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History of Mathematics: A Global Cultural Approach (online meeting)

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Abstract. The primary purpose of this workshop was to take account of progress on an ongoing six-volume cultural history of mathematics from antiquity to the present. This project is led by nine editors working with a large team of authors. Since the workshop had to be held remotely, it took the form of various group meetings held throughout the week. The final session involved assessments by editors of the six volumes with an eye toward completing the project by the end of 2021. The abstracts below summarize the contents of the individual chapters in the entire project, which will be published in Bloomsbury's cultural history series.

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Introduction by the Organizers

Culture, history, and mathematics are words that seldom appear together. Yet as the six volumes outlined below will demonstrate, mathematical activity has played a pervasive role in diverse cultures throughout human history. Indeed, these volumes represent merely an introduction to a vast and highly diverse range of developments that form an important component within and integral part of human civilization. Written by some fifty leading scholars, *A Cultural History of Mathematics* traces the many ways in which individuals and societies have interacted with mathematical phenomena from antiquity to the present. It also addresses the wide range of meanings and associations that have attached to the word mathematics. Originally, in the ancient Greek language, máthēma simply meant "that which one learns," though by the time of Plato and Aristotle the discipline of mathematics was clearly associated with the study of numbers (arithmetic) and figures (geometry).

Since this project aims to provide an overall global history based on current scholarship, volume editors had to confront difficult choices about which topics and cultures to include. In doing so, they have tried to strike a balance between representative features of the period, on the one hand, and particular developments of long-term significance, on the other. All six volumes, the first two of which stretch over the ancient and medieval worlds, have placed special emphasis on the material culture of mathematical activities as well as on how mathematical knowledge changed over time alongside new forms of communication and circulation.

At the same time, each volume has its own distinctive character as described in the introductory essays written by their respective editors. Yet all six address seven general themes in as many chapters, which provide the overall structure for this project. Thus, each volume begins with three chapters – Everyday Numeracy, Practice and Profession, Inventing Mathematics – that deal with three different levels of mathematical expertise within the period under study. These chapters thus explore mathematics per se, as the notion evolved over time, whereas the three that follow – Mathematics and Worldviews, Describing and Understanding the World, and Mathematics and Technological Change – consider the larger impact of mathematics on other dimensions of human culture. Finally, the broader understanding of how mathematics manifests itself in other cultural spheres and what sorts of activities are deemed mathematical forms the subject of the final chapter on Representing Mathematics.

Workshop (online meeting): History of Mathematics: A Global Cultural Approach

Table of Contents

Abstracts

Volume 1: Introduction, A Cultural History of Mathematics in Antiquity

Michael N. Fried

Volume I concerns the cultural history of mathematics in the ancient world. Every word in that description contains subtleties and complexities. Culture, history, and mathematics, each by itself is far less clear than one might think, not to speak of the combinations, "cultural history," "history of mathematics," and even "ancient world." Part of the collective goal of the chapters in this volume is to bring out the subtleties and complexities of these words and their associated meanings, most of all the difficulty of grasping the idea of mathematics itself in the context of the ancient world. The chief aim of the chapters individually, though, will be to present concrete aspects of mathematics in ancient times in ways that will be new, informative, and eye opening for the reader.

The problem of mathematics in the ancient world is a particularly difficult one since it is not clear what mathematics was before there was a name for it, and even after the Greek name was adopted. "Mathematics" obviously included less of what might be called mathematics today, but also included many things that are no longer considered to be "mathematics" at all (music, for example). This is all the more so true for the ancient cultures of Mesopotamia, China, and India, that which never used the Greek name for their own versions of "mathematics." It might be said that mathematics arose in all cultures from a basic observation of nature and various practical necessities. But this suggests other difficulties, if only because, unlike today, the separation between mathematics and its applications was not so sharply drawn in ancient times.

Adopting a cultural view of mathematics means beginning with this indefiniteness as to what mathematics was taken to be. A cultural point of view also has implications for the historiography of mathematics. And historiography brings us back to epistemological questions about how one should understand mathematics: is it something eternal, essentially unchanging, perhaps only waiting to be discovered, or is mathematics a historical phenomenon, always changing, never quite defined, or rather always redefining itself? Is it one thing, or as varied almost as humanity itself? Such questions are always in the background of any cultural history of ancient mathematics.

Volume 1: Everyday Numeracy in Antiquity. Mathematics in the Everyday Lives of Citizens in Greco-Roman Times

Merav Haklai

Numeracy is the ability to count, calculate, and measure. It is based on natural abilities of human beings, such as, a basic ability to estimate size and small clusters. Yet, numeracy may also be understood in terms of cultural practices and beliefs building on these basic abilities. Tools and artifacts are also culturally conditioned. This chapter is concerned with these cultural and historically-located aspects of ancient numeracy. It focuses chiefly on Greco-Roman civilizations, though it includes some remarks on Mesopotamian and Egyptian cultures. For other, 'non-western ancient societies readers are referred to the corresponding chapter in Volume II of this series. A working hypothesis of this chapter is, that numeracy comprises a spectrum of abilities of varying degrees, moving from a basic capacity to appreciate sizes, to an acquaintance with numerals and counting systems, to being able to do sums, to performing complicated calculations. Based on historic texts and artifacts, the chapter analyzes the variety of cultural practices which manifest this spectrum of mathematical knowledge in the everyday lives of the general public, that is, of non-professionals, in the ancient world.

A difficulty here is that both 'general public and 'non-professionals' are ambiguous categories. While 'professionals' may be viewed as high ranking state officials or persons allocated with complicated tasks, such as, land-surveying, architecture, or financial book-keeping, it is harder to define 'the unskilled, and not easy to say always where one begins and the other ends. In ancient societies, work was carried out by different individuals, from unskilled laborers to those with practical experience, from those with some expertise to those with specialist knowledge. It is also difficult to say who along this chain should be considered a practitioner with special mathematical knowledge. For example, an inscription from Rome preserves the self-testimony of an imperial freedman who took pride for being a "dealer (negotiator) in food and wine at [the fountain of] the Four Scauri (a IIII scaurisaram) in the Velabrum". One may assume that his job did not require extraordinary mathematical knowledge; yet, even its description demonstrates an acquaintance with numerals. In fact, at least in Greco-Roman antiquity, almost everyone had to deal regularly with numerical notions and often with highly numerate professionals; hence, would need some level of numeracy to handle their affairs to the best of their interest or to have close, trust-based relationship with a person who had such knowledge.

References

[1] M. Bailey, *Numeracy, Greek and Roman*, in Bagnall, R. (ed.) The Encyclopedia of Ancient History, Boston, 2015.

[2] S. Cuomo, *Exploring ancient Greek and Roman numeracy*, Journal of the British Society for the History of Mathematics **27(1)** (2012), 1–12.

[3] K.A. Menninger, *Number words and number symbols: a cultural history of numbers*, London, 1969.

[4] I. Morley and C. Renfrew, eds., *The Archaeology of Measurement: comprehending Heaven, Earth and Time in Ancient Societies*, Cambridge, 2010.

[5] K.J. Nemet-Nejat, *1, 2, 3, . . . Numeracy and Mathematics in Ancient Mesopotamia*, CSMS Bulletin **39** (2005), 43–49.

[6] R. Netz, *Counter Culture: Towards a History of Greek Numeracy*, History of Science **40** (2002), 321–52.

Volume 1: Practice and Profession in Antiquity. Mathematics and its Transmission in Mesopotamia and Egypt

Cécile Michel

This chapter concerns the practice of mathematics in the Ancient Near East and to the professionals involved. It covers a long period of three thousand years, from the invention of writing on clay tablets in Southern Mesopotamia to the Achaemenid Empire, but it focuses on the Old Babylonian period (ca. 2000–1600), from which we have substantial cuneiform educational material. Data from ancient Egypt are more scarce, due to the use of perishable writing media (papyri). However, several papyri, presumably written for educational purposes, illustrate practical uses of mathematics.

There are mainly two categories of available texts that provide us with data on ancient mathematics. The mathematical texts are problems, procedure texts and tables entirely dedicated to the topic. The corpus of these texts for both Mesopotamia and Egypt is well delimited and has been the subject of many detailed studies. We will present it briefly, linking the texts to their general historical contexts.

The use of metrology and mathematics is quite widespread in the Ancient Near East, and not restricted to 'mathematicians'. Thus, practitioners of mathematics come from a great variety of milieus. The mathematical methods they use vary according to milieus and professions. These methods are taught in 'schools', in private context by masters, or passed on from generation to generation. Significantly, most of the documents containing mathematical work are unsigned: while the type of mathematics involved can be seen, the identity of mathematical practitioners has to be guessed through their professions and activities. Only then can we consider their role and status in society.

This chapter opens with a presentation of the different types of texts involving mathematics both in Mesopotamia and in Egypt. A description of the various professions requiring some mathematical knowledge will then move on to a discussion of the social role and status of mathematical practitioners. The chapter ends with some considerations concerning the transmission of mathematical knowledge and the place of mathematics in education.

References

[1] M. Clagett, *Ancient Egyptian Science. Volume III: Ancient Egyptian Mathematics*, Philadelphia, American Philosophical Society, 1999.

[2] A. Imhausen, *Mathematics in Ancient Egypt. A Contextual History*, Princeton, Princeton University Press, 2016.

[3] C. Michel, *Computation Practices of the Assyrian merchants during the 19th century BCE*, K. Chemla & C. Michel, eds., Mathematics and Administration in the Ancient Worlds, Berlin/Heidelberg: Springer, 2021.

[4] K.R. Nemet-Nejat, *Cuneiform Mathematical Texts as a Reflection of Everyday Life in Mesopotamia*, New Haven: American Oriental Society, 1993.

[5] G. Schubring, *Handbook on the History of Mathematics Education*, Berlin: Springer, 2014.
[6] I. Shaw, *Ancient Egyptian Technology and Innovation. Transformations in Pharaonic Material Culture*, London: Bloomsbury, 2012.

Volume 1: Inventing Mathematics in Antiquity. Contexts of Invention in Mesopotamia and The Greek Invention of the Author

Christine Proust, Reviel Netz

The chapter considers the idea and problem of mathematical invention in the cuneiform culture that flourished in Southern Iraq and in the Classical world of Greece.

In the case of the cuneiform culture, transmissions, continuities or conceptual shifts between moments or places of invention can in some instances be described. But in general, it is relatively arbitrary to decide that a particular text contains a specific invention. As far as cuneiform texts are concerned, we do not know which results or practices were perceived as innovative by their authors, nor exactly when notions or methods that appear at a given time in a given document were invented. This part of the chapter presents mathematical texts that present novelties or breaks from those attested in previous periods, at least in our eyes as modern historians. Among other things, it presents the principles of the sexagesimal place value system and notation which are, in one form or another, at the heart of mathematical concepts and practices in the Near East.

The decisive feature of Greek mathematics, by contrast with that of Mesopotamia, is its break away from the state. Neither bureaucrats nor merchants, Greek mathematicians, instead, were authors – making their name via their individual contributions. This put a premium on originality and made 'invention', for once, a useful metaphor. This, of course, does not mean that Greek mathematicians acted alone. Their very individuality was predicated on particular social conditions and their endeavor soon give rise to a widely shared set of written practices – a kind of literary genre – that would remain stable and define mathematics, with no more than local variations, for two millennia, well beyond Greek antiquity itself. As these genres came to encompass mathematics, a new kind of mathematical interest was put to the fore: one that aimed simultaneously at effect (hence, aiming for more surprising results) as well as persuasion (hence, aiming at proof). The mathematical treatise was a series of structured proofs presented via a formulaic language and a diagram of a labeled network of lines and points. The origins of the genre are difficult to pin down but its consequences for deduction are enormous.

References

[1] J. Friberg, *A Remarkable Collection of Babylonian Mathematical Texts*, vol. I. New York, Springer, 2007.
[2] J. Høyrup, *Lengths, Widths, Surfaces. A Portrait of Old Babylonian Algebra and its Kin*, Berlin/Heidelberg, Springer, 2002.
[3] W.R. Knorr, *The Ancient Tradition of Geometrical Problems*, Boston, 1986.
[4] G.E.R. Lloyd, *Demystifying Mentalities*, Cambridge, 1990.

[5] R. Netz, *The Shaping of Deduction in Greek Mathematics: A Study in Cognitive History*, Cambridge, 1999.
[6] R. Netz, *Ludic Proof: Greek Mathematics and the Alexandrian Aesthetic*, Cambridge, 2009.
[7] C. Proust, *Tablettes mathématiques de la collection Hilprecht*, avec la collaboration de Manfred Krebernik et Joachim Oelsner. Wiesbaden, Harrassowitz Verlag, 2008.
[8] C. Proust, *Masters' writings and students' writings: school material in Mesopotamia*, Gueudet, G., Pepin, B. & Trouche, L., eds., From Text to 'Lived' resources: Mathematics Curriculum Material and Teacher Development, Berlin/Heidelberg, Springer, 2012, pp. 161–179.
[9] C. Proust, *A mathematical collection found in the "House of āšipus": the art of metrology in Achaemenid Uruk*, C. Proust and J. Steele, eds., Scholars and Scholarship in Late Babylonian Uruk, Heidelberg, New York: Springer, 2019.

Volume 1: Mathematics and Worldviews in Antiquity. The Distinctiveness and Diversity of Greco-Roman Mathematics and the Visions of the World It Underpinned

G.E.R. Lloyd

The interactions between 'mathematicians' and 'cosmologists' are complicated by the fact that neither dealt with an agreed and well-defined subject-area. Nevertheless, in Greece, and in other ancient civilisations, many were sufficiently impressed by the prestige and power of 'mathematics' to see it as in some sense the key to the understanding of the world in general. Yet the way in which that was imagined differed profoundly. In the Greco-Roman world what came to impress many commentators in this regard the certainty of mathematics. Given self-evident axiom and valid inferences from them, the conclusions are incontrovertible. Yet the notion and practice of axiomatic-deductive demonstration were not a necessary condition for the preeminent prestige of mathematics. Without this kind of axiomatic-deductive approach, ancient Egypt, Mesopotamia and India still engaged in sophisticated mathematical investigations. Another ancient culture, China, where again no equivalent to axiomatic-deductive demonstration existed, provided texts explicitly proclaiming that mathematics enables one to solve many puzzling questions. It is possible, for instance, to determine the height of distant objects by taking angular measurements from two positions at a given distance and using similar right angled triangles. In a text that dates from the turn of the millennium, the Zhoubi suanjing, this technique was even used to assess the height of the sun - on the assumption of a flat earth.

How mathematics was brought to bear on world-views differs not only across ancient civilizations, but also within the Greco-Roman civilization. Topics on which different Greek and Roman writers engaged in vigorous debate included the following fundamental questions. (1) What does mathematics study? Are there separate mathematical objects and if so how do these relate to the rest of the phenomena of experience? (2) Do different branches of mathematics have indeed different such objects? How do the objects of 'arithmetic' relate to those of 'geometry', not to mention those of such 'applied' studies as 'harmonics', 'astronomy', statics, dynamics, mechanics and cartography? (3) What modes of cognition does mathematics secure? How far can a model of axiomatic-deductive demonstration be reasonably applied? (4) What understanding of natural phenomena and the cosmos as a whole, including of the place of humans in it, can mathematics yield? While a mathematical approach gives robust-seeming results over a certain range, other areas of experience appear to resist such an approach. How far do the claims for mathematics as the best or even the only sound method have to be qualified in the light of the difficulties it encounters? (5) For many, mathematics was good for more than just knowledge and understanding. By providing access to the profoundest truths it contributed to happiness and fulfilment. On that view this discipline had moral and even religious implications. Training in mathematics was compared with, even seen as an example of, a kind of initiation, whose aim was to reveal the mysteries of the universe. Yet once again this point of view was contested. These five issues must be borne in mind as we tackle some of the rich, if often problematic, evidence in our sources.

References

[1] A. Barker, *Greek Musical Writings: Vol 2 Harmonic and Acoustic Theory*, Cambridge, 1989.
[2] S. Cuomo, *Ancient Mathematics*, London, 2001.
[3] C.A. Huffman, *Philolaus of Croton: Pythagorean and Presocratic*, Cambridge, 1993.
[4] C.A. Huffman, *Archytas of Tarentum: Pythagorean, Philosopher and Mathematician King*, Cambridge, 2005.
[5] W.R. Knorr, *The Evolution of the Euclidean Elements*, Dordrecht, 1975.
[6] G.E.R. Lloyd, *The Revolutions of Wisdom*, Berkeley, 1987.
[7] G.E.R. Lloyd, *Aristotelian Explorations*, Cambridge, 1996.
[8] G.E.R. Lloyd, *Principles and Practices in Ancient Greek and Chinese Science*, Aldershot, 2006.

Volume 1: Describing and Understanding the World in Antiquity. From Counting and Measuring to Mathematical Modeling in Babylonian and Greco-Roman Worlds

Francesca Rochberg, J. Lennart Berggren

The long tradition of numeracy in cuneiform culture with its consequent mastery of advanced arithmetic and its algorithms enabled the Babylonian scribes to manage a sophisticated administrative bureaucracy and a complex economy. It also lay at the basis of the first empirical and quantitative astronomy, which included relations for the cyclical movements of the Sun, Moon and planets, the introduction of the system of degrees for recording zodiacal positions, and records of observations going back to the early eighth century. All of these were ingredients for astronomical modelling by linear mathematical methods which were imported to the Greek world during the Hellenistic period (after the 2nd century BCE) and adapted for both astronomy and astrology. Greek astronomy, however, focused on predicting positions of heavenly bodies at any given time instead of the Babylonians' interest in synodic moments. Most importantly the Greek astronomers and mathematicians expanded their methods to include geometry and trigonometry. This Greek development in geometrical methods and modelling enabled new mathematical treatments of the dimensions of the cosmos. Some of the same individuals who worked in this geometric tradition also applied astronomical data and geometrical methods to create planar maps of the known world that gave a reasonable approximation to the earth's curved surface and were, even in the 15th century, the best basis for scientific geography.

References

[1] A. Aaboe, *Episodes in the Early History of Astronomy*, Berlin, Springer, 2001.
[2] J.P. Britton, C. Proust, S. Shnider, *Plimpton 322: A Review and a Different Perspective*, Archives for History of the Exact Sciences **65** (2011), 519–566.
[3] S. Chrisomalis, *Numerical Notation: A Comparative History*, Cambridge and New York, Cambridge University Press, 2010.
[4] H. Hunger, D. Pingree, *Astral Sciences in Mesopotamia*, Leiden and Boston, Brill, 1999.
[5] O. Neugebauer, *A History of Ancient Mathematical Astronomy*, Berlin, Springer, 1975.
[6] E. Robson, *Mesopotamian Mathematics. 2100–1600 BC. Technical Constants in Bureaucracy and Education*, Oxford, Clarendon Press, 1999.
[7] F. Rochberg, *The Heavenly Writing: Divination, Horoscopy, and Astronomy in Mesopotamian Culture*, Cambridge and New York, Cambridge University Press, 2004.
[8] F. Rochberg, *Before Nature: Cuneiform Knowledge and the History of Science*, Chicago and London, University of Chicago Press, 2016.
[9] J.M. Steele, *Rising Times Schemes in Babylonian Astronomy, Springer Briefs in the History of Science and Technology*, Berlin, Springer, 2017.

