the former kind of motion neither presupposes rectifications nor material curves in Leibniz's sense. Tractional motion was conceived, therefore, as the most useful and natural extension of Cartesian machines, and allowed the construction of transcendental curves using one continuous motion.

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Instruments for impossible problems: Around the work of Ljubomir Klerić (1844-1910)

Dominique Tournès

I am interested in mathematical problems that are non-constructible with a certain set of instruments, but that may be constructed by creating new devices. After introducing this issue, I will present a case study centred on a specific instrument conceived at the end of the 19th century by the Serbian engineer, Ljubomir Klerić.

In the first three postulates of Euclid, no physical instrument is mentioned, but everyone understands that they constitute an idealization of the use of a ruler and a compass. In his famous commentary on the first book of the *Elements*, Proclus insists that a straight line, a circle and, more generally, any line, is the trace of a moving point. He describes without any ambiguity the mechanical movement that generates a circle, but for the straight line he offers a more vague "uniform and undeviating flowing", which is not as well-defined.

This flaw has been noticed by Alfred Kempe and many engineers such as Watt and Tchebycheff, when they wanted to mechanically replicate perfect linear motion in steam engines. In his stimulating book entitled *How to draw a straight line*, Kempe says ([2], p. 2): "If we are to draw a straight line with a ruler, the ruler must itself have a straight edge; and how are we going to make the edge straight? We come back to our starting-point." The first satisfactory answer was found by the French engineer Peaucellier and, independently, by the Lithuanian Lipkin: one can construct a straight line with a linkage transforming by inversion a circular motion into a perfect straight-line motion.

Due to its first three postulates, Euclid's geometry concerns the problems that can be solved by constructing a finite number of straight lines and circles. However, in the Antiquity and the Middle Ages, in particular in the Greek and Arabic worlds, mathematicians encountered problems that they could not solve with a ruler and a compass: the most famous of these are the duplication of the cube, the trisection of an angle, and the quadrature of the circle. To solve these problems, they had to introduce new devices and new curves, starting with the conic sections.

In his *Géométrie* of 1637, Descartes remarked that ruler and compass are machines, and so there was no reason to refuse the use of other machines in geometry, as long as they generated curves by a simple, continuous motion. By this definition, he accepted the use of what we call today algebraic curves, which can be mechanically traced, at least locally, by linkages. In a certain sense, we can view a linkage as analogous to a combination of a finite number of compasses.

At the end of the 17th century, algebraic curves were no longer a source of great interest within the realm of infinitesimal calculus, hence Leibniz expended much effort exploring new kinds of continuous movements that could generate transcendental curves. Eventually deciding that tractional motion was the best candidate to revitalize geometry, Leibniz imagined a universal integraph for quadratures and more general differential equations. His idea was to take a tense string and, by a suitable mechanical device, as it moved, impose the concomitant slope given by the differential equation. Thus the motion of the string traced a curve whose tangents are given, in other words, an integral curve solution of the inverse tangent problem. This idea gave birth to a complete theory developed later by Euler and Vincenzo Riccati, and is also at the origin of the conception and the making of actual integraphs, a few in the 18th century, but the majority at the end of the 19th century [6].

Throughout the 19th century, new abstract methods of reasoning were implemented to study the classical problems that remained unsolved. For the first time, the impossibility of defining a solution to a given problem was rigorously established, and the set of problems that could be solved with a given procedure, clearly characterized. Among the results, we should mention here that Wantzel proved in 1837 the impossibility of the duplication of the cube, and the trisection of an angle with a ruler and a compass; also Lindemann established in 1882 the transcendence of π, the consequence of which was to be able to confirm the impossibility of squaring the circle with a ruler and a compass.

In fact, all the problems that were proven as impossible to solve at that time were subsequently solved by the introduction of new instruments. I want to illustrate this by considering the case of Ljubomir Klerić. The starting point for this study was a curious paper published in 1897 in the *Dinglers polytechnisches Journal*, which announced an ambitious program: the construction of the numbers π and e, and all regular polygons [4].

Julius Klery was born in Subotica, Austria-Hungary, on June 29, 1844. His family was of German origin. When he arrived in Belgrade, he decided to adopt a Serbian form for his name: Ljubomir Klerić (or Kleritj). After graduation from high school, he studied engineering at the Belgrade College. In 1865, having received a state scholarship, he was sent to the mining academies in Freiberg and Berlin, and to the Zürich polytechnical school. From 1870 to 1875, he worked for mining companies in Westphalia, Saxony, Upper Silesia and Bohemia. In 1875, he became a professor of the Belgrade College. In 1887, he was elected as a full member of the Serbian Royal Academy. During 1894-1895, he was Minister of education and ecclesiastical affairs, and in the period 1896-1897, he was Minister of the national economy. He died in Belgrade on the 21st of January, 1910 [5].

Klerić fits the definition of an "ingénieur-savant", that is an engineer with a strong training in mathematics, able to create new mathematics by himself for the needs of his practice, as well as being active in the scientific institutions of his country and publishing in scientific journals. Between 1872 and 1907, he actually produced 48 articles and books in several domains of the engineering sciences and in mathematics. In particular, his work included the invention of some new instruments: a new typewriter named a "polypantograph"; a new compass named a "tractoriograph" or "logarithmograph", and several measuring instruments including a precision curvometre and a logarithmometre.

The second of these instruments concerns us. Klerić describes it in these terms ([4], p. 234): "In 1891, I invented a very simple instrument with which, for all kinds of plane curves, one can describe their tractrix at a constant distance, that is with a constant tangent. I called this instrument 'tractoriograph'. This instrument is made at the Mechanical Institute of Oskar Leuner in Dresde, and costs 22 M." In fact, Klerić's device (see Fig. 1) is a variant of the famous Prytz planimeter, a very simple instrument that allows calculation of areas, not exactly, but to a very good approximation sufficient for most practical applications. This planimeter was a great success because it was cheap and easy to use.

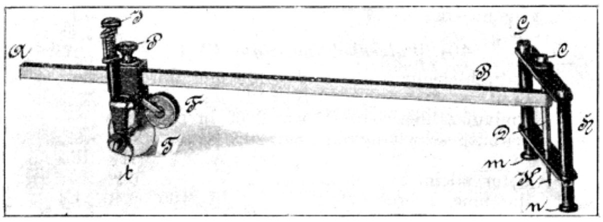

FIGURE 1. Klerić's tractoriograph ([4], p. 234)

Klerić's originality is that he does not use his instrument for calculating areas of surfaces but, in a purely theoretical sense, for the construction of impossible mathematical problems. The major part of these constructions is based on the "circular tractrix", that is the tractrix of a circle traced with the condition that the length or the tractoriograph is equal to the radius of the circle. A spectacular property of this curve is that, if it is joined to a ruler and a compass, it allows the rectification of any arc of the circle. From that, it is easy to construct the number π, to rectify and to square the circle, and to inscribe in it a regular polygon with any number of sides. Incidentally, by using the tractrix of a straight line, Klerić proved that the number e is also constructible with his tractoriograph.

It is amazing to note that at the same time, a different solution of the quadrature of the circle was published independently by Felix Klein in his *Famous Problems of Elementary Geometry* ([4], p. 78): "An actual construction of π can be effected only by the aid of a transcendental curve. If such a construction is desired, we must use besides straight edge and compasses a 'transcendental' apparatus which shall trace the curve by continuous motion. Such an apparatus is the *integraph*, recently invented and described by a Russian engineer, Abdank-Abakanowicz, and constructed by Coradi of Zürich."

It is true that in the solution of classical problems, some instruments like Descartes' linkages or Leibniz's integraphs seem to be thought experiments or imaginary, ideal instruments conceived to solve theoretical problems, but never constructed for actual use. From a different perspective, we have seen that other people, such as Klerić or Klein, have exploited actual physical devices to solve the same problems.

In the 1970s, the Russian engineer Ivan Ivanovitch Artobolevsky published an encyclopedia of mechanisms in five volumes [1]. The aim of this treatise is to provide an inventory and describe all the elementary mechanisms that engineers can use and combine to create complex machines. In this collection, we find algebraic mechanisms: linkages to trace the three conic sections, a linkage to extract cubic roots (which is in fact the old device attributed to Plato), the trisector of Descartes, a linkage to trace the conchoid of Nicomedes, and another to trace the cissoid of Diocles. We can also find transcendent mechanisms: the polar planimeter of Amsler, the integraph of Abdank-Abakanowicz, tractional instruments to trace the logarithmic curve, and the spiral of Archimedes.

In this inheritance accumulated in mechanical engineering practice, we recognize the major part played by the famous devices that have been conceived since Antiquity to solve the classical problems. Most of them were probably ideal machines when they were first used by mathematicians. However, in Artobolevski's catalogue, they are also physical and perfectly efficient machines. All these devices are clearly at the crossroads of mathematics, mechanics and technology. They can be both imaginary and physical, and it seems important to me to study them from both of these perspectives.

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Tractional constructions as foundation of differential equations: Ancient open issues, new results, possible fallouts

Pietro Milici

Machines play various roles in mathematics: they can embody mathematical concepts to be transferred to real-world applications and foster deeper understanding (while conceiving, constructing and using them). But devices can also play a very relevant foundational role, as seen in the geometry of Euclid or Descartes: "simple" machines can be idealized to become the quintessence of fundamental concepts still keeping a strict contact with concrete experience and allowing manipulation (so with a wonderful cognitive richness). The fundamental question behind this work is, can machines constitute a foundation also of advanced mathematics, avoiding abstract concepts such as infinite objects or processes?

Since its birth, Infinitesimal Analysis required the concept of infinity. Is it possible then to provide a rigorous but also concrete/sensitive foundation of this subject? From this perspective I propose a new setting of calculus based on some historical geometric insights. In making such an attempt to rephrase the subject, the first step is to clearly define the required tools and their constructive limits, and this is the focus of this talk; but previously we should consider some history.

In the 17th century, curves were generally introduced as traces of ideal machines. A balance between algebra and construction of curves was provided by Descartes's *Géométrie* thanks to a suitable class of machines (cf. [3]). Soon after the spread of the Cartesian canon, polynomials were no longer considered as formalizations of geometric problems, but as solutions. Thus the foundational role of machines continued only to justify non-algebraic curves (not treatable by Cartesian tools). In particular, a general problem that affected a wide range of transcendental curves was the *inverse tangent problem* (in the modern setting, it is found in the geometrical solution of differential equations). The first documented appearance is attributed to Perrault (late 17th century): such constructions were termed "tractional."

