Mathematisches Forschungsinstitut Oberwolfach

Report No. 14/2022

DOI: 10.4171/OWR/2022/14

New Mathematical Techniques in Information Theory

Organized by
Amos Lapidoth, Zürich
Prakash Narayan, College Park

13 March – 19 March 2022

Abstract. Information theory is the richer for a surge of recent advances in relevant mathematical techniques. The workshop fostered an exchange of ideas on new mathematical tools which are typically outside the classical toolbox of information theorists and that are yet useful in solving classical and modern problems in information theory and related areas. The focus was on mathematical techniques that are of a general nature and that could benefit a wide class of problems. A number of broad mathematical areas were identified that held promise with established early successes, and key contributors were invited to make presentations and initiate discussions with an emphasis on emergent topics. The areas were: information measures, measure concentration, hypercontractivity and correlation measures, Shannon theory and extremal combinatorics, advanced tools for proving converse results in coding theorems, and recent techniques for proving Gaussian optimality entailing new characterizations of Gaussian distributions.

Mathematics Subject Classification (2020): 94A17.

Introduction by the Organizers

The workshop *New Mathematical Tools and Techniques in Information Theory* was organized by Amos Lapidoth (ETH, Zürich) and Prakash Narayan (University of Maryland, College Park).

Striking progress has been made in recent years in developing mathematical tools and techniques for solving classical as well as a new generation of emerging problems in information theory and related areas. These methods, which lie typically outside the classical and venerable toolbox of information theorists, have led to early successes and hold the potential for enabling significant breakthroughs. A driving objective of the workshop was to foster an exchange of ideas among key players – main contributors to the development of said mathematical techniques as well as scientists whose research serves to motivate them. Participants were invited to make presentations with emphasis on emerging developments in a variety of interrelated topics: (1) information measures; (2) measure concentration inequalities and information theory; (3) hypercontractivity and correlation measures; (4) product-channel and min-max interchange techniques for proving converse coding theorems (impossibility results); (5) optimality of Gaussian distributions in network information theory; and (6) extremal combinatorics and information theory. While Topics (1) - (5) were mentioned in the original proposal, Topic (6) was included newly, spurred by recent exciting developments in an area of wide interest to information theorists and theoretical computer scientists, and of special appeal to the participants of the workshop.

The workshop drew 28 enthusiastic participants with broad geographic representation from Europe, Asia and North America. In inviting participants, special emphasis was placed on young researchers (including several women of whom 5 eventually attended). There were 16 online and 12 in-person participants, respectively. The ongoing Covid pandemic and unpredictable travel restrictions worldwide prevented the former group from attending in person.

The talks were categorized roughly equally into mathematical techniques in the making and their application to specific classical as well as modern problems in information theory and statistical inference. Each talk lasted between 40 minutes and an hour, with adequate time allowed for discussion during and after the presentation.

Falling in the category of mathematical advances, the inaugural talk of the workshop by Chandra Nair was on the derivation and identification of appropriate information functionals whose optimized values characterized capacity regions arising in network information theory. Such functionals possess subadditivity properties that are crucial in obtaining desirable "single-letter" expressions for capacity regions. In this spirit, Amin Gohari's talk examined the role of mutual-f-information for suitable convex mappings f, as an extension of Shannon's mutual information, in establishing infeasibility or converse results for network communication models.

Shannon entropy inequalities including the entropy power inequality, its variants, and connected family of inequalities, e.g., Brunn-Minkowski, are enduring tools used in the proofs of many coding theorems in information theory. Recent developments in generalizing these inequalities, by means of an enhanced framework of linear maps for Gaussian comparisons, were described by Thomas Courtade. Mokshay Madiman's talk discussed an analogous class of inequalities for minentropy that entailed comparisons with uniform distributions on Euclidean balls and also nonEuclidean spaces with some algebraic structure such as locally compact groups. These inequalities will be potentially important in establishing a new generation of multiuser coding theorems. On a related theme, Jingbo Liu explained how methods from convex geometry, when used anew to extract lower bounds for the "soft-max" of a stochastic process supplemented by mixed-volume inequalities, led to a complete solution of a conjecture of Thomas Cover for the capacity of a discrete memoryless relay channel.

The notorious difficulty faced in tackling the many open problems in network information theory has motivated a radically different approach in this realm that uses automated theorem proving techniques. Using as a springboard the linear programming approach of Zhen Zhang and Raymond Yeung for verifying linear information inequalities that is buttressed by a specific concept of such relationships, Cheuk Ting Li explained his new algorithm for automatically generating human-readable proofs of inner and outer bounds for capacities of multiuser models.

Zero-error information theory and extremal combinatorics have long enjoyed a symbiotic relationship. Igal Sason presented new Shannon entropy-based proofs of known or refined combinatorial bounds for bipartite graphs, which hold promise for shedding new light on these problems of consequence in graph theory, coding theory and information theory. On a variant theme, Shannon's zero-error channel capacity problem, combinatorial at its heart, remains a longstanding open challenge. Anelia Somekh-Baruch's talk dealt with the notoriously defiant information theoretic characterization of channel capacity under mismatched decoding, of which zero-error capacity is a special case. Her talk explained a new technique for developing improved converse bounds for mismatch capacity; it entails augmenting the channel with another output and decoder, and could be potentially of independent interest in other settings.

Two flavors of broadly applicable new techniques based on measure concentration were presented by Yihong Wu and Himanshu Tyagi, respectively, for analyzing reconstruction and estimation errors arising in statistical learning contexts. Wu discussed phase transition effects displayed by reconstruction errors in bipartite graph matching, while Tyagi addressed information theoretic lower bounds for statistical estimation and testing hampered by incomplete knowledge of measurement samples. Showing yet another flavor of learning in graph-based models, Yury Polyanskiy's talk examined properties of the posterior probabilities of an Ising model on a rooted tree conditioned on a noisy version thereof, by analyzing belief-propagation distributional recursions and its fixed point. This method also was shown to resolve a number of earlier open conjectures.

