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Homogeneous Structures: Model Theory meets Universal Algebra (online meeting)

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Abstract. The workshop "Homogeneous Structures: Model Theory meets Universal Algebra" was centred around transferring recently obtained advances in universal algebra from the finite to the infinite. As it turns out, the notion of homogeneity together with other model-theoretic concepts like ω-categoricity and the Ramsey property play an indispensable role in this endeavour.

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Introduction by the Organizers

The workshop *Homogeneous Structures: Model Theory meets Universal Algebra*, organised by Manuel Bodirsky (Dresden), Joanna Ochremiak (Talence) and Michael Pinsker (Vienna/Prague), was held 2021, January 3-9. Due to the COVID-19 restrictions, the 54 participants had to meet virtually. The program featured 3 tutorials, each consisting of two talks, 12 one-hour talks as well as an open problem session.

Many fundamental mathematical first-order structures, such as the order of the rational numbers or the random graph, are *homogeneous* in the sense that isomorphisms between finite or finitely generated substructures can be extended to automorphisms of the entire structure. Homogeneous structures can be built systematically as limits of classes of finite structures. That way, a class of finite structures is stored in a single homogeneous structure, and can be investigated via that structure from the viewpoint of model theory or, more recently, universal algebra. This perspective has found many applications in computer science, for example in constraint satisfaction, automata theory, and verification.

Because of their high degree of symmetry, homogeneous structures tend to have large *automorphism groups*. Their abstract and topological properties make these groups an extremely interesting topic in topological dynamics with many recent developments, for example the connection between extreme amenability and the Ramsey property of classes of finite structures due to Kechris, Pestov, and Todorčević.

For some of the applications of homogeneous structures in computer science, the automorphism group does not store enough information about the homogenous structure, and one has to instead study the richer *polymorphism clone* of the structure. This is the place where universal algebra enters the picture.

Universal algebra has achieved a series of milestone results in the past years regarding the equational structure of polymorphism clones of *finite* structures. This development culminated in 2017 in a proof of the *Feder-Vardi dichotomy conjecture* from theoretical computer science, obtained independently by Bulatov and Zhuk. Their result states that every finite structure either has a constraint satisfaction problem that can be solved in polynomial time, or is NP-complete. A similar conjecture exists in the context of homogeneous structures, but is open in spite of emerging results on the structure of their polymorphism clones.

The aim of the workshop was to carry the recent successful structural advances of universal algebra from finite structures to infinite structures with large automorphism groups. Such an enterprise requires a joint effort of model theory and universal algebra.

The addition of universal algebra to model theory allows for a finer distinction between structures than is classically envisaged in model theory. Such a finer-grained analysis of structures is, on the one hand, necessary for applications of homogeneous structures in theoretical computer science, but has on the other hand also led to the development of new useful methods for obtaining results which belong to classical model theory, such as the classification of reducts of a fixed structure.

Conversely, model theoretic concepts and methods appear naturally in universal algebra when investigating the equational structure of infinite algebras rather than finite ones. This is, in particular, true for the polymorphism algebras which are naturally induced by homogeneous structures.

Finally, very recent results have shown that the equations which hold in the polymorphism clone of a structure are directly linked to the structure of its automorphism group. In other words, the connection between model theory and universal algebra not only consists of applications of one field in the other one, but certain seemingly unrelated properties of structures from the two fields turn out to be tied closely together. This tight connection is an astonishing new discovery whose full meaning is yet to be unveiled.

Achieving the fusion between model theory and universal algebra has many facets, touching infinite permutation groups, topological dynamics, Ramsey theory, and core questions in universal algebra.

The following is a list of key concepts which defined the scope of the workshop:

- Equational theory of polymorphism clones of homogeneous structures;
- Equations as fixed points of certain actions of algebras (so-called "loop lemmata"); possible links with fixed point phenomena of group actions;
- Connections between the equational theory of polymorphism clones of a structure and the structure of its automorphism group;
- Reconstruction of the topology of automorphism groups, endomorphism monoids, and polymorphism clones of homogeneous structures;
- Classifications of homogeneous structures in various restricted signatures;
- Precompact Ramsey expansions of homogeneous structures ;
- First-order reducts of homogeneous structures with finite relational signature;
- Classifications of structures with small orbit growth.