Many mathematicians worked on clarification and definition of tractional motion from both practical and pure mathematical perspectives (cf. [10]). Physically, the component solving the inverse tangent problem has to avoid the lateral motion of a point with respect to a given direction. This can be accomplished by something that, like the blade of a pizza-cutter or the front wheel of a bike, guides the direction of the motion (for a contemporary example of such machines see also [5]).

Leibniz was particularly interested in these constructions, so the historical role that tractional motion played as a tangible insight to the origin of calculus is clear. However, the analytic tools that Leibniz used to solve such problems involved the introduction of infinity. As happened with Cartesian machines, the geometric synthetic counterpart gradually fell out of favour, even for tractional constructions, and it soon became obsolete in mathematical practice.

Even though almost forgotten, I would assert that tractional constructions can provide an alternative foundation to calculus, cognitively based no longer on the metaphor of infinity (as suggested in [4]), but on a more concrete metaphor, e.g. "the wheel direction defines the tangent to a curve." Such a metaphor is present in everyday experience (to turn in bike, we turn the direction of the handlebars).

Given these premises, it is interesting to go back and delve into some extant, ancient issues. Firstly, constructive limits of tractional motion have never been clearly defined; not only this, the various attempts at defining a canon of such machines never achieved a generally accepted end point.

So begins my mathematical work. Once such a canon is defined (see [6]), it is possible, thanks to 20th century differential algebra (a branch of computer algebra, cf. [8]), to define the limits of tractional constructions (cf. [7]). Specifically, all the constructible curves are the ones that locally can be parametrized by differentially algebraic functions: we can note that such functions are the same as those obtained in Shannon's GPAC (cf. [9]) many centuries after the introduction of tractional constructions.

From a foundational perspective, it is also interesting that the analysis of tractional machines does not need infinity: tractional machines can be investigated in a purely symbolical way using differential algebra (restricted to ordinary differential equations) without the need for infinite objects or processes. This can be considered as an extension of Descartes's foundational balance (synthesis with ideal geometric machines, analysis without infinity and a well-defined class of obtainable objects), but far beyond polynomial algebraic boundaries.

Out of the theoretical model, tractional machines can be useful for didactic purposes, in particular to foster a deep and conscious understanding of calculus and differential equations. Such topics pose several challenges because they involve the manipulation of infinite objects. Research in mathematics education, which has focused on this topic for a long time, has highlighted obstacles and proposed different approaches in this field of mathematics.

Indeed, the actual manipulation of an artefact can help students to experience and internalize the underlying mathematical concepts if suitably introduced into educational pathways (as suggested in [1], which focuses on the use of artefacts to transmit mathematical knowledge).

The adoption of tractional tools in laboratories to improve the learning of students has already been seen in the Italian tradition: we may recall Giovanni Poleni (Padua, 18th c.) and Ernesto Pascal (Naples, early 20th c.). An interdisciplinary commitment will consist of developing suitable didactic activities concerning tractional motion with the aid of both physical machines and digital tools, as already realized for algebraic machines (cf. [2]).

To conclude with a view to the future, we can note that the geometric "legitimation" of analytic results is somehow present today in some fields of advanced mathematics as *fractional calculus*, that still requires a widely accepted geometric interpretation. Fractional calculus analytically involves the use of Euler's gamma function, that is not a differentially algebraic function, so this class of problem cannot be tackled using tractional motion. An exciting problem is thus to overcome tractional constructions in order to include such class of problems, extending the power of analogue computation whilst still avoiding the introduction of infinity or approximation.

This reminds us of the lack of a mathematical definition for the "exactness" of a constructive/computational framework (in both digital and analogue paradigm), where exactness means that the framework does not involve infinity or approximations. An initial notion that could be further explored might be that a computational framework is exact if and only if the equality test between any two constructed objects is computable. From this perspective, we have that even in the more "concrete" approach of analysis (*Computable Analysis*, mathematical analysis approached through computability theory) the equality test is not computable, while with the tractional motion model the equality test is computable despite certain open questions existing regards boundary conditions. The long quest for exactness has just begun, again.

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Mathematical instruments in medieval monasteries

Seb Falk

This presentation was based on a new research project on the study of sciences in late medieval monasteries. Its focus so far has been on mathematics and astronomy, as studied and practised by Benedictine monks, particularly in England. A tentative early conclusion is that monks were – in comparison with their contemporaries in university settings – unusually interested in instruments.

The presentation began by discussing the considerable problems of evidence. Even where monastic instruments survive, it is hard to be certain about their origins. Since few do survive, we are reliant on written descriptions and lists; instruments are sometimes included in library catalogues. When they are described in monastic books, we often face the problem that a book may have been produced outside a monastery. In the medieval period books and instruments moved easily between different settings such as monasteries and universities, and could change hands frequently.

In addressing the overall question "Was there a distinctive interest in instruments in late medieval monasteries?", my research investigates several underlying questions, such as: what was the status of mathematics and astronomy in monasteries? What motivations did monks have to study instruments? How were instruments studied and used in monasteries? Monasteries were central to the European reception of scientific knowledge from the Islamic world in the eleventh and twelfth centuries. Even after the rise of universities, monasteries remained centres of scholarship: some monks studied at university and continued their studies in the cloister. But how was monastic astronomy different from the science studied in universities? In the early Middle Ages, monks were motivated to study astronomy and mathematics by their interest in problems of calendrical computation; this interest was less evident in monasteries by the thirteenth century.

In the later Middle Ages, monks valued instruments for practical functions such as timekeeping and astrology; for education and study as models of the heavens; and for their devotional potential as symbols of the order of Creation. Most instruments performed multiple functions. For example, Richard of Wallingford, abbot of St Albans (c.1292-1336) highlighted the multifunctionality of his Albion instrument (see Fig. 1), and emphasised its potential to "direct the minds of many people to higher things". He stated that there was nothing new in his instrument, which was a device able to perform a wide range of calculations in planetary astronomy; although he may have been right that it added nothing to astronomical theory, it was hugely innovative in the ways it presented astronomical models accessibly for easy calculation.

Few monks could have grasped the full complexity of Richard's work, but it is clear that his successors at St Albans were proud of his achievements. They had an ambivalent attitude to the expensive clock he designed for the abbey church, but the Albion symbolised his devotion to science and success in its pursuit. Later copies of his work were made as acts of devotion to his memory, as well as (in at least one case) charity to the daughter house to which it was donated.

Another mathematical-astronomical instrument was discussed: the planetary equatorium designed by John of Westwyk (c.1358-c.1397), a St Albans monk and disciple of Richard of Wallingford. Its designer's priorities were simplicity of construction and user-friendliness. The treatise which describes its manufacture was written in Middle English and contains a wealth of practical advice to the reader who might try to make it – and stresses the importance of constructing it at a large size to ensure the greatest possible precision.

Monasteries, therefore, do seem to have accommodated distinctive interests in instruments. Particular religious concerns may have motivated the design of instruments or the copying of texts describing them; for charitable reasons monks may have valued simplicity and user-friendliness more than university scholars; and with the great resources of some monasteries (and the vow of stability taken by monks) they may have had more reason to make instruments large, and less need to make them portable. Perhaps above all, the construction of large instruments, especially clocks, were evocative statements of the expertise and authority housed in monasteries.

Figure 1. Albion (ø = 325 mm), made in central Europe in the 15th century, following the instructions in Richard of Wallingford, *Tractatus Albionis* (1326). Rome, Museo Astronomico e Copernicano

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Geometrical tools and mathematical practices: An exploration of BSB Cod.icon. 182 (1520s)

Richard Kremer

Most extant 15-16th century texts on mathematical instruments are what we might call "algorithmic". They instruct readers, by commands in the imperative voice, to follow a series of steps, first to make the instrument, and then to use it. Draw this line, inscribe that circle, set the rule on this point. Or to determine how many equal hours have passed since sunrise, note the day of the year, turn the rete of the astrolabe to this place, read the hours from the lines at that place. A good example of such algorithmic prose appears in J. Stoeffler's widely distributed *Elucidatio fabricae ususque astrolabii*, first printed in 1513. Such texts offer no explanations for the geometry of stereographic projection, no proofs that the techniques of two-dimensional construction replicate geometrical relations on the sphere, etc.

A quite different approach to mathematical instruments appears in a Munich manuscript, BSB Cod.icon. 182, dated on its cover by an early hand to "um 1508". Written on paper, inexpensively bound in thicker paper half covered with leather, this codex contains two parts, 80 folios of drawings (no text apart from short rubrics above the drawings) of what I will call "geometrical tools" and 15 folios of an abbreviated version of Companus of Novara's 13th-century *Theorica planetarum* or equatoria for the planets [1]. Written on different paper by different hands, these two parts were brought together in the 1520s when the codex was bound [10]. A sixteenth-century hand has entitled the codex: Astrolabia / Quadrantes / Annulus horarius / Theoricae planetarum / ex Companio. We shall focus on the first part, filled with drawings related not only to astrolabes, quadrants and ring sundials but also to projection schemes for maps, nomograms for day lengths over the course of the year, and several astrological themes (house divisions).

Rarely, if ever, do the drawings depict a finished instrument. Never do they include "construction lines" or features emphasized in the algorithmic texts to guide construction. Instead, the drawings in our codex isolate parts of an instrument or distinct geometrical ideas into what I previously have called geometrical tools, i.e.:

> a particular configuration of graphical elements that allow users to solve a discrete geometrical problem. As suggested by the word, such tools are mobile and easily handled. They can be added to existing instruments, combined to form new instruments, or simply interrogated on their own terms. Early modern mathematicians, I shall argue, play with geometrical tools... I shall suggest that geometrical tools rarely if ever required geometrical proofs to justify their efficacy ([7, p. 105]; see also [2, p. 248]).

Fig. 1, for example, shows the plate of an astrolabe marked only with circles for the equator and tropics and a set of unsteadily drawn, looping curves that mark the rising of half signs (15-degree intervals) of the ecliptic above the horizon. I have found no earlier examples, on paper or brass instruments, of such lines [3, 15, 8]. In the drawing, the scribe of Cod.icon. 182 has isolated these lines, presumably novel to him as well, from the other features of planispheric astrolabes. Nothing is explained; no hints are provided for how the irregular loops are generated. In Fig. 2, our scribe isolates one feature of a basin sundial, one of three drawings related to this instrument. None of the individual drawings provide a visual representation of the full set of lines required for this dial. The codex is filled with such sketches of discrete ideas, fragmentary representations not found in the usual "making and using" instrument texts of the period.