Several application-motivated talks ranged over classical as well as newer models in information theory and its interface with statistical decision theory. Distributed hypothesis testing under rate-constrained communication among decision centers in multihop network settings, was discussed by Michèle Wigger with emphasis on ensuing optimal tradeoffs between error performance at various centers and communication rates. In a similar setting, Shun Watanabe showed the suboptimality of a specific quantization-and-random-binning scheme and proposed a new technique with improved rate performance. Solutions to the problem of guessing the value of a random variable, especially when standard connections to data compression cannot be exploited, were presented by Robert Graczyk. The underlying approach based on restriction-of-type-class and change-of-measure arguments could be useful when classical information theoretic arguments do not suffice.

Universal lossless compression of large-scale data modeled by a labeled graph, with the compression scheme not depending on the underlying probability distribution of the model, is motivated by many modern data science applications. Lele Wang proposed a new algorithm for such compression for graphs generated by a stochastic block model, employing a block decomposition of the graph adjacency matrix. Its compression performance is theoretically optimal for a certain range of model parameters; beyond this range, counterpart schemes that operate optimally constitute an open challenge. In a different setting, Ligong Wang presented a lossy compression scheme for a data sequence where the output of the encoder must be kept concealed from an observer with access to side information that is correlated with the data; such methods are pertinent in applications involving information security.

In the realm of communication channel transmission, Ramji Venkataraman addressed a multiple-access channel where the receiver's measurement consists of the sum of the transmitters' signals and Gaussian noise, with the number of transmitters growing linearly with codeword blocklength. In the limit of a large transmitter population, he showed how spatially coupled coding combined with approximate message passing decoding come close to achieving optimal tradeoffs between energy-per-bit and the transmitter density for a fixed error rate. Yossef Steinberg provided an overview of current progress in determining the capacity of compound channels and arbitrarily varying channels in specific network settings; these models describe transmission channels with uncertain statistics.

Abstracts of all the talks below are accompanied by open problems selected by the speakers.

At the conclusion of the workshop, the unanimous opinion of the participants, including the Workshop Organizers, was that it had met all of its objectives successfully, was hugely beneficial and should be conducted again. The Organizers recall the grand tradition of erstwhile Oberwolfach Workshops on Information Theory organized in the 1980s and 1990s by Professors Rudolf Ahlswede (Bielefeld), James Massey (ETH - Zurich) and Jack van Lint (Eindhoven). Those workshops served to instruct, elevate and inspire a current generation of leading information theorists. The Organizers nurture the hope that our recent workshop will mark the revival of a rich tradition.

Workshop: New Mathematical Techniques in Information Theory

Table of Contents

Abstracts

On Subadditivity, Global, and Local Optimizers of Information Functionals

Chandra Nair

(joint work with Sida Liu, Lingxiao Xia, Mehdi Yazdanpanah, Wai Ho David Ng, Yannan (Dustin) Wang, Chin Wa Ken Lau, Amin Gohari, Venkat Anantharam, Abbas El Gamal)

Upon expressing the capacity region for a memoryless multiuser communication setting as the optimal value of associated information functional (expressed in terms of a single use of the channel), the value satisfies a certain single-letterization property. This usually translates to a subadditivity property of the underlying information functional (or a slight modification thereof). Given this observation, it is of interest to derive and identify information functionals that satisfy the subadditive property. While there has been exciting progress and results in the direction of deriving functionals that satisfy the subadditivity property (which consequently has led to upper bounds for the capacity region for channel coding settings), this talk will focus mainly on ideas for identifying information functionals that satisfy the subadditive property. This latter pursuit is critical in understanding the optimality of certain inner bounds in multiuser settings.

Open problem:
An open problem that is of considerable interest is the following: *Does local tensorization imply global tensorzation* for a family of information functionals.

The motivation of this open problem is to see if a locally-testable criteria can determine whether functionals have the magical "single-letterization" property in information theory. We shall assume that all random variables take values in finite alphabet spaces. Below $H(X)$ refers to the Shannon entropy of a random variable and $A \otimes B$ refers to the Kronecker product of the matrices A and B.

Let X denote the input of a channel and let the output be denoted by $(Y_1, .., Y_k)$. Denote the stochastic matrix corresponding to the channel as $W_{Y|X}$. Given a set of parameters $\alpha_S \in \mathbb{R}$, for $S \subseteq [1:k]$, define a family of functions, parameterized by a real valued function $\gamma_X : \mathcal{X} \mapsto \mathbb{R}$, according to

$$\hat{F}(\gamma_X, W) := \sup_{q_X} \left\{ \sum_{S \subseteq [1:n]} \alpha_S H_q(Y_S) - \mathrm{E}_{q_X}(\gamma(X)) \right\}.$$

Definition: A functional $\sum_S \alpha_S H(Y_S)$ is said to satisfy a *global tensorization property* if a product distribution maximizes $\hat{F}^{(\gamma_1,\gamma_2)}(W_1 \otimes W_2, p_{X_1X_2})$ for all $\gamma_1(X_1), \gamma_2(X_2)$, where

$$\hat{F}^{(\gamma_1,\gamma_2)}(W_1 \otimes W_2, p_{UX_1X_2}) := \sum_S \alpha_S H(Y_{1S}, Y_{2S}) - \mathrm{E}(\gamma_1(X_1)) - \mathrm{E}(\gamma_2(X_2))$$

Definition: A functional $\sum_S \alpha_S H(Y_S)$ is said to satisfy a *local tensorization property* if the product of local maximizers of $\hat{F}_1^{(\gamma_1)}(W_1, p_{UX_1})$ and $\hat{F}_2^{(\gamma_2)}(W_2, p_{UX_2})$ is a local maximizer of $\hat{F}_{12}^{(\gamma_1,\gamma_2)}(W_1 \otimes W_2, p_{X_1X_2})$ for all $\gamma_1(X_1), \gamma_2(X_2)$, where

$$\hat{F}_1^{(\gamma_1)}(W_1, p_{UX_1}) := \sum_S \alpha_S H(Y_{1S}) - \mathrm{E}(\gamma_1(X_1))$$

$$\hat{F}_2^{(\gamma_2)}(W_2, p_{UX_2}) := \sum_S \alpha_S H(X_{2S}) - \mathrm{E}(\gamma_2(X_2))$$

$$\hat{F}_{12}^{(\gamma_1,\gamma_2)}(W_1 \otimes W_2, p_{UX_1X_2}) := \sum_S \alpha_S H(X_{1S}, X_{2S}) - \mathrm{E}(\gamma_1(X_1)) - \mathrm{E}(\gamma_2(X_2))$$

Conjecture: If a functional $\sum_S \alpha_S H(Y_S)$ satisfies the local tensorization property, then it also satisfies the global tensorization property.