Three tutorials brought together the two communities assembled by the workshop: L. Barto outlined the fundamental concepts of universal algebra as well as more sophisticated tools which were frequently used or referenced in subsequent talks. P. Simon covered the model theory side, focusing on the concept of ω-categorical structures (which becomes central as soon as algebraic methods are to be employed) and in particular on NIP structures which are homogeneous in a finite language and hence ω-categorical. The third tutorial given by M. Bodirsky and M. Pinsker already showcased the interplay between the two fields, introducing oligomorphic clones: The notion of a clone is an algebraic one, whereas oligomorphicity can be seen as arising from model theory via ω-categorical structures. If the clone additionally contains the automorphism group of a structure which is homogeneous in a finite language, particularly strong results arise.

P. Mayr talked about the connection between varieties generated by finite simple Mal'cev algebras and Boolean powers as well as about Boolean powers arising as Fraïssé limits. S. Braunfeld surveyed several results in which cellular structures form a dividing line in combinatorial problems and reported on connections to other model-theoretic properties. Ramsey classes were treated by J. Hubička who gave new conditions for a class of structures to be Ramsey or to at least have a Ramsey expansion. M. Valeriote's talk was algebraic in nature treating a correspondence between near unanimity terms and systems of projections of a single subalgebra of a direct product, and also giving an application in constraint satisfaction. I. Kaplan reported on additional evidence to support a positive answer to a question by D. Macpherson about the connection between NIP and the strict order property of a structure and its automorphism group not having ample generics. Equationally complete theories were taken up by K. Kearnes who surveyed several new results, in particular partial classifications. A. Mottet presented a new technique in the algebraic approach to constraint satisfaction problems on infinite templates and showed how it can be used to confirm the dichotomy conjecture for certain classes of structures as well as to recognise within those classes the CSPs solvable by local consistency checking. A. Pongrácz suggested tackling the Thomas Conjecture – any countable homogeneous structure has only finitely many reducts up to first-order interdefinability – by looking at the stronger assertion that the number of reducts up to existential interdefinability be finite as well. G. Paolini spoke about the reconstruction of structures from their automorphism groups and gave an overview of results involving the strong small index property. Returning to computational problems, A. Bulatov's talk focused on the Ideal Membership Problem and showed how the algebraic approach to CSPs can be translated to IMPs. D. Zhuk explained how strong subalgebras, a central notion in his proof of the CSP dichotomy conjecture, can be used to obtain results about the complexity of CSPs as well as universal algebraic results not directly related to CSPs. Finally, D. Macpherson reported on a Fraïssé construction of a specific type of Jordan permutation group which has not been exhibited before.

Workshop (online meeting): Homogeneous Structures: Model Theory meets Universal Algebra

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Abstracts

Tutorial on Universal Algebra (2 talks)

Libor Barto

The aim of the tutorial is to explain the fundamental concepts and theorems of universal algebra, and to sketch main ideas of the more advanced tools. The selection of material is heavily biased toward the mathematics of constraint problems over finite templates.

The first part of the tutorial is devoted to the basics. We start by discussing two viewpoints on the subjects: universal algebra can be regarded as model theory without relational symbols (this is the standard, textbook presentation) but also as a generalization of the permutation group theory from unary functions to functions of higher arity. We focus on the second viewpoint and describe some fundamental concepts and theorem in this language: clones, free algebras, the connection between clones and pp-definitions, and the connection among clone homomorphisms, algebraic constructions, and pp-interpretations [2].

The second part of the tutorial introduces the main concepts and ideas of four more advanced tools: the tame congruence theory [4] that studies the structure of algebras using small images of unary polynomial operations, the commutator theory [5] that generalizes concepts related to abelianess from group theory to general algebras, Bulatov's theory [3] that studies algebras via their two-generated subalgebras, and the absorption theory [1] that is based on a concept resembling ideals in rings.

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Tutorial on Model Theory of ω-categorical Structures (2 talks)

Pierre Simon

The study of ω-categorical structures is an important topic in model theory that has led to many important developments in pure model theory, in particular geometric stability theory and simplicity theory. This tutorial focussed on the classification of structures homogeneous in a finite relational language that are *NIP*. In this context, the NIP property can be defined simply as saying that the number of orbits under the automorphism group fixing a finite subset of size n grows polynomially in n. If the theory furthermore does not interpret an infinite linear order, then it is *stable*.