Volume 1: Mathematics and Technological Change in Antiquity. Technology, Expertise, and Numerical Knowledge in Cuneiform Procedural Texts

Eduardo Escobar

'Technology', despite its global ubiquity, is among the most plastic terms in use today. Referencing anything from machinery, language, and communication, to religion, it may come as some surprise to learn that 'technology' was introduced to the English lexicon only as recently as the nineteenth century, and the adaption of the term 'technology' into anglophone academic studies was slow and incremental. But the goal of this chapter is not to explore the rich history of technology as a concept. Rather, in considering the linguistic, cultural, and intellectual scope of technology viz-a-viz ancient mathematics, this chapter aims to challenge one particular scholarly definition of technology as it relates to science.

Like the term technology itself, the 'applied science' model is also historically recent and has led to a semantic shift making 'technology' nearly synonymous with 'applied science'. Using textual sources from ancient Iraq, this chapter will argue that such uses of the terms 'technology' and 'applied science' constitute in fact an epistemic claim regarding the dependence of knowledge on another; it is also a claim subdivides knowledge types in a way incompatible with the non-Western intellectual cultures we will examine.

Conceptualizing technology as dependent on, or as the 'handmaiden' of the sciences can be traced to attitudes of figures like Aristotle, who, for example, drew a stark boundary between knowing-how, i.e., technē, and the mental process of reasoning and abstraction and knowing-that, i.e, epistēmē. This fundamental bifurcation of knowledge types resonates in modern philosophical and historical scholarship where technological advances are taken as fundamentally distinct from scholarly ones. On the one hand, the study of ancient technology (technē) is focused on the reconstruction of ancient craft practices and somatic know-how, in sum: making things. Studies of ancient science, on the other hand, deal in numbers, axioms and abstract claims (epistēmē) made by literate scholarly communities.

Overcoming the inherited boundary that separates the head from the hand is challenging precisely because it is deeply cultural. Cultural reifications of the practical and theoretical knowledge dichotomy are found all over our society: making the intuitive leap that they must have also existed in past knowledge cultures comes rather naturally. As we shall find, these epistemological boundaries are consequential for how we understand the intellectual interaction of technology and mathematical knowledge in cuneiform cultures.

References

[1] P.R.S. Moorey, *Ancient Mesopotamian Materials and Industries: The Archeological Evidence*, Oxford, The Clarendon Press, 1994.

[2] O. Neugebauer, A.J. Sachs, *Mathematical Cuneiform Texts*, New Haven, American Oriental Society, 1945.

[3] M. Ossendrijver, *Babylonian Mathematical Astronomy: Procedure Texts*, Heidelberg, Springer, 2012.

[4] E. Robson, *Technology in Society: Three Textual Case Studies from Late Bronze Age Mesopotamia*, A.J. Shortland, ed., The Social Context of Technological Change: Egypt and the Near East, 1650–1550 BC, Oxford, Oxbow Books, pp. 39–57.

[5] E. Robson, *Mathematics in Ancient Iraq: a Social History*, Princeton, Princeton University Press, 2008.

[6] E. Schatzberg, *Technology: Critical History of a Concept*, Chicago, The University of Chicago Press, 2018.

Volume 1: Representing Mathematics in Antiquity. Depictions of Ancient Mathematics in Word and Image

Liba Taub

Ancient verbal and visual depictions of mathematics – including activities involving mathematical skills, as well as individuals identified as mathematicians – are considered here, with a focus on Greek and Roman culture. Mathematics was part of broader culture and society, not confined only to specialists. Mathematics is represented as practical, theoretical (or philosophical), as well as recreational, including riddles and games. There is evidence that a good number of people, mainly but not only men, took pleasure in displaying – even flaunting – their mathematical knowledge. The practical desirability of certain calculations contrasted with the special intellectual ability and training required by such tasks. Some notable accounts of mathematical problems and solutions were quite literally legendary.

Terms for the branches of mathematics, including arithmetic, geometry, astronomy, harmonics, logistikē (calculation), stereometry, and spherics carry other, non-mathematical meaning, such as measurement of the earth or land (geometry), knowledge of the stars (astronomy), and musical theory (or how things are joined together=harmonics). These other meanings color the representation of mathematics. For example, in (ca. 446 – ca. 386 BCE) play *Clouds*, Aristophanes jokes about whether geometry (Greek gē = earth; metron = measure) refers to measuring a farm or the entire Earth.

This chapter examines ancient representations of mathematical work and of mathematical practitioners. The nature and character of mathematical texts in different cultures is considered elsewhere in this volume. Here, the focus is on how mathematics and mathematicians were portrayed not only to those reading specifically mathematical works, but also texts which – for a variety of reasons – depicted mathematics and those engaged in mathematical work.

Those texts that describe and portray mathematics and mathematicians were not always written by mathematicians themselves. Plato is a case in point. The genre of dialogue, particularly in the form of the Socratic model devised by Plato, is well suited to the presentation and consideration of problems. However, very few of the 'problems' presented in the Socratic dialogues are concerned with mathematics; rather, Plato was concerned with philosophical issues. So, for example, in the *Theaetetus* the problem of incommensurability is touched upon only briefly (e.g. at 148a), while the question of the nature of knowledge underpins the work as a whole. Nevertheless, historically there has been a strong sense of a link between Plato and his valorization of mathematics, for a variety of purposes, including the creation of the world. Plutarch (ca. 46 – ca. 120 CE), one of the few ancient authors to compose a dialogue concerned with scientific and mathematical questions, did present some mathematics in his dialogue *On the face on the Moon.* Yet, surprisingly, of all the interlocutors named in the dialogue, it is only the one described as a mathematician, Theon, who never himself speaks. Plutarch presents the mathematician as a silent participant in the discussion of the problems posed.

References

[1] J. Evans, *Images of Time and Cosmic Connection*, A. Jones, ed., Time and Cosmos in Greco-Roman Antiquity, Princeton, Princeton University Press, 2017, pp.143–170.
[2] R. Hannah, *Time in Antiquity*, London, Routledge, 2009.
[3] A. Imhausen, *Traditions and myths in the historiography of Egyptian mathematics*, E. Robson and J. Stedall, eds., The Oxford Handbook of the History of Mathematics, Oxford, 2009, pp.781–800
[4] A. Jones, *A Portable Cosmos: Revealing the Antikythera Mechanism, Scientific Wonder of the Ancient World*, Oxford University Press, 2017.
[5] G.E.R. Lloyd, *What was mathematics in the ancient world? Greek and Chinese perspectives*, E. Robson and J. Stedall, eds., The Oxford Handbook of the History of Mathematics, Oxford, 2009, pp. 7–26.

[6] E. Robson, *Mathematics education in an Old Babylonian Scribal School*, E. Robson and J. Stedall, eds., The Oxford Handbook of the History of Mathematics, Oxford, 2009.
[7] D.G. Romano, *The archaeology of mathematics in an ancient Greek city*, E. Robson and J. Stedall, eds., The Oxford Handbook of the History of Mathematics, Oxford, 2009.
[8] C. Rossi, *Mixing, building, and feeding: mathematics and technology in ancient Egypt*, E. Robson and J. Stedall, eds., The Oxford Handbook of the History of Mathematics, Oxford, 2009.
[9] K. Saito, *Reading ancient Greek mathematics*, E. Robson and J. Stedall, eds., The Oxford Handbook of the History of Mathematics, Oxford, 2009, pp. 801–826.
[10] N. Sidoli, *Greek Mathematics*, A. Jones and L. Taub, eds., Cambridge History of Science, vol. 1, Ancient Science. Cambridge, 2018.
[11] N. Sidoli, *Ancient Greek Mathematics*, L. Taub, ed., Cambridge Companion to Ancient Greek and Roman Science. Cambridge, 2019.

Volume 2: Introduction, A Cultural History of Mathematics in the Medieval Age

Joseph W. Dauben, Clemency Montelle, Kim Plofker

This volume will offer a culturally balanced view of global mathematics in the period extending from the aftermath of the expansion of ancient empires to the beginnings of early modern European imperial enterprises, as reflected in main literate mathematical traditions preserved primarily in Chinese, Sanskrit, Arabic and Persian, Greek, and Latin (CSAPGL) texts.

It very briefly surveys the surviving traces of contemporary regional mathematical cultures in, e.g., North and South America, sub-Saharan Africa, Polynesia, etc., that are not included in this treatment, and stress the importance of their historical investigation although the scope of our volume precludes it.

We also discuss briefly the role of vernacular languages and so-called “adjacent” literate traditions, such as late medieval Italian and German vis-a-vis Latin, Dravidian literature vis-a-vis Sanskrit, Hebrew vis-a- vis Arabic, Latin and Byzantine Greek, Japanese vis-a-vis Chinese, etc. Although all of these languages include corpora of mathematical knowledge in some form, due to the lack of space we will treat them intermittently and usually in the context of the dominant linguistic traditions they interacted with.

Volume 2: Everyday Numeracy in the Medieval Age

Joseph W. Dauben, Clemency Montelle, Kim Plofker

The main connecting theme is the cross-cultural spread of decimal place-value numerals and their arithmetic techniques. The rival systems they paralleled/replaced, vernacular arithmetic education, vocational requirements, etc., will be treated here. So will the general topic of class and gender roles in medieval mathematical practice, and the mathematical contributions and/or cultural expectations of women. Basic numerical literacy was required of those engaged in trades, commerce, arts and crafts, as well as bureaucrats and various officials throughout

China. We compare this documentation to mid-first millennium sources on education and numeracy in Latin and Greek, Sanskrit, Arabic, and adjacent cultures.

The elements that helped shape the eventually universal system of Indo-Arabic numerals (Mesopotamian sexagesimal place-value and the use of zero tokens in its descendants, glyphs for the first nine natural numbers, the decimal place-value principle) will be dealt with in Volume 1. Here the focus will be on its creation and spread as well as its absorption into everyday numeracy in the various cultures which it reached. Part of the story of innovations is the introduction of zero as an actual symbol (in China this seems to have occurred first in astronomical tables in texts recovered from Dunhuang). Broadly, we'll need to address the problem of how gaps or empty spaces may represent the concept of zero early on in Babylonian cuneiform tablets, on the Chinese counting board, etc. (presumably covered in vol. 1), and how the introduction of actual symbols for zero affected that. In Chinese one needs to differentiate between kong (empty, vacant, appearing in astronomical tables) and ling (zero, in mathematical texts). The early appearance of zero as a written symbol in Indian texts as well and its significance will be discussed here, and in particular, how it was passed along with decimal place-value arithmetic and crucial paper technology from Indian sources through Arab mathematics to the Latin and Greek worlds.

References

[1] S. Chrisomalis, *The Comparative History of Numerical Notation*, Cambridge, Cambridge University Press, 2010.

[2] R. Kaplan, *The Nothing That Is: A Natural History of Zero*, Oxford, Oxford University Press, 2000.

Volume 2: Practice and Profession in the Medieval Age

Clemency Montelle

This chapter will discuss the primary professional contexts of medieval mathematics, especially astrology, commerce, and a range practical-oriented activities such as surveying, architecture and construction. For instance, Chinese and Islamic state and local engineering projects such as canals, construction of state buildings and observatories, etc., required mathematics, as did the elaborate decoration on Indian temples. Mathematics was indispensable for the Medieval European *agrimensores*, as well as in military logistics and architecture. Various administrative demands required computation, as did everyday financial transactions and trade.

We also explore the education and training, status, and support of professionals using and developing the mathematical sciences during this time. What were the demographics of these professionals in various contexts and how did that affect their modes of operation as well as collaboration and communication between them? Where did practitioners carry out their professions and how were they supported, be it from patronage, government salaries, returns from small business enterprises, and the like? What was their professional status with respect to other disciplines and how did public attitudes towards them impact their profession?

References

[1] C. Montelle, *Chasing Shadows: Mathematics, Astronomy, and the Early Reckoning of Eclipse Theory*, Baltimore, Johns Hopkins University Press, 2011.
[2] C. Montelle, *The 'Well-Known' Calendars: al-Khazin's Description of Significant Chronological Systems for Medieval Mathematical Astronomy in Arabic*, J.S. Steele, ed., Calendars and Years II: Astronomy and Time in the Ancient and Medieval World, Oxford, Oxbow Books, 2011, pp. 107–125.

Volume 2: Inventing Mathematics in the Medieval Age

Kim Plofker

This chapter stresses how advanced research emerged organically from interest in particular types of problems and the social/cultural phenomena that provided incentive and impetus for understanding them. In turn, individual breakthroughs eventually reshaped the standard mathematical background of practitioners. These can be tentatively specified using the following general categories of problems as matrices of innovation:

Techniques for facilitating computation with known quantities More sophisticated methods (e.g., classical prosthaphairesis in trigonometry) joined ancient tools like "function" tables and arithmetic techniques. Sophisticated place-value computation methods in decimal and sexagesimal bases were developed for rapid and efficient calculations, including complicated tasks like root extraction. Numbers were encoded verbally and graphically in tables for rapid retrieval and convenient use.

Techniques for facilitating computation with unknown quantities This category includes various methods comprising what we now call algebra, linear algebra, indeterminate equations, etc. Elaborate classification schemes for equations and algorithms for solving them led to solutions for equations of higher order or in more than one variable, Diophantine problems, and systems of multiple equations, involving subtle concepts of algebraic sign, surds, and so on. Examples include Chinese systems of simultaneous linear equations, Indian techniques for first- and second-order indeterminate equations, and techniques of Arabic algebra as adopted by late medieval cossists in Europe.

Properties of numbers, including series, primality, harmonics, etc. Different mathematical traditions emphasized different numerical properties. For instance, integer characteristics like primality and amicability were eagerly studied by the Islamic and European direct heirs of the Euclidean tradition, while Sanskrit and Chinese mathematics investigated types of series and equations.

Techniques of enumeration, combinations etc. The number of possible ways to choose a particular outcome from a set of options was a line of inquiry encountered again and again in a variety of contexts, from counting metrical patterns in Sanskrit verse, to algorithms for enumerating all the possible Arabic words that can be formed from a given number of root letters, to the so-called "Pascal triangle" organizing the coefficients of equations.

Properties of figures in general Ancient geometry knowledge provided the foundations for innumerable explorations and demonstrations of the nature of geometric shapes. These shapes might model real-world phenomena, like cyclic quadrilaterals inscribed in circles of planetary orbits, or polygons representing quantities such as distance or time in the late medieval scholastic discourse of "latitude of forms". Or they might be highly abstract refinements of earlier results, such as constructions of the regular heptagon and similar findings in Islamic Euclidean geometry.

Solutions of triangles and relations between triangles and circles, trigonometry Sharing a common ancestor in Hellenistic Greek trigonometry of chords, various methods for quantitatively relating triangle sides to angles and/or circular arcs developed together with geometric study of circles and triangles. Their origin in Hellenistic spherical astronomy is reflected in their 360-degree circles, in their accompanying celestial coordinate systems and in their focus on the mathematization of space. Examples here include the development of Indian trigonometry of sines from Greek trigonometry of chords, relations between circle measurement in Chinese geometry and trigonometric techniques from Indian and Islamic sources, and the cross-fertilization of Indian trigonometry with Greco-Islamic spherical trigonometry.

Techniques for refining approximate solutions: interpolation, iterative methods, etc. Many practical computations, especially of astronomical quantities involving irregular planetary motions, were not amenable to exact solutions. Simple linear approximations were often employed instead, and were also corrected by various numerical methods. Not being easily analyzable by rigorous mathematical demonstration, such methods were not always admitted as acceptable practices. Examples of this are "false position" methods in Chinese, Islamic, and European sources, plus iterative techniques in Sanskrit and Arabic mathematics.

Mathematics of infinitely large and small quantities The nature of processes infinitely repeatable or divisible allowed for additional ways to refine approximations of, e.g., the value of the ratio of the circumference to the diameter. Concepts of enumerability and infinity were also applied to extrapolating beyond known large numbers, and the problems of how to express numerical quantity at such scales.

References

[1] J.L. Berggren, *Episodes in the Mathematics of Medieval Islam*, New York, Springer, 1986; repr. 2003.

[2] J.L. Berggren, *Mathematics in Medieval Islam*, V. Katz, ed., A Sourcebook in the Mathematics of Egypt, Mesopotamia, China, India, and Islam, Princeton, Princeton University Press, 2007, pp. 525–675.

[3] J.W. Dauben, *Chinese Mathematics*, V. Katz, ed., A Sourcebook in the Mathematics of Egypt, Mesopotamia, China, India, and Islam, Princeton, Princeton University Press, 2007, pp. 187–384.
[4] A. Heeffer, *The Tacit Appropriation of Hindu Algebra in Renaissance Practical Arithmetic*, Gaṇita Bhāratī **29(1/2)** (2007), 1–60.
[5] Lam L.Y., Ang T.S., *Fleeting Footsteps. Tracing the Conception of Arithmetic and Algebra in Ancient China*, Singapore, World Scientific, 1992; rev. ed. 2004.
[6] Li Y., Du S., *A Concise History of Chinese Mathematics*, John N. Crossley and Anthony W.-C. Lun, trans. Oxford, Clarendon Press, 1987.
[7] J.-C. Martzloff, *A History of Chinese Mathematics*, Stephen S. Wilson, trans. Berlin: Springer Verlag, 1997; rev. ed. 2006.
[8] Y. Mikami, *The Development of Mathematics in China and Japan*, Abhandlungen zur Geschichte der mathematischen Wissenschaften **30** (1913); repr. New York: Chelsea, 1961.
[9] K. Plofker, *Mathematics in India*, V. Katz, ed., A Sourcebook in the Mathematics of Egypt, Mesopotamia, China, India, and Islam, Princeton, Princeton University Press, 2007, pp. 385–514.
[10] K. Plofker, *Mathematics in India*, Princeton, Princeton University Press, 2009.
[11] B.L. van der Waerden, *Geometry and Algebra in Ancient Civilizations*, Berlin, Springer, 1983.

Volume 2: Mathematics and Worldviews in the Medieval Age

Joseph W. Dauben

Mathematics and its relation to world views throughout the medieval period are interlinked with general philosophical outlooks and the most fundamental relations between the individual, larger social frameworks, correspondences between the terrestrial and celestial realms of nature, and connections between the micro- and macro-cosms. Mathematics as it relates to the basic organizing principles of the universe, in religious and philosophical contexts, will also be explored in this chapter. These involve mathematics in so far as it relates to general matters of cosmology and conceptions of time and space, cyclical versus linear views of time, specific details like calendars and almanacs, as well as abstract questions of infinity and infinitesimals. These themes can be tentatively divided into the following categories:

- Modifications in geometric models of the universe, especially "two-sphere" with emphasis on Ptolemaic along with non-spherical alternatives;
- Celestial divination, astrology, and calendric timekeeping;
- Abstract concepts such as sacred shapes/numbers, periodicity, predictability, and the role of the divine in worldviews.

These categories suggest a kind of thesis-anthesis-synthesis setup: the infinite regularity of astronomical models contrasts with the supposed unpredictability of divination and the untidiness of physical time, which metaphysical views on the ultimate meaning of the universe are somehow supposed to resolve.

One challenge will be to distinguish between traditions that clearly originated much earlier and will already have been covered in Volume 1 versus those that are uniquely medieval. Here the emphasis will be on innovations or reinterpretations that significantly changed during the Middle Ages due to new discoveries or ways of thinking about mathematics.

References

[1] S.L. Jaki, *Science and Creation: From Eternal Cycles to an Oscillating Universe*, New York, University Press of America, 1990.

[2] A. Koyré, *From the Closed World to the Infinite Universe*, Baltimore, Johns Hopkins University Press, 1957.

[3] G.E.R. Lloyd, *Adversaries and Authorities. Investigations into Ancient Greek and Chinese Science*, Cambridge, Cambridge University Press, 1996.

[4] G.E.R. Lloyd, N. Sivin, *The Way and the Word. Science and Medicine in Early China and Greece*, New Haven, Yale University Press, 2002.