References

[1] A. Gohari and C. Nair, *Outer bounds for multiuser settings: the auxiliary receiver approach*, IEEE Transactions on Information Theory, **68** (2022), 701–736.

[2] V. Anantharam, A. Gohari and C. Nair, *On the evaluation of Marton's inner bound for two-receiver broadcast channels*, IEEE Transactions on Information Theory, **65** (2019), 1361–1371.

[3] C. Nair, *On Marton's achievable region: local tensorization for product channels with a binary component*, 2020 Information Theory and Applications Workshop (ITA) (2020), 1–7, doi: 10.1109/ITA50056.2020.9244997.

Source Coding with Information Obfuscation

Ligong Wang

(joint work with Gregory Wornell)

A memoryless source sequence X^n contains some sensitive attributes, which are modeled as a sequence of random variables S^n that have some joint distribution with the source. The encoder must perform lossy compression on X^n in such a way that its output is nearly independent of S^n. We characterize the minimum achievable distortion on a discrete source under such a requirement, and provide necessary and sufficient conditions under which nontrivial encoding is possible.

Open problems:

1. Relax the "information obfuscation" constraint to an "information masking" constraint where, instead of being nearly independent, U^n and S^n are allowed to have a limited amount of mutual information.

2. Find out if there exist direct links between various notions of "common information" to the studied problem or to problems on the "privacy funnel." The latter is a class of problems where maximization is sought of the (single-letter) mutual information between a chosen random variable U and a given random variable X, while minimizing the mutual information between U and another random variable S, where S has a given joint distribution with X.

References

[1] G. Wornell and L. Wang, *Source coding with information obfuscation*, in prepration (2022).

Takeaways from the Guessing Problem

Robert Graczyk

(joint work with Amos Lapidoth, Yiming Yan, Neri Merhav, Christoph Pfister)

We consider two variations on the Massey-Arikan guessing problem that cannot be solved directly by reduction to a data compression problem. Instead, a restriction-of-type-class argument and a change-of-measure argument are employed; both are generic and may prove useful when classical information-theoretic arguments fall short.

Open problem:

As demonstrated in [1, 2] below, there is an intimate relation between guessing a chance variable X using the least number of guesses in expectation, and compressing X using the least number of bits. Loosely speaking, the analysis in [1, 2] suggests that any guessing problem can be reduced to a corresponding compression problem, with the latter solvable using classical information theoretic techniques. We propose verifying this conjecture as a key open problem in guessing. If true, the vast research body on compression should readily yield corresponding results on guessing.

References

[1] R. Graczyk, A. Lapidoth and Y. Yan, *Guessing a tuple*, 2021 IEEE International Symposium on Information Theory (ISIT) (2021), 1997–2001. doi: 10.1109/ISIT45174.2021.9517798

[2] R. Graczyk, A. Lapidoth, N. Merhav and C. Pfister, *Guessing based on compressed side information*, IEEE Transactions on Information Theory, to appear (2022). doi 10.1109/TIT.2022.3158475, arXiv:2106.13341.

Universal Graph Compression: Stochastic Block Models

Lele Wang

(joint work with Alankrita Bhatt, Ziao Wang, Chi Wang)

Motivated by prevalent data science applications of processing and mining large-scale graph data such as social networks, web graphs, and biological networks, as well as the high I/O and communication costs of storing and transmitting such data, this work investigates lossless compression of data appearing in the form of a labeled graph. An efficient universal graph compression scheme is proposed, which does not depend on the underlying statistics/distribution of the graph model. For graphs generated by a stochastic block model, which is a widely used random graph model capturing clustering effects in social networks, the proposed scheme achieves the optimal theoretical limit of lossless compression without the need to know edge probabilities, community labels or the number of communities. The key ideas in establishing universality for stochastic block models include: 1) block decomposition of the adjacency matrix of the graph; and 2) generalization of the Krichevsky-Trofimov probability assignment, which was initially designed for i.i.d. random processes. In four benchmark graph datasets (protein-to-protein interaction, LiveJournal friendship, Flickr, and YouTube), the compressed files from competing algorithms (including CSR, Ligra+, PNG image compressor, and Lempel-Ziv compressor for two-dimensional data) take 2.4 to 27 times the space needed by the proposed scheme.

Open problem:

In our work, the submatrix decomposition is asymptotically i.i.d. for block size k with $\omega(1) \leq k \leq o(n)$, and the compression algorithm is universal for $k \leq \sqrt{\delta \log n}$, with n being the size of the adjacency matrix. For $k > \sqrt{\delta \log n}$, finding a universal compressor for large/unknown alphabets is an open problem.

References

[1] A. Bhatt, Z. Wang, C. Wang and L. Wang, *Universal block compression: Stochastic block models*, (2021). arXiv:2006.02643 [cs.IT]

[2] A. Bhatt, Z. Wang, C. Wang and L. Wang, *Universal block compression: Stochastic block models*, 2021 IEEE International Symposium on Information Theory (ISIT) (2021), 3038–3043.

On the Suboptimality of Random Binning for Distributed Hypothesis Testing

Shun Watanabe

We investigate the quantization-and binning scheme known as the Shimokawa-Han-Amari (SHA) scheme for distributed hypothesis testing. We develop tools to evaluate the critical rate attainable by the SHA scheme. For a product of binary symmetric double sources, we provide a sequential method that improves on the SHA scheme.