Stable finitely homogeneous structures are very well understood, due to work of Cherlin, Harrington, Lachlan and Hrushovski [1], [2], [3]. It is known in particular that there are countably many (up to inter-definability) and that they are all interpretable in dense linear order. We know much less about finitely homogeneous NIP structures, though it is conjectured that there are also countably many. We do have results for the case where the structure is interpretable in a binary homogeneous structure. In particular, primitive structures in this class are classified. They are essentially built from linear and circular orders [4].

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On varieties generated by finite simple groups

Peter Mayr

A *variety* is a class of algebraic structures (*algebras* for short) of the same type that is defined by equations. By Birkhoff's Theorem the *variety generated by a class of algebras C* is the class of all homomorphic images of subalgebras of products of elements in C. We refer to [4] for background on general algebra.

Varieties generated by a finite simple group have been studied by Neumann [9], Apps [1] and others. Many of the applied techniques carry over from groups to Mal'cev algebras in general. Here an algebra $\mathbf{A}$ is *Mal'cev* if it has a ternary term operation m satisfying $m(x, x, y) = y = m(y, x, x)$. Note that groups, loops, rings as well as their expansions are all Mal'cev.

For the remainder let $\mathbf{A}$ be a finite simple non-abelian Mal'cev algebra (see [7] for a generalization of the commutator theory from groups to algebras). Let V denote the variety generated by $\mathbf{A}$, V_{fin} the class of its finite members, and W the variety generated by all proper subalgebras of $\mathbf{A}$. Then

- every element in V_{fin} is isomorphic to some $\mathbf{A}^k \times \mathbf{B}$ for $k \geq 0$ and $\mathbf{B} \in W$ (cf. [9] for groups, [6] for primal algebras);
- if $\mathbf{A}$ is a group (loop, ring), then the countable free algebra in V has a normal subgroup (subloop, ideal) isomorphic to the Boolean power $\mathbf{A}^{\mathbf{R}_0}$ for $\mathbf{R}_0$ the countable atomless Boolean ring without 1 (cf. [3]) and a complement isomorphic to the countable free algebra in W.

Note that V_{fin} has the joint embedding property (JEP) and the hereditary property (HP) but in general not the amalgamation property (AP). On the other hand its subclass K of finite direct powers of $\mathbf{A}$ has JEP and AP but in general not HP.

Still by a generalization of Fraïssé's Theorem [8] there exists a unique (up to isomorphism) countable algebra $\mathbf{D}$ such that (i) every finitely generated subalgebra of $\mathbf{D}$ embeds into some element of K, (ii) $\mathbf{D}$ is a direct limit of algebras in K, and (iii) every isomorphism between subalgebras of $\mathbf{D}$ that are isomorphic to some element in K extends to an automorphism of $\mathbf{D}$. We call such a $\mathbf{D}$ the *Fraïssé limit* of K and denote it by $\mathrm{Flim}(K)$.

Then $\mathrm{Flim}(K)$ is ω-categorical, universal for V_{fin} and is isomorphic to a *filtered Boolean power* as defined by Arens and Kaplansky [2] (see also [5]).

Theorem. *Let* $\mathbf{A}$ *be a finite simple non-abelian Mal'cev algebra with* $n \geq 0$ *singleton subalgebras generated by* $a_1, \ldots, a_n \in A$*, respectively. Let* X *be the Stone space of the countable atomless Boolean algebra, and let* $x_1, \ldots, x_n \in X$ *be distinct.*

Then $\mathrm{Flim}(\{\mathbf{A}^k : k \geq 1\})$ *is isomorphic to the subalgebra of* $\mathbf{A}^X$ *with universe*

$$\{f\colon X \to A : f \text{ is continuous and } f(x_1) = a_1, \ldots, f(x_n) = a_n\},$$

where A *is endowed with the discrete topology.*

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Cellularity and Beyond

Samuel Braunfeld
(joint work with Michael C. Laskowski)

Cellular structures are a class of very simple ω-categorical structures that yield a dividing line in many combinatorial problems concerning hereditary classes and countable structures, such as when counting the number of structures bi-embeddable with a given countable structure [2], or counting the finite substructures of a homogeneous structure [1]. We will discuss where cellularity appears and its relation to the more general model-theoretic properties of mutual algebraicity, monadic stability, and monadic NIP.

The main intuition for cellular structures is that their ages encode neither an infinite linear order nor an infinite equivalence relation (i.e. an equivalence relation with infinitely many infinite classes). The first part of the intuition can be formalized by the fact that cellular structures are stable, while the latter will be partially formalized by the next theorem.