[5] J. Needham, C. Ronan, *Chinese Cosmology*, N. S. Hetherington, ed., Cosmology: historical, literary, philosophical, religious, and scientific perspectives, New York, Garland, 1993, pp. 25–32.
Needham, Joseph, and Ling Wang,

[6] J. Needham, L. Wang, *Science and Civilisation in China. Vol. 3. Mathematics and the Sciences of the Heavens and the Earth*, Cambridge, Cambridge University Press, 1959.

[7] E. Nicolaidis, et al., *Science and Orthodox Christianity: An Overview*, Isis **107(3)** (2016), 542–566.

[8] K. Plofker, *Derivation and Revelation: The Legitimacy of Mathematical Models in Indian Cosmology*, T. Koetsier and L. Bergmans, eds., Mathematics and the Divine, Amsterdam, Elsevier, 2005, pp. 61–73.

Volume 2: Describing and Understanding the World in the Medieval Age

Jeff Chen

A primary focus of mathematics in China in the Medieval period was on astronomical modeling and astrological prediction, particularly trigonometric techniques in models. While the computations of the distance between stars or planets in the heaven are fairly common in astronomy in China, it is pre-mature to categorize the usage as techniques in spherical trigonometry. Even in the well-studied calendar reform in the 13th century, modern historians cannot agree whether the computational methods can be qualified as a version of spherical trigonometry.

The geomancy and astrology in the Chinese context appear to be different from those practiced in Europe or Africa and have more to do with the year and dates in the sexagenary cycle (ganzhi). The numerical mysticism in China is usually associate with the "magic squares", the arrangement of numbers in various shapes that result the equal sum of numbers and the images of the hexagrams in the studies of the Book of Change (Yijing). In several 14th- and 15th-century mathematical treatises, these magic "shapes" symbolize the mysterious origin of numbers and mathematics. In several mathematical works can be seen a procedure of numerical counting that predicts the gender of an unborn baby. A trend can be observed that the numerical mysticism diverged gradually from the treatment of legitimate mathematics while the astronomical bureau continued to conduct astronomical observations and computations as well as to perform as the royal diviners in determining the auspicious hours, dates, and sites for important events.

Map making, also greatly advanced in ancient China, was also developed in the medieval period. The arrival of the Jesuits and the famous maps of Matteo Ricci at the end of the 16th century precipitates an advance in cartography in China although they fall out of the time frame in volume 2 and should be treated in volume 3. Closely related to the mathematical problems of map-making are various means Chinese inventors devised to measure actual ground distances, a subject pioneered in the Han dynasty when it was necessary to measure accurately distances travelled throughout the empire, for which a mechanical odometer of sorts was constructed.

References

[1] C. Cullen, *Heavenly Numbers. Astronomy and Authority in Early Imperial China*, Oxford, Oxford University Press, 2017.

[2] K. Plofker, T.L. Knudsen, *Calendars in India*, J.S. Steele, ed., Calendars and Years II: Astronomy and Time in the Ancient and Medieval World, Oxford, Oxbow Books, 2011, pp. 53–68.

[3] N. Sivin, *Granting the Seasons: The Chinese Astronomical Reform of 1280, With a Study of Its Many Dimensions and a Translation of its Records*, New York, Springer, 2009.

[4] N. Sidoli, G. Van Brummelen, eds., *From Alexandria Through Baghdad. Surveys and Studies in the Ancient Greek and Medieval Islamic Mathematical Sciences in Honor of J.L. Berggren*, Heidelberg, Springer, 2014.

Volume 2: Mathematics and Technological Change in the Medieval Age

Joseph W. Dauben, Clemency Montelle, Kim Plofker

This chapter will survey important and more modest advances in a number of different directions, among which are the following.

Observational, navigational, and timekeeping devices The ancient mathematical devices of the astrolabe, armillary sphere, sighting tube, compass, water- and sand-clocks, and various forms of sundials were refined and modified in innumerable ways during this period, and spread throughout much of the Eurasian world. This period ends before the development of glass lenses for effective high-power magnification and the concurrent proliferation of early modern mathematical instruments. Important innovations included Islamic and Chinese astronomical instruments as well as varied types of non-astronomical clocks and other navigational devices.

Architecture, construction and mechanics A substantial body of knowledge from antiquity (Vitruvius etc.) was greatly expanded during this period. The "practical geometry" of European stonemasons, the elaborate tilings and vaults of Islamic buildings, the śilpa formulas used by Indian builders, and so forth, all attest to conscious reliance on mathematics in developing these technologies as well as profound “subscientific” mathematical understanding in traditional designs and techniques.

Transport and military science This includes methods of medieval shipbuilding, military architecture and fortifications. Most applications of mathematics to ballistics and fort design occurred near or beyond the end of the period. Key technological innovations such as the riding stirrup were influential but not explicitly mathematical.

Agriculture, manufacture and mining/metallurgy Mathematical texts contain classic formulas and problems involving mensuration in agriculture and manufacture. Geometrical optics likewise discusses the physical behavior of lenses and mirrors. But the specific material innovations that had the greatest impact (e.g., the medieval European development of the heavy plough) were not necessarily mathematical in nature; similarly for mechanical mills, pumps and looms. This illustrates the familiar scholar/artisan divide,” which suggests that the impact of mathematics on technological developments came later.

References

[1] A.Y. al-Hassan, D.R. Hill, *Islamic Technology: An illustrated history*, Cambridge: Cambridge University Press, 1986.
Grafton, Anthony.
[2] A. Grafton, *Leon Battista Alberti: master builder of the Italian Renaissance*, New York, Hill and Wang, 2000.
[3] D.R. Hill, *A History of Engineering in Classical and Medieval Times*, London, Croom Helm & La Salle, Illinois, Open Court, 1984.
[4] D.R. Hill, *Islamic Science and Engineering*, Edinburgh: Edinburgh University Press, 1993.
[5] R. Tavernor, *On Alberti and the Art of Building*, New Haven, Yale University Press, 1998.
[6] L.T. White, *Medieval Technology and Social Change*, Oxford, Oxford University Press. 1962.
[7] L.T. White, *Medieval Religion and Technology*, Berkeley, University of California Press, 1978.

Volume 2: Representing Mathematics in the Medieval Age

Sonja Brentjes, Nathan Sidoli

Structuring mathematical texts through various devices such as titles, tables of content, headlines, subdivisions, foliations, formula, diagrams and other organisational means seems to be an ordinary representational practice when looked at it from modern writing, printing and reading practices. But this was not always the case. In our period, the overwhelming number of mathematical treatises were handwritten on various kinds of material such as parchment or vellum, papyrus, paper, birch bark and palm or banana leaves. The exception to this rule is found in East Asia where wood-block printing was introduced at the end of the sixth century.

East Asian print culture and West Asian manuscript culture were materially linked through their use of different sorts of paper. Most treatises meant for the practical use of experts, or in teaching, were written or printed on cheaper types of paper, while expensive, gilded and patterned papers served for works produced for courts, including mathematical texts. Parchments and vellums were used in the eastern Roman Empire as well as western Christian Europe for written work in all intellectual fields until about the fifteenth century, including mathematical works.

Around the middle of this century, paper finally replaced those two media as the main material carrier of scholarly productions at universities, scribal workshops and private homes in western Europe, while the eastern Roman Empire was conquered by the Ottoman dynasty (r. c. 1300-1922). In North Africa, parchment or vellum served as the primary writing material, in particular for sacred books, until about the thirteenth century and paper seems to have entered into book production after 1100, when the first paper mill is recorded for Fez. In the Islamic parts of the Iberian Peninsula, al-Andalus, paper seems to have made its entrance some decades later and soon acquired a special reputation for its quality.

Birch bark, palm or banana leaves became standard material for text production in South and possibly also Southeast Asia in the first millennium BCE. While birch bark was mostly used in northern South Asia, palm or banana leaves dominated in the South as well as the Southeast. Paper spread there centuries later. It was the preferred medium of Muslim scholarly and administrative cultures in South and Southeast Asia since the establishment of Turkic dynasties and the conversion of local rulers soon after the tenth century.

In addition to those dominant material carriers of mathematical knowledge in post-Antiquity, metals, in particular copper and brass, silk, bamboo and stones were used in the forms of plates, dishes, temple walls or rocks for royal and other institutional inscriptions of various lengths, containing calendar information, astronomical data, astrological interpretations and, occasionally, even calculations or drawings. Those different material carriers of mathematical texts shaped substantially the knowledge that we have today about representational practices in the mathematical sciences and that we can possibly acquire.

References

[1] S. Brentjes, *Safavid Art, Science, and Courtly Education in the Seventeenth Century*, N. Sidoli, G. Van Brummelen, eds., *From Alexandria Through Baghdad. Surveys and Studies in the Ancient Greek and Medieval Islamic Mathematical Sciences in Honor of J.L. Berggren*, Heidelberg, Springer, 2014, pp. 487–502.

[2] J. Chabás, B.R. Goldstein, *A Survey of European Astronomical Tables in the Late Middle Ages*, Brill, Leiden, 2017.

[3] K. Chemla, *Numerical Tables in Chinese Writings Devoted to Mathematics: From Early Imperial Manuscripts to Printed Song-Yuan Books*, East Asian Science, Technology and Medicine **44** (2016), 69–121.

[4] G. De Young, *Diagrams in the Arabic Euclidean tradition: a preliminary assessment*, Historia Mathematica **32** (2005), 129–179.

[5] K. Saito, N. Sidoli, *Diagrams and arguments in ancient Greek mathematics: Lessons drawn from comparisons of the manuscript diagrams with those in modern critical editions*, K. Chemla, ed., The History of Mathematical Proof in Ancient Traditions, Cambridge, Cambridge University Press, 2012, pp. 135–162.

[6] J. Sesiano, *Magic Squares in the Tenth Century: Two Arabic Treatises by Anṭākī and Būzjānī*, Cham, Springer, 2017.

[7] J. Sesiano, *Magic Squares: Their History and Construction from Ancient Times to 1600*, Cham, Springer, 2019.
[8] N. Sidoli, *Mathematical tables in Ptolemy's Almagest*, Historia Mathematica **41** (2014), 13–37.
[9] W. Stahl, *Martianus Capella and the Seven Liberal Arts*, vol. 2, New York, Columbia University Press, 1977.

Volume 3: Introduction, A Cultural History of Mathematics in Early Modern Europe

Jeanne Peiffer, Volker Remmert

The Early Modern Period, from the fall of Constantinople (1453) to the publication of Isaac Newton's seminal *Philosophiae Naturalis Principia Mathematica* (1687) is a period of tremendous change esp. in Western Europe. In the history of science it is often labeled "Scientific Revolution," although serious nuances have been formulated, also in this volume. In the mathematical sciences this is a period of transition from the medieval quadrivium (arithmetic, geometry, astronomy, music) to the invention of the new analysis towards the end of the seventeenth century. It is difficult to assess the meaning of mathematics in the narrow sense and the mathematical sciences in a wider sense in the period. Both are characterized by a great heterogeneity, strongly depending on the time period, local contexts and demands, and individual approaches. Scholars working on commented editions (and translations) of ancient Greek mathematical texts, mathematical practitioners such as astrologers, mapmakers, surveyors, seamen, stonemasons, court mathematicians like Galileo, Kepler and many others, Jesuits teaching mathematics throughout the world, university based mathematicians – they all share practices and passions, for which we would be hard pressed to find a common denominator. At some point during the early modern period there was a turn to mathematics in as well as outside the universities. When precisely is subject to discussion. Many factors contributed to this development: the rise of the printed book clearly stimulated mathematics and facilitated growing libraries in universities, but also in courts, cities and in individual households. The spreading of the mathematical sciences and their applications profited from networking through letters as well as the exchange of books and instruments. The mathematical sciences facilitated and profited from encounters with other kinds of knowledge (missions to China, discovery of the new world,...). The so-called practical mathematics – surveying, building, navigating, gauging – developed in opposition to or in collaboration with scholarly bookish knowledge. Mathematics was increasingly considered highly useful (first for astrology, medicine, civil administration in the fifteenth century then for crafts, bankers, ...). The volume will shed light on when this happened and where and who did it. At the same time mathematics was increasingly deemed the most certain kind of knowledge humanely attainable. The debate about its certainty shaped philosophies and worldviews. The seventeenth century was a period of intense mathematical invention. In a context characterized by the debate between Ancients and Moderns, new curves were discovered, like the cycloid for instance; new fields opened to mathematical investigation like the theory of probabilities; new methods brought about tremendous change, like Vieta's symbolic algebra which Descartes applied to geometry, and infinitesimal methods inherited from Archimedes which eventually led to the invention of the calculus by Newton and Leibniz. These innovations were crucial for a better understanding of the world in all its complexity.

Volume 3: Everyday Numeracy Through the Mirror of Arithmetic Textbooks

Maryvonne Spiesser

The central topic of the chapter concerns the evolution/transformation of the different systems of numeration inherited from the past and whose features are described in Volume 2 (taking into account different geographical areas and chronological evolutions). During the medieval period and long after, different methods of calculation were used. These were linked to material tools and to the type of number system employed. In Europe, there were two principal methods:

- calculations with counters on calculating boards (of different kinds), associated with Roman numeration;
- calculations by writing (on paper or another medium) with Indo-Arabic numerals (positional decimal system).

These systems will be discussed in connection with geographic areas, users' needs, and with some attention to technical improvements (such as printing techniques). The positional decimal system spread first in Southern Europe, from the 12th century onward, and progressively supplanted calculation with counters, though for a long time both methods continued to coexist, as did finger counting during the period.

In early modern Europe, during a period of great technological changes, those "who know by doing and those who know by thinking" were not in opposition. More and more people were using mathematics for technical or practical purposes – trade, surveying, navigation, etc. – and many educated men were widely interested in theory as well as practice. The study of numeracy therefore takes place in the context of a rising interest in practical mathematics.

Numbers representations are at the core of numeracy. During the Renaissance, Hindu-Arabic calculation, already well established in the Islamic world, was spreading unevenly throughout Europe but had yet to supplant its competitors. The ongoing evolution of the various methods of calculation was crucial, but also complex. For this reason we will focus on Western European science. Even within this limited framework, part of the difficulty lies in the heterogeneity of numerical skills. The local shopkeeper does not have the same needs as the international merchant. Accounting techniques were more sophisticated in large Italian cities than in most other European areas, and the literacy rate was higher among the urban population.

The first two sections of this chapter tackle the following questions: what were the techniques and devices in use to solve problems involving numbers? How could people become numerate? The third section addresses the issue of numeracy in daily (professional or private) life. The most accessible information available on everyday arithmetic has been passed down to us through textbooks. From the fifteenth to the seventeenth centuries, these sources multiplied and diversified. It is chiefly through the analysis of these texts that we will address the issue of everyday numeracy.

References

[1] S. Brentjes, *Teaching and Learning the Sciences in Islamicate Societies (800-1700)*, Brepols, 2018.
[2] L. Corry, *A brief history of numbers*, Oxford, Oxford University Press, 2015.
[3] J. Deniss, *Learning arithmetic: textbooks and their users in England 1500-1900*, E. Robson and J. Stedall, eds., The Oxford Handbook of the History of Mathematics, Oxford, 2009, pp. 448–467.
[4] M. Folkerts, *The development of mathematics in Medieval Europe*, Aldershot, 2006.
[5] V. Katz, ed., *The mathematics of Egypt, Mesopotamia, China, India, and Islam: a sourcebook*, Princeton, Princeton University Press, 2007.
[6] K. Plofker, *Mathematics in India*, Princeton, Princeton University Press, 2008.
[7] J. Wess, *Avoiding arithmetic, or the material culture of not learning mathematics*, BSHM Bulletin: Journal of the British Society for the History of Mathematics, **27(2)**(2012), 82–106.

Volume 3: Practice and Profession. Range and Change in Mathematical Lives

Jim Bennett

The question of the disciplinary position and setting of mathematics must explain that it is a not a 'given' but a set of assumptions, conventions and patterns that are modulated over time and place and renegotiated by those who identify themselves as mathematicians. Further, these mathematicians seek to instantiate their understanding of such matters through academies, teaching institutions, professional guilds and positions in public administration. Characteristic tools of mathematical activity – books and instruments – are used on both sides of this distinction: to give disciplines shared mathematical identity and to regulate teaching and institutional character.

The period witnessed a confident spread of mathematics through a number of fields of practice. This was commonly presented as the reform of manual or mechanical arts into mathematical arts by re-founding their practice on mathematical science. A realm of (in English) 'speculative' mathematics was recognised, where notions were pursued for purely intellectual exercise, but it is mistaken to characterise mathematical practice in the various disciplines with the modern idea of being 'applied'. This implies that there is an independent mathematics that is applied as a separate step, whereas practitioners saw themselves doing mathematics through the exercise of their disciplinary practice. Astronomy played a major role in this story, as it served as the exemplar for the general development. Other instances include dialing, warfare – offensive (artillery) and defensive (fortification) – surveying, cartography, commerce, architecture, perspective, navigation, etc.

Organisational factors must also be taken into account. In addition to teaching and learning (schools, universities, private tuition, subscription courses), one finds examples of mathematical societies alongside widespread courtly patronage. Mathematical practitioners had relations with religious institutions, held positions in public/civic administration (excise, military and naval boards, standards, metrology, etc.), and were members of professional bodies, such as guilds, corporations, and trading companies. The tools of their trade were reflected in a wide variety of books and scientific instruments. The Latin title "Mathematicus", or its linguistic equivalents, might thus derive from an official position, office or responsibility, whether at court, in a university or in a civil administration. Alternatively an individual might come to be recognised as a mathematician through activates undertaken outside any office or employment.

Almost every adult member of society uses mathematics, while only a few are acknowledged as mathematicians. By reviewing a wide range of examples, this chapter offers an account of how mathematicians achieved and sustained their status in early-modern Europe. The transcendent claims of mathematicians notwithstanding, mathematical practice is located in time and place, and to affirm this we will begin with a particular time (May, 1547), followed by a more extended account of activity in a particular place (Nuremberg). By invoking the more abstract concept of 'locus', we then consider types of places, such as observatories, printing-houses and colleges, that are found in many locations. Finally, generalising further, we look at principles, tools and methodologies that are common to mathematical practice across time and location.

References

[1] M. Biagioli, *The Social Status of Italian Mathematicians, 1450–1600*, History of Science, **27(1)** (1989), 41–95.

[2] S. Brentjes, *Teaching and Learning the Sciences in Islamicate Societies (800-1700)*, Brepols, 2018.

[3] L.B. Cormack, S.A. Walton, J.A. Schuster, eds., *Mathematical practitioners and the transformation of natural knowledge in early modern Europe*, Springer, 2017.

[4] S. Johnston, *Mathematical practitioners and instruments in Elizabethan England*, Annals of Science, **48**(1991), 319–344.

[5] S. Johnston, *The identity of the mathematical practitioner in 16th-century England*, Irmgard Hantsche, ed., Der 'mathematicus': Zur Entwicklung und Bedeutung einer neuen Berufsgruppe in der Zeit Gerhard Mercators, Duisburger Mercator-Studien, volume 4, Bochum, 1996.

[6] P.R. Rose, *The Italian Renaissance of mathematics: Studies on humanists and mathematicians from Petrarch to Galileo*, Geneva, 1975.

[7] E.G.R. Taylor, *The mathematical practitioners of Tudor & Stuart England*, Cambridge, 1954.

[8] R.S. Westman, *The Astronomer's Role in the Sixteenth Century: A Preliminary Survey*, History of Science, **18**(1980), 105–147.