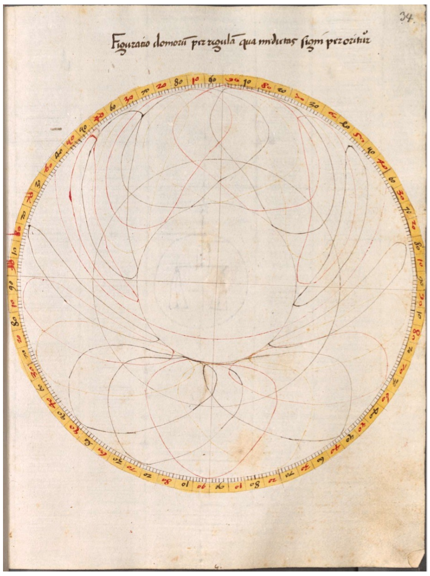

FIGURE 1. Figuratio domorum per regulam qua medietas signi per oritur, Munich, BSB Cod.icon. 182, f. 34r

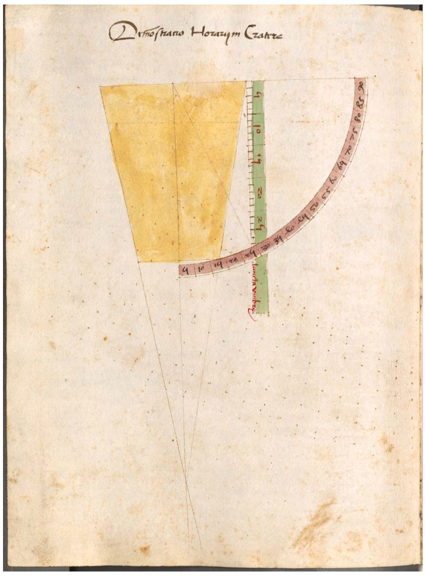

FIGURE 2. Demonstratio horarij in cratere, Munich, BSB Cod.icon. 182, f. 66v

How are we to situate Cod.icon. 182 within the visual and mathematical cultures around 1500? Who might have drawn the sketches and why? Contemporary artists' **sketchbooks**, such as Albrecht Dürer's "buchlein" of silverpoint drawings made in 1520-21 during his trip to the Netherlands, usually feature fully finished pictorial motifs, recordings intended to be used as components in later, larger artworks. Some studios produced what art historians have called **pattern books**, such as the early 15th-century boxed set of small, complete drawings of various types of human heads and some animals, copied by apprentices learning to draw (or collected by wealthy patrons?). The only other contemporary example I know of a manuscript filled with drawings of mathematical instruments is Georg Hartmann's so-called *Astrolabes*, Nuremberg, 1527, a codex of 75 folios on parchment, finely drawn (a presentation copy?), showing various instruments in drawings far more finished than those of Cod.icon. 182 [14, 13, 9, 5].

Several physical clues in the codex, plus some of the sketches, situate Cod.icon. 182 in a quite different context, viz., amidst the Vienna university astronomy lectures of Andreas Stiborius (c. 1464-1515). The bottom right corners of the recto sides of many folios are heavily thumbed, suggesting frequent use of the loosely bound codex. Nearly every folio bearing a sketch of an astrolabe base (28 in all) has on its verso side a small, square slip of paper pasted on the sheet, presumably to provide support for a thread to be attached through the center of the circle to anchor a rotating vovelle (rete in this case). Yet only several times are the sheets actually pierced in the center. We might guess that the astrolabe sheets had been "mass produced" before being assembled into the codex. Likewise, most of the sketches are pricked, suggesting that the designs were copied from a pattern and not constructed free-hand on the folios. Many sketches are clearly unfinished, displaying divided arcs without numerals, circles not divided, etc. These clues seem to place Cod.icon. 182 in a pedagogical context.

One of the sketched globe projections (f. 22r), heart-shaped, had been invented by the mathematicians Johannes Werner and Johann Stabius in Nuremberg around 1500. In 1502, Stabius and Stiborius together moved from the university in Ingolstadt, where both had been teaching, to Vienna to join the circle of Konrad Celtis [11]. Another sketched astrolabe plate (f. 37r) divides the day into four equal segments for the four humors (a design I have never seen before), a topic discussed by Stiborius in his lectures on the astrolabe. Notes have also survived for his lectures on a universal astrolabe called the saphea and on the Ptolemaic organum. A recent analysis of these notes has speculated (without knowing about Cod.icon. 182) that "Stiborius probably used inexpensive, paper instruments to illustrate his lectures" [6]. I would add that he probably encouraged his students to prepare their own sketches of the ideas or geometrical tools in those instruments by copying paper patterns he distributed. Indeed, one of the earliest scholarly appraisals of Cod.icon. 182, made without any knowledge of Stiborius's lectures, concluded that the codex "probably was an astronomical compendium [Grundriß] given to students in manuscript copies by a university lecturer as a teaching aid" [12, 4].

A thorough study of Stiborius's lectures and Cod.icon. 182 may illustrate how university lectures on instruments differed from the usual "making and using" texts preserved so frequently in 15th and 16th-century manuscripts, by breaking down the instruments into geometrical tools.

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Comparing astrolabes

Petra G. Schmidl

"Comparing astrolabes" presented two case studies that demonstrate how productive comparing astrolabes can be, and how difficult this task still is. In a brief conclusion this talk develops some very preliminary ideas regarding a helpful tool for comparing astrolabes.

An astrolabe captures the movements of celestial objects around a terrestrial observer and simulates this three-dimensional experience within a two-dimensional model. There is a celestial part, a star map, called the *rete*, and a terrestrial part, the *plates*, that fit in the *mater*, the box that holds them. Astrolabes might be used for calculation, observation, and timekeeping. Although multifunctional astronomical instruments and analogue computers, astrolabes also serve other purposes such as demonstration, decoration, amusement, education, and entertainment.

Many astrolabes are undated and unsigned, in particular the earliest instruments made in medieval Europe. The earliest in this group that is signed and dated was made, most probably, in Barcelona in 1375 by Petrus Raimundus (IIC #3053)[6]. But there is more than sufficient evidence to indicate that the astrolabe became known in medieval Europe centuries before. One example is the undated and unsigned astrolabe Marcel Destombes called "l'astrolabe carolingien" (IIC #3042); this, incidentally, is probably the most intensively studied European astrolabe, and demonstrates perfectly how difficult it is to date and locate these early European astrolabes.

Therefore, comparing astrolabes can bring new facts to light, for example their places and dates of construction, that can help us learn more about their relationships, origins, and dispersal, as well as their possible movements over time. This comparison is, though, hindered because of the lack of easily available and reliable research tools. The following two case studies will illustrate this assertion.

[6] Astrolabes are identified by their IIC (International Instrument Checklist number). IIC numbers #0001 to #0336 according to [1], following numbers, although not consecutively, according to [5], instruments #4000 onwards according to [3], see also [2, p. 1054-1060].

The first case study deals with an undated and unsigned European astrolabe preserved in Oxford (IIC #0191), made of brass and 146 mm in diameter. Besides a detailed investigation and description of the instrument, recent research concentrated on the question of if and how it is possible to establish a plausible medieval record of its geographical movements. In sum, the result reads: The astrolabe most probably began its voyage in Valencia before finding its way to Saragossa. It then moved north and spent some time in Paris; it is plausible that it even made a detour to Italy before coming back to France.

Of the evidence supporting this possible journey, that pointing to Valencia is the most eye-catching. The rete of IIC #0191 is nearly the exact twin of IIC #0121, an Andalusī astrolabe (see Fig. 1) that bears an inscription "At the end of the year 478 (April 1086) – Ibrahīm b. al-Sahlī made it in Valencia." Both astrolabes are approximately of the same size.

Figure 1. On the left: The front of the undated and unsigned European astrolabe in Oxford (IIC #0191; photo courtesy of the Museum of the History of Science, Oxford). On the right: The front of Ibn al-Sahlī's astrolabe in Kassel (IIC #0121; photo courtesy of the Staatliche Museen, Kassel)

Although, there are slight differences, the most striking being that the star pointers of α Sco (Antares) and δ Cap are missing on the European model, and the pointer for α Leo (Regulus) only hits the ecliptic on the Andalusī rete. Nevertheless, the Andalusī looks like a template for the European rete. Further evidence, such as the notation of numbers, indicate an Andalusī influence on the European astrolabe, that is supported by a plate for Saragossa. This plate is most probably older than a second one for Paris. Again, the design of the plates and the notation of numbers supports the argument. The conjecture regarding the detour via Italy and the astrolabe's return to Paris is primarily based on two observations.

The first concerns two early European astrolabes (IIC #0168 and #0410), whose bold majuscule resembles the majuscule on the back of the mater of the European astrolabe (IIC #0191). The problem, however, is not so much that there are similarities and differences, but rather that the descriptions found in the literature of these two astrolabes are very ambiguous – only an agreement of an Italian connection is apparent. The second observation takes into account the fact that the European astrolabe possesses a second plate for Paris, without any obvious reason. One might speculate if a longer absence of the instrument from Paris was somehow responsible for this odd plate doubling [4].

To conclude this first case study, two points are worth stressing. Firstly, the initial and final resting places of the early European astrolabe are not defined by the instrument itself, but rather by the widely accepted narrative of knowledge transfer from the Iberian Peninsula to Europe in these times. Secondly, with tools at hand that could simplify a comparison of astrolabes, how much easier it would have been to track down its movements or to find answers regarding the origins and whereabouts of IIC #0168 and IIC #0410?

The second case study is still "work in progress" in collaboration with a student of Frankfurt University, Convin Splettsen, and concerns, again, an undated and unsigned astrolabe, this time preserved in Florence (IIC #0101). Made of brass, it measures about 161 mm in diameter and is, therefore, approximately 15 mm larger than the astrolabe in the first case study (IIC #0191). In earlier descriptions it was attributed to Gerbert of Aurillac, later Pope Silvester II (d. 1003), a title that gave the astrolabe its unlikely but enduring nickname.

FIGURE 2. On the left: The front of the undated and unsigned astrolabe in Florence (IIC #0101; taken from [2, p. 491]). On the right: The back of the same astrolabe (IIC #0101; taken from [2, p. 492])

Again, research focuses on establishing a possible medieval itinerary for the movement of this astrolabe. So far, indications are that the astrolabe originated in Baghdād, later moving to Europe following a stopover in al-Andalus. Concerning its initial whereabouts, primary evidence is provided by the design of the rete that shows similarities with those of Arabic astrolabes from early ʿAbbāsid times. Most characteristic are the star pointers that comprise acute isosceles triangles on rectangular pedestals whose upper two angles can be raised up. They resemble little daggers (see Fig. 2). This characteristic is found on other early Arabic astrolabes, for instance on those made most probably in the 9th century, for example, Baghdād by al-Khafīf (IIC #1026 and IIC #4030). King places this instrument (IIC #0101) into a group of early ʿAbbāsid astrolabes, most probably because of these similarities, and deals with the other features as "later additions".