Theorem. [3, 4] *If M is mutually algebraic but non-cellular, there is $N \succ M$ adding infinitely many new pairwise-isomorphic infinite components.*

If M is not mutually algebraic, there is $N \succ M$ such that infinitely many quantifier-free k-types over N support infinite arrays.

Thus we may prove that cellularity is a dividing line in the following steps, exploiting our main intuition by passing through mutual algebraicity.

(1) If M is unstable, we encode $(\mathbb{Q}, <)$ in an age-preserving extension and reproduce its wild behavior.
(2) If M is stable but non-mutually algebraic, we use the infinitely many infinite arrays to mimic the classes of an infinite equivalence relation.
(3) If M is mutually algebraic but non-cellular, we do the same with the infinitely many pairwise-isomorphic infinite components.

A technical complication in this strategy is caused by the configurations possibly appearing on tuples instead of singletons. Passing through the even more general property of monadic stability allows us to encode configurations on singletons in a unary expansion. This would be useful, if it could first be shown that if a unary expansion of T is on the bad side of a dichotomy, then so is T itself. But we know of few cases where this can be done.

Finally, we mention that monadic stability has appeared in a dichotomy about growth rates of homogeneous structures, allowing a description of the subexponential growth rates [1]. There is some evidence that monadic NIP will correspond to the condition of at most exponential growth rate.

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Tutorial on Oligomorphic Clones, Part 1 of 2

Michael Pinsker

This is the first part of a tutorial the second part of which is delivered by Manuel Bodirsky. We discuss which results about finite algebras discussed in Libor Barto's tutorial on universal algebra can be lifted to algebras on a countably infinite domain.

If we wish to investigate an infinite algebra $\mathbf{A}$ via its invariant relations (e.g., its congruences), then we really study the closure $\overline{\mathrm{Clo}(\mathbf{A})}$ of the set $\mathrm{Clo}(\mathbf{A})$ of its term functions in the topology of pointwise convergence; this topology is discrete in the finite case. The object $\overline{\mathrm{Clo}(\mathbf{A})}$ forms a *topological clone* [BPP17]. It bears an algebraic structure, provided by composition of functions or equivalently, the equations that are satisfied between the functions of the clone, as well as a topological structure given by pointwise convergence. In the countable case, this topology is induced by a complete metric. Every closed function clone is the *polymorphism clone* $\mathrm{Pol}(\mathbb{A})$ of a relational structure $\mathbb{A}$, just like every closed permutation group is the automorphism group of a relational structure. Polymorphism clones allow for a finer-grained study of relational structures than automorphism groups.

If $\mathbb{A}$ is a countable ω-categorical structure, then $\mathrm{Pol}(\mathbb{A})$ is *oligomorphic*, i.e., it contains (among its unary functions) a permutation group which acts with finitely many orbits on n-tuples, for all $n \geq 1$. In this case, a lot of the basic theory for finite clones applies: the relations invariant under $\mathrm{Pol}(\mathbb{A})$ are precisely those which have a primitive positive definition in $\mathbb{A}$, and *primitive positive interpretations* in $\mathbb{A}$ as well as *primitive positive constructions* can be characterized via the structure of $\mathrm{Pol}(\mathbb{A})$ [BP15, BOP18, BN06].

Some of the deepest results on finite algebras have been obtained under the assumption of *idempotency*, which requires the unary functions of the term clone to contain only the identity function. This assumption is in stark contradiction with oligomorphicity for clones on countable sets, a fact we refer to as the *First dilemma of the infinite sheep.* For closed oligomorphic function clones, an approximation to the notion of idempotency is that of a *model-complete core*: a structure $\mathbb{A}$ is a model-complete core if the unary functions of $\mathrm{Pol}(\mathbb{A})$ consist only of the closure of its automorphism group. If $\mathbb{A}$ is ω-categorical, then it is homomorphically equivalent to a model-complete core $\mathbb{A}'$, and $\mathbb{A}'$ is again ω-categorical and unique up to isomorphism [Bod07].

There exist so far basically two general algebraic results on polymorphism clones of ω-categorical structures that go beyond the basic theory. Both results concern the characterization of different notions of triviality for clones, and are of particular interest in the context of applications to Constraint Satisfaction Problems, where they often describe the border between tractability and NP-completeness.