Volume 3: Inventing Mathematics. Between Tradition and Modernity

Sébastien Maronne

The role of invention in the mathematics of the Early Modern Period was closely tied with various efforts to recover or restore classic works from Greek Antiquity but also to supplement them, either by producing "upgrades" or to go beyond the Ancients by inventing entirely new methods. Thus, the Moderns were deeply aware of how their own approaches to mathematics were linked with the Greek tradition. On the one hand, they relied on classical methods (Pascal's mechanical quadrature method was inspired by Archimedes) but also Greek foundational principles (classical proportion theory, geometry of indivisibles, and the method of exhaustion). On the other hand, early modern mathematicians were not content with classical synthetic demonstrations but felt the need to develop analytic and algebraic (Descartes) methods of invention which would reveal the presumed hidden analysis of Ancient geometers. In this way, they re-conceptualized the classical topos of analysis and synthesis.

Early modern mathematicians placed a strong emphasis on the resolution of mathematical problems rather than demonstrating theorems. Here, too, they took inspiration from Greek geometers, who according to Pappus took great care to discover whether a given problem could be solved by straightedge and compass constructions or required more sophisticated tools. The resolution of mathematical problems was thus submitted to methodological constraints. Such constraints might originate in tradition or derived from philosophical prerequisites (such as Descartes' notion of "clear and distinct" ideas).

Mathematical innovations can be found in various kinds of texts, both printed as well as manuscript sources. By the end of the Early Modern Period, scientific journals began to appear, such as the *Mémoires de l'Académie des Sciences* and the the *Acta Eruditorum.* Prior to this, apart the printed books, scientific correspondence played a crucial role, mediated by key figures, such as Mersenne. As regards texts, our period begins with the first printed edition of Euclid's *Elements*, which appeared in 1492. It was based on a Latin translation made by Campanus, a medieval scholar. It ends, two centuries later, with the publication in 1684 of a very different type of work, namely Leibniz's rather abstruse "Nova Methodus pro Maximis et Minimis" in the journal *Acta Eruditorum.*

Infinitesimal calculus, which was the most famous and important mathematical invention of this era, mainly developed in the first decades of eighteenth century. It originated with Leibniz's publications and Newton's contemporary works on fluxions. The 1492 text of Euclid's *Elements*, on the other hand, initiated the erudite tradition of printed editions of ancient Greek works. These objective boundaries point to a long-lasting and continuous process from tradition to invention, classicism to modernity, which reflects many of the developments in early modern mathematics. These are some of the main paths we will travel to gain an overview of mathematical invention in the early modern period. At its end, Isaac Newton's *Philosophiae Naturalis Principia Mathematica* (1687) – which borrowed heavily from classical geometry while adding to it modern infinitesimal considerations –

will offer an exemplary epilogue to our journey passing from tradition to modernity.

References

[1] H.J.M. Bos, *Redefining Geometrical Exactness. Descartes' Transformation of the Early Modern Concept of Construction*, New York, Springer, 2001.

[2] G. Desargues, *The Geometrical Work of Girard Desargues*, J.V. Field, J. Gray, eds., New York, Springer, 1987.

[3] N. Guicciardini, *Isaac Newton on mathematical certainty and method*, Cambridge, MA and London, MIT Press, 2009.

[4] V. Jullien, ed., *Seventeenth-Century Indivisibles Revisited*, Cham, Birkhäuser, 2015.

[5] P. Mancosu, *Philosophy of mathematics and mathematical practice in the seventeenth century*, New York, Oxford University Press, 1996.

Volume 3: Mathematics and Worldviews. Mathematics as a Touchstone for "Certainty"

David Rabouin

The Early Modern Period is often seen as a time where the very criterion of truth changed. Whereas the Ancient tradition tends to emphasize the fact that truth should be considered as a way in which knowledge corresponds to reality (*adaequatio rei et intellectus*), the Modern Age would be marked by the overwhelming power of the criterion of *certitudo.* In this development, mathematics plays a crucial role precisely because it offers a model in which no correspondence to reality is needed. It is no surprise then when explaining the famous Rules of his method, Descartes immediately refers to mathematics. However, it is very important to keep in mind the role of the debate on the certainty of mathematics to assess the way in which early modern authors related the standard of mathematics with the search of a method. Indeed, there are many ways to refer to mathematics and what Descartes has in mind is, in many aspects, the kind of *mos geometricus* one can find in authors such as Spinoza or Leibniz. In this chapter, this diversity will be emphasized as well as the danger attached to a retrospective and univocal meaning of what is supposed to be behind the "mathematical method."

In assessing the relationship between mathematics and worldviews in Early Modern Europe, one immediately confronts the role played by the "Scientific Revolution" and, in particular, the Copernican Revolution that preceded it. By expelling the Earth from the centre of the world and by seeing the universe as a geometrical infinite space, a new era, it has been said, had begun. Mathematics suddenly entered, for better or worse – and not without resistance – in the way the so-called Western world looked at the universe. As expressed by Galileo in a famous passage of the *Assayer (1623)*: "Philosophy [nature] is written in that great book which ever lies before our eyes – I mean the universe – but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language." Following a time when Aristotelian scholasticism stood against the impulse to mathematize nature, the time had come, as Alexandre Koyré put it, for 'Plato's revenge'.

According to Koyré, the Early Modern period is an era in which the Universe was, for the first time, identified with the infinite geometrical Euclidean space. As mathematics entered into physics, it destroyed the old image of a 'closed world' and gave its place to infinity. This launched several questions about the way in which this mathematical infinity could relate to the traditional place for infinity – and until then the only legitimate one: theology. The late debate between Newton – speaking through the theologian Samuel Clarke – and Leibniz represents a good example of how hotly the nature of physical space was contested. Koyré's view, however, is very simplistic and, as will be shown in this chapter, it does not accord with the way in which philosophers and mathematicians addressed the issues surrounding infinity.

References

[1] P. Descola, *Beyond nature and culture*, University of Chicago Press, 2013.
[2] M. Foucault, *The Order of Things: An Archaeology of the Human Sciences*, trans. Sheridan, London, Tavistock, 1970.
[3] N. Guicciardini, *Isaac Newton on mathematical certainty and method*, Cambridge, MA and London, MIT Press, 2009.
[4] D. Jesseph, *Squaring the Circle. The War between Hobbes and Wallis*, Chicago, The University of Chicago Press, 1999.
[5] A. Koyré, *From the Closed World to the Infinite Universe*, Baltimore, Johns Hopkins University Press, 1957.
[6] P. Mancosu, *Philosophy of mathematics and mathematical practice in the seventeenth century*, New York, Oxford University Press, 1996.
[7] J. Needham, L. Wang, *Science and Civilisation in China. Vol. 3. Mathematics and the Sciences of the Heavens and the Earth*, Cambridge, Cambridge University Press, 1959.
[8] P.R. Rose, *The Italian Renaissance of mathematics: Studies on humanists and mathematicians from Petrarch to Galileo*, Geneva, 1975.
[9] G. Schubring, *Conflicts between generalization, rigor, and intuition. Number concepts underlying the development of analysis in 17th–19th Century France*, New York, Springer, 2005.

Volume 3: Describing and Understanding the World in Early Modern Europe

Antoni Malet

Mathematics fundamentally contributed to optics, mechanics, cosmography, and map-making in Early Modern times. Mathematical knowledge not only provided inspiration to design new scientific instruments but many original mathematical results and new curves were invented or gained currency and legitimacy for being needed in describing or understanding the world. In what follows we will focus on chosen exemplary problems that provide insights in the ways mathematics contributed to understand nature, or in which mathematical inventions appeared in answer to natural philosophical questions.

Up to the 15th century the relations between mathematics and the description and understanding of the material world were essentially circumscribed to the science of optics and to Ptolemaic geometrical models describing the motions of the heavenly bodies. Outside of these disciplines, mathematics also played a role in the medieval science of weights (*scientia de ponderibus*), a discipline that was not central to the natural philosophical medieval curriculum. As we enter into the early modern period things will strikingly change. New geometrical techniques are introduced in optics, and in kinematics and mechanics, all sciences that expand enormously while undergoing radical reformulations. Moreover, new fields and problems appeared that incorporate mathematical methods to their standard tools – map making, artificial perspective, and games of chance being the most substantial ones. These new fields generate in turn new mathematical results.

The chapter is organized thematically. First it presents the mathematics of cartography, a science that dramatically changed in our period and which posed substantial mathematical problems involving what is now called integration and rectification. Next we will turn to optics, where geometry underpinned a major shift in painting. Mathematics also crucially contributed to Kepler's explanation of the optical properties of the eye, and the first theoretical explanations of the properties of optical instruments. We will turn next to the mathematics of motion and impact, paying particular attention to Huygens' work on the cycloid, directly inspired by his interest in inventing a perfectly isochronous pendulum clock. We will also include an overlook of the first mathematical analyses of games of chance and their application to actuarial problems.

References

[1] K. Andersen, *The Geometry of an Art: The History of the Mathematical Theory of Perspective from Alberti to Monge*, New York, Springer, 2006.
[2] A. Chalmers, *One hundred Years of Pressure. Hydrostatics from Stevin to Newton*, New York, Springer, 2018.
[3] L. Cormack, *Charting and Empire. Geography at the English Universities*, Chicago, University of Chicago Press, 1997.
[4] I. Hacking, *The Emergence of Probability*, Cambridge, Cambridge University Press, 1975; rev. ed. 2006.
[5] J.B. Harley, D. Woodward, eds., *The History of Cartography, Volume 1. Cartography in Prehistoric, Ancient, and Medieval Europe and the Mediterranean*, Chicago, University of Chicago Press, 1987.
[6] B. Stephenson, *Kepler's Physical Astronomy*, Princeton, Princeton University Press, 1987.
[7] N.J.W. Thrower, *Maps & civilization : cartography in culture and society*, Chicago, University of Chicago Press, 1996.
[8] D. Woodward, ed., *The History of Cartography, Volume 3. Cartography in the European Renaissance*, Chicago, University of Chicago Press, 2007.
[9] J. Yoder, *Unrolling Time. Christiaan Huygens and the Mathematization of Nature*, Cambridge, Cambridge University Press, 1989.

Volume 3: Mathematics and Technological Change. The Silent Rise of Practical Mathematics

Thomas Morel

The role of technology changed considerably in the Early Modern Period. Conflicts and epidemics had plagued Europe in the Late Medieval Age, and the somewhat calmer recovery period correlated with an accelerated rate of technological innovation. This development, marked in Western and Central Europe, exerted a crucial influence on a global scale due to overseas exploration and subsequent colonisation. As a reference work on the history of technology puts it, "the technical level that was reached and mastered put the European States in the position to define the course of World history" in the coming centuries, with far reaching consequences.

The exact role played by mathematical sciences in this process, difficult to assert as it may be, will be the subject of this chapter. A few preliminary remarks should be made, however, in order to qualify any further claim about the relationship between science and technology in the Early Modern Period. First of all, most of the technical fields were free from mathematisation or from any scientific formalisation of knowledge. In other words, most early modern technicians and artisans never considered using mathematics, be it numerical tables or technical drawing, simply because it could not be expected to be useful in any way.

This was true for many prevalent activities of the time such as agriculture, mining, textile manufacturing or dyeing and tanning, but also for innovations such as the printing press, whose development did not involve formal exact knowledge. Even technical achievements such as the medieval cathedrals were based on craft knowledge that was almost completely non mathematical. It would be hard to overstate how much this approach differs from our Modern World, where the relationship between mathematics and technology have been deeply reshaped during the Industrial Revolution. This observation corroborates, in the technical world, conclusions drawn in the previous chapter: in the Early Modern Period, mathematizing nature was still an ambitious project, not yet a general reality.

Among historians, there has been a marked discrepancy with regard to their views on the importance of mathematics for early modern technicians. This mainly comes down to the versatility of what "mathematics" has meant, but also to its uneven applicability to various technological fields. This chapter uses the category of "practical mathematics" to denote a knowledge mainly produced by technicians, engineers, artists or merchants for their own practical needs. In other words, it forms a disparate set of useful formulas, methods or techniques used in civil life. While these persons cared about accuracy and steadiness, little attention was given to abstraction or demonstration. This use of mathematics thus differs markedly from both our modern conception of science based technology and from the academic disciplines of the contemporary quadrivium.

This chapter presents a nuanced view of the impact mathematics had on technical activities. Some classic and well-known examples are given, together with more specific or esoteric case studies, in order to underline the diversity of actors, methods and possible outcomes. We deliberately begin with the unsuccessful attempts to improve the design of water wheels using mathematical theories. We then turn to the various uses of surveying. From Dutch polders to German mines, and from the draining of the English Fens to map-making, it was one of the most tangible successes for early geometry. A growing use of tables, data collection and formulas was a much less visible evolution, but its impact ultimately proved even more transformative. Finally, a selection of major architectural enterprises will underline the growing scale of engineering projects developed in the burgeoning nation-states of Western Europe.

References

[1] J. Bennett, *The Challenge of Practical Mathematics*, S. Pumfrey, P.L. Rossi and M. Slawinski, eds., Science, Culture and Popular Belief in Renaissance Europe, Manchester University Press, 1991, pp. 176–190.

[2] C.H. Cotter, *Early tabular, graphical and instrumental, methods for solving problems of plane sailing*, Revista da Universidade de Coimbra, **26**, 3–20.

[3] J. Grabiner, *Some Disputes of Consequence: Maclaurin among the Molasses Barrels*, Social Studies of Science **28(1)**(1998), 139–168.

[4] E. Knobloch, *Mathematical Methods in medieval and renaissance technology, and machines*, I. Grattan-Guinness, ed., Companion Encyclopedia of the history and philosophy of the Mathematical sciences, London, 1994, pp. 251–258.

[5] P. Long, *Engineering the Eternal City. Infrastructure, Topography, and the Culture of Knowledge in Late Sixteenth Century Rome*, Chicago, Chicago University Press, 2018.

[6] T. Morel, *De Re Geometrica: Writing, Drawing and Preaching Mathematics in Early Modern Mines*, Isis, **111(1)**(2020), 22–45.

[7] C. Mukerji, *Impossible Engineering*, Princeton and Oxford, Princeton University Press, 2009.

[8] T.S. Reynolds, *Stronger than a Hundred Men: A History of the Vertical Water Wheel*, Baltimore, Johns Hopkins University Press, 2003.

[9] F. Willmoth, *Sir Jonas Moore: Practical Mathematics and Restoration Science*, Woodbridge, Suffolk, UK; Rochester, NY, Boydell Press, 1993.

Volume 3: Representing Mathematics. Selling the Mathematical Sciences

Robert Goulding, Volker Remmert

The Early Modern Period was a time of transition for mathematics. Mathematicians in the universities could no longer rely on the established instruction in the *quadrivium* (i.e. arithmetic, geometry, music and astronomy). They were compelled to define their place with respect to both the ever more dialectical curriculum of the arts faculty, and the new challenge of the humanistic disciplines. Beyond the universities, there were opportunities for mathematical entrepreneurship in princely courts and in crafts and industries – for mathematicians, at least, who were able to persuade others of the utility of their expertise – as well as in the growing worldwide cosmos of Jesuit Colleges reaching into Asia and the Americas. And as mathematical printing was perfected through the sixteenth century, the works of ancient and modern mathematicians vied in the bookstore for the educated reader's attention. In such a climate, mathematicians developed strategies to represent their art and expertise to interested parties – to sell it in a highly competitive marketplace. This chapter will examine ways in which mathematics was represented to enhance its desirability and prestige as well as to raise its epistemological (cf. chapter 4: *quaestio de certitudine mathematicarum*) and social status.

References

[1] N.H. Clulee, *John Dee's Natural Philosophy: Between Science and Religion*, London and New York, Routledge, 1988.

[2] S. Cuomo, *Shooting by the Book: Notes on Niccolò Tartaglia's Nova Scientia*, History of Science **35**(1997), 155–188.

[3] J. Ellinghaus, V. Remmert, *Visual Worlds on Early Modern Scientific Instruments: Types and Messages*, R. Noyes, ed., Reassessing epistemic images in the early modern world, Amsterdam, 2021.

[4] R. Goulding, *Defending Hypatia: Ramus, Savile, and the Renaissance rediscovery of mathematical history*, New York, Springer, 2010.

[5] R. Goulding, *Five Versions of Ramus's Geometry*, M. Meserve and A. Ossa-Richardson, eds., Et Amicorum: Essays on Renaissance Humanism and Philosophy in Honour of Jill Kraye, Leiden, Brill, 2018, pp. 355–387.

[6] I. Imhausen, V. Remmert, *The Oration on the Dignity and the Usefulness of the Mathematical Sciences of Martinus Hortensius (Amsterdam, 1634): Text, Translation and Commentary*, History of Universities **21**(2006), 71–150.

[7] A. Marr, ed., *The worlds of Oronce Fine: mathematics, instruments, and print in Renaissance France*, Donington: Shaun Tyas, 2009.

[8] E. Reeves, *Galileo's glassworks: The telescope and the mirror*, Cambridge, MA, Harvard University Press, 2008.

[9] V. Remmert, *Picturing the Scientific Revolution: Title Engravings in Early Modern Scientific Publications*, Philadelphia, 2011.

[10] A.M. Smith, *From Sight to Light: The Passage from Ancient to Modern Optics*, Chicago, University of Chicago Press 2014.

[11] J. Williams, *Robert Recorde: Tudor Polymath, Expositor and Practitioner of Computation*, New York, Springer, 2011.

Volume 4: Introduction, A Cultural History of Mathematics in Enlightenment Europe

Maarten Bullynck

The long 18th century or the period of (European) Enlightenment is in many ways a period of transition. The late 17th century had marked the end of the religious wars in Europe to make place for increasing tensions over economic issues, both locally and globally, putting the existing often still feudal structures under pressure. The slow transformation of the societal and economic structures mobilized old and new practices of mathematics and redefined the role(s) of mathematics in society. Though mathematics certainly played its role in the many discussions (religious and otherwise) in the Republic of Letters, it became increasingly interwoven with the factual reorganization of society. Reform of the taxation system, advances in military technology, engineering design or data gathering for the centralized administration, all required people with varying mathematical competences.

The body of mathematical knowledge is equally reshaped during this period. While the great inventions of symbolic algebra and the calculus properly belong to the 17th century, they come to full fruition and application in the 18th century. The success of these new branches put the old classic build-up of mathematics and its status within the sciences in question. While the discussions over the status of mathematics certainly had theoretical bearings, its practical counterpart was the integration of mathematics into society and, in particular, in the educational system. As professional groups, be they engineers, officers or civil servants, increasingly made use of mathematics, they needed access to more advanced and recent mathematical knowledge, whether in the form of advanced courses, manuals or specialized journals.

Volume 4: Everyday Numeracy in Enlightenment Europe. The Slow March towards Universal Numeracy and Universal Measurement

Maarten Bullynck

Over the course of the 18th century, mathematics, and in particular arithmetic, acquired considerable social prestige. Instead of being mostly associated with the clerical work of writing, copying and accounting, its virtues for society as a whole and for improving the human mind began to be extolled. The trope of the utility of mathematics for the human mind and for mankind was not altogether new, but it gained much momentum during this era. The success of mathematical methods in the natural sciences made their mark on literature, philosophy and theology and the usefulness of mathematics for science and society were quite widely promoted in Enlightenment literature. The influential philosopher and manual writer Christian Wolff praised mathematics both for its intellectual virtues as well as its practical utility. Baron Anne-Robert-Jacques Turgot, who tried to reform the French tax system by placing it on a more data-based and scientific foundation, vaunted mathematics because it led to truth and avoided errors.