Open problem:

Determine the optimal distributed hypothesis testing scheme for cases other than testing against (conditional) independence. Is the Shimokawa-Han-Amari testing scheme optimal when both the null hypothesis and the alternative hypothesis are binary symmetric double sources?

References

[1] S. Watanabe, *On suboptimality of random binning for distributed hypothesis testing*, 2022 International Symposium on Information Theory (2022).

Shannon Entropy and Bipartite Graphs

Igal Sason

Combinatorial properties of bipartite graphs are of considerable interest in graph theory, combinatorics, modern coding theory, and information theory. In light of the key role of Shannon theory in combinatorial and graph-theoretic aspects, this talk considers new entropy-based proofs of some known, or otherwise refined, combinatorial bounds for bipartite graphs. These include upper bounds for the number of the independent sets and lower bounds on the number of walks of a given length in bipartite graphs. The proofs of these combinatorial results rely on basic properties of the Shannon entropy, and some open problems will be addressed.

Open problems:

1. Our information theoretic bound on the number of independent sets is tight if G is a bipartite graph that is regular on one side, and it may be completely irregular on the other side. This extends the information theoretic approach by Kahn for regular bipartite graphs (on both sides). In the most general case of an arbitrary bipartite graph (which is irregular on both sides), our information theoretic approach gives a loose bound. It is left for future work to study if our analysis can be adapted to yield a tight bound on the number of independent sets of a bipartite graph when both sides of the graph are irregular. Also, it is left to examine a possible adaptation of our analysis to get bounds on the size of a random independent set.

2. We provide in our talk improved and refined lower bounds on the number of walks of a given length for bipartite graphs with given degree distributions. It is left for a future study to examine an adaptation of our analysis to yield similar bounds on the number of k-length trails (i.e., walks with no repeated edges), and k-length paths (i.e., walks with no repeated edges and vertices).

References

[1] J. Kahn, *An entropy approach to the hard-core model on bipartite graphs*, Combinatorics, Probability and Computing, **10** (2001), 219–237.

[2] I. Sason, *A generalized information-theoretic approach for bounding the number of independent sets in bipartite graphs*, Entropy, **23** (2021), 1–14.

[3] I. Sason, *Entropy-based proofs of combinatorial results on bipartite graphs*, Proceedings of the 2021 IEEE International Symposium on Information Theory (2021), 3225–3230.

Forms of Uncertainty in Communication Networks: Results and Open Problems

Yossef Steinberg

(joint work with Uzi Pereg, Dor Itzhak, Wasim Huleihel)

Channels with uncertainty have been an extensive topic of research since their introduction in the 1960s. Two main models are common: the compound channel, in which the communication channel depends on a parameter that is chosen once and remains fixed during transmission; and the arbitrarily varying channel, where the parameter varies arbitrarily during transmission, with or without a statistical law. Since their introduction in the context of single-user channels, the models have been extended to communication networks, with various assumptions on the form of uncertainty. This talk will provide an overview of the state-of-the-art in network communication systems with uncertainty, and introduce open problems in these topics.

Open problems:

1. It is well-known that for the AVC with causal side information (state sequence) at the encoder, nonsymmetrizability in the single-letter sense is a sufficient condition for the deterministic code capacity to coincide with the random code capacity. In addition, there are examples of AVC with causal SI that are symmetrizable, yet their deterministic code capacity if positive. *Problem*: Find a better characterization of the conditions under which the random and deterministic code capacities coincide. Is it necessarily a multiletter condition?

2. In [2] below, inner and outer bounds on the (random and deterministic code) capacity region of the broadcast AVC were introduced. The bounds differ only in the order of (infinite) intersection and union operations: in the outer bound the intersections are the outer operation, and in the inner bound the intersections are the inner operation. A natural question that arises is: Under what conditions the order of intersections and unions can be interchanged, thus yielding the capacity region? This can be viewed as a set extension of the minimax theorem in convex optimization. In [2] below sufficient conditions were derived under which the order can be interchanged. We still do not have a complete understanding of this problem, and in particular, whether these sufficient conditions are also necessary. Any result on necessary conditions would be interesting (even if they are not tight).

References

[1] U. Pereg and Y. Steinberg, *The arbitrarily varying channel under constraints with side information at the encoder*, IEEE Transactions on Information Theory, **65** (2019), 861–887.

[2] U. Pereg and Y. Steinberg, *The arbitrarily varying broadcast channel with causal side information at the encoder*, IEEE Transactions on Information Theory, **66** (2020), 757–779.

[3] D. Itzhak and Y. Steinberg, *The broadcast channel with degraded message sets and unreliable conference*, IEEE Transactions on Information Theory, to appear.

Minoration via Mixed Volumes and Cover's Problem for General Channels

Jingbo Liu

We give a complete solution to an open problem of Thomas Cover in 1987 about the capacity of a relay channel in the general discrete memoryless setting without any additional assumptions. The key step in our approach is to lower bound a certain soft-max of a stochastic process by convex geometry methods, which is based on two ideas. First, the soft-max is lower bounded in terms of the supremum of another process, by approximating a convex set with a polytope with bounded number of vertices. Second, using a result of Pajor, the supremum of the process is lower bounded in terms of packing numbers by means of mixed-volume inequalities (Minkowski's first inequality).

Open problems:

1. *An information inequality*: Motivated by problems in multiuser information theory and analogous results in empirical processes, we conjecture the following inequality: there is a constant $c > 0$ such that for any distribution P_X on $\mathbb{R}^n$, we have the following bound relating the mutual information and the partition function:

$$I(X+G;X) \le c\,\mathbb{E}_G\left[\log \int e^{\langle G,X\rangle} dP_X\right], \tag{1}$$

where G is standard normal in $\mathbb{R}^n$ independent of X and $\langle\cdot,\cdot\rangle$ denotes the inner product. If this inequality is true, some interesting consequence in multiuser information theory can be derived.