But one has always to keep in mind that this could simply be retro style. The Yemenī astrolabe of al-Ashraf ʿUmar from 13th c. (IIC #0109) includes star pointers that have the characteristic form of those found on early ʿAbbāsid retes, and others that do not. This reminds one to be very careful when making deductions from a single source of evidence.

The astrolabe's stopover in al-Andalus is indicated by its circular scale on the back, although incomplete and not inscribed (see Fig. 2). It resembles one presented on the back of all three astrolabes al-Sahlī made in mid-11th century al-Andalus (IIC #0117, IIC #0118 and IIC #0123). At the moment, this scale appears to be unique to these astrolabes, but further research is required.

The astrolabe's arrival in Europe is obvious because of the Latin inscriptions, but this has been of secondary concern until now. Only an initial list of early European astrolabes has been prepared to find possible *comparanda*.

While the results of these two case studies are instructive, the method of obtaining them was not; it happened more or less by accident. A better tool for comparing astrolabes is a real *desideratum* for the history of astronomy and astronomical instruments.

Needless to say, since the Frankfurt catalogue project began, many descriptions, catalogues, overviews and webpages have been published. Nevertheless, finding objects for comparison is a difficult enterprise given the current tools at hand. Standardized descriptions based on a strict hierarchic structure as components of a searchable online data base would be a possible solution, that might be accompanied by printed fascicles.

If one takes as an example the astrolabe in Oxford (IIC #0191) as seen in the first case study, it would easily be possible to search quickly for star pointers with bases referred to as "small rings"; thus its movements might be easier to establish, or new facts might come to light. The same holds for the second case study, the astrolabe in Florence (IIC #0101). These are, though, only very preliminary ideas that will be constrained by the affordability of such a database. All manner of applications are conceivable, however; the sky is the limit!

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What were portable astronomical instruments used for in late-medieval England, and how much were they actually carried around?

Catherine Eagleton

Deceptively simple, but challenging to answer, the title of this paper was a question that runs through much scholarship on astronomical and timekeeping instruments in the later Middle Ages. The link between instruments and astronomy is clear in manuscripts of the period, so there can be no question that they were seen as connected. However, a combination of theoretical knowledge and instruments provides no clear evidence of how the instruments were used, and what for. Portable sundials could be carried around to tell the time, but were they?

Derek J. de Solla Price, whose scholarship included studies of a range of different instrument types, published an influential article that outlines the uses of medieval astronomic instruments, suggesting that their practical value may have been that they were "ideas made brass" rather than that they were used for observation:

> "These devices [...] were tangible models that served the same purpose as geometric diagrams or mathematical or other symbolism in later theories. They were embodied explanation of the way that things worked [...] I suggest that tangible modelling as a species of comprehension comes nearer to the 'purpose' of armillary spheres or star and earth globes than to imagine they had a prime utility as devices for teaching or for reference." [10, p. 76]

Francis Maddison, who was Curator of the Museum of the History of Science in Oxford, took a similar view, and in an article based around a typology of the most important medieval instruments explained that "none of the [...] instruments discussed above was of much use to any practical profession, except that of teaching astronomy" and that they were instead (with the exception of the magnetic compass and mechanical clock) an application of theoretical astronomical knowledge [8, p. 20].

However, as I argued in my presentation, there is some evidence that suggests we might need to reconsider the practical uses of some types of instruments, and consider whether some were carried around to tell the time, or used for practical purposes alongside the symbolic, teaching, and other functions that Price and

Madison outline. Instruments could be "ideas made brass" but they could also be of practical use, and perhaps this combination was part of what gave them their importance in the period.

Texts. The challenges around answering questions about what instruments were used for relate in part to the available evidence, which can be patchy. Documents from the medieval period include scattered references that shed light on ownership of astronomical and timekeeping instruments. For example, the 1434 will of John de Manthorp, vicar of Hayton, East Yorkshire, in northern England, includes an astrolabe and a calendar, and the will of John Hurt in 1476 bequeathed to the Cambridge University Gotham Loan Chest a book about astronomical instruments that was already in the chest as security against a loan ([2, p. 561–562], [3, p. 704]). However, without biographical or other information about these two Johns, it is impossible to know more about their interest in astronomical instruments, and what their ownership of instruments and books might indicate in terms of their uses.

Literary references provide clues, but these, too, can be difficult to interpret. Geoffrey Chaucer wrote a *Treatise on the Astrolabe*, and he also put references to astronomy and astronomical instruments into some of his works of fiction, including the *Canterbury Tales*, which tells the story of a group of pilgrims travelling to Canterbury. These and other literary references perhaps indicate an understanding of astronomy and its instruments among the courtly audiences who read or heard the works, but they are rarely unambiguous evidence for people using instruments to tell the time or observe the heavens [9].

Among the most numerous sources that help us to understand the types and functions of medieval astronomical instruments are manuscript texts describing how to make and use them. These technical works describe a wide range of types of instruments, and many different practical uses to which they could be put. Chaucer's *Treatise on the Astrolabe*, for example, describes in English more than 40 calculations and observations that the astrolabe could perform. The instructions are clear, but the diagrams, if they are present, are usually more geometrical than instructional – these manuscripts are more than simply handbooks for the practical use of the instruments they describe. Interestingly, there is strong evidence from medieval England that manuscripts were kept together with the instruments they describe in libraries – and, moreover, they are the only objects other than books that appear in the surviving late-medieval English booklists. I have argued in a previous publication that this is the case because both are regarded as sources of information about astronomy and the achievements of great astronomers, with the instruments complementing the books [5].

Instruments. Today, the surviving medieval astronomical and timekeeping instruments are no longer in libraries, but in museums and collections around the world [6]. Some museums have particular strengths in their holdings of medieval English astronomical instruments, and their holdings can give an idea of the types of objects that survive. However, few of these instruments have detailed provenance recorded, and few are associated with specific places or people, making it rather difficult to consider what they might (or might not) have been used for. Comparing instruments preserved in museum collections to the surviving manuscript texts shows also that there are instruments made and used that tend not to have survived, for example cylinder dials, which according to a commonly-copied text should be made from boxwood [7]. Some complex instruments may have been only rarely constructed, and on the other hand there are instruments that may have been widely used that were not written about, like sandglasses and simple compass dials [1].

Archaeology. Pointing to the potential for archaeological evidence to contribute to our understanding of astronomical instruments in the medieval period, there was an extraordinary find in 2005, of an astrolabe quadrant. This instrument is especially rare in that it was found during a planned dig, and therefore recorded with full context and documentation. It was found in a dig at an inn in Canterbury, the *House of Agnes*, dating back to the thirteenth century. The instrument itself was dated from examination of the scales on it to 1388, opening up the tantalising possibility that pilgrims did indeed take these kinds of instruments with them when travelling on pilgrimage [4].

For England in particular there is a resource that can be drawn on to investigate what contribution a consideration of archaeological evidence can make to our understanding of the uses and users of medieval astronomical and timekeeping instruments: the Portable Antiquities Scheme (PAS). This voluntary scheme records small finds that are not required to be processed as Treasure, finds that are often made by metal detectorists. The objects recorded are often "stray finds" of objects that were lost as people moved around the country, rather than things that were deliberately concealed. The scheme not only records finds, but promotes good practice among metal detector communities, including the need to gain permission from the landowner, and the importance of recording context and precise locations for finds [11].

Among the objects recorded in the PAS database are hundreds of thousands of coins, more than 75,000 buckles and brooches, and a small group of 110 objects recorded as sundials from the medieval and post-medieval periods. Among the most interesting objects recorded is another quadrant, found by a metal detectorist in a hedgerow – he said that he knew it was an important object because he had seen the news stories about the finding of the Canterbury quadrant. In most cases, the find spot for these objects is recorded, along with basic information identifying the object. These finds provide new evidence that can be compared with the surviving astronomical and timekeeping instruments in museum collections to begin to assess whether portable instrument were actually carried around, or whether they more often stayed inside the libraries, institutions, or houses in which they were kept.

Conclusions. I argue that this analysis of archaeological information, in combination with written sources and objects in museum collections, points to a more complex pattern of uses and usage than has previously been understood. There is little evidence from the limited available archaeological data that astrolabes, that quintessential astronomical instrument in the period, were carried around, and this is perhaps unsurprising given the size and weight of the brass astrolabes that are preserved in museums. It is possible that paper or wooden astrolabes were more portable, but these are less likely to have survived, whether in museums or buried in the ground. On the other hand, there are a number of finds indicting that simple compass dials may have been fairly commonly used, and carried around, although these are rarely written about in manuscript texts.

My paper concluded by suggesting that between these two more clear-cut cases there is interesting evidence suggesting that two other types of English medieval astronomical instrument – quadrants and *navicula* sundials – were both carried around and written about in manuscripts and studied in learned contexts like monasteries and universities. Combining the archaeological evidence with written sources and close study of surviving objects suggests that they were perhaps instruments that in medieval England provided examples of the kind of tangible modelling that Price discussed, but were also practical objects that people took with them and which did not stay in libraries and private studies. There remains work to do to assess what people were doing with these instruments when they carried them around – whether they were decorative and symbolic, or of practical use for timekeeping, but there is some manuscript evidence that suggests the latter, which I will expand in future work, and in the longer version of this paper, for publication.

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Were early modern optical diagrams mathematical instruments?

Arianna Borrelli

What is a mathematical instrument? The answer to this question is, of course, up to a point, arbitrary. If we define a mathematical instrument primarily as a material, three-dimensional artefact that can be displayed on a museum shelf, then optical diagrams will be excluded. However, if mathematical instruments are rather defined on the grounds of their function, then optical diagrams may fit the bill, because they play the same role as a broad number of apparatuses that are widely recognized as mathematical instruments. Like surveying tools, mechanical clocks, maps, or astrolabes, optical diagrams are "instruments" because they constitute a way to conceptualize phenomena such as landscapes, time, or celestial motion that makes them cognitively and practically manipulable. They are "mathematical" in that the conceptualization makes use of pre-existing notions that are situated within the realm of (somehow defined) mathematical practice.