The first result states that if $\mathbb{A}$ is an ω-categorical model-complete core, then some *stabilizer* of Pol($\mathbb{A}$) has a continuous clone homomorphism to the clone Proj of projections on a Boolean domain if and only if Pol($\mathbb{A}$) does not satisfy the *pseudo-Siggers* equation [BP16, BP20]. The second result states that if the number of orbits of the action of the automorphism group of $\mathbb{A}$ on n-tuples grows less than doubly exponentially in n, then the above is moreover equivalent to Pol($\mathbb{A}$) having a uniformly continous mapping to Proj which preserves equations of height 1 [BKO$^+$19, BKO$^+$17].

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Ramsey classes using parameter spaces

Jan Hubička

Given (relational) structures $\mathbf{A}$ and $\mathbf{B}$, we denote by $\binom{\mathbf{B}}{\mathbf{A}}$ the set of all embeddings from $\mathbf{A}$ to $\mathbf{B}$. We write $\mathbf{C} \longrightarrow (\mathbf{B})^{\mathbf{A}}_k$ to denote the following statement: For every colouring χ of $\binom{\mathbf{C}}{\mathbf{A}}$ with k colours, there exists an embedding $f : \mathbf{B} \to \mathbf{C}$ such that χ is constant on $\binom{f(\mathbf{B})}{\mathbf{A}}$. Class $\mathcal{K}$ of finite structures is *Ramsey* if for every $\mathbf{A}, \mathbf{B}$ there exists $\mathbf{C}$ such that $\mathbf{C} \longrightarrow (\mathbf{B})^{\mathbf{A}}_2$.

The most versatile technique to prove that a given class is Ramsey is the *partite construction* developed by Nešetřil and Rödl in series of papers (see, for example, [NR89, HN19]). While alternative proof techniques exists [Prö13, PTW85, Maš18] they can not be applied for classes with forbidden substructures (such as for the class of ordered triangle-free graphs).

We discuss a new technique based on parameter spaces [Hub20] that can be applied to triangle-free graphs, metric spaces with additional triangle constraints and other examples. This technique originates in recent developments in infinitary structural Ramsey theory [Dob20a, Dob19, Dob20b, Zuc19, Zuc20] and leads to further consequences such as the upper bounds on big Ramsey degrees.

This is a joint project with Martin Balko, Natasha Dobrinen, David Chodounský, Matěj Konečný, Jaroslav Nešetřil, Stevo Todorcevic, Lluis Vena and Andy Zucker.

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Tutorial on Oligomorphic Clones, Part 2 of 2

Manuel Bodirsky

I will present some techniques to study closed oligomorphic clones that contain the automorphism group of a finitely homogeneous structure. A central concept in this context are canonical functions. I will discuss links with topics that have been studied intensively in model theory, such as Thomas' conjecture, the small index property, and the Ramsey expansion conjecture. An additional interesting finiteness condition that is of particular relevance for the study of the CSP are finitely bounded homogeneous structures.

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Near unanimity terms and the Baker-Pixley Theorem

Matt Valeriote
(joint work with Libor Barto, Marcin Kozik and Johnson Tan)

Given a subalgebra A of a direct product $\prod_{i\leq n} A_i$ of algebras A_i from some variety V, one can consider $\mathrm{Proj}_k(A)$, the system of projections of A onto all k-element sets of coordinates. In general, A is not uniquely determined by $\mathrm{Proj}_k(A)$, but if V happens to have a $(k+1)$-near unanimity term, then Kirby Baker and Alden Pixley show that this is the case [1]. They also show that if a variety V satisfies this uniqueness property for all subalgebras of direct products of its members, then it must have a $(k+1)$-ary near unanimity term.

In this talk I will consider the following existence question: Given a system of k-fold projections Γ over some direct product $\prod_{i\leq n} A_i$ of algebras A_i from a variety V, under what circumstances will there exist a subalgebra A of $\prod_{i\leq n} A_i$ such that $\Gamma = \mathrm{Proj}_k(A)$? An answer will be given that settles a question posed by George Bergman [3].

This is joint work with Libor Barto, Marcin Kozik, and Johnson Tan [2].