2. *A conjecture from the literature*: In solving Cover's problem [1] we applied a result of Pajor which upper bounds the packing number using the expected maximum of a stochastic process, where the process is defined using a general convex cone volume measure. Pajor's bound is dimension-dependent, and is sufficient for solving Cover's Problem. However, it has been conjectured that a certain dimension-independent minoration inequality exists for general log-concave measures; see the following paper and the references therein:
"Generalized Dual Sudakov Minoration via Dimension Reduction - A Program," S Mendelson, E Milman, G Paouris, 2016.

References

[1] J. Liu, *Minoration via mixed volumes and Cover's problem for general channels*, Probability Theory and Related Fields (2022).

[2] J. Liu, R. v. Handel and S. Verdú, *Second-order converses via reverse hypercontractivity*, Mathematical Statistics and Learning, **2** (2020), 103–163.

[3] J. Liu and A. Özgür, *Capacity upper bounds for the relay channel via reverse hypercontractivity*, IEEE Transactions on Information Theory, **66** (2020), 5448–5455.

Gaussian Comparisons

Thomas Courtade

(joint work with Efe Aras)

It is well-known that the entropy power inequality has an equivalent statement as a Gaussian comparison: for independent random vectors X_1, X_2, we have $h(X_1+X_2) \ge h(Z_1+Z_2)$ for independent isotropic Gaussian vectors Z_1, Z_2 with $h(Z_i) = h(X_i)$. In this talk, we explain how this comparison framework can be significantly generalized. Toward this end, let $X_1, \dots, X_k$ be marginally-specified random vectors (of varying dimensions) with finite second moments, and write $X \in \Pi(X_1, \dots, X_k)$ to denote that the vector X is a coupling of the X_i's. For any non-negative scalars $(d_j)_{j=1}^m$ and linear maps $(B_j)_{j=1}^m$ with suitably dimensioned domains, it can be shown that there exist Gaussian random vectors $Z_1, \dots, Z_k$ with $\dim(Z_i) = \dim(X_i)$ and $h(Z_i) = h(X_i)$ such that

$$\max_{X \in \Pi(X_1,\dots,X_k)} \sum_{j=1}^m d_j h(B_j X) \geq \max_{Z \in \Pi(Z_1,\dots,Z_k)} \sum_{j=1}^m d_j h(B_j Z). \tag{1}$$

We'll explore the many consequences of this inequality in the talk. To state a few examples, it extends the one-dimensional version established by Aras and Courtade [2], gives an entropic strengthening of the Brunn–Minkowski inequality, and implies the Gaussian saturation property of the forward-reverse Brascamp–Lieb inequalities on Euclidean spaces due to Courtade and Liu [3]. Of note, (1) admits a self-strengthening that allows one to restrict the maxima to be taken over information-constrained couplings. The singleton set consisting of the independent coupling is one special case; as such, (1) also implies the Anantharam–Jog–Nair inequality [1], and the Zamir–Feder inequality in the instance where each X_i is one-dimensional.

Open problem:

While (1) gives a concise unification of many entropic and geometric inequalities found in the literature, there remain several that are not obviously implied. To give one easily-stated example, if X, Y, Z are independent random vectors of the same dimension n, and Z is Gaussian, it is known that

$$N(X+Z)N(Y+Z) \geq N(X)N(Y) + N(Z)N(X+Y+Z),$$

where $N(\cdot) := e^{2h(\cdot)/n}$ denotes the entropy power of its argument. This does not appear to be explained by (1). Is there a common framework that does?

References

[1] V. Anantharam, V. Jog, and C. Nair. Unifying the Brascamp-Lieb inequality and the entropy power inequality. *arXiv:1901.06619*, 2019.

[2] E. Aras and T. A. Courtade. Sharp Maximum-Entropy Comparisons. in Proc. of the 2021 IEEE International Symposium on Information Theory (ISIT). IEEE, 2021.

[3] T. A. Courtade and J. Liu. Euclidean forward-reverse Brascamp–Lieb inequalities: Finiteness, structure, and extremals. The Journal of Geometric Analysis 31.4 (2021): 3300–3350.

Automated Theorem Proving for Network Information Theory

Cheuk Ting Li

We describe an algorithm for automatically generating human-readable proofs of inner and outer bounds in network information theory, based on the linear programming approach to verifying linear information inequalities by Zhang and Yeung, and an existential version of the concept of linear information inequalities. Using the theorems in the textbook Network Information Theory by El Gamal and Kim as a benchmark, the algorithm successfully proves 30.2% (32/106) of the theorems in the book. This software is potentially useful for both research and pedagogical purposes.

Open problem:

An open problem in this area is the decidability of the linear information inequality, i.e., whether there exists an algorithm that can determine whether a given linear information inequality holds for all sequences of random variables. This problem has applications in network coding and secret sharing.

References

[1] C. T. Li, *An automated theorem proving framework for information theoretic results*, 2021 IEEE International Symposium on Information Theory (ISIT) (2021), 2750-2755.

Near-Optimal Coding for Many-User Multiple Access Channels

Ramji Venkataraman
(joint work with Kuan Hsieh, Cynthia Rush)

We consider communication over the Gaussian multiple-access channel in the asymptotic regime where the number of users grows linearly with the code length. We propose efficient coding schemes based on random linear models with approximate message passing (AMP) decoding, and study the tradeoff between energy-per-bit and achievable user density for a fixed target error rate. It is demonstrated that in the large system limit, a spatially coupled coding scheme with AMP decoding achieves near-optimal tradeoffs for a wide range of user densities.

Open problems:

This work discussed achievable regions and converse bounds for the Gaussian multiple-access channel in the 'many-user' regime, where the number of users grows proportionally to the code length. There is a gap between the inner and outer bounds – it is likely that the inner bound (achievable region) is loose, and possibly the outer bound as well. A natural open problem is to this close this gap by obtaining improved bounds in both directions. Another open problem is to obtain inner and outer bounds for the problem when practical constraints such as random access are included.

References

[1] K. Hsieh, C. Rush and R. Venkataramanan, *Near–ptimal coding for many-user multiple access channels*, IEEE Journal on Selected Areas in Information Theory **3** (2022), 21–36.