This case study is presented to support the more general thesis that the notion of "mathematical instrument" can be productively extended beyond material artefacts or ideal machines such as compass, clock, or computer, to also indicate apparently less "material" tools such as diagrams or symbolic formalism. My key assumption is that, in mathematics as in all other cultural activities, there is no such thing as disembodied knowledge. Even the most abstract notions can be the object of scientific, historical or philosophical discussion only if expressed in some sensually perceivable way, such as spoken or written words, formulas, diagrams, computer programs, or material instruments. A translation between one form and the other is of course possible, but in most cases it entails an adaptation of the content to the new form, as often discussed in the case of geometrical and algebraic practices of the early modern period. Looking at mathematical practices from this perspective allows us to bridge the gap between material tools and modes of representation, since both the former and the latter function as instruments to construct, manipulate, and represent mathematical knowledge and are, as such, intuitively linked to it. Such epistemic constellations should be taken into account in the study of all areas of mathematics, but they are of paramount importance when reconstructing processes of "mathematization" or "geometrization" of natural phenomena, since the interplay between experience and the specific mathematical instruments employed to study it often shaped the emerging physical-mathematical frameworks. In the case of late Renaissance optics, new and highly non-trivial optical experiences were slowly conceptualized in terms of a small number of rules for drawing optical diagrams. The rules had their genesis in ancient and medieval optical tradition, but the diagrams soon took up an epistemic life of their own. Eventually, these developments led to what we today call "geometrical optics", whose central notions can only be grasped in terms of diagrammatic rules, although they may appear to us today as immediately abstracted from physical phenomena.

Optical diagrams already existed in Antiquity [5]. They were geometrical constructions representing and conceptualizing optical experiences to make them compatible with Euclidean geometry. A simple example of how ancient optical diagrams worked is the so-called "cathetus rule" for locating reflected or refracted images, which we see represented in Fig. 1, where the reflected or refracted image O' of an object O is seen by the eye E at the intersection of two lines: (a) the (extension of the) reflected or refracted ray that reaches E (continuous line in the figures) and (b) the "cathetus line", i.e. the perpendicular to the reflecting/refracting surface which passes through the object O (dotted line in the figures). The cathetus rule helps explain why we see an object in a mirror as though it stood at the same distance from the mirror, but beyond it, and why we see objects underwater as though they were nearer to the surface than in reality. Of course, modern optical theory regards the rule as meaningless, but this does not detract from the fact that, for these specific cases, it appears to work quite well, so that the relevant diagrams functioned in their contemporary cultural and historical context as mathematical instruments that allowed the conceptualization of, and explanation for, the simple phenomena of reflection and refraction from a plane surface.

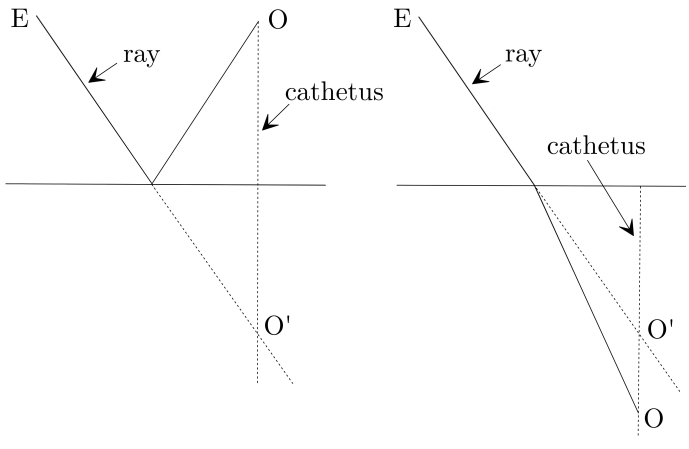

Figure 1. The cathetus rule for plane reflection (on the left) and plane refraction (on the right). E = eye, O = object; horizontal line = plane of reflection or refraction; full line = visual ray

Until the Middle Ages, only reflection and refraction on plane surfaces were systematically investigated, but in the late Middle Ages transparent, homogeneous "crystal" glass was developed, and optical devices made out of it, such as spherical mirrors and lenses, which generated new visual experiences such as inverted, larger or smaller, or "hanging in the air" images. In the Renaissance, various scholars and practitioners attempted to adapt optical diagrams to conceptualize the new experiences, working with an experimental spirit analogous to that which, in the same period, led to a broad range of new astronomical-astrological and surveying or measuring tools. Three key authors contributing to these developments are discussed here: Francesco Maurolico (1494-1575), Giovanni Battista Della Porta (ca. 1535-1615) and Johannes Kepler (1594-1630).

Following the analysis by Riccardo Bellé [1] I have argued that Maurolico adapted optical diagrams relating to refraction in the glass sphere with the aim of applying Euclidean geometry to the analysis of the new optical phenomena. Della Porta, on the other hand, in his treatise *On Refraction* (1593) and in the manuscript *On the Telescope* (ca. 1610-15), was not so much interested in applying Euclidean geometry, but rather in formulating new diagrammatic rules capable of conceptualizing the new optical experiences, such as the inverted images produced by glass lenses. The cathetus rule had already been successfully adapted to the treatment of spherical mirrors around 1550 ([4], [5]), and Della Porta built upon this result to deal with glass spheres and lenses. Although such a step may, a posteriori, appear quite straightforward, anyone familiar with the great variety of optical experiences possible with these artefacts knows that they go well beyond those which can be easily grasped in today's geometrical-optical terms of "real" and "virtual" image. As in the case of the original cathetus rule, trying to conceptualize in geometrical terms what one sees through a lens is all but obvious, and Della Porta introduced some empirically successful, if limited, rules to take the initial steps along this path ([2], [3]). Both in his treatise and in the manuscript, he showed great skill as a communicator, guiding his readers along a new perceptual and cognitive path which opened up when using his new, adapted optical diagrams, in which he tried to formulate a cathetus rule applicable to bi-spherical lenses. A particularly significant passage in this sense is found at the beginning of the last extant version of the manuscript on the telescope. In this passage, Della Porta describes the experience of looking at an object through a biconvex lens whilst moving it from the observer towards the object, but he does so using not one, but three different strategies, one after the other: (1) a verbal description of the qualitative impressions of what one sees (first object upright, then blurred, then inverted), (2) an optical diagram, and (3) a verbal description relating the experience and the diagram. In this way, the visual and bodily experience is finally conceptualized in terms of a standardized optical-geometrical procedure. This was a fundamental conceptual innovation which transformed optical diagrams in more powerful mathematical instruments and paved the way for all later developments. Johannes Kepler, in his treatise *Dioptrice* (1611), employed a mixture of Maurolico's and Della Porta's methods in that he combined diagrams aimed at Euclidean demonstrations with diagrams that geometrically expressed experience, but were not used as basis for proofs, rather providing a geometrization of experience.

It is my conviction that the suggested extension of the notion of mathematical instrument may not only represent a fruitful heuristic method in the historical study of mathematical and physical practices, but also provide a deeper understanding of the tensions which have emerged in the last decades between computational methods and practices of mathematics still linked to symbolic formalism, which since modernity has been regarded as a "transparent", "objective" tool for representing mathematical knowledge.

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From academic to practical areas in Germany: The use and development of mathematical instruments in the first decades of the 20th century

Renate Tobies

This lecture discussed mathematical instruments as a component of the reformed program to promote applied mathematics at German universities. It particularly considered the training programs at the University of Göttingen and the University of Jena, where participants were actively encouraged to engage with the theory of instruments. Well-trained students in this field then went on to use instruments in industrial laboratories and subsequently used their expertise to develop new devices [5].

Felix Klein (1849-1925), who was influenced by his experiences in the United States and by developments in applied mathematics in France and Great Britain, vehemently promoted this field in the 1890s, having already realised the importance of mathematical instruments during his time as a young professor at the University of Erlangen. After attending the meeting of the British Association for the Advancement of Science in 1873, Klein took advantage of his mathematical seminar [2] to provide details of some new devices: Olaus Henrici's models, a tide-predicting machine, and a mechanical device for transforming circular motion into linear motion. Some months later he managed to secure funds to purchase a mechanical calculator, the Arithmomètre, designed by Charles Xavier Thomas in 1868, the operation of which he then expounded to his students. He also presented a Polar planimeter, designed and constructed by Jakob Amsler (1823-1912), and promoted its wider use in other institutes.

Approximately twenty years later, Klein had a further opportunity to promote this field of interest. He had recognised that numerical, graphical and instrumental methods had a major role to play in solving problems of different disciplines. He inspired the book *Über die Nomographie von M. d'Ocagne*, written by his former doctoral student, Friedrich Schilling (1868-1950). Published in 1900, Schilling's offering appeared just one year after Maurice d'Ocagne's (1862-1938) seminal work, *Traité de nomographie.* Klein also stressed the importance of John Perry's (1850-1920) works, *Practical Mathematics* (3rd edition, London, 1899; German edition, Wien, 1903), and *Calculus for Engineers* (3rd edition, London, 1899; German edition, Leipzig: B. G. Teubner, 1902). Perry had propagated a laboratory method with the training of the use of tables and mechanical instruments that was developed further in other places. In his book *Practical Mathematics*, for example, Perry explained the use of a slide rule for a wider audience. The so-called "Perry movement" certainly influenced the teaching of applied mathematics at German universities, particularly in Göttingen, and Jena.

Klein's program included some key elements that were very important to the development of mathematical instruments:

– The launching of the great undertaking *Encyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen* [Encyclopedia of Mathematics and its Application], published in German and French editions, both containing articles on instruments in the volumes I and II.

– The foundation of a new kind of society combining scientists and financially strong industrialists, the Göttinger Vereinigung zur Förderung der angewandten Physik und Mathematik [Göttingen Association for the Promotion of Applied Physics and Mathematics] in 1898, which could support the use of instrumental methods.

– The introduction of new examination regulations for prospective teachers including, for the first time, applied mathematics (numerical, graphical and instrumental methods).

– The transformation in 1901 of the *Zeitschrift für Mathematik und Physik* [Journal of Mathematics and Physics] into a journal exclusively for applied mathematics, with instruments given the status of a sub-component of this genre.

– The creation of a professorship for applied mathematics at the University of Göttingen in 1904, the initial incumbent being Carl Runge (1865-1927), who effectively became the first full professor in this field at a German university.

– The initiation of interdisciplinary research seminars for applied mathematics at the University of Göttingen, including the use and discussion of instruments.

Carl Runge was fully committed to Klein's reformed program to promote applied mathematics. He designed his applied mathematics courses along the lines of the laboratory sessions common to physics and chemistry, and would instruct his students not only in the application of numerical methods, but also in the use of plotting tables and drawing boards, compasses, slide rules, four-digit logarithm tables, other tables, mechanical calculators, and various other instruments. For demonstration purposes, large slide rules decorated the lecture halls, not only at the University of Göttingen, but also in Jena, where the famous Carl Zeiss foundation supported institutes of the university.