On the whole, this positive attitude towards mathematics, in combination with changing economic circumstances, made numeracy slowly appear as a prominent goal on the political agenda of newly modernizing states. Indeed, both the internationalization of trade and the rise of industrialization created a new need for people conversant in (elementary) mathematics. Also the state's role changed and in many places the court and administration evolved from being a class-based system to a mixed system where merit played in increasing role alongside class in the recruitment and advancement of civil servants. As European states tried slowly (and often unsuccessfully) to abolish old feudal privileges and install more modern ways of administrating, they needed civil servants who could readily apply a number of mathematical techniques. As a consequence, state authorities took steps to organize some form of teaching to promote numeracy as well as literacy by supporting, supplementing, or even supplanting earlier initiatives. Although mostly limited to cities or regions that were tied in to international and/or industrial developments, these measures eventually spawned a general trend toward promoting numeracy throughout much of Europe.

If there was one watershed moment in the history of numeracy during the 18th century, this came in the year 1773. That year witnessed the suppression of the Jesuit order and, as a consequence, the reorganization of the school systems in larger parts of Continental Europe. For many states, this was an opportune moment to seize control of education and redefine its contents. This modernization of education promoted a new élan, both institutionally and methodologically, with the goal of attaining universal literacy. As part of this process, concrete steps towards establishing universal numeracy were spelled out as well. Throughout the 18th century, many local initiatives had begun to spread numeracy, but only from 1773 onwards did a more general movement to establish a truly universal numeracy grow and endure, even though the ambitions of progressive law-makers and educators would only be generally realized in practice far into the nineteenth century. While the larger horizon for 18th-century numeracy was the inclusion of elementary arithmetic into universal literacy, the landscape of numeracy throughout the period was filled with a bewildering variety of experiments that defy easy categorization and generalization. The geographical and social distribution of numeracy reflects a number of global societal divisions, but the scene is mainly dominated by local practices and individual trajectories. There were also numerous continuities, so the the traditions and practices of previous centuries persist and many earlier forms of numeracy coexist throughout long 18th century.

References

[1] M. Bullynck, *The transmission of numeracy. Integrating reckoning in protestant North-German Education (1770-1810)*, Paedagogica Historica, **44(5)**(2008), 463–485.

[2] J. Deniss, *Learning arithmetic: textbooks and their users in England 1500-1900*, E. Robson and J. Stedall, eds., The Oxford Handbook of the History of Mathematics, Oxford, 2009, pp. 448–467.

[3] N.F. Ellerton, M.A. Clements, *Rewriting the history of school mathematics in North America 1607-1861*, New York, Springer, 2012.

[4] A. Favre, *Les origines du systeme metrique*, Paris, 1931.

[5] C. Gilain, A. Guilbaud, eds., *Sciences mathématiques 1750-1850: continuités et ruptures*, Paris, CNRS editions, 2015.

[6] A. Karp, G. Schubring, eds., *Handbook on the history of mathematics education*, New York, Springer, 2014.

[7] B. Wardhaugh, *Poor Robin's Prophecies. A curious almanac and the everyday mathematics of Georgian Britain*, Oxford, Oxford University Press, 2012.

[8] J. Wess, *Avoiding arithmetic, or the material culture of not learning mathematics*, BSHM Bulletin: Journal of the British Society for the History of Mathematics, **27(2)**(2012), 82–106.

Volume 4: Practice and Profession in Enlightenment Europe. Embedding Mathematical Practices in the Modernization of the State

Maarten Bullynck

In many ways the 18th century can be seen as a continuation of the centuries before, but there are quite some changes happening in the social fabric and in the communication patterns that underly and support mathematical practices and professions. The accumulation and confluence of those changes slowly prepare the way for the 19th century institutionalization and professionalization of mathematics featuring the new social phenomenon of the professional mathematician (see chapter 2 in volume 5). In previous centuries, it is more accurate to speak of mathematical practitioners or of trades and professions that make use of (some) mathematical knowledge and skill. In this chapter we will therefore focus on those changes that made this transition possible. We will contrast especially the time before and after Austrian Succession War (1740-1748) and the Seven Years' War (1756-1763), because it will allow us to describe mathematical practice in motion while embedding them in their immediately practical, economic and political contexts.

A rather new and typical institutional creation of the 18th century are the academies of sciences. They are institutions that award a limited number of pensions to some of the most prominent scientific minds in all branches of the sciences, ranging from the belles-lettres (history and literature) to the exact sciences. In general, there were three categories of academicians: the full members, the associate members and the corresponding and/or foreign members. One had to ascend in the ranks from the lower to the higher categories, with only the full members receiving a full pension, the other positions being less financially rewarding. In most academies these pensions did not suffice for a living and had to be supplemented with other sources and opportunities of income, or rather vice versa, the academy pension in most cases was just a supplement. Most of the academicians had a job within the government, many in the educational system (the collèges, the universities, the officer's schools etc.), but frequently also a consulting or supervisory function in more technical endeavors such as the Mint, the mines, the navy, the military. the observatory etc. (Hahn 1975) Apart from their rôle in the government, the academicians also played an important part in the Republic of Letters, the 18th century public space of opinion that was constituted of letter correspondences, journals, pamphlets and books.

References

[1] E. Brian, *La mesure de l'Etat. Administrateurs et geometres au XVIIIe siècle Paris*, Albin Michel, 1994.

[2] M. Croarken, *Providing Longitude for All: The Eighteenth Century Computers of the Nautical Almanac*, Journal of Maritime Research, October 2002.

[3] R. Hahn, *Scientific Research as an Occupation in Eighteenth-Century Paris*, Minerva , 13(4)(1975), 501–513.

[4] T. Morel, M. Bullynck, *Une revolution peut en cacher d'autres. Le paysage morcele des mathematiques dans l'espace germanophone et ses reconfigurations (1750–1850)*, C. Gilain,

A. Guilbaud, eds., *Sciences mathématiques 1750-1850: continuités et ruptures*, Paris, CNRS editions, 2015, pp. 181–206.

[5] J. Peiffer, J.-P. Vittu, eds., *Les journaux savants dans l'Europe des XVIIe et XVIIIe siècles. Formes de la communication et agents de la construction des savoirs*, Archives internationales d'histoire des sciences, 63, nos. 170–171, June–December 2013.

[6] R. Taton, ed., *Enseignement et diffusion des sciences en France au dix-huitième siècle*, 1964.

[7] E.G.R. Taylor, *The Mathematical Practitioners of Hanoverian England, 1714–1840*, Cambridge, 1966.

Volume 4: Inventing Mathematics in Enlightenment Europe. Mathematical Challenges and their Disciplinary Offspring

Jeanne Peiffer

In the seventeenth century, mathematics underwent profound transformations affecting methods of proof and objects of investigation (as we have seen in volume 3). The eventual invention of the calculus – namely of a method for finding tangents and calculating areas (or quadratures) – may be seen as the starting point of a new phase of mathematical research, at the origin of an emerging mathematical field, analysis, one of the outstanding inventions of the 18th century. Indeed, the Ancients considered mainly two mathematical disciplines: arithmetic and geometry. With the Arabs, the Italians and Vieta, algebra or a new "ars analytica" came into being. Descartes applied algebra to geometry, later in the 17th century the invention of the calculus was mainly understood as an extension of algebra to infinitely small quantities. The development of eighteenth-century analysis is inextricably linked to mechanics. Indeed, mathematicians striving to test the newly invented methods applied them to problems often taken from mechanics, put in an idealized form that changed them into mathematical entities and problems. The solution of those problems often led to the invention of new mathematical techniques which in turn allowed to solve questions hitherto considered inaccessible in mechanics, and more generally in physics.

In the following, we will investigate the main changes that intervened in eighteenth century mathematics. We will distinguish three sometimes overlapping time periods. The first concerns the turn of the century after the publication in 1687 of Isaac Newton's seminal Philosophiae naturalis principia mathematica. Deeply anchored in the problem solving tradition of the seventeenth century, mathematicians tended to extend the methods of the newly created calculus by means of challenge problems, a frequent source of innovation. The second period, centred on the middle of the century, may be characterized (even if the polemics and contests among mathematicians did not disappear) by a greater effort of systematization of the acquired knowledge, which was considerable. When put in order, the map of knowledge showed lacunas, lack of foundations, contradictions which had to be treated. Mathematics was by then considered part of the enlightened culture that came to dominate whole parts of the European continent. Finally in the last period, we will concentrate on what can be seen as emerging disciplinary developments which were to give birth to distinct mathematical disciplines in the nineteenth century.

References

[1] D. Bertoloni Meli, *Equivalence and Priority. Newton versus Leibniz*, New York, Oxford University Press, 1993.
[2] H. Bos, *Differentials, higher-order differentials and the derivative in the Leibnizian calculus*, Archive for History of Exact Sciences, **14**(1974), 1–90.
[3] R.S. Calinger, *Leonhard Euler. Mathematical Genius in the Enlightenment*, Princeton and Oxford, 2016.
[4] H.H. Goldstine, *A History of the Calculus of Variations, from the 17th through the 19th century*, New York/ Heidelberg/ Berlin, 1980.
[5] N. Guicciardini, *Isaac Newton on Mathematical Certainty and Method*, Cambridge, MA, 2009.
[6] N. Guicciardini, *A brief introduction to the mathematical work of Isaac Newton*, R. Iliffe and G.E. Smith, eds., The Cambridge Companion to Newton, 2nd ed., Cambridge, 2016, pp. 382–420.
[7] A.R. Hall, *Philosophers at War. The quarrel between Newton and Leibniz*, Cambridge, 2002.
[8] T.L. Hankins, *Jean d'Alembert. Science and the Enlightenment*, Oxford, 1970; 2nd ed., New York, 1990.
[9] J.E. Hofmann, *Leibniz in Paris 1672-1676. His growth to mathematical maturity*, Cambridge, 1974.
[10] H.N. Jahnke, ed., *A History of Analysis*, American Mathematical Society/ London Mathematical Society, 2003.
[11] A. Malet, *Newton's Mathematics*, E. Schliesser and C. Smeenk, eds., The Oxford Handbook of Newton, Online Publication Date: July 2020, DOI: 10.1093/ oxfordhb/ 9780199930418.013.24.
[12] H. Pulte, S. Mandelbrote, eds., *The Reception of Isaac Newton in Europe*, London etc., Bloomsbury Academic, 2019.
[13] T. Sonar, *The History of the Priority Dispute between Newton and Leibniz. Mathematics in History and Culture*, Basel, Birkhäuser, 2018.

Volume 4: Mathematics and Worldviews in Enlightenment Europe

David Bellhouse, Jip van Besouw

The dominant physical worldview of the eighteenth century was based on Newtonian mechanics, a theory greatly refined by leading mathematicians of the period, including d'Alembert, the Bernoullis, Euler, and Lagrange. On the cosmic level, this was a refined version of the mechanical clockwork universe dating back to the early modern period (see Volume 3). The update came about through a new physical theory that opened the way to celestial mechanics. Newton's law of universal gravitation now regulated the movements of heavenly bodies, thereby exposing the mechanism behind the divine machine as designed by its Creator.

Whereas the religious implications of Newtonian natural philosophy dominated discussion in various eighteenth-century circles, the actual development of celestial mechanics fell to an array of talented Continental mathematicians, beginning with Jakob and Johann Bernoulli, the foremost followers of Leibniz. They, together with Leonhard Euler, another native of Basel, elaborated the mathematical methods of the Leibnizian differential and integral calculus. This new mathematical language soon became the modern vernacular for solving problems in mechanics.

Within the realm of celestial mechanics, Pierre-Simon Laplace published the five-volume *Traité de mécanique celeste* between 1798 and 1825. During these years, he also published major works on probability theory, including A Philosophical Essay on Probabilities (1814). This contains a famous passage, often cited as "Laplace's demon," which captures the determinism underlying the Deist's worldview. Laplace imagined an intellect capable at a certain moment in time of grasping the position of all particles in the universe and the forces acting on them, but also able to subject this data to mathematical analysis. For such a being, he asserted, the future as well as the past would lie open before its eyes. Laplace and other leading Deists broke entirely with Newton's own theological and metaphysical views. His celestial mechanics aimed to prove the long-term stability of the solar system, a central tenet for Deists, who espoused a clockwork universe, just as Leibniz had earlier. Samuel Clarke, speaking for Newton, strongly opposed this view in his correspondence with Leibniz, calling this the "Notion of Materialism and Fate, [which] tends, (under pretense of making God a Supra-mundane Intelligence,) to exclude Providence and God's Government in reality out of the World."

The providentialist Newtonian philosophy, however, held little persuasive power over those who did not already share most of its assumptions. It is well documented that many of the later readers of for example Clarke's works took a different direction. To name just two, both Benjamin Franklin and David Hume digested Clarke's works with enthusiasm, but in the end decided that the camp that Clarke set out to refute actually had the better arguments. Deists, too, found much to their liking in the *Principia* as the book implied that the motions of all the heavily bodies could be explained mathematically via the laws of mechanics – this was not Newton's view to be sure – and thus dispensed with a God needing to balance things everywhere at all times.

Indeed, throughout the eighteenth century natural philosophers gradually moved away from metaphysics, as a certain strand of Newtonianism arose that was averse to such reasoning. Its proponents were committed to a search for the laws of nature expressed in terms of forces that emulated Newton's laws of motion and gravity. Initially, most Newtonian philosophers of this brand argued that the laws of nature in general, and the law of gravity in particular, depended on God in some mysterious manner. One of these was Willem Jacob 's Gravesande, a professor in Leiden, who argued convincingly that none of the available metaphysical groundings of the laws could be confirmed. Like Newton, he claimed we could only know the laws that God had created, not the ultimate principles behind them. Instead of searching for these, natural philosophy should seek to uncover as many laws as possible through hard experimental investigations.

By downplaying the metaphysical debates, 's Gravesande and others managed to make Newtonianism the uniting banner for Christian natural philosophers of all kinds of religious denominations. These included Dissenters, Huguenots, various atypical forms of Protestantism in the Netherlands, and even Catholicism later in the century. In this form, Newtonianism quickly became the dominant natural philosophical worlview, first in Great Britain and the Netherlands, then over most of Protestant Europe, reaching as far as the Spanish Americas in the last third of the century.

References

[1] D.R. Bellhouse, *Leases for Lives: Life Contingent Contracts and the Emergence of Actuarial Science in Eighteenth-Century Britain*, Cambridge: Cambridge University Press, 2017.
[2] L. Daston, *Classical Probability in the Enlightenment*, Princeton: Princeton University Press, 1988.
[3] A. Desrosières, *The Politics of Large Numbers*, Camille Naish, trans., Cambridge, MA, Harvard University Press, 1998.
[4] M. Feingold, *The Newtonian Moment: Science and the Making of Modern Culture*, Oxford: Oxford University Press, 2004.
[5] N. Guicciardini, *Reading the Principia: The Debate on Newton's Mathematical Methods for Natural Philosophy from 1687 to 1736*, Cambridge: Cambridge University Press, 1999.
[6] N. Guicciardini, *Isaac Newton and Natural Philosophy*, London: Reaktion Books, 2018.
[7] A. Hald, *A History of Probability and Statistics and Their Applications before 1750*, New York, John Wiley, 1990.
[8] S.B. McGrayne, *The Theory that Would not Die: How Bayes' Rule Cracked the Enigma Code, Hunted Down Russian Submarines & Emerged Triumphant from Two Centuries of Controversy*, New Haven: Yale University Press, 2011.
[9] I. Newton, *The Principia: Mathematical Principles of Natural Philosophy; a new translation by I. Bernard Cohen, Anne Whitman; assisted by Julia Budenz; preceded by A guide to Newton's Principia by I. Bernard Cohen*, Berkeley: University of California Press, 1999.
[10] E. Schliesser, *On reading Newton as an Epicurean: Kant, Spinozism and the changes to the Principia*, Studies in History and Philosophy of Science, **44**(2013), 416–428.
[11] E. Schliesser, *Newton and Newtonianism*, A. Garrett, ed., The Routledge Companion to Eighteenth Century Philosophy. London: Routledge, 2014, pp. 62–90.
[12] S.M. Stigler, *The History of Statistics: The Measurement of Uncertainty before 1900*, Cambridge, MA, Belknap Press, 1986.
[13] M. Terrall, *The Man Who Flattened the Earth: Maupertuis and the Sciences in the Enlightenment*, Chicago: The University of Chicago Press, 2002.
[14] E. Vailati, *Leibniz and Clarke: A Study of Their Correspondence*, Oxford: Oxford University Press, 1997.
[15] E. Watkins, ed., *The Divine Order, the Human Order, and the Order of Nature: Historical Perspectives*, Oxford: Oxford University Press, 2013

Volume 4: Describing and Understanding the World in Enlightenment Europe

Helmut Pulte

Our understanding of science in the Age of Enlightenment has probably been influenced by no other discipline so strongly as by mathematics. This can be said for both aspects of mathematics as it was understood during this era. During the Enlightenment, mathematics was a science of reason that made new knowledge possible (*mathesis pura*) as well as a tool that enables a better understanding of the physical world and a better human life within this world (*mathesis mixta* or *mathesis applicata*). Although the utilitarian thinking of the Enlightenment favoured applications of mathematics to a considerable extent, most mathematicians, scientists and philosophers were aware that practical use could not be achieved without new basic developments of the discipline.

The new infinitesimal calculus, discovered independently by Newton and Leibniz, was in this respect paradigmatic for the 18th century. It became the decisive mathematical instrument of science under the rule of mechanism and it served – in quite different forms – as the key technical achievement that made this new approach to natural philosophy possible. While in the early development of rational mechanics geometry still played a role both as a source of intuition (synthetical thinking) and as a scientific ideal due to its axiomatic foundation, later developments strove for a purification of the calculus as the one and only means for a mathematical understanding of the physical world. The achievements of Euler and, to an even greater extent, Lagrange are typical in this respect. Lagrange understood a 'higher form of the calculus, the so-called calculus of variations, as 'clé universelle' (universal key) for a proper understanding of the whole of nature. This general process was accompanied by the development of further mathematical tools such as potential theory and the theory of differential equations. By the end of the 18th century, rational mechanics was a largely mathematised science clothed in 'analytical garments'. In close connection with the more analytical method, which Newton had advocated in his philosophical commentaries, this new mechanics sought to decipher the last secrets of nature. Areas such as optics, electricity or magnetism – even chemistry and biology – were believed to be reducible to mechanical laws that could be understood completely by means of the latest mathematical principles and techniques.

This chapter will mainly focus on describing these developments using as little technical jargon as possible, though one cannot avoid formulas completely. Nevertheless, a more or less non-technical presentation of the developments in question can be achieved. Certain philosophical aspects will also be addressed, such as the 'semantical unloading' of mathematical concepts from rational mechanics, which went so far that by the end of the 18th century mechanics could hardly be called a natural science. Mathematical physics served as a model that spread to new disciplines, like the social sciences and economics. Alongside this core theme stands another of great relevance for cultural history of mathematics, namely, the applications of mathematics in engineering and craftsmanship, which were in many ways closely connected with developments in mathematical physics.

Volume 4: Mathematics and Technological Change in Enlightenment Europe

Jane Wess

Focusing on Western Europe, and in particular Britain, this chapter touches on a range of activities which were mathematised during the period. It explores areas in which the new calculus, or as Leonard Euler termed it 'sublime mathematics, was first applied, as well as fields that involved large numbers of people who became mathematically competent for the first time. The 18th century saw developing industrialisation and imperialism, which was changing the nature of the physical and cultural landscape in Europe. For both industrial and imperial purposes mathematics was increasingly applied to technology. Mathematical literacy was becoming a requirement for significant sections of society, and relatively new branches of mathematics were starting to be applied to a range of physical problems, although in most cases without real tangible benefits.