Distributed Hypothesis Testing under Expected Resource Constraints

Michèle Wigger
(joint work with Mustapha Hamad, Mireille Sarkiss, Sadaf Salehkalaibar)

This talk presents an information-theoretic study of the rate-performance tradeoff in multi-hop distributed hypothesis testing systems as encountered in the context of the Internet of Things where communication is of low power and thus of low range. A particular focus is on systems with expected communication-rate constraints, under which different applications can share the available resources such as bandwidth. As we show in this talk, in such systems an interesting tradeoff arises between the optimal Type-II error exponents that are simultaneously achievable at the different decision centres, meaning that one has to sacrifice the performance at certain decision centres to attain the best possible performance at others. The optimal tradeoff is achieved by a novel scheme that multiplexes different instances of the optimal scheme under variable-length coding. Converse results are also presented, showing that no other scheme performs better than the proposed multiplexed scheme.

Open problems:

The most prominent open problem in distributed hypothesis testing is the characterization of the maximum type-II error exponent of the simple single-sensor, single decision center problem where the sensor communicates with the decision center over a rate-limited noise-free link. The problem is closed for special cases such as testing against independence, but not for general hypothesis testing. In our talk we considered a variation of this classical problem in that we considered a multi-hop scenario and also in that not the maximum rate but the expected rate of communication between adjacent sensors and between sensor and decision center are limited. The optimal type-II error exponent for general hypotheses (not necessarily testing against independence) is still open. Another interesting open problem is to determine the optimal type-II error exponent for an interactive scenario where two sensors can interactively communicate with each other before deciding on the hypothesis.

References

[1] M. Hamad, M. Wigger, and S. Sarkiss, *Optimal exponents in cascaded hypothesis testing under expected rate constraints*, 2021 IEEE Information Theory Workshop (2021).

[2] S. Salehkalaibar and M. Wigger, *Distributed hypothesis testing with variable-length coding*, IEEE Journal on Selected Areas in Information Theory **1** (2020), 681–694.

The Multicast Transmission Approach to Converse Theorems for Mismatched Decoding

Anelia Somekh-Baruch

The question of characterizing the fundamental limits of channel coding when the receiver uses a suboptimal decoding rule, also known as “mismatched decoding”, has been a longstanding open problem. This question has many applications in communications, information theory and computer science; for example, the zero-error capacity of a channel is a special case of a mismatched channel coding setup. This talk provides a brief overview of the problem, and presents the principles and refinements of a new bounding technique called ”the multicasting approach” which yields several nontrivial computable (single-letter) converse bounds. Further, exciting new developments in this research area will be described with a discussion of conditions for the tightness of the bounds as well as equivalence classes of isomorphic channel-metric pairs that share the same fundamental limits.

Open problems:

The work presented relates to the following two open problems.

1. Find a single-letter expression for the mismatch capacity of the discrete memoryless channel, i.e., the supremum of rates for which there exists a sequence of codebooks with vanishing average probability of error when the decoder uses a suboptimal additive metric q. The zero error capacity introduced by Shannon is an instance of this problem.

2. Find an exact single-letter expression for the reliability function of the discrete memoryless channel (with or without mismatch), i.e., the supremum of exponents E for which there exists a sequence of rate-R block-codes such that the resulting average probability of error in channel coding $P(error)$ where the decoder uses metric q vanishes at least as fast as e^{-nE}. The reliability function is known for the maximum likelihood metric for a certain range of high rates, but it is unknown for low rates (except zero-error codes) and for general additive metrics q.

References

[1] A. Somekh-Baruch, *A single-letter upper bound on the mismatch capacity via multicast transmission*, IEEE Transactions on Information Theory, **68** (2022), 2801–2812.

[2] A. Somekh-Baruch, *Robust multicasting and an upper bound on the mismatch capacity of the DMC*, 2021 IEEE International Symposium on Information Theory (ISIT) (2021), 1130–1135.

[3] A. Somekh-Baruch, *New converse bounds on the mismatched reliability function and the mismatch capacity using an auxiliary genie receiver* (2022). arXiv:2203.08524[cs.IT]

Impossibility Results for Planted Matching Problems

Yihong Wu

(joint work with Jian Ding, Jiaming Xu, Dana Yang, Haoyu Wang and Israel Yolou)

Motivated by applications such as particle tracking, a statistical model was proposed by [Chertkov *et al* 2010] for the bipartite matching problem, where the goal was to reconstruct a perfect matching planted in a randomly weighted bipartite graph, whose planted and unplanted edge weights are independently drawn from two different distributions. We determine the sharp threshold at which the optimal reconstruction error (fraction of mismatched nodes) exhibits a phase transition from imperfect to perfect. Furthermore, in the special case of exponential weight distributions which is the planted version of the celebrated random assignment model, this phase transition is shown to be of infinite-order, confirming the conjecture by physicists in [Semerjian, Sicuro, and Zdeborov 2020]. More recent results on geometric models will also be discussed. The common proof technique for both models is by analyzing the typical behavior of the posterior distribution. We show that, below the threshold, the posterior distribution is concentrated away from the hidden matching by constructing exponentially many augmenting cycles. This will be contrasted with the mutual information method (based on capacity and rate-distortion function calculations) which yields suboptimal results.

Open problem:

The most significant open problem in the area of graph matching at large is to theoretically justify the success of a natural and commonly used convex relaxation. Specifically, consider a uniform random permutation π on $[n] = \{1, ..., n\}$ and two symmetric $n \times n$-random matrices A and B (with iid, e.g., standard normal or Bernoulli, entries in the upper triangular part), such that $(A_{ij}, B_{\pi(i)\pi(j)} : 1 \leq i < j < n)$ are iid pairs with some correlation coefficient ρ. Consider the following convex program: $\min ||AX - XB||_F^2$ s.t. X is doubly stochastic. Show that when the correlation ρ exceeds some constant, the solution X^* can be rounded to a permutation matrix that coincides with high probability of the ground truth π.