Professor of mathematics, Robert Haußner (1863-1948), was not the only academic to support Klein's program at the University of Jena. August Gutzmer (1860-1924), Haußner's predecessor, had already established new examination regulations for prospective teachers that included applied mathematics, two years after the Prussian regulation. Following some temporary appointments at Jena in the post of professor of applied mathematics, including a two year stint by Wilhelm Kutta (1867-1944), famous for the Runge-Kutta procedure for solving ordinary differential equations, the former doctoral student of Klein, Max Winkelmann (1879-1946), became a more long-term and successful incumbent in 1911 [1].

In research seminars at German universities, several mathematical, mechanical, and physical instruments were discussed. One of the main instruments in this program was the harmonic analyzer, which had applications in Fourier analysis. The instrument was prohibitively expensive at the time, however, and so could not be afforded by all German universities. Those that managed to acquire an analyzer generally enjoyed some form of external financial support connected with industry, the Göttingen Association for the Promotion of Applied Physics and Mathematics a typical sponsor.

Thus, at the University of Göttingen, Klein was able to explain in some detail the operation of the harmonic analyzer using a lecture during the summer semester in 1902 specifically for this purpose; interestingly, this lecture became the basis for volume 3 of his notable work, *Elementary Mathematics from a Higher Standpoint*. This instrument was also at the centre of Klein's seminar in applied mathematics (electrical technology), which he headed together with Carl Runge, Ludwig Prandtl (1875-1953), professor of applied mechanics, and Hermann Theodor Simon (1870-1918), professor of applied electricity [6].

The University of Jena was the second German university where a harmonic analyzer was used in the teaching program of applied mathematics. Clemens Thaer (1883-1974), who completed his post doctoral degree (*Habilitation*) at the University of Jena in 1909, and later became famous for his translation of Euclid's *Elements*, published a paper on the accuracy of the harmonic analyzer [3]. He wrote that the Mathematical Institute of the University of Jena had received its harmonic analyzer as a valuable present in 1909. Here the important regional optical industry, specifically the Carl Zeiss Foundation mentioned above, supported the facilities of the local university institutes.

Because professional mathematicians in Germany had long neglected such matters, industrial engineers had already independently developed instrumental methods for solving technical problems during the 19th century. In order to learn and better understand these methods, during the summer vacation of 1907, Carl Runge spent nine days in an industrial environment. His host was a bridge construction company, an affiliate of a famous machine factory in Nuremberg, whose director was one of the founding members of the aforementioned Göttingen Association for the Promotion of Applied Physics and Mathematics. Here Runge learned, first-hand, new special methods, and integrated them in his vision of applied mathematics.

Together with colleagues at his mathematical school, he developed table algorithms to solve equations that also built a basis for several *Rechenschablonen* [calculating templates] for harmonic analysis, tools and methods that later would be used by scientists and engineers for the treatment of special problems.

The American mathematician George A. Campbell (1870-1954) who had studied with Felix Klein in Göttingen, with Ludwig Boltzmann (1844-1906) in Vienna, and also with Henri Poincaré (1854-1912) in Paris, became a pioneer in developing and applying quantitative mathematical methods to the problems of long-distance telegraphy and telephony (Bell Telephone System). His female co-worker Edith Clarke (1883-1959) became famous for designing an important device named after her, the "Clarke calculator" (Patent Number: 1552113, in 1921/1925). As a member of General Electric, she had designed this graphical device that solved equations involving electric current, voltage, and impedance in power transmission lines. Remarkably, this device could solve line equations ten times faster than previous methods.

Industrial researchers also used and developed special instruments and devices including special slide rules in German electrical corporations; for example, special slide rules were designed at the OSRAM Corporation in Berlin. For further development of industrial mathematics at electrical corporations in Berlin starting in the 1920s to the end of the Second World War, see [4].

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From analogue to digital mathematical instruments: Examples of mathematical practices with machines in the 1950s and 1960s

Loïc Petitgirard

The rise of digital computers has transformed mathematical practices at large, but in the 1950s this instrument was only accessible to a few scientists around the world; primarily the mathematicians and physicists who invented computers for their own use. At that time then, a "digital revolution" was barely on the agenda, it was neither an immediate nor general transformation.

Here we want to address the "collateral" transformations in mathematical instruments, right from the beginning of the modern computer, in two different domains: the theory of "dynamical systems" and "Fourier transform" optics. We chose these domains because they are not strictly "mathematical" disciplines: both fields deal with very practical problems, requiring intense calculations, as well as highly mathematical theories. The communities gathering around these problems were heterogeneous, including essentially mathematicians, physicists and engineers. Here we shall focus on the use, role, and invention of dedicated mathematical instruments that were commonly analogue devices, and their evolution when facing the digital revolution.

(1) Dynamical system theory. We focus on a milestone in the broad and long-term history of dynamical systems, in which mathematical instruments have been used regularly for calculating and visualizing dynamics. At the turn of 1950s a small group of physicists, involved in mathematical developments as well as engineering problems, gathered around Theodore Vogel in a laboratory in the south of France (Marseille) [7]. This group devised a program they called "theoretical dynamics" which included many ingredients: a clear empirical epistemology (that is to say, "experimental mathematics"), the development of specific (analogue) instruments for their mathematical investigations, and an interest in practical dynamical problems. Their instruments could be mechanical or electronic and they rendered topological, geometrical information about dynamical systems, particularly "phase portraits" that were unobtainable by other means.

A good example of this analogue practice is to be found in the work of Michel Jean (a PhD student of Vogel) who conceived and used a mechanical analogue of the Duffing equation[7]:

$$I\,\frac{d^2x}{dt^2} + 2f\,\frac{dx}{dt} + ax + bx^3 = C\cos(\omega t + \varphi).$$

In his PhD, Jean is looking for periodic solutions to certain differential equations. This is one of the machine's strengths: to be able to render graphical details of the dynamics in the phase plane, to give some geometrical insights regarding the dynamics and assist in the identification of periodic solutions. The measurements

[7] Following Jean notation, x is the angle of rotation of the disk; I stands for inertia momentum; f = damping factor; ω = forcing beat; a, b and C are determined by the characteristics of the machine including the fact that the disk is subjected to a magnetic field that generates Foucault currents, thereby restraining rotation.

on the working oscillator, without any supplementary calculations, are reported graphically. In this phase portrait appear two focal points A_1 and A_2 (asymptotically stable) and a saddle point C (conditionally stable).

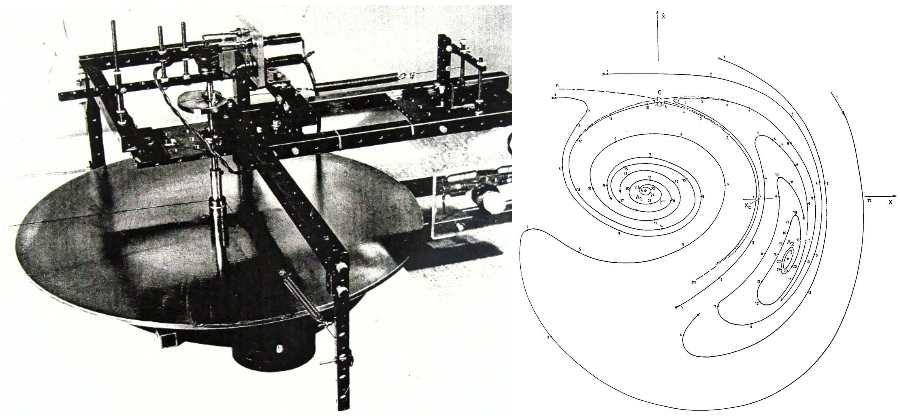

FIGURE 1. The mechanical oscillator and a phase portrait

Facing the growing competition from digital computers (and we must remember that analogue computers were, at that time, generally more efficient than digital computers), the "analogue practices" of the group must be emphasized, since they were based on a trade-off: no need for "high" precision results but rather quick graphical representations. When the first digital computers entered their lab, they quickly developed new techniques for use in their investigations into dynamics; a new tool with new modelling possibilities to explore different aspects of the dynamics, enhancing but not replacing the extant analogue systems. Their adoption of the digital was essential to maintain continuity in their practices and their comprehension of machines.

This work on "theoretical dynamics" has remained relatively unknown until recently, but analogue calculating methods and strategies have been around since the 1930s. I have worked on the ideas of Nicolas Minorsky in the mathematics of control and his "dynamical analogue" calculating devices inspired by the V. Bush "differential analyser" [6]; the meteorological (and essentially mathematical) work of E. Lorenz later in the 1960s was, in some sense, a dialog between mathematical theories on differential systems and digital calculations, leading to the so called "Lorenz attractor" in chaos theory; the analogue computers rendering of chaotic attractors by O. Rössler after 1975, etc. [5]. The whole history of dynamical systems is comprised of mathematical theories associating calculating/visualizing practices developed by mathematicians and many other professions such as physicists, engineers, chemists, and biologists.

(2) Fourier transform spectroscopy. Our second focus deals with a different domain of physics, but over the same time period: optics or, more precisely, infrared spectroscopy. Here we focus on an instrumental practice, developed throughout the 20th century, with a crux in the 1950s, when "Fourier transform" spectroscopy in the infrared was conceived independently by P. Fellgett (Cambridge, UK) and P. Jacquinot (CNRS Bellevue, France) [4]. Such a spectroscope comprises an infrared reference source passing through a Michelson interferometer that illuminates an analysis sample. Obtaining the infrared spectrum gives a measure of how much the sample diffuses the incident infrared light at every apposite wavelength.

This spectrum gives a molecular "fingerprint" of the sample, which was the main incentive for the development of such instruments[8]. In the 1950s, however, this was still a "dream". One of the main reasons is that the spectrum analysis requires the calculation of the complete Fourier transform of all the measures (because the beam passes through the interferometer that combines different wavelengths); there was no definitive theory asserting that this FTIR spectroscope would provide the expected spectrum (although Fellgett and Jacquinot paved the way).

The mathematical theory of Fourier transform was, at that time, no mystery, but the reasonably practical calculation of FT to obtain a spectrum was a challenge. However, the culture and practice of Fourier analysis was not limited to mathematics, especially among the French physicists involved, who were familiar with "Fourier optics", developed shortly after the WWII [3]. Fourier optics offered ways to calculate analogically (by optical means) this Fourier transform. It could do the job, but was a very complicated device to build. The incentive to compute the inverse Fourier transforms led to an instrumental "creativity", using "off the shelf" analysers (optical or even mechanical systems, as Strong and Vanasse used in 1958[9]), optical devices, and also, of course, electronic analogue devices; later some hybrid systems were used[10].