Several authors concur with Eric Ash, who wrote: "One of the most characteristic driving impulses of state formation in early modern Europe was the need to quantify, rationalise, and exploit the natural environment." Europe's population was growing, a market economy had developed, and the increasing pressures on resources gave rise to new attitudes towards their efficient use. It is thus natural to ask: what technical and technological problems were posed by the newly-developing economy, and what knowledge and mathematical skills did these developments generate?

The needs of an industrialised society were considerable: surveying for new land use, improved agriculture to feed an increasing population, civil engineering, efficient transport systems, and water supply on an unprecedented scale. Power sources of men, animals, wind, and water, and later steam engines, needed to be evaluated and compared. Imperial interests also required mathematical skills. Navigational techniques needed to be extended from the local surveys of coastlines on the basis of the largely Portuguese portolans, or coastal charts, to meet the demands of those sailing further afield. Expanding rival nation states were in competition, hence the need for effective artillery which was increasingly mathematised to optimise results. To pay for this expansion and increasing expenditure on wars, taxes on alcohol had been introduced in England and France in the 17th century. Assessing this tax spawned an army of excise men performing quite complicated mathematics by the 18th century. These examples suggest the larger context of social, economic, and political themes that set the stage for the developments described in this chapter.

References

[1] K. Alder, *Engineering the Revolution*, Princeton, Princeton University Press, 1999.

[2] R.S. Calinger, *Leonhard Euler. Mathematical Genius in the Enlightenment*, Princeton and Oxford, 2016.

[3] H.W. Dickenson, *James Watt, Craftsman and Engineer*, London, Macmillan, 1936.

[4] T. Frangsmyr, J.L. Heilbron, R.E. Rider, eds., *The Quantifying Spirit in the Eighteenth Century*, Berkeley, University of California Press 1990.
[5] M.C. Jacob, L. Stewart, *Practical Matter; Newton's Science in the Service of Industry and Empire*, Cambridge, MA, Harvard University Press, 2004.
[6] N. Kollerstrom, *Newton's forgotten Lunar Theory: His contribution to the Quest for Longitude*, New Mexico, Green Lion Press, 2000.
[7] T.S. Reynolds, *Stronger than a Hundred Men: A History of the Vertical Water Wheel*, Baltimore, Johns Hopkins University Press, 2003.
[8] J.B. Shank, *The Newton Wars and the Beginning of the French Enlightenment*, Chicago and London, 2008.
[9] M.B.W. Tent, *Leonhard Euler and the Bernoullis: Mathematicians from Basel*, Natick, MA, AK Peters, 2009.
[10] C. Truesdell, *Essays in the History of Mechanics*, Berlin, Springer, 1968.
[11] V.S. Varadarajan, *Euler through Time: A new look at old Themes*, Providence, RI, American Mathematical Society, 2006.

Volume 4: Representing Mathematics in Enlightenment Europe

Franziska Bomski

The ways in which philosophers, writers, and artists in the 18th century engaged with mathematics are numerous and diverse. In the following, important aspects of these images of mathematics will be highlighted, without claiming to exhaust the subject. Further, the development of the history of ideas, even if it deals with mathematics, does not strictly adhere to the caesuras marked by dates. Therefore, there will be some retrospection into the late 17th century as well as outlooks into the early 19th century in order to trace important lines of tradition. The essay is divided into two main sections. The first section deals with philosophical images of mathematics conceived by Leibniz, D'Alembert, Mendelssohn, Kant and others. Their reflections on mathematics pursued the question of the nature of mathematical truth in general, its position in the order of knowledge and, closely related, its potential as a leading discipline and a universal role model for epistemic progress.

The question of the relationship between the philosophical view of nature and the mathematical description of the cosmos assumed a central place in the poetical thinking of German Romanticism in the late 18th and early 19th centuries. Rejecting more rigorous forms of argumentation in favor of aesthetic knowledge and sensory intuition, German Romantics probed new perspectives, embraced literature as a means of philosophizing, played with semantic ambiguity, and employed metaphors along with other forms of associative thinking. The second part of this essay will discuss the poetic reflections on mathematics of Friedrich Schlegel, Novalis, and Goethe. Like the aforementioned philosophers, they have been chosen not only because they are important representatives of the literary resp. intellectual life in the 18th century, but also because each exemplifies important tendencies of the images of mathematics, though of course there are many other instances and authors as well.

References

[1] F. Bomski, *Die Mathematik im Denken und Dichten von Novalis. Zum Verhältnis von Literatur und Wissen um 1800*, Berlin, Akademie/ de Gruyter, 2014.

[2] M. Bullynck, *Vom Zeitalter der Formalen Wissenschaften. Anleitung zur Verarbeitung von Erkenntnissen anno 1800, vermittelst einer parallelen Geschichte*, Gent, 2006.

[3] R. Darnton, *Philosophers Trim the Tree of Knowledge. The Epistemological Strategy of the Encyclopédie*, R. Darnton: The Great Cat Massacre and other Episodes in French Cultural History, New York, Basic Books, pp. 191–213.

[4] M. Dyck, *Novalis and Mathematics. A Study of Friedrich von Hardenbergs Fragments on Mathematics and its Relation to Magic, Music, Religion, Philosophy, Language and Literature*, Chapel Hill, NC, University of North Carolina Press, 1960.

[5] K. Hamburger, *Novalis und die Mathematik. Eine Studie zur Erkenntnistheorie der Romantik*, Deutsche Vierteljahresschrift für Literaturwissenschaft und Geistesgeschichte **16**(1929), 113–184.

[6] M. Illi, *Sprache in Wissenschaft und Dichtung. Diskursive Formationen von Mathematik, Physik, Logik und Dichtung im 17. und 18. Jahrhundert*, Berlin/ Boston, 2017.

[7] D. Koriako, *Kants Philosophie der Mathematik. Grundlagen – Voraussetzungen – Probleme*, Hamburg, 1999.

[8] S. Krämer, *Berechenbare Vernunft. Kalkül und Rationalismus im 17. Jahrhundert*, Berlin/ New York, 1991.

[9] J. Maatsch, *Naturgeschichte der Philosopheme. Frühromantische Wissensordnungen im Kontext*, Heidelberg, 2008.

[10] P. Mancosu, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century*, New York/ Oxford, 1996.

[11] J. Neubauer, *Symbolismus und symbolische Logik. Die Idee der Ars Combinatoria in der Entwicklung der modernen Dichtung*, Munich, 1978.

[12] H. Pollack, *Novalis and Mathematics Revisited. Paradoxes of the Infinite in the Allgemeine Brouillon*, Athenäum **7**(1997), 113–140.

[13] C.J. Posy, ed., *Kant's Philosophy of Mathematics. Modern Essays*, Dordrecht, 1992.

[14] B. Specht, *Physik als Kunst. Die Poetisierung der Elektrizität um 1800*, Berlin, 2010.

Volume 5: Introduction, A Cultural History of Mathematics in the Long Nineteenth Century

Tom Archibald, David E. Rowe

A cultural history of mathematics, like this one, is predicated on the assumption that mathematical practice, like any other form of human activity, has been deeply affected by social, political, and technological developments, but particularly so over the last two centuries. Even well before the onset of the electronic age and the IT revolution, the dominant modes of communication used by mathematicians underwent a profound transformation, which had major repercussions for mathematical research. This involved not only written forms of communication but also oral exchanges, which became increasingly important over the course of the long nineteenth century. As professional societies proliferated, their members contributed to shaping new communities with their own distinctive cultural identities. Along with these developments came new mathematical practices, some based on group endeavors rather than activities pursued by a handful of geniuses working in splendid isolation. In lecture halls and seminar rooms, the spoken word gradually assumed a central place in making as well as for disseminating mathematical knowledge. At the same time, the first scientific journals specializing in mathematics arose as outlets for disseminating and circulating newly found knowledge. A few of them served as hubs for networks of researchers well before the formation of national mathematical societies.

Volume 5: Everyday Numeracy in the Long Nineteenth Century. Numeracy in the Age of Mass Education

Adrian Rice

The nineteenth century was the period when the current conception of the utility of basic arithmetic began to emerge. Although everyday numeracy had long been the ability to understand and compute with numbers, it was during this era that the fields in which these numerical skills could be applied increased dramatically, driven by rapidly-evolving developments in technology, economics, trade and industry. Yet, while the notion of what was meant by a basic competence in the use and manipulation of numbers would have been largely understood throughout the nineteenth century, no single term existed to describe such a skill. It would not be until the word 'numeracy was coined in a 1959 report for the British Ministry of Education that the English language was provided with a term analogous to the meaning of 'literacy' – itself a concoction of late-nineteenth-century American English. This chapter will thus discuss a concept for which no name existed at the time. It is therefore perhaps unsurprising that some basic parameters will be necessary before we begin.

For the entirety of the nineteenth century, everyday numeracy would have mainly comprised the ability to perform numerical calculations correctly. But since no understanding or appreciation of the underlying reasoning or logic was required, such mechanistic skills were largely considered to be beneath the higher concerns of true mathematicians. As Leibniz had lamented in 1685: 'It is unworthy of excellent men to lose hours like slaves in the labor of calculation which could be relegated to anyone else if machines were used.' Such sentiments were common among the educated elites in the eighteenth and nineteenth centuries – in 1847, an article in the Dublin Review observed: 'We suppose it will hardly be disputed, that to speak the opinion of mankind, we must say that of all disgusting drudgery, numerical calculation is the worst: a combination of all the worry of activity with all the tediousness of monotony and all the fear of failure.'

But numeracy was not all mere number crunching. Although nineteenth-century education in arithmetic continued the eighteenth-century emphasis on memory, rote-learning and reckoning, the demands of a changing society ultimately required increasing sections of the populace to be able to use, interpret and apply the results of numerical computations in an ever expanding variety of settings, from household management to commerce, demography to social reform. To understand this nineteenth-century expansion in the applicability of numeracy, we will attempt to answer a variety of questions. To what extent was education in arithmetic available at this time, and to whom? What form did this education take, and what skills were imparted by contemporary textbooks? What systems of weights, measures and currency were in place, and what efforts were made to facilitate their use? What other resources were available to facilitate everyday numerical computations? And to what extent were people exposed to data and numerical information in everyday life?

Clearly, the answers to such questions are highly location-dependent and, even if all the necessary historical sources were available, it would still be impossible to provide satisfactory answers on a global scale. However, an illustrative case study provides an excellent alternative. And since Britain was the world's foremost economic, industrial and military power throughout this period, it makes sense to use Victorian Britain as a geographical focus for this chapter, as developments there were often highly influential elsewhere around the globe, particularly in contemporaneous and former colonies, such as the United States of America. During the nineteenth century, the close cultural connection between Britain and the United States, which had existed since colonial times, remained strong. Thus, since basic numerical mathematics was a fundamental component of this shared cultural relationship, this chapter will focus on aspects of everyday numeracy in Britain and the United States in the nineteenth century, comparing and contrasting the provision, application, and manifestations of numeracy in the daily life of both cultures.

References

[1] P.C. Cohen, *A Calculating People: The Spread of Numeracy in Early America*, Routledge, 1999.
[2] J. Denniss, *Learning arithmetic: textbooks and their users in England 1500–1900*, E. Robson and J. Stedall, eds., The Oxford Handbook of the History of Mathematics, Oxford University Press, 2009, pp. 448–467.
[3] R. Flood, A. Rice, R. Wilson, eds., *Mathematics in Victorian Britain*, Oxford University Press, 2011.
[4] M. Harris, *Common Threads: Women, Mathematics and Work*, Trentham Books, 1997.
[5] A.G. Howson, *A History of Mathematics Education in England*, Cambridge University Press, 1982.
[6] W. Kula, *Measures and Men*, trans. R. Szreter, Princeton University Press, 1986.
[7] M. Poovey, ed., *The Financial System in Nineteenth-Century Britain*, Oxford University Press, 2003.

Volume 5: Practice and Profession in the Long Nineteenth Century. National Models for Professionalization

Tom Archibald, David E. Rowe

Professionalization in mathematics, as we know it today, largely evolved over the course of the nineteenth century, a development brought about by institutions that not only promoted research but also trained a new generation of researchers. Innovation in admissions and in teaching methods were hallmarks of the newly founded École Polytechnique in Paris, an elite school for training engineers, who had to master large doses of mathematics. The social status of higher mathematics had already taken a dramatic turn upward over the course of the French Enlightenment, whose proponents helped mold the political ideals of the Revolution. Mathematics, as taught at the École Polytechnique, thus came to serve as a gateway to the higher echelons of the French meritocracy. Under Napoleon, who cultivated close personal relations with leading mathematicians, in particular Gaspard Monge and Joseph Fourier, France embraced the spirit of a new society dominated by technocratic elites. This message, reflecting the significance of mathematics as a harbinger of social change, echoed throughout Europe during the first decades of the century. A very different educational model emerged, however, in Prussia, an eastern German state that acquired territories along the Lower Rhine in 1815.

Berlin University spearheaded an educational movement in Prussia, which eventually led to widespread reforms at other universities during the period that culminated with the political unification of the German states in 1871. This educational system aimed not so much to foster the acquisition of technical knowledge and skills, but rather to promote spiritual development, or Bildung, mainly by pursuing neo-humanist ideals. According to this conception, there was no better way to build character and discipline in young men than by immersing them in the world of classical antiquity. At the Gymnasien, which were elite secondary schools, Greek and Latin dominated the curriculum; mathematics was valued as well, but mainly as a tool for developing the mind. Thus, whereas the French created a new elite comprised of scientist-engineers, the Germans placed humanists, especially philologists and philosophers, at the pinnacle of quite different type of intellectual elite. For mathematics, both of these competing models enjoyed impressive success and each exerted a strong influence on institutions in other nations, both within Europe and beyond, as part of a general modernization process. Out of this international mix emerged a kind of template for what we recognize today as the professional mathematician.

In surveying these developments, we begin with social and institutional innovations that took place in France, the dominant mathematical culture in Continental Europe up until 1848. Its École Polytechnique served as a kind of model for the polytechnical schools founded in various German states (Karlsruhe, Munich, and Dresden) as well as in Zurich and in the Italian states, notably Piedmont. French textbooks were also quite popular in Germany, largely because professors at the universities lectured without using a text and often on material that went beyond this literature. By mid-century, mathematicians at leading German universities had begun to spawn various schools of mathematics with a strong accent on pure research in analysis, geometry, algebra, and number theory.

Professionalization also went hand in hand with growing specialization, as witnessed by a proliferation of journals devoted exclusively to the publication of research in pure and applied mathematics. This situation contrasts sharply with the role played by professional mathematicians during the early modern period (volume 3), which covered mathematical practices ranging from conducting ballistics tests to composing horoscopes. The locus of activities in higher mathematics shifted during the eighteenth century to the learned academies in Paris, Berlin, and St. Petersburg. The many leading mathematicians who worked at these institutions often undertook projects of a more mundane nature, whereas celestial mechanics was one of their dominant theoretical interests. By the end of the nineteenth century, the notion of a natural philosopher in the tradition of Isaac Newton had disappeared giving way to a new breed: the natural scientist. The new term aptly fits figures like Hermann von Helmholtz or Lord Kelvin, but it no longer applies to Henri Poincaré or David Hilbert; even if their interests were exceptionally broad: they were mathematicians. By 1900, the trend toward specialization thus led to the formation of separate national societies for the disciplines of mathematics, astronomy, and physics. At the same time, these trends led to the growth of a class of professional mathematicians, paid to do and to disseminate mathematical research while imparting their ideas to others.

References

[1] I. Grattan-Guinness, *Convolutions in French Mathematics, 1800–1840*, 3 vols., Basel, Birkhäuser, 1990.

[2] J. Gray, *Henri Poincaré: A Scientific Biography*, Princeton: Princeton University Press, 2013.

[3] C. Jungnickel, R. McCormmach, *The Second Physicist: On the History of Theoretical Physics in Germany*, New York: Springer, 2016.

[4] A.H. Koblitz, *A Convergence of Lives: Sofia Kovalevskaia: Scientist, Writer, Revolutionary*, Boston, Birkhäuser, 1983.

[5] U. Merzbach, *Dirichlet. A Mathematical Biography*, Basel, Birkhäuser, 2018.

[6] K.H. Parshall, D.E. Rowe, *The Emergence of the American Mathematical Research Community, 1876–1900. J.J. Sylvester, Felix Klein, and E.H. Moore*, Providence: American Mathematical Society, 1994.

[7] D.E. Rowe, *A Richer Picture of Mathematics: the Göttingen Tradition and Beyond*, New York, Springer, 2018.

[8] R. Tobies, *Felix Klein. Visionen für Mathematik, Anwendungen und Unterricht*, Heidelberg, Springer, 2019.

[9] B. Yandell, *The Honors Class: Hilbert's Problems and their Solvers*, Wellesley, MA: AK Peters, 2002.

Volume 5: Inventing Mathematics in the Long Nineteenth Century. Rigor, Creation, and the Unexplored

Jeremy Gray

The profound changes that mathematics underwent in the long 19th century can usefully be reviewed under three headings: new subjects, better proofs, and different answers. Included among the new subjects are topics which became mathematised that were not simple extensions of what was already known. The idea of better proofs refers to a new, more abstract and autonomous sense of rigour that changed what could be said and what had to be done. Improved rigour was not an insistence on spelling out the obvious, but a tool for making new discoveries and for moving mathematics away from a reliance on naive intuition – one reason higher standards of rigour were required was to legitimize some of these new research areas. Different answers refers to answers of a qualitatively different kind, often more conceptual and less explicit, answers that might, for example, prove that a certain equation has a solution but do nothing to exhibit it explicitly (or show that an equation has no solutions of a given kind at all). It will become clear that the ideas of better proofs and different answers are linked, much in the way that new means of discovering things lead to new kinds of things being discovered.

The net effect was to make mathematics a more self-contained, autonomous, and abstract subject, one that was increasingly independent of contemporary science and had its own criteria for merit, its own definitions, and its own methods. Loosely, one could say that it made mathematics more 'pure', or perhaps 'pure' in a different way, but ultimately it is not helpful to try to divide the subject into 'pure' and 'applied' halves. The reasons for these changes go beyond the inclinations of a few influential mathematicians and into the creation of a mathematics profession and the evolving study of physics, and beyond that into broader social changes. These are discussed elsewhere in this volume: here we concentrate on the invention of new mathematics in the long 19th century.

To establish the importance of new standards of rigour we will look chiefly at how the calculus became rigorized. Mathematical analysis, as the rigorized calculus became known, can be said to start with Augustin Louis Cauchy, who reformulated the ideas of his 18th-century predecessors, but only to reach a stable and generally accepted form by the end of the century with the work of Karl Weierstrass, Richard Dedekind, and others. We can look at this process here only selectively. One particularly interesting area is its interaction with the ideas of Joseph Fourier, who claimed to have proposed a dramatic and highly effective way of representing any function as an infinite series. The extent to which he was correct and his representations actually useful was to remain unclear for a long time, as we shall see. The clearest examples of mathematicians creating new fields come from algebra and geometry. In algebra the theory of numbers was wholly redefined by Carl Friedrich Gauss in a book of 1801. Thirty years later, the theory of polynomial equations was given wholly novel foundations by Évariste Galois, and once these foundations were accepted, in the 1860s, they became a cornerstone of the new theory of groups. Likewise, in geometry, the description of a radically different but physically plausible geometry of space that differed from Euclidean geometry, the so-called non-Euclidean or hyperbolic geometry, also took a generation to be accepted, but it helped open the way to further novel formulations of geometries of many kinds when it was connected to Gauss's reformulation of the geometry of surfaces. By the end of the century, the drive for autonomy and rigour was leading to surprising, and sometimes contentious new branches of mathematics, often of an axiomatic kind, and often with novel foundations in an emerging theory of sets. This set the stage for the production of a distinctively modern mathematics in the 20th century.