References

[1] J. Ding, Y. Wu, J. Xu, D. Yang, *The planted matching problem: Sharp threshold and infinite-order phase transition*, Probability Theory and Related Fields, in review (2021). https://arxiv.org/abs/2103.09383

[2] H. Wang, Y. Wu, J. Xu, I. Yolou, *Random graph matching in geometric models: the case of complete graphs*, Conference on Learning Theory, in review (2022). https://arxiv.org/abs/2202.10662

Uniqueness of BP fixed point for Ising models

Yury Polyanskiy
(joint work with Qian Yu)

In the study of Ising models on large locally tree-like graphs, in both rigorous and nonrigorous methods, one is often led to understanding the so-called belief propagation distributional recursions and its fixed point (also known as Bethe fixed point, cavity equation, 1RSB, etc.). In this work, we prove there is at most one nontrivial fixed point for Ising models for both zero and certain random external fields. As a concrete example, consider a sample A of Ising model on a rooted tree. Let B be a noisy version of A obtained by independently perturbing each spin as follows: B_v equals to A_v with some small probability δ and otherwise taken to be a uniform $+-1$ (alternatively, 0). We show that the distribution of the root spin A_ρ conditioned on values B_v of all vertices v at a large distance from the root is independent of δ and coincides with $\delta = 0$. Previously this was only known for sufficiently "low-temperature" models. Our proof consists of constructing a metric under which the BP operator is a contraction (albeit nonmultiplicative). The proof is technically rather simple. This simultaneously closes the following 5 conjectures in the literature: uselessness of global information for a labeled 2-community stochastic block model, or 2-SBM (Kanade-Mossel-Schramm, 2014); optimality of local algorithms for 2-SBM under noisy side information (Mossel-Xu, 2015); independence of robust reconstruction accuracy to leaf noise in broadcasting on trees (Mossel-Neeman-Sly, 2016); boundary irrelevance in broadcasting on trees (Abbe-Cornacchia-Gu-Polyanskiy, 2021); and characterization of entropy of community labels given the graph in 2-SBM (ibid).

Open problem:

Suppose that n vertices are randomly colored using q colors. Given the coloring, a random graph is constructed by connecting any two vertices by an edge independently with probability a/n if the vertices have the same color or b/n otherwise. The goal is given a graph to reconstruct the original vertex coloring (up to a permutation of colors, of course). It is known that for certain values of (q, a, b), it is not possible to reconstruct the original coloring better than just randomly partitioning the vertices and disregarding the observed graph. An open problem is to find the precise boundary of bad values of (q, a, b) from good ones.

References

[1] E. Mossel, J. Neeman and A. Sly, *Reconstruction and estimation in the planted partition model*, Probability Theory and Related Fields **162** (2015), 431–461.

[2] E. Mossel, J. Neeman and A. Sly, *Belief propagation, robust reconstruction and optimal recovery of block models*, Proceedings of The 27th Conference on Learning Theory, ser. Proceedings of Machine Learning Research, M. F. Balcan, V. Feldman, and C. Szepesvari, Eds. **35** (2014), 356–370.

[3] V. Kanade, E. Mossel and T. Schramm, *Global and local information in clustering labeled block models*, IEEE Transactions on Information Theory, **62** (2016), 5906–5917.

Min-Entropy Inequalities on Algebraic Structures

Mokshay Madiman
(joint work with James Melbourne, Cyril Roberto, Peng Xu)

For the Shannon entropy, a vast array of inequalities are known for linear images of random vectors in a Euclidean space, beginning with the entropy power inequality and culminating in very general inequalities due to Courtade and Liu (that are dual to the Brascamp-Lieb functional inequalities). At the heart of these inequalities for Shannon entropy is comparison with Gaussians. We explore an analogous class of inequalities for the min-entropy, where comparison with uniform distributions on Euclidean balls plays the central role. These distributions emerge because uniform measures are extreme points in sets of measures with compact support and fixed min-entropy, combined with the power of spherically symmetric rearrangements. Consequences include a general inequality for the min-entropy of linear images that simultaneously sharpens and generalizes earlier results due to Bobkov-Chistyakov and Livshyts-Paouris-Pivovarov. Moreover, the technique we use is flexible and carries over to nonEuclidean spaces with some algebraic structure – for example, we develop min-entropy inequalities on locally compact groups, and present special cases such as the integers where our results can be seen as generalizations of the Littlewood-Offord-Erdos Theorem.

Open problems:

We presented a min-entropy power inequality for sums of N independent random vectors in R^d, with a constant that is independent of N but sharp for any fixed dimension d. This inequality potentially can be tightened by the use of a constant that depends on both the number N of summands and d – the determination of such is open, but involves delicate geometric questions about certain projections of Cartesian products of Euclidean balls. More generally, one can also ask about Rényi entropy power inequalities for orders other than $0, 1$ and ∞ (which are the only cases where sharp inequalities are known). In particular, it is unknown what extremal distributions look like when we compare the Rényi entropies of order p of X, Y, and $X + Y$, when X and Y are independent random vectors and p is not one of the special orders. A conjecture that a natural class of generalized Gaussians should be the extremal distributions was posed in Madiman-Wang (2014), but was shown to be false for $p = 2$ (though in a very delicate fashion) by Jaye-Livshyts-Paouris-Pivovarov (2019).

References

[1] M. Madiman, J. Melbourne and P. Xu, *Forward and reverse entropy power inequalities in convex geometry*, Convexity and Concentration, E. Carlen, M. Madiman and E. Werner (eds.), The IMA Volumes in Mathematics and its Applications, Springer, **161** (2017), 427–485. arXiv:1604.04225

[2] M. Madiman, J. Melbourne and P. Xu, *Rogozin's convolution inequality for locally compact groups*, (2017). arXiv:1705.00642

[3] M. Madiman, J. Melbourne and C. Roberto, *Bernoulli sums and Rényi entropy inequalities*, Bernoulli, to appear (2022). arXiv:2103.00896

Lower Bounds for Inference Under Information Constraints

Himanshu Tyagi
(joint work with Jayadev Acharya, Clément Canonne, Ziteng Sun)

For the past four years, starting from ITA 2018 in San Diego, we have been developing information theoretic lower bounds for statistical estimation and testing when only limited information is available about each independent sample. This limitation on information can be, for instance, due to a communication constraint or a privacy constraint. In fact, we use an abstract model to represent information constraints using a channel family, which covers many other constraints beyond communication and privacy. How difficult is it to solve statistical inference problems under such constraints? In this talk, we will summarize our progress so far in answering this question, highlighting the results for discrete distributions and high-dimensional continuous distributions, for both interactive and noninteractive communication models. This is work in progress and raises several interesting open problems.