The capability of "digital computers" was very different. At the beginning of the 1950s it was impossible to calculate, digitally, a complete spectrum, even with the most powerful mainframe of the day. This changed quickly, of course, culminating in the "Fast Fourier transform" algorithm in 1966 (that made, incidentally, the FTIR spectroscope a fast, easy-to-use, accessible instrument). For years, the French physicists, as they recalled, had been "sadly innocent of digital techniques" [2].

But once the digital hurdle had been negotiated (as they met the international community of spectroscopists, trying to use computers), they integrated and transformed FTIR into the digital: J. Connes (PhD student of Jacquinot) gave the first complete mathematical theory of the instrument, particularly focusing on the way to calculate Fourier transform, in 1960. The transition from analogue devices to

[8] The FTIR spectroscope is today a widespread, easy-to-use, and cheap instrument that can be found in laboratories all over the world (chemical, biological, industrial laboratory, and so forth).

[9] From 1956, the group led by John Strong and Georges Vanasse (USA) promoted many different computing systems. With Idealab as an instrument maker, they fostered and commercialised the "Idealab Fourier Transform analogue computer".

[10] The hybrid special purpose Fourier Transform computer FTC-100 was released in 1967.

digital instruments was not linear, it was made of cumulative expertise, a material culture of optics, and a series of mathematical, numerical and physical breakthroughs.

(3) Open questions. These two case studies throw up many questions regarding the fundamental concept of what actually defined a mathematical instrument around 1950. Analogue instruments, analogue practices and culture were widespread and diverse. The "impact" of the digital computer was also very different from one context to another. Was it a radical change? The temporality of this evolution is also an important question to address.

Comparing analogue and digital sounds very "retrospective", as C. Care addresses in his book [1], where he clearly emphasizes the continuities in modelling practices from analogue to digital during the 1950s through to the 1970s. Altogether, it appears that the "analogue culture" was not obliterated as soon as digital techniques were available. It seems that it was not a simple matter of change from one to the other, but more a process of hybridization and adaptation. The history of FTIR is often focused on the FFT algorithm: but one must remember that the interferometer at the heart of FTIR is the instrument that first produces a mathematical (analogical) transform of the (input) light beam (the transform that is to be inverted with numerical calculation at the output!). It illustrates that the material base of such instrument is hybrid, that long and sinuous theoretical and mathematical developments were necessary, indicating a growing importance of mathematics, even in domains far removed from pure mathematics.

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A brief history of the slide rule

Marc Thomas

The slide rule has been in use for three and a half centuries, and its history can be divided in four periods.

1) Great Britain 1614-1815: The beginning. After the computation of logarithms by John Napier in 1614, Edmund Gunter in 1620 invented the Gunter's line, which is a logarithmic scale used with a pair of dividers. In 1624, William Oughtred made the first slide rule, a circular one, closely followed in 1627 by Edmund Wingate, who built the first linear slide rule. It is quite extraordinary that logarithms gave birth to such an instrument during a period of barely 15 years but, in many ways, it was simply a necessary response to a pressing requirement.

In these early days, slide rules were specialised, the most important applications being found in the timber and gauging industries. In the timber yard the carpenter's slide rule (Coggeshall) was commonplace, and in gauging, primary use was by excise officers to calculate taxes on alcoholic liquids (Everard). Some contemporary sailors used essentially what were Gunter's lines in naval applications.

Until the beginning of 19th century, slide rules were used almost exclusively in Great Britain where there were about 50 makers. They were known in Continental Europe, but scarcely used, and there were certainly no slide rule manufacturers elsewhere.

Around 1780, in his Soho factory, James Watt began to use improved slide rules known as 'the SOHO rule'. This date is very important, because it marks the point when the slide rule became an instrument for general calculation rather than specific application.

2) Arrival in France: 1815-1851. In the spring of 1815, Edme-François Jomard, an engineer geographer, was sent to London to discuss the Egyptian antiques that returned to England following Bonaparte's Egypt campaign, 1799-1801. Jomard noticed the significant industrial progress being made in England, and came across the slide rule. He was very enthusiastic about the potential of this new instrument so, upon his return to France, he presented an example of the rule to some French scientists and also asked Lenoir, one of the best instrument makers in France, to manufacture and market these slide rules at an affordable price. In 1820, the first French slide rules were available. Jomard's genius was that he recognised that the rules needed to be inexpensive enough that they could be afforded by all, and his aspiration was that all school children should be taught to use the instrument.

Lenoir, then Mabire and Gravet (after Lenoir's death) made slide rules of very good quality, but their use was not fully appreciated and exploited until 1851, when two significant developments changed things. Amédée Mannheim, a young officer in the army corps of engineers transformed the position of the scales on the slide rule so that it became a more precise and efficient instrument; this enhancement is, indeed, the basis for all the modern slide rules (Mannheim type). Also, the French government decided that knowledge of the use of a slide rule would be a prerequisite of entry into the École polytechnique and the Saint-Cyr military academy. These two events propelled the slide rule to popularity in France, and its general use increased significantly during the second half of the nineteenth century, going hand-in-hand with the industrialisation of France over the same period. Gravet continued to be the main manufacturer of high quality slide rules in France, followed by Tavernier-Gravet, Tavernier being Gravet's son-in-law.

3) Diffusion: 1851-1900. Dennert and Pape (Aristo) began to import slide rules into Germany from France in 1865, and then began making their own rules after the war against France of 1870-71. In 1878, Nestler, and in 1892, Faber (now Faber-Castell), began to make rules. At this time, some rules were covered with celluloid, making them cheaper and easier to work with than wood.

From 1887, Keuffel and Esser imported rules into the USA from France and Germany, and then began making their own rules from 1895 onwards. In Japan, around 1895, Hemmi began to produce slide rules in bamboo, European woods being incompatible with the Japanese climate.

Thus we can see that the diffusion of the slide rule was almost perfectly linked with the industrialisation of each country; it was clearly an instrument of the industrial era.

4) Twentieth Century: Slide rule universally used in all industrialised countries. The greatest makers were Keuffel and Esser in USA, Hemmi-Sun in Japan, and Faber-Castell in Germany, all using industrial methods of manufacture to produce rules of great quality. There were though, hundreds of makers around the world. The rules that were made in the 20th century were adapted to follow the progress of science and technology: there were rules to specifically deal with calculations in the fields of concrete science, chemistry, radio, electronics, aircraft, atomic weapons, and so forth.

After WWII, the most prolific manufacturers were producing hundreds of thousands of rules each year. Even the astronauts flying the Apollo missions were equipped with slide rules in case of computer failure, although it appears, thankfully, they didn't need to use them. In France, the company Graphoplex became the best-known manufacturer.

In the 1970's, however, the electronic calculator appeared. Some initially paid homage to the slide rule in their designators, for example, with the TI SR-50, the 'SR' stood for 'Slide Rule', but in only a few years, as electronic components became relatively inexpensive, the slide rule was forgotten and its electronic equivalent took over.

Conclusion. The slide rule can be considered as the calculating instrument of the industrial era. Born in 17th century, enhanced by James Watt, its diffusion followed and accompanied the industrialisation of the world until the invention of electronic machines that now symbolize the post-industrial, digital era.

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Subterranean geometry and its instruments: About practical geometry in the early modern period

Thomas Morel

Underground surveyors (known as *Markscheider*) were active in the metallic mines of early modern German states. They had to solve difficult problems regarding water exhaustion, digging of shafts and galleries, and used geometrical instruments in this quest. My aim is to compare what scholars wrote with what mathematical practitioners did, focusing here on one specific task: calculating the *Seigerteuffe* or perpendicular depth of a measurement.

Analysis of an instrument described in the *De Re Metallica* (1556). Georgius Agricola (1494-1555) was no mathematician, neither was he a mining engineer. He was a physician active in Saxony during the first half of the 16th century, remembered today for his *De Re Metallica Libri XII.* This work not only gathers knowledge about mining, smelting, metallurgy and earth sciences, but also about subterranean geometry.

His general principle is simple: "Each method of surveying depends on the measuring of triangles. A small triangle should be laid out, and from it calculations must be made regarding a larger one" [1]. The given illustration (see Fig. 1) looks very precise and Agricola describes the process at length. However, the whole procedure is not very realistic for a number or reasons. It seems abundantly clear to me that Agricola inscribes his work, here, in the tradition of *geometria practica*: the reader is shown a geometry of the triangle, not the actual geometry of the mines.

If one discards the hypothesis that Agricola's work was completely disconnected from existing practices, we can formulate a more generous hypothesis and see his work as metaphorical. Practitioners had, indeed, to solve triangles, obtaining the *cathetus* ("Seigerteuffe" in their language) and the *basis* ("Sohle") from the *hypotenusa* and a vertical angle. Things get complicated, however, when trying to find out how this knowledge was produced. In his *De Re Metallica*, Agricola describes and shows a picture of a measuring stick, saying that it was actually used in conjunction with a half-circle.

Figure 1. General principle of the *geometria subterranea* according to G. Agricola [1, p. 90]

The somewhat obscure explanation given by the author makes sense if you consider, as his English (modern) translator did, that the stick's graduated scale can be seen as a cosine table inscribed in wood. Twentieth-century analysis of these instruments has reached conflicting conclusions about their status and existence; one view regards them as a product of Agricola's imagination and scholarship, whilst the other reasons that Agricola divulged a well-kept secret [8, 2].

It seems unlikely that such an instrument was routinely designed by surveyors. But an instrument could be used in surveying procedures without knowing any trigonometry or mastering the embedded algorithm. This is, for example, what Menso Folkerts has shown for wine gauging, another discipline of early modern practical mathematics [3].

Tables as instruments: the sine table in the *Markscheider*-tradition. Agricola's text is therefore not a mere description of what he saw, as has sometimes been said. It looks like a complex mixture of pseudo-physical situations borrowed from the *geometria practica* tradition and instruments that indeed existed, or were at least plausible. The central question remains: what did practitioners do when they needed to solve triangles, that is to find the *Sohle* and *Seigerteuffe* from the *hypotenusa*? A caveat is that no precise description exists from the 16th century, and later manuscripts never mention Agricola.

To this end, a *Markscheider* would need several things. First of all, he would need to write things down. The measurement can be seen as a broken line, and one had to collect data for each point. This operation is called the *Gruben-Zug* and is summed up in a table. To process it, one needs a trigonometric table. In the 17th century, we do know for certain that they used sine tables, more precisely tables computed by Simon Stevin (1548-1620) and Ludolf van Ceulen (1540-1610).