References

[1] U. Bottazzini, *The Higher Calculus. A History of Real and Complex Analysis from Euler to Weierstrass*, Springer, 1986.
[2] R. Cooke, *The Mathematics of Sonya Kovalevskaya*, Springer, 1984.
[3] C. Goldstein, N. Schappacher, J. Schwermer, *The Shaping of Arithmetic after C.F. Gauss's Disquisitiones Arithmeticae*, Springer, 2007.
[4] J.V. Grabiner, *The Origins of Cauchy's rigorous calculus*, MIT Press, 1981.
[5] J.J. Gray, *Worlds out of Nothing; A Course on the History of Geometry in the 19th Century*, 2nd ed., Springer, 2011.
[6] J.J. Gray, *A History of Abstract Algebra: From Algebraic Equations to Modern Algebra*, Springer, 2018.
[7] G.H. Moore, *Zermelo's Axiom of Choice: Its Origins, Development and Influence*, Springer, 1982.
[8] S. Russ, *The Mathematical Works of Bernard Bolzano*, Oxford University Press, 2004.
[9] G. Schubring, *Conflicts between Generalization, Rigor, and Intuition*, Springer, 2005.
[10] H. Wussing, *The Genesis of the Abstract Group Concept*, tr. A. Shenitzer, MIT Press, 1984.

Volume 5: Mathematics and Worldviews in the Long Nineteenth Century. Worldviews in Collision

Ivahn Smadja

To approach this topic, it is useful to begin with some natural questions. What exactly is meant by "worldviews"? Should this be understood as an actors' category or as an observers' category? And why? Within the context of German cultural history, these conceptions were entangled with the natural and human sciences in complex ways. The notion of *Weltanschauung* has a rich history of its own, mostly in the nineteenth and the twentieth-century, with specific inflexions that one should constantly keep in mind. Wilhelm von Humboldt's term, for instance, was significantly *Weltansicht*, not *Weltanschauung*. He conceived of each national tongue as an "organic totality", that is a dynamic activity that shaped the thought of the speaking subject, not a dead product. The Humboldt scholar Jürgen Trabant emphasized this distinction, noting that *Weltanschauungen* came to mean "visions of the world" in the sense of ideologies, hence it carried an assertive component that affirms the nature of the world and our place within it. Humboldt's *Weltansichten*, on the other hand, affirmed nothing about the world, insofar as languages do pose no claim as to the ultimate truths of the world, they merely re-present the world through the mediation of the autonomous, constructive powers inherent in national tongues. Another significant milestone in the history of the word *Weltanschauung* came in the 1930s when the term "lost its solemnity and acquired an everyday, business-like ring" (Victor Klemperer). The semantic demotion of the term in ordinary Nazi parlance reduced it to a certain form of staunchness.

This chapter is organized to address worldviews on two levels by jointly considering a variety contexts and topics. The institutional and intellectual contexts include:

- the École Polytechnique together with Comtean positivism;
- the Cambridge Analytical Society and debates over the relationship of mathematics with the natural sciences;
- the methodological and philosophical commitments of mathematicians in Berlin;
- tentative unified worldviews (from Fechner to Riemann).

Among the topics considered are:

- Laplacian determinism vs. statistical and probabilistic views;
- the invention of pure mathematics and the consequent demotion of applied mathematics;
- the divide *Natur- und Geisteswissenschaften* and its impact on the conception of mathematics;
- the ethos of precision in industrial cultures: quantification, accuracy, calculation;
- Neohumanism vs. a scientific and mechanistic worldview (Helmholtz, Du Bois Reymond).

References

[1] M. Epple, *Die Entstehung der Knotentheorie*, Braunschweig, 1999.
[2] M. Epple, C. Zittel, eds., *Science as Cultural Practice*, Berlin, Akademie Verlag, 2010.
[3] J. Ferreirós, *Labyrinth of Thought*, Basel, Birkhäuser, 1999.
[4] G. Gigerenzer, et al., *The empire of chance: How probability changed science and everyday life*, Cambridge, 1989.
[5] J. Gray, *Plato's Ghost*, Princeton, 2008.
[6] M. Heidelberger, *Nature from Within: Gustav Theodor Fechner and his Psychophysical Worldview*, Princeton University Press, 2004.
[7] H.N. Jahnke, M. Otte, *Epistemological and Social Problems of the Sciences in the Early Nineteenth Century*, Dordrecht, Reidel, 1981.
[8] H. Mehrtens, *Moderne – Sprache – Mathematik*, Frankfurt/Main, Suhrkamp, 1990.
[9] H. Mehrtens, H. Bos, I. Schneider, eds., *Social History of Nineteenth-century Mathematics*, Basel, Birkhäuser, 1981.

Volume 5: Describing and Understanding the World in the Long Nineteenth Century. From Probability and Statistics to Field Theory

Scott A. Walter

The final quarter of the eighteenth century saw its share of revolutions. Among the changes brought about by the new French state was a closer alignment of science and technology with state interests, including the establishment of a variety of standards enforced by the power of the state. Among the most celebrated of the new French standards was the metric system of decimalized weights and measures. The meter, gram, and liter were prescribed, with the assistance of members of the Paris Academy of Sciences, for the measurement of length, mass, and liquid volume. The early years of the French Republic, soon to be overthrown by Napoleon, produced important institutional changes in the organization of science and technology with an impact that soon spread throughout Continetal Europe and beyond.

If the origins of this movement are to be found in France, the gathering of statistical data was also gaining momentum in Britain. Part of the motivation for this movement was provided by the Napoleonic Wars, as accurate census data were required in order to obtain satisfactory results from orders of conscription designed to expand the armed forces. In part, the British data-collection efforts were motivated by the social dislocation brought about by industrialization, and a concomitant fear of a French-style revolution on British soil. In Britain, as in France, the "experts in calculation" took an interest in the numbers produced by the new data-collection schemes. Led by Malthus, Charles Babbage, and others, a statistical section was formed in the British Association for the Advancement of Science in 1833.

Quantitative measurements concerned not only the natural but also the human realm, as the state apparatus collected "statistics", with which it meant to predict and control populations, from gathering data on birth, death, crime, education level, revenue, and the like. The acquisition of such statistics, combined with the use of mathematical modeling, gave rise, by the end of the century, to "economics". A new and important domain of application for probability arose in the aftermath of Darwin's theory of natural selection, which inspired his cousin Francis Galton to examine the statistics of human heredity. In a tangential way, statistical studies of the early 19th century further motivated the introduction of statistical reasoning in physics, a movement marked by the mid-century invention of kinetic gas theory by Maxwell and Boltzmann, and by Henri Poincaré's new methods of celestial mechanics. At the end of the century, Max Planck put probabilistic arguments to use in order to express his law of blackbody radiation, setting the scene for the 20th-century revolution of quantum mechanics.

Alongside these developments, the 19th century, also saw the discovery of new forces and fields. By mid-century, Maxwell, building on Faraday's notion of "lines of force", proposed a unification of optics and electrodynamics, in a new theory of the "electromagnetic field". Although it took another twenty years to win over physicists, Maxwell's theory eventually opened up broad new horizons for physics and technology. Most notably, the propagation of electromagnetic waves in air was demonstrated by Hertz, giving rise not only to wireless telegraphy, but to two revolutions of the 20th century: broadcast radio and radio astronomy. These two streams, electromagnetic field theory and probability, capture broad swaths of new mathematical thinking about the natural and social worlds in the 19th century. In some ways they were naturally antagonistic (particle vs. field), and yet they introduced a fruitful tension for a deeper understanding of the natural world.

References

[1] S.G. Brush, *The Kind of Motion We Call Heat: A History of the Kinetic Theory of Gases in the 19th Century, Volume 1: Physics and the Atomists*, Amsterdam, North-Holland, 1976.

[2] O. Darrigol, *Electrodynamics from Ampère to Einstein*, Oxford, Oxford University Press, 2000.
[3] O. Darrigol, *Atoms, Mechanics, and Probability: Boltzmann's Statistico-Mechanical Writings, An Exegesis*, Oxford: Oxford University Press, 2018.
[4] J.D. North, *Cosmos: An Illustrated History of Astronomy and Cosmology*, Chicago, University of Chicago Press, 2008.
[5] T.M. Porter, *The Rise of Statistical Thinking, 1820–1900*, Princeton, Princeton University Press, 1986.
[6] C. Smith, M.N. Wise, *Energy and Empire: A Biographical Study of Lord Kelvin*, Cambridge, Cambridge University Press, 1989.
[7] F. Steinle, *Exploratory Experiments: Ampère, Faraday, and the Origins of Electrodynamics*, Pittsburgh, University of Pittsburgh Press, 2016
[8] S.M. Stigler, *The History of Statistics: The Measurement of Uncertainty before 1900*, Cambridge, MA, Harvard University Press, 1986.
[9] M.N. Wise, *The Values of Precision*, Princeton: Princeton University Press, 1995.

Volume 5: Mathematics and Technological Change in the Long Nineteenth Century. Connecting the World through Observation, Calculation, Transportation, and Communication

June Barrow-Green, Tony Royle

Mathematics underpinned much of the technological change witnessed throughout the nineteenth and early twentieth centuries. Advancements in transportation and innovative applications of electromagnetism provided a great deal of the impetus that would revolutionise many aspects of society. These were the two predominant threads within the fabric of technology that permeated and most influenced other areas of technological progress. Few would argue that the static steam engine was the salient device that powered the Industrial Revolution of the early 1800s, but it was its application to transport that sparked a fundamental change in methods of communication. The ability to move people and goods more efficiently over land and sea had a remarkable impact on life in general and radically influenced the way information was disseminated and exchanged. The ability to move information, goods, and people over ever-increasing distances in a timely fashion fostered efficient communication of ideas, bolstered economies, and enabled more frequent gatherings of scientific communities. Emergent from these exchanges and collaborations were the nascent propositions and designs that would provide the stimuli and ingredients for transformation.

Some of the associated mathematics was new, but much that already existed found novel application. The improving precision of mathematical instruments also played a key role in progress by enabling the accumulation of more accurate and abundant data and by allowing observations that offered insight to inform theory and design. Mathematicians themselves were prime movers in the transformation, many diversifying or adapting to become leaders in the various strands of engineering and physics that lay at the heart of this flux. Some would become household names whilst others remained more anonymous, but whatever their profile the hallmark of their influence is stamped on almost every facet of the innovation that drove technological advancement as the nineteenth century unfolded.

References

[1] R. Flood, M. McCartney, A. Whitaker, *James Clerk Maxwell: Perspectives on his Life and Work*, Oxford University Press, 2014.

[2] J.S. Gordon, *A Thread Across the Ocean: The Heroic Story of the Transatlantic Cable*, Perennial, 2003.

[3] T. Royle, *The Flying Mathematicians of World War I*, McGill-Queen's University Press, 2020.

[4] W.G.S. Scaife, *From Galaxies to Turbines*, Taylor & Francis, 2000.

[5] D. Swade, *The Cogwheel Brain: Charles Babbage and the Quest to Build the First Computer*, Little, Brown & Company, 2000.

[6] C. Woolmar, *Fire & Steam*, Atlantic Books, 2008.

Volume 5: Representing Mathematics in the Long Nineteenth Century. Material Practices of Producing, Visualizing and Circulating Mathematical Knowledge

Anja Sattelmacher

The term representation, when used in combination with mathematics, needs to be carefully specified. In this chapter, it will employed in the sense of "to show" or "to demonstrate" a particular concept or result connected with mathematical knowledge. This usage is closely connected with educational practices and mainly for two reasons. First, the ways in which mathematical knowledge evolved has always been linked with the ways in which mathematics was taught – at colleges and universities, as well as at primary schools. Second, when analysing how mathematical ideas were disseminated throughout the 19th century, we find many borrowings from the preceding "century of pedagogy", with its diverse approaches, ideas, and concepts. The focus of this chapter will be the history of mathematics teaching through drawing, modeling, and model demonstrations at universities and technical colleges, mainly in France and Germany.

An important concept that relates to the word "representation" and that accompanied mathematical thinking throughout the entire 19th century was mathematical intuition ("Anschauung"). This refers to a mode of visualization that goes beyond the sense of vision, embracing the haptical sense as well. This term intuition/Anschauung has different roots in philosophy as well as pedagogy during the 18th century. When used by German mathematicians of the 19th century it referred to a certain type of thinking that could be trained with the help of suitable tools, such as models or drawings. These ideas were advanced by influential pedagogues, especially Friedrich Fröbel and Johann Heinrich Pestalozzi, who developed these ideas for elementary instruction within the German speaking countries and beyond. Their approach involved a two-step process: first, children should gain direct experience with physical objects, like cubes or spheres, and then in a second step, they should build on this experience to develop a more abstract form of geometrical thinking.

Throughout the 19th century, many educators used concrete teaching aids, such as models or drawings, to promote mathematical understanding. A leading advocate of this approach was Felix Klein, who initiated collections of mathematical models wherever he taught. For Klein, models had a twofold importance. In one sense, they represented the visual or material embodiment of a mathematical concept or formula. In a second sense, their representative value conveyed a more general meaning. Mathematicians – and particularly geometers like Klein who wanted to defend the importance of intuition – could point to such models as symbols of their particular scientific approach. The models they used in their teaching were in this broader sense status symbols that they could attach to their scientific reputations.

References

[1] J. Ferreirós, *Mathematical Knowledge and the Interplay of Practices*, Princeton, Princeton University Press, 2016.
[2] M. Friedman, *A History of Folding in Mathematics: Mathematizing the Margins*, Heidelberg, Springer, 2018.
[3] O. Gaycken, *Devices of curiosity*, Oxford University Press, New York; Oxford, 2015.
[4] P.A. Kidwell, A. Ackerberg-Hastings, D.L. Roberts, *Tools of American mathematics teaching, 1800–2000*, Smithsonian Institution, Washington, D.C., 2008.
[5] M. Lynch, S. Woolgar, eds., *Representation in Scientific Practice*, MIT Press, 1990.
[6] H. Münsterberg, *The Photoplay. A Psychological Study*, Hamburg, Tredition, 2011.
[7] C. von Oertzen, *Science, gender, and internationalism*, New York, Palgrave Macmillan, 2014.

Volume 6: Introduction, A Cultural History of Mathematics in the Modern Global Age

Tom Archibald, David E. Rowe

This volume of essays, covering the period from the end of WWI up to the present, poses challenges quite unlike any of the five that precede it. During these years, the role of mathematics in the world diversified enormously as the size of the communities of mathematical researchers and practitioners grew perhaps 1000-fold. The images of mathematical manifestations offered in the chapters that follow must necessarily be both incomplete and tentative. Despite this, one can easily discern ways in which mathematical activities took place alongside more familiar currents of political and cultural history.

During the latter half of the long nineteenth century leading up the First World War, forces of modernization reshaped life in Western Europe and by extension numerous other societies around the world when global imperialism dominated world politics. As shown in Volume 5, mathematics, which forged ever-stronger links with innovations in science and technology, played a major role in this accelerating modernization. Its practitioners shaped these forces just as they and their practices were molded by the various social, economic, and political contexts in which they lived. In a word, mathematics and modernity were inextricably interwoven, a trend that only intensified during the period covered by the present volume.

Although these general factors illustrate the entanglement of mathematics with major scientific and technological innovations of the last century, they tell us relatively little about the typical working environments of professional mathematicians and the ethos that informed those spheres of activity. One pervasive and long-lasting influence of modernization has been the proliferation of specialized knowledge, as reflected in the wealth of highly esoteric mathematical research that grew exponentially throughout the last century and into the present one. Looking back to the middle of the nineteenth century – the period during which our modern idea of a professional research mathematician first arose – it seems remarkable how tiny that world of "mathematical inventors" really was. Even in Germany, one of the leading countries in the production of mathematical research, there were well into the last century only around 100 full professorships in mathematics at all of the nearly 40 universities and institutes of technology throughout the land. By contrast, an estimate a few years ago by Jean-Pierre Bourgignon, a French mathematician and national research director, put the number of research mathematicians worldwide around 80,000.

Volume 6: Everyday Numeracy in the Modern Global Age. Numeracy in the Information Age

Christopher J. Phillips

In 1914, mathematics was one way among many of knowing about the world and was taught in schools largely as rudimentary reckoning and rote deductive reasoning. Just over a century later, the field has become the paradigmatic mode of rigorous analysis. In schools around the world, success in mathematics is seen not just as a general indicator of intelligence but also as a reliable path to employment across many sectors. This transition occurred alongside an increase in the number of fields relying on the analysis of numerical data, from marketing to professional sports. At the same time, the rise of numbers in many domains necessitated a level of numeracy simply to visit the doctor or enter the voting booth. A philosophical and scientific shift from a generally deterministic world to one of probability and chance was felt by everyday people as well. Though relatively few would ever understand the mechanisms behind formal statistical tools or the new technologies of machine learning, preparation in a wide range of fields increasingly includes numeracy as a prerequisite. If calculating with weights, measures, and money were the key elements of everyday mathematics in the nineteenth century, by the end of the twentieth, the focus was on being able to function in a world of statistical averages, data trends, and probabilistic predictions. Numeracy became more than a basic ability to reckon with integers. It was a set of skills crucial for understanding the algorithms and uncertainties central to everyday life.

Even more profound than this change in the definition of numeracy was the way access to numerical skills expanded around the globe in this period. In 1914, only a tiny percentage of humans had access to even rudimentary mathematical education. Only a handful of countries provided their citizens any conception of free universal schooling, and beyond those places, mathematical education was reserved for the elite. The expansion of schooling throughout the century meant that mathematics increasingly was included as part of what everyone should know.

The very invention of the English term "numeracy" in 1959 was directly tied to this expansion, in that it drew attention to global disparities in levels of educational attainment. That is, the very idea of measuring numeracy didn't exist until mathematical competence was something that might be reasonably expected for most people to obtain. After the creation of the International Bureau of Education in 1925, and the United Nations Educational, Scientific, and Cultural Organization (UNESCO) after World War II, global data started to be kept systematically on mathematical attainment as a general condition of educational expansion. Though disparities in access persisted (even as late as 1975, few countries outside of the largest economic powers educated girls and boys at similar rates), the general expansion of access to formal education meant that nearly all children were exposed to basic numeracy education at some point. Global statistics on numeracy and mathematical achievement are not as robust as those for literacy, but what evidence does exist suggests that both minimum standards and competency rates have increased slowly but steadily over time. Mathematics beyond basic arithmetic was no longer a specialized subject, limited to those training to become scientists, navigators, or traders; ordinary people around the globe could be expected to have some knowledge of mathematics. Only in the second half of the twentieth century could the very notion of "numeracy" as a necessary component of education on par with literacy be conceivable.

References

[1] D. Bouk, *How Our Days Became Numbered: Risk and the Rise of the Statistical Individual*, Chicago, University of Chicago Press, 2015.

[2] H. Gispert, G. Schubring, *Societal, Structural, and Conceptual Changes in Mathematics Teaching: Reform Processes in France and Germany over the Twentieth Century and the International Dynamics*, Science in Context **24(1)**(2011), 73–106.

[3] L. Gitelman ed., *"Raw Data" Is an Oxymoron*, Cambridge, MA, MIT Press, 2013.

[4] D. Lupton, *The Quantified Self*, Cambridge, MA, Polity, 2016.

[5] B. Moon, *The "New Maths" Curriculum Controversy*, London, Falmer Press, 1986.