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On the automorphism group of the universal homogeneous meet-tree

Itay Kaplan

An element g of a Polish group G is *generic* if the conjugacy class g^G is comeager in G. There are some nice corollaries of having a generic element, for instance having no non-trivial normal subgroup of index $< 2^{\aleph_0}$. Having *ample generics* is much stronger: for every $n < \omega$ there is a comeager orbit in the action of G on G^n given by conjugation coordinate-wise. This implies in particular the small index property. We are interested in the case where $G = \mathrm{Aut}\,(M)$ where M is some ω-categorical ultrahomogeneous structure (and the topology is that of pointwise convergence). There are many examples of natural homogeneous structures which have ample generics such as the random graph or the infinite set, but order seems to be an obstacle:

Example 1. $\mathrm{Aut}\,(\mathbb{Q},<)$ has a generic automorphism (and there is a precise description of the generic automorphism) [Tru92][KT01]. But does not have a generic pair of automorphisms by [Tru07] (based on a proof of Hodkinson), and reproved by Siniora [Sin17].

Example 2. Let $(P, <)$ be the generic poset. Then $\mathrm{Aut}(P)$ has a generic automorphism [KT01] but no generic pair [KP20].

These examples lead to a very natural question by Macpherson: assuming that M has the *strict order property* (M has a definable partial order with infinite chains), then is it true that M has no ample generics? This is not true, since the countable atomless Boolean algebra does have ample generics (and the strict order property) [Kwi12], so we may try to rephrase the question by adding NIP.

This work started as an attempt to find a counterexample to Macpherson's question (inspired also by the fact that examples with trees can exhibit strange phenomenons in the model theoretic study of NIP, e.g., [KS14]) but turned out to give yet another evidence to it.

Definition 3. A *tree* is a partially ordered set $(A, \leq)$ which is *semilinear* (that is, for every $a_0 \in A$, the set $A_{\leq a_0} = \{a \in A \mid a \leq a_0\}$ is linearly ordered) and such that every pair of elements has a common lower bound.

A *meet-tree* (or $\wedge$-tree) $(A, \leq, \wedge)$ is a tree which is also a lower semilattice, i.e a tree $(A, \leq)$ together with a binary (meet or infimum) function $\wedge : A^2 \to A$ such that for every $a, b \in A$, $a \wedge b$ is the largest element of $A_{\leq a} \cap A_{\leq b}$.

Fact 4. *The class of all finite meet-trees is a Fraïssé class (in the language of meet-trees). Consequently, there is a countable generic meet-tree, $\mathbb{T}$, which is ω-categorical and ultrahomogeneous.*

The main result of this talk is the following:

Theorem 5. $\mathrm{Aut}(\mathbb{T})$ *has a generic automrophism but no generic pair.*

To prove the first part we used a characterization of having a generic automorphism established by Ivanov [Iva99] and [KR07] given in terms of the amalgamation property in a class of structures with partial automorphisms. Following [KT01], we define *determined* partial automorphism and prove that they form amalgamation bases. We then prove their existence borrowing ideas from the model theoretic notion of existential closeness: these will be finite partial automorphisms in which any behavior appearing in an extension, already appears (in a precise sense). This involves classifying the possible types of oribts of automorphisms in meet-trees, and studying their behavior in detail.

To prove that there is no generic pair, we use the fact that for any point $a \in \mathbb{T}$, $\mathbb{T}_{\leq a}$ is densely and linearly ordered, which allows us to adapt the proof from the case of $\mathrm{Aut}(\mathbb{Q}, <)$ (as presented in [Sin17]).

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Can we classify equationally complete theories?

Keith Kearnes

In this talk we discuss the problem of classifying equationally complete theories Σ, or equivalently the problem of classifying minimal varieties $\mathcal{V} = \mathrm{Mod}(\Sigma)$.

We begin by referencing the result in [11], which describes an algorithm to decide whether a finite algebra $\mathbf{A}$ in a finite language generates a minimal variety. This algorithm gives no information about $\mathbf{A}$. We next survey the results in [6, 7, 12, 13, 14]. The paper [6] describes a new algorithm for deciding if a finite algebra in a finite language generates a minimal variety, and provides structural information about the variety. This work is based on [12], which classifies the clones of finite term minimal algebras. The papers [6, 7, 12, 13, 14] include a complete classification of minimal abelian varieties containing a finite nontrivial member. The paper [10] introduces a modification of the construction in [9] to show that the general problem of determining if a finite algebra in a finite language generates a minimal variety is 2EXPTIME complete.