Balthasar Rösler (1605-1673), an important *Markscheider*, seems to have computed a table (see Fig. 2) which could be directly used for field work. It features two main differences from standard sine tables. First, it uses the actual measuring units in the Saxon mines, the *Berglachter*. Secondly, the result of the sine is given not only in an abstract way for one *Lachter* (as one would expect in reference to the *sinus totus*) but the commonly used multiples and sub-parts are calculated (while no original table from Rösler seems to have been preserved, we have numerous copies from his students, for example [6]).

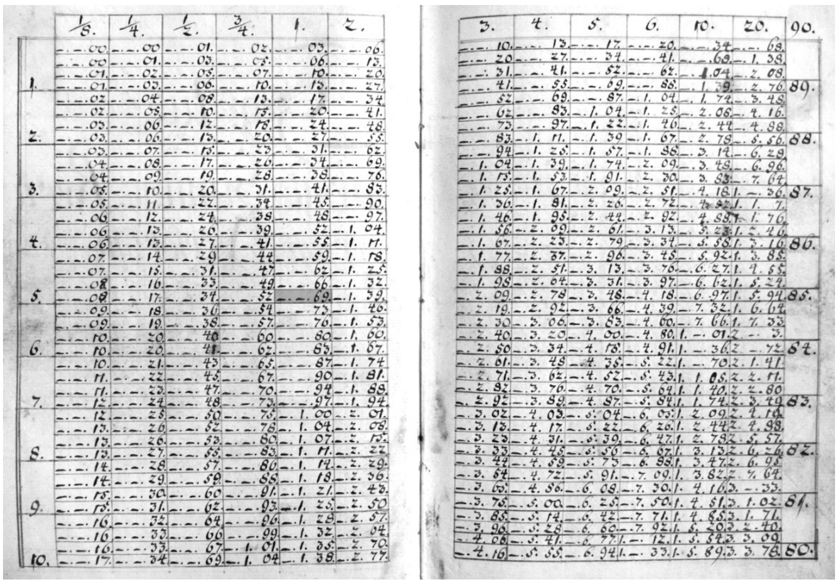

Figure 2. Sine table computed by B. Rösler to be used in the mines [6, f. 8v-9r]

After this transformation, the trigonometric table has become an instrument adapted for subterranean geometry. Just as the regular *Kompass* had been modified in the early 17th century into a suspended compass (*Hängekompass*) perfectly suitable for mining works, the table made by S. Stevin was then adapted by B. Rösler. This might seem trivial, but I do think it is crucial. It comes with a very precise instruction in practical mathematics. The table is actually described as an instrument: the name of the parts are given (*spatia*, *columna*) together with a set of instructions enabling anyone to use it. With this table in their *vade mecum*, surveyors could swiftly "process" or "solve" the measurement and obtain the *Resolutio*. With minimal computing and drawing skills, they could then produce an accurate map of the mines.

Conclusion. This example shows how theoretical problems, in our case the computation of the *cathetus* and the *basis* from the *hypotenusa* and an angle, can provide various practical solutions. Practitioners tended to use instruments to improve efficiency, and these devices took many forms. Agricola's solution was an elaborate material set of instruments, but practitioners of the 17th century seemed to prefer using sine tables adapted to their specific units and needs. Another practical solution, developed at the turn of the 18th century, would be a paper instrument proposed by Nicolas Voigtel (1658-1713) [7] and Jacob Leupold (1674-1727) [4].

This case study is meaningful since it illustrates how the standard model, in which serious science is brought to practitioners by scholars, can be completely wrong. Underground surveyors were able to find something they considered useful, and adapt it to their specific needs [5]. They were constantly looking for ways to improve their own methods or adapting existing knowledge to suit their particular requirements.

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The nomogram: An artifact changing during the First World War

Nathalie Daval

In the *Encyklopädie der mathematischen Wissenschaften*, the German mathematician Rudolf Mehmke presents in 1902 a classification of mathematical instruments used at that time [3]. Three main types of computational tools are distinguished: numerical tables, graphical tables and mechanical machines. We will focus on nomograms – another name for graphical tables – by studying their use by the French artillery during the First World War.

In battle, undirected cannon fire is ineffective and costly, hence there is a requirement for artillerymen to meticulously prepare each shot in order that it is as accurate and effective as possible. To facilitate this, several important factors must be taken into account: wind speed and direction, variations in air density, weight of projectiles, and so forth. Preparation requires a significant number of calculations to be made in the shortest possible time. At the beginning of the war, the use of basic numerical tables was commonplace, but new, more intricate, versions were gradually introduced: these were graphical tables made up of graduated lines or points which, by simple graphical reading, made it possible to determine quickly the result of a calculation. The French engineer Maurice d'Ocagne (1862-1938) named these graphical tables "nomograms", from the Greek *nomos* (law) and *grammè* (traced).

Among the multitude of nomograms used during the war, we find concurrent-line abaques. Such tables graphically represent the relationship between three or more variables by means of graduated curves. In his treatise *Nomographie. Les calculs usuels effectués au moyen des abaques* [2], d'Ocagne sets out the general method involved in using these abaques: a given equation $F(\alpha, \beta, \gamma) = 0$ is considered as the result of the elimination of two auxiliary variables x and y from the system of three equations $F_1(x, y, \alpha) = 0$, $F_2(x, y, \beta) = 0$, and $F_3(x, y, \gamma) = 0$. Then, a solution of the initial equation corresponds to an intersecting point of three curves traced in the Cartesian plane Oxy.

The book *Carnet de graphiques pour le canon de 75* [1] contains the ballistic elements useful for preparing a shot so that an artilleryman has only simple operations to perform when using a nomogram. Adjustments are made in relation to two main components: direction and inclination. The calculation of these parameters is completed using several different nomograms in succession: firstly a nomogram to calculate the angle wind-firing plan (see Fig. 1), which then makes it possible to calculate the transverse wind correction (action on the drift) and the longitudinal wind correction (action on the range). For even greater accuracy and efficiency it was necessary to calculate, in a similar way, a correction for many other factors such as air density, initial velocity due to powder temperature, the site correction according to the difference in altitude of the cannon and the objective, the convergence in the case of a battery of several cannons, and so on.

Later, d'Ocagne made a major breakthrough in the field of graphical tables by introducing simpler, more readable and complete graphs than the previous offerings. His idea was to replace, by duality, each line by a point so that the concurrence of three lines was transformed into the alignment of three points. Reading was done by simple alignment of two values to get the third one. These nomograms are called "alignment nomograms". For example, such a nomogram was used during the war by artillerymen who used an auxiliary target upon which batteries of cannons had already fired, to then deduce from this information the corresponding firing elements for the final target. Whilst such calculations would have been long and fastidious in traditional numerical form, the use of an alignment nomogram made them possible in a few seconds.

D'Ocagne, the "father of nomography", was a past student of the École polytechnique in Paris. He mainly worked as an engineer at the Ponts et Chaussées,

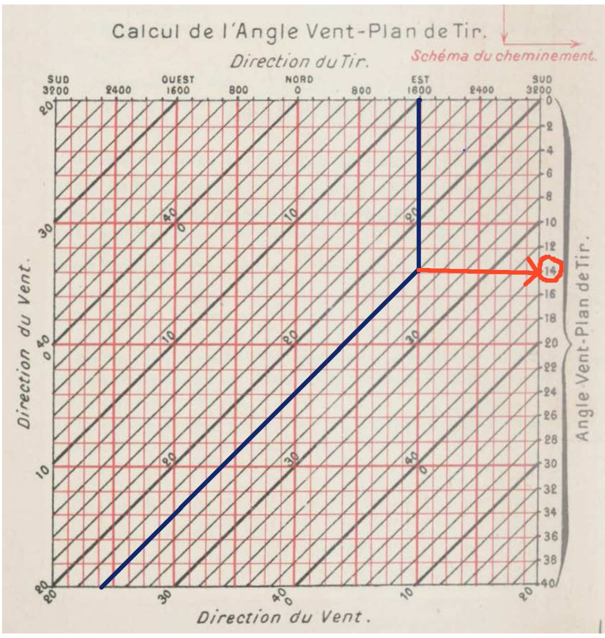

FIGURE 1. Nomogram for the calculation of the angle wind-firing plan ([1], p. 8). Example of use: knowing the direction of the shot (East) and the direction of the wind (24 décagrades = 240 decimal degrees), we find that the angle wind-firing plan is 14 décagrades.

notably in the levelling of France. During the war he became head of a nomographic bureau created especially for him. But his real passion was mathematics, in particularly geometry. He became a teacher of astronomy, geodesy and topography at the École nationale des ponts et chaussées and at the École polytechnique. This dichotomy of interests sometimes led him to neglect his engineering career in favour of a mathematical one in which he did his best to be recognised through the invention and diffusion of nomography.

At the beginning of the Great War, d'Ocagne helped officers and engineers construct firing nomograms, but he quickly realized that a more formal organisation needed to be set up. In February 1916, Paul Painlevé, Minister of Public Instruction and Inventions concerning National Defense charged him with the development of nomographic work for the needs of artillery and aviation. In January 1917, when the Undersecretariat of State for Inventions was created, a special section of nomography was formed under the direction of d'Ocagne. One of his goals was the establishment of individual nomograms for the many different calibre guns and charges in use. Very quickly, he received many letters from the front that confirmed the usefulness of nomograms; they made it possible to significantly shorten the time taken to determine the initial parameters for a shot (less than 5 minutes, instead of the previous 15 to 20 minutes). Many nomograms were then created in response to this positive feedback. D'Ocagne offered training to artillery batteries and was gifted young officers who had been wounded at the front to help him in his task. He scrupulously made lists of the people to whom he sent the precious calculators, and by May 1918, the approximate number of nomograms for shooting correction dispatched had reached almost 2 000 in France and 300 in America.

The benefits of the use of nomograms seemed obvious, but their application during the war was not always possible due to a lack of training and fear of the consequence of an errant calculation. Indeed, the nomographic bureau received many letters expressing the disadvantages of attempting to use nomograms if not trained in their use. The fact that some batteries had already been trained in other techniques meant that it was safer to stick with what they knew. A separate criticism was that the paper upon which nomograms were depicted seemed not particularly robust and, in battlefield conditions, quickly became dirty and damaged. Finding practical solutions to these drawbacks became another focus of the bureau's work.

D'Ocagne received many letters of thanks for his methods and, in 1922, he was elected into the French Académie des sciences at the age of 60, his work at the nomographic bureau certainly contributing to this honour.

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