[6] J.Z. Muller, *The Tyranny of Metrics*, Princeton, Princeton University Press, 2018,

[7] C. O'Neil, *Weapons of Math Destruction: How Big Data Increases Inequality and Threatens Democracy*, New York, Crown, 2016.

[8] C.J. Phillips, *The New Math: A Political History*, Chicago, University of Chicago Press, 2015.

[9] T.M. Porter, *Trust in Numbers: The Pursuit of Objectivity in Science and Public Life*, Princeton, Princeton University Press, 1986.

Volume 6: Practice and Profession in the Modern Global Age. The Mathematical Sciences and Globalization

Tom Archibald, David E. Rowe

This chapter is structured around three periods familiar from political histories. The vivid human drama that played out during the course of two world wars touched the lives of mathematicians in numerous ways. This naturally sets off the period ending in 1945 from the post-WWII era, dominated in political history by the Cold War conflict between the USA and the USSR. Throughout the period 1920 to 1933, international relations among mathematicians from the leading nations gradually improved, as highlighted by the 1932 International Congress held in Zurich. Four years later, at the ICM in Oslo, political tensions largely overshadowed the mathematical festivities. The year 1945 marked the starting point of a new economic world order, one in which mathematical models played a key role in both government and private economic policy and decision-making. Throughout much of the period up until 1990, competition between the two superpowers, the United States and the Soviet Union, created tensions across the international mathematical landscape. In research mathematics, the USA strengthened its dominant position, a process already begun before WWII but augmented by a large mathematical immigration to that country, which produced an explosion of educational and research developments during the postwar period. Meanwhile, the Soviet school continued to turn out brilliant researchers not only in pure mathematics, but also in scientific fields that depended on sophisticated mathematical methods. This period thus saw a vastly expanded use of quantitative techniques in the physical sciences, medicine, and engineering together with the widespread development and adoption of statistical methods as well as deterministic models to advance these fields.

The demise of the Soviet Union in 1990 hardly bears comparison with the collapse of Nazi Germany in 1945. Nevertheless, that year offers a convenient demarcation point for our present era since it coincides quite closely (and not coincidentally) with the revolution in communications associated with the rise of the internet. In 1990 Tim Berners-Lee, then working at the European nuclear research agency CERN, produced the first web browser and editor, later named WorldWideWeb. Three years later, it entered the public domain, launching one of the most massive transformations in human history. From a wider perspective, the Web stands out as the most visible byproduct of the IT revolution with its roots in electronic computing, a field whose ties with the mathematical world have been far from incidental. Today, hardly any field of mathematical research has remained untouched by the fast-growing developments in computer science and technology.

References

[1] P. Duren, ed., *A Century of Mathematics in America*, 2 vols., Providence, RI, American Mathematical Society, 1988–89.

[2] J. Green, J. LaDuke, *Pioneering Women in American Mathematics: The Pre-1940 PhD's*, Providence, RI, American Mathematical Society, 2008.
[3] O. Lehto, *Mathematics without Borders: a History of the International Mathematical Union*, New York Springer, 1998.
[4] K. Parshall, A. Rice, eds., *Mathematics Unbound, The Evolution of an International Mathematical Research Community, 1800–1945*, Providence, RI, American Mathematical Society, 2002.
[5] J.-P. Pier, ed., *Development of Mathematics 1950–2000*, Basel, Birkhäuser, 2000.
[6] D.E. Rowe, *A Richer Picture of Mathematics: the Göttingen Tradition and Beyond*, New York, Springer, 2018.
[7] R. Siegmund-Schultze, *Rockefeller and the Internationalization of Mathematics between the Two World Wars: Documents and Studies for the Social History of Mathematics in the 20th Century*, Basel, Birkhäuser, 2001.
[8] R. Siegmund-Schultze, *Mathematicians Fleeing from Nazi Germany: Individual Fates and Global Impact*, Princeton, Princeton University Press, 2009.
[9] B. Yandell, *The Honors Class: Hilbert's Problems and their Solvers*, Wellesley, MA, AK Peters, 2002.
[10] S. Zdravkovska, P.L. Duren, eds., *Golden Years of Moscow Mathematics*, Providence, RI, American Mathematical Society, 1993.

Volume 6: Inventing Mathematics in the Modern Global Age. The Programs of Bourbaki and the Soviet School

Roger Cooke and Leo Corry

Over the course of the twentieth century, mathematical invention was profoundly influenced by two somewhat idiosyncratic, but also quite distinct research programs that arose in Eastern and Western Europe, respectively. The first was associated with the Soviet school of mathematics, which took shape between 1917 and 1945. Its influence continued throughout the century and extended well beyond the borders of the Soviet Union to nearby countries, like Poland and Hungary, but also beyond to the United States and Israel. The second research program, developed by a far smaller group of mathematicians that first formed in France during the mid-1930s, later became famous through its collective works, Éléments de mathématique, published under the pseudonym Nicolas Bourbaki. Many of the group's members were graduates of the École normale supérieure (ENS), where the Seminar Bourbaki was founded after the Second World War. As a movement that aimed to establish new foundational principles for modern mathematics, Bourbaki's impact gradually spread throughout Europe, the United States, and Latin America.

These two programs involved large-scale efforts to reform the ways in which mathematics was both taught but also understood. By drawing on set-theoretically formulated abstract methods, they helped shape as well as spread new forms of mathematical knowledge that led to important new innovations and sub-disciplines. Together, the Soviet school and the Bourbaki movement provided a framework for the pursuit of mathematical research at several leading centers around the world. Both embodied modernist impulses that developed into major cultural movements shaped by some of the century's most charismatic personalities, talented mathematicians with personal achievements of the first magnitude, but who also expressed clear guiding visions for the future course of the discipline at large. Apart from these shared characteristics, however, the two programs differed quite sharply regarding major issues, such as the essence of mathematical knowledge or its role in science and society. As such, each reflected a distinct vision of what inventing mathematics ought to mean.

Two main centers dominated mathematics in the Soviet Union: Moscow and Petersburg. Each had formidable leaders whose work and methodological preferences strongly colored the research activity in their respective localities. In the wake of the Russian Revolution, both centers came under political pressure to reorganize their educational programs in accordance with the principles of a Communist society. They responded, in turn, by adopting eclectic, vaguely Marxist orientations predicated on materialist principles. Above all, Soviet mathematicians excelled in the art of problem solving in work that ranged over problems at all levels of difficulty and that covered virtually every area of mathematical knowledge. They cultivated this special talent, in particular, by drawing on a wide variety of tools and methods taken from several very different disciplines, including probability theory, number theory and descriptive set theory.

Problem solving, in the concrete sense practiced by Soviet mathematics, played only a secondary role in the Bourbaki project, which was instead driven by an ambitious effort to present an overview of what mathematical knowledge had come to be. With this goal in mind, the group placed a heavy accent on the systematic development of a few fundamental theories. Their members designed these to serve as the basic architectonic foundation for all mathematical knowledge, out of which all future developments should arise. Two main ideas provided the central organizing principles for Bourbaki's image of mathematics: formal axiomatics and the notion of mathematical structures. These general principles guided the work of Bourbaki as the group gradually developed and promoted a universal program that would play an instrumental role in shaping the course of mathematical research.

References

[1] A.D. Aleksandrov, A.N. Kolmogorov, M.A. Lavrent'ev, eds., *Mathematics: Its Content, Methods, and Meaning*, Cambridge, MA, MIT Press, 1963.

[2] G.J. Borjas, K.B. Doran, *The collapse of the Soviet Union and the productivity of American mathematicians*, The Quarterly Journal of Economics, **127(3)**(2012), 1143–1203.

[3] N. Bourbaki, *The architecture of mathematics*, American Mathematical Monthly, **57**(1950), 221–232.

[4] L. Corry, *The origins of eternal truth in modern mathematics: Hilbert to Bourbaki and beyond*, Science in Context, 12(1997), 137–183.

[5] L. Corry, *Modern Algebra and the Rise of Mathematical Structures*, Basel, Birkhäuser, 2004.

[6] L. Corry, *Writing the Ultimate Mathematical Textbook: Nicolas Bourbakis Éléments de mathématique*, E. Robson and J. Stedall, eds., The Oxford Handbook of the History of Mathematics, Oxford, 2009.

[7] S.S. Demidov, B.V. Lëvshin, et al., *The Case of Academician Nikolai Nikolaevich Luzin*, trans. R. Cooke, Providence, RI, American Mathematical Society, 2016.

[8] G.G. Lorentz, *Mathematics and politics in the Soviet Union from 1928 to 1953*, Journal of Approximation Theory, **116**(2002), 169–223.
[9] M. Mashaal, *Bourbaki: a Secret Society of Mathematicians*, Providence, RI, American Mathematical Society, 2006.
[10] A.N. Shiryaev, ed., *Kolmogorov in Perspective*, Providence, RI, American Mathematical Society, 2000.
[11] V.M. Tikhomirov, *Moscow mathematics 1950–1975*, J.-P. Pier, ed., *Development of Mathematics 1950–2000*, Basel, Birkhäuser, 2000, pp. 1107–1120.

Volume 6: Mathematics and Worldviews in the Modern Global Age. Worldviews in the Age of Ideologies

José Ferreirós

This chapter follows a chronology based on three periods: the era of the two world wars, 1914 to 1945, the post-war years from 1945 to 1990, and the thirty years 1990 to 2020. The first period witnessed the rise of mathematical modernism and the clash of worldviews manifested in the foundational debates. This conflict was both a power struggle within the discipline as well as part of an effort to define normative standards for mathematical research. During the second period, a strong consensus emerged in the West, partly due to the influence of Hilbert and of Bourbaki, a pseudonym adopted by a group of young French mathematicians, most of whom were *normaliens* (graduates of the École normale supérieure). The research norms that came to fore in the West reflected disciplinary autonomy alongside a unified conception of mathematical structures anchored by stringent conditions for mathematical proof.

The purism of this trend had little influence on mathematical research in the Soviet Union, however, where such ideals were antithetical to the state ideology of Marxism. Soviet mathematics was, in fact, highly eclectic, but its practitioners advocated socialist ideals and rejected bourgeois trends identified with the capitalist West. With the fall of the Soviet Union, competing ideologies no longer played a leading role in the world of mathematics, which became highly diverse and increasingly interdisciplinary during the third period. On the one hand, this era saw several unanticipated conceptual developments that served to undermine the former foundational picture; on the other, the IT revolution exerted a profound influence on nearly every sphere of mathematical research. If, throughout most of the last century, much mathematical work was largely invisible, in today's highly digitalized world many are well aware that their lives have been deeply affected by the products and byproducts of mathematical knowledge.

References

[1] A. Connes, J.-P. Changeux, *Conversations on Mind, Matter and Mathematics*, Princeton University Press, 1998.
[2] J. Ferreirós, J. Gray, eds., *The Architecture of Modern Mathematics*, Oxford University Press, 2006.
[3] L.R. Graham, J.-M. Kantor, eds., *Naming Infinity*, Harvard University Press, 2009.
[4] J. Gray, *Plato's Ghost*, Princeton University Press, 2008.

[5] H. Mehrtens, *Moderne – Sprache – Mathematik*, Frankfurt/Main, Suhrkamp, 1990.
[6] E. Seneta, *Mathematics, religion, and Marxism in the Soviet Union in the 1930s*, Historia Mathematica, **31**(2004), 337–367.
[7] A. Vucinich, *Soviet mathematics and dialectics in the post-Stalin era: new horizons*, Historia Mathematica, **29**(2002), 13–39.
[8] H. Weyl, *Philosophy of mathematics and natural science*, Princeton University Press, 1949, new ed. 2009.

Volume 6: Describing and Understanding the World in the Modern Global Age. Quantum Physics and Relativity

Marta Jordi Taltavull, Tilman Sauer

The relationship between mathematics and physical theory in the twentieth century was decisively shaped by the occurrence and outcome of two major conceptual transformations. Quantum mechanics, the fundamental theory of matter, was established in the years 1925-27, and the general theory of relativity, the fundamental theory of space and time, and gravitation and cosmology, was established in 1915. Quantum mechanics changed the way we understand the microscopic world. One of the key differences between classical and quantum physics is that, according to quantum physics, motions and energy exchanges are discontinuous and fundamentally unpredictable at a microscopic level. Relativity theory changed our understanding of the macroscopic world. Space and time turned out to be intertwined in ways physicists had never before realized, and spacetime was included in the dynamics of physical laws. Both transformations of our world understanding implied fundamental mathematical and philosophical changes in our way of describing physical reality.

In both cases, mathematical concepts and theories played a major role in adapting physical theory to new experimental findings and empirical evidence as well as in reacting to problems that had arisen as inner-theoretical difficulties. The outcome in either case was a novel foundational equation, which to this day govern our understanding of the physical world, both small and large. The Dirac equation or its non-relativistic counterpart, the Schrödinger equation, represents the basis for all of quantum physics, including atomic and molecular physics, nuclear as well as elementary particle physics. The Einstein equation is the foundation of the general theory of relativity and of modern gravitation theory, including our understanding of the cosmos at large and its historical development. Both equations captured major transformations in the basic physical concepts underlying our understanding of the world. Both equations went along with the introduction of new mathematical techniques and formalism into physical theory. And in both cases, it took many years and much effort in elaborating the consequences and implications inherent in those equations to fully understand their impact on our present world view. In this, process, new mathematical concepts and a new physical semantics evolved together to reflect the new ways of describing and understanding physical reality. In the quantum case, most prominent among these new concepts were the notion of a Hilbert space and its vectors or wave functions.

In the case of relativity theory, the most prominent new concepts were the metric tensor and affine connection and its associated spacetime curvature.

References

[1] O. Darrigol, *From c-Numbers to q-Numbers. The Classical Analogy in the History of Quantum Theory*, University of California Press, 1993.
[2] H.S. Kragh, *Quantum Generations: A History of Physics in the Twentieth Century*, Princeton: Princeton University Press, 1999.
[3] T.S. Kuhn, *Black-Body Theory and the Quantum Discontinuity, 1894–1912*, University of Chicago Press, 1978, 2nd ed. 1987.
[4] J. Mehra, H. Rechenberg, *The Historical Development of Quantum Theory*, 6 vols., New York, Springer, 1982–2001.
[5] J. Renn, ed., *The Genesis of General Relativity*, 4 vols., Boston, Springer, 2007.
[6] S. Schweber, *QED and the Men Who Made It: Dyson, Feynman, Schwinger, and Tomonaga*, Princeton, NJ, Princeton University Press, 1994.

Volume 6: Mathematics and Technological Change in the Modern Global Age. Programming Mathematics

Liesbeth De Mol

This chapter will deal the role of modern computing as a mathematical technology. A central concern will be the following question: how do the histories of computing and mathematics intertwine (or not) and what does this tell us about the history of mathematics itself and how it is perceived today? While the rise of the modern computer should not be reduced to developments in mathematics, it is also clear that the field was an important driving force for the early history of large-scale machine computation. So, how is it that mathematics and mathematicians became involved with the making and use of this technology?

One important impetus came from an increased need for mathematics research in the context of military science. In that context it became clear that brute-force computation would be a necessary tool to develop and deploy ever more sophisticated weapons, a development dating back to the First World War. This is part of a very broad and complex history, much of which remains to be written. It concerns the changing relationship between, on the one hand, mathematics – its practices, results, and the self-understanding of its practitioners – and, on the other, computational technologies and the academic fields which are anchored in them historically. In order to give focus to the present account, this chapter will concentrate on four major topics as these developed in the United States. These are 1) the early history of computers in relation to mathematics; 2) the use of computational technology within mathematics; 3) mathematics as a tool for computation; and 4) the changing disciplinary identity of mathematics from the perspective of computational technologies.

References

[1] W. Aspray, *John von Neumann and the origins of modern computing*, Cambridge, MA, MIT Press, 1990.
[2] T. Archibald, D. Dumbaugh, D. Kent, *U.S. mathematicians in World War I*, (Aubin and Goldstein 2014), pp. 229-271.
[3] D. Aubin, C. Goldstein, eds., *The war of guns and mathematics. Mathematical practices and communities in France and its Western Allies around World War I*, Providence, RI, American Mathematical Society, 2014.
[4] L. De Mol, M. Bullynck, *Making the History of Computing. The History of Computing in the History of Technology and the History of Mathematics*, Revue de Synthèse **139(3/4)**(2018), 361–80.
[5] L. De Mol, *Turing Machines*, The Stanford Encyclopedia of Philosophy, Edward N. Zalta, ed., Winter 2019 Edition.
[6] S. Gerovich, *From Newspeak to Cyberspeak: A History of Soviet Cybernetics*, Cambridge, MA, MIT Press, 2002.
[7] D.A. Grier, *When computers were human*, Princeton, Princeton University Press, 2005.
[8] D. MacKenzie, *Mechanizing proof. Computing, risk and trust*, Cambridge, MA, MIT Press, 2004.
[9] M. Priestley, *A science of operations. Machines, logic and the invention of programming*, Berlin, Springer, 2011.
[10] M. Tedre, *The science of computing. Shaping a discipline*, Boca Raton, CRC Press, 2014.

Volume 6: Representing Mathematics in the Modern Global Age. Mathematics in Literature: Between Rationalization and Creative Freedom

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In many literary texts of the last century, mathematics serves as a short hand for reason, while reflecting concerns or revaluations of rationality seen against the possibilities for individuality and freedom. Almost all texts to some degree acknowledge the common juxtaposition of mathematics, on the one side, and life, literature, and creative freedom, on the other, even if only to take this as a basis from which ultimately to denounce the contrast and the reductive image of mathematics. Yet, these two aspects need not stand in opposition. In George Orwell's dystopian novel *Nineteen Eighty-Four* (1948), the protagonist Winston Smith attempts to hold on to a simple mathematical truth in a society that manipulates facts, memories, and perceptions to suit the purposes of its leaders. In this repressive system, which almost annihilates individual liberty, Winston takes a simple equation as a symbol for the existence of truth that the totalitarian Party cannot touch: “Freedom is the freedom to say that two plus two make four.”

This chapter is dedicated to exploring the representation of mathematics in works with literary ambition that have neither a mainly didactic nor a purely entertaining intention. It will discuss only in passing topics such as mathematics in popular culture, scientists' biographies, or hard science fiction. The focus lies instead on literary texts that represent mathematics in more complex terms, revealing surprising aspects of uncertainty, freedom, and creativity connected with it. These works suggest certain similarities between mathematics and literature, while showing that mathematics applied to life can also yield fascinating insights into its many mysteries.

References

[1] Y. Gingras, *What Did Mathematics Do to Physics?*, History of Science, **39**(2001), 383–416.

[2] F.T. Marinetti, *Technical Manifesto of Futurist Literature (1912)*, L. Rainey, C. Poggi, and L. Wittman, eds., Futurism: An Anthology, New Haven, Yale University Press, 2009, pp. 119–125.

[3] F.T. Marinetti, *Geometric and Mechanical Splendor and the Numerical Sensibility (1914)*, L. Rainey, C. Poggi, and L. Wittman, eds., Futurism: An Anthology, New Haven, Yale University Press, 2009, pp. 175–180.

[4] R. Musil, *The Mathematical Man (1913)*, B. Pike and D.S. Luft, eds. and trans., Precision and Soul: Essays and Addresses, Chicago, University of Chicago Press, 1990, pp. 39–43.

[5] T. Pynchon, *Gravity's Rainbow*, London, Vintage, 1st ed. 1973, 2000.

[6] T. Pynchon, *Mason & Dixon*, New York, Picador, 1997.

[7] T. Stoppard, *Arcadia*, London, Faber and Faber, 1st ed. 1993, 2000.

[8] Y. Zamyatin, *We (1921)*, trans. Clarence Brown, London, Penguin, 1993.

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