Next we turn to minimal abelian varieties with no finite nontrivial members. The main new results are:

(1) (Reference [3]) Every minimal abelian variety is affine or strongly abelian.
(2) (Reference [4]) The category of affine clones is categorically equivalent to a variety of 2-sorted structures, and the simple 2-sorted structures are classified. This effects a classification of minimal affine varieties up to a determination of finite simple rings.
(3) (Reference [5]) The minimal strongly abelian varieties of finite essential arity are exactly the varieties categorically equivalent to minimal essentially unary varieties. The proof makes essential use of the fact, proved

in [8], that minimal strongly abelian varieties of finite essential arity are Hamiltonian.

(4) (Reference [5]) An essentially unary variety is minimal if and only if it is definitionally equivalent to the subvariety of the variety of all M-sets that is axiomatized by $0(x) \approx 0(y)$, where M is some simple monoid with 0.

(5) (Reference [5]) A construction from [2] of an infinite dimensional matrix power can be used to produce minimal strongly abelian varieties unlike those in any of the above cases. It is proved in [1] that the variety of Jónsson-Tarski algebras is a minimal variety. This is another example of a strongly abelian minimal variety unlike those in any of the above cases.

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Smooth approximations

Antoine Mottet
(joint work with Michael Pinsker)

I present the novel machinery of smooth approximations. The method of smooth approximations gives information about a polymorphism clone $\mathscr{D}$ from a subclone $\mathscr{C} \subseteq \mathscr{D}$. When so-called smooth approximations do exist, one obtains information about uniformly continuous actions of $\mathscr{D}$ on specific finite clones, such as the clone of projections on a 2-element set of clones of affine maps over a finite module. When these approximations do not exist, one obtains information about specific operations in $\mathscr{D}$. The approximation technique was used recently to confirm the CSP dichotomy conjecture for first-order reducts of the random tournament, various homogeneous graphs including the random graph, and for expansions of the order of the rationals. For all except the last class, the technique was moreover used to give a characterization of CSPs solvable by local consistency methods.

This talk is based on a joint work with Michael Pinsker [1].

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The existential Thomas Conjecture

András Pongrácz

Simon Thomas conjectured [11] that every countable homogeneous structure over a finite relational language has finitely many reducts up to first-order interdefinability. This classical model theoretic problem is open for thirty years, and the proofs of all partial results make use of an ordered Ramsey expansion of the structure [8, 11, 12, 9, 6, 2]. To date, there is no counter-example to the strictly stronger and less understood assertion that there are finitely many reducts up to existential interdefinability. In my talk, I will argue that it is more natural to study the stronger statement for ordered Ramsey structures, and potentially easier to attack. The proposed method is illustrated on a concrete structure, a binary branching semilinear order, whose reducts are of interest in theoretical computer science [5, 3, 4].

The method is robust; it is a work in progress to use analogue arguments to different structures, and some rudimentary general results are also going to be presented in the talk [10]. We build on a very effective technique introduced by Manuel Bodirsky, Michael Pinsker and Todor Tsankov [7], which makes it possible to find reducts systematically by using so-called canonical functions; see also [1]. The idea is to use a combinatorial lemma, also observed recently by Michael Pinsker and Antoine Mottet in a different area, which shows that a reduct is either model-complete or its self-embedding monoid contains a very special type of canonical function. The existence of such a canonical function reduces the reduct classification problem to a similar problem with a simpler input structure. Hence, we can prove assertions concerning reducts of Ramsey structures by using an inductive argument. Moreover, it is demonstrated how reduct classification results up to first-oder interdefinability can be refined to classification up to existential interdefinability, making use of the Galois connections obtained by universal algebraic methods, see [7, 3, 1].

At the end of the talk, plausible open problems are proposed to achieve partial results in the existential Thomas Conjecture.

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Homogeneous structures, reconstruction theory and non-Archimedean Polish groups

Gianluca Paolini

(joint work with Saharon Shelah)

We survey some recent results joint with S. Shelah on the problem of reconstruction of countable structures from their automorphism groups (under suitable assumptions). Central to our approach is the notion of strong small index property.

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On the Complexity of CSP-based Ideal Membership Problems

Andrei Bulatov

In this paper we consider the Ideal Membership Problem (IMP for short), in which we are given real polynomials $f_0, \vec{f_k}$ and the question is to decide whether f_0 belongs to the ideal generated by $\vec{f_k}$. In the more stringent version the task is also to find a proof of this fact. The IMP underlies many proof systems based on polynomials such as Nullstellensatz, Polynomial Calculus, and Sum-of-Squares. In the majority of such applications the IMP involves so called combinatorial ideals that arise from a variety of discrete combinatorial problems. This restriction makes the IMP significantly easier and in some cases allows for an efficient algorithm to solve it.

In 2019 Mastrolilli initiated a systematic study of IMPs arising from Constraint Satisfaction Problems (CSP) of the form CSP(Γ), that is, CSPs in which the type of constraints is limited to relations from a set Γ. He described sets Γ on a 2-element set that give rise to polynomial time solvable IMPs and showed that for the remaining ones the problem is hard. We continue this line of research.

First, we show that many CSP techniques can be translated to IMPs thus allowing us to significantly improve the methods of studying the complexity of the IMP. We also develop universal algebraic techniques for the IMP that have been so useful in the study of the CSP. This allows us to prove a general necessary condition for the tractability of the IMP, and three sufficient ones. The sufficient conditions include IMPs arising from systems of linear equations over GF(p), p prime, and also some conditions defined through special kinds of polymorphisms.

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Strong Subalgebras and the Constraint Satisfaction Problem

Dmitriy Zhuk

In 2007 it was conjectured that the Constraint Satisfaction Problem (CSP) over a constraint language Γ is tractable if and only if Γ is preserved by a weak near-unanimity (WNU) operation. After many efforts and partial results, this conjecture was independently proved by Andrei Bulatov and the author in 2017. In this talk we consider one of two main ingredients of author's proof, that is, strong subalgebras that allow us to reduce domains of the variables iteratively.

The idea of the approach is that every idempotent algebra has a sualgebra of one of the following five types: binary absorbing, central, PC, projective or linear. These subalgebras have additional strong properties, which allow us to reduce the domain iteratively maintaining some property. Since we consider only finite algebras, finally we get a one-element subalgebra, for which the required fact is obvious.

To explain how this idea works we show the algebraic properties of strong subalgebras and present a simple proof of several important (and known) facts about the complexity of the CSP. First, we prove that if a constraint language is not preserved by a WNU operation then the corresponding CSP is NP-hard. Second, we characterize all constraint languages whose CSPs can be solved by local consistency checking. Additionally, we characterize all idempotent algebras not having a WNU term of a concrete arity n, not having a WNU term, having WNU terms of all arities greater than 2.

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Jordan permutation groups and limits of treelike structures

Dugald Macpherson
(joint work with Asma Almazaydeh)

If G is a transitive permutation group on a set X, then $A \subset X$ is a *Jordan set* if $|A| > 1$ and the pointwise stabiliser $G_{(X\setminus A)}$ is transitive on A. It is a *proper* Jordan set if it is not the case that $|X \setminus A| = k \in \mathbb{N}$ and G is $(k+1)$-transitive, and is a *primitive* Jordan set if $G_{(X\setminus A)}$ is primitive on A. A *Jordan group* is a transitive permutation group with a proper Jordan set.

Finite primitive Jordan groups are 2-transitive, and were classified by Peter Neumann in [7], with related work around the same time in [6] (with a model-theoretic application) and by W.M. Kantor. Examples of infinite primitive Jordan groups include: (i) the automorphism groups of $(\mathbb{Q}, <)$ and its reducts (i.e. the automorphism groups of the corrresponding linear betweenness relation, circular order, and separation relation); (ii) automorphism groups of certain 'treelike' structures (semilinear orders, betweenness relations, C and D relations); (iii) automorphism groups of projective and affine spaces and more generally of certain Steiner systems (possibly with infinite block size); and (iv) 'limits' of Steiner systems, betweenness relations, and D-relations.

Adeleke and Neumann in [3] gave a structure theorem for primitive Jordan groups with primitive Jordan sets (they are of type (i) or (ii) above, or highly transitive, that is, k-transitive for all k). It was shown in [2] that any primitive Jordan group preserves a structure of one of the types (i)-(iv) above or is highly transitive. Constructions of types (iv) have been given by Adeleke (see [1] for Jordan groups preserving limits of betweenness and D-relations). In [5] Bhattacharjee and I gave a construction of an oligomorphic group preserving a limit of betweenness relations (but not a structure of type (i)-(iii)), describing it as the automorphism group of an ω-categorical treelike object (a Fraïssé limit of finite trees with vertices labelled by finite betweenenss relations). In this talk I describe analogous work with Almazaydeh [4] which constructs a Jordan group preserving a limit of D-relations, built as a Fraïssé limit. These constructions of new treelike objects may give interesting examples of other phenomena.

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