Open problem:

Optimal Gaussian mean testing under information constraints: We need new algorithms as well as a new approach for lower bounds. We have been trying to resolve this using our approach, with only partial success so far.

References

[1] J. Acharya, C. Canonne, Z. Sun and H. Tyagi, *Unified lower bounds for interactive high-dimensional estimation under information constraints*, (2020). arXiv: 2010.06562

On Infeasibility Proofs via Mutual-f-Information

Amin Gohari
(joint work with Mohammad Saeed Masiha, Mohammad Hossein Yassaee)

Shannon's mutual information is widely utilized in writing upper bounds for capacity regions owing to its nice properties such as nonnegativity and chain rule. The question arises whether other measures of information can be used instead of mutual information to write upper bounds for capacity regions. This question has been answered affirmatively for a few examples. In particular, measures of information based on maximal correlation or hypercontractivity have been shown to be helpful in problems involving distributed sources. In this presentation, we focus on a natural extension of Shannon's mutual information, namely the mutual-f-information between two random variables X and Y defined as $I_f(X;Y) = D_f(p_{X,Y}||p_Xp_Y)$ where f is a convex function and D_f is the f-divergence of Ali and Silvey, Csiszár and Morimoto. We discuss whether mutual-f-information can be used to derive new infeasibility results in network information theory; we provide some positive and negative answers to this question. In particular, even though mutual-f-information does not satisfy the chain rule property, given random variables $X_1, \ldots, X_n$, certain useful inequalities involving mutual-f-information terms between different subsets of X_is hold (if the function f is restricted to a suitable class of convex functions). These inequalities allow us to derive explicit and easy-to-compute bounds for the rate-distortion function for a given blocklength n. We also discuss whether mutual-f-information can be used to obtain new dependence-balance bounds (for two-way channels or MAC with feedback) and give a negative answer to this question.

Open problems:

A measure of dependence $\rho(A;B)$ assigns a nonnegative real number to a joint distribution $p(a,b)$. It satisfies the data processing property. Moreover, $\rho(A;B) = 0$ if A and B are independent.

(i) Prove or disprove: Shannon's mutual information is the only measure of dependence $\rho(.;.)$ for which $\rho((A,B);C) - \rho(A;C)$ and $\rho(A;(B,C)) - \rho(A;C)$ are concave in $p(a,c)$ for any fixed $p(b|a,c)$.

(ii) Can we characterize (or find new examples of) measures of dependence satisfying $\rho(X;Y) = \rho(Y;X)$, and $\rho(X;Y)$ being concave in $p(x)$ for every fixed $p(y|x)$?

References

[1] M. M. Mojahedian, S. Beigi, A. Gohari, M. H. Yassaee and M. R. Aref, *A correlation measure based on vector-valued L_p-norms*, IEEE Transactions on Information Theory, **65** (2019), 7985–8004.

[2] S. Beigi and A. Gohari, *Phi-entropic measures of correlation*, IEEE Transactions on Information Theory, **64** (2018), 2193-2211.

Reporter: Prakash Narayan

Participants

Sagnik Bhattacharya
Electrical Engineering Department
College of Engineering
University of Maryland
2353 A.V. Williams Bldg.
8223 Paint Branch Drive
College Park 20743
UNITED STATES

Prof. Dr. Holger Boche
LST für Theoretische
Informationstechnik
Technische Universität München
(LTI)
Theresienstraße 90/IV
80333 München
GERMANY

Prof. Dr. Thomas Courtade
Dept. of Electrical Engineering and
Computer Sciences
University of California at
Berkeley
265 Cory Hall
Berkeley, CA 94720
UNITED STATES

Prof. Dr. Valérie Girardin
Laboratoire de Mathématiques Nicolas
Oresme
Université de Caen Normandie
14032 Caen Cedex
FRANCE

Prof. Dr. Amin Gohari
Tehran Institute for Advanced Studies
Tehran 11155-8639
IRAN, ISLAMIC REPUBLIC OF

Dr. Robert Graczyk
Telecom Paris
16, Bergstrasse
8044 Zürich
SWITZERLAND

Dr. Albert Guillen i Fabregas
Department of Engineering
Information Engineering Division
University of Cambridge
Trumpington Street
Cambridge CB2 1PZ
UNITED KINGDOM

Prof. Dr. Ioannis Kontoyiannis
Centre for Mathematical Sciences
University of Cambridge
Wilberforce Road
Cambridge CB3 0WB
UNITED KINGDOM

Prof. Dr. Amos Lapidoth
ETH Zürich
D-ITET
Institut für Signal- und
Informationverarbeitung
Sternwartstrasse 7
8092 Zürich
SWITZERLAND

Dr. Cheuk Ting Li
The Chinese University of Hong Kong
Department of Information Engineering
Shatin, N.T., Hong Kong N. T.
CHINA

Prof. Dr. Jingbo Liu
Dept. of Statistics
University of Illinois
Room 228, Computing Applications
Building
605 E Springfield Ave
Champaign IL 61820
UNITED STATES

Prof. Dr. Mokshay Madiman
Department of Mathematical Sciences
University of Delaware
501 Ewing Hall
Newark, DE 19716-2553
UNITED STATES

Ajay Nageswaran
Electrical Engineering Department
College of Engineering
University of Maryland
2353 A.V. Williams Bldg.
8223 Paint Branch Drive
College Park, MD 20742
UNITED STATES

Prof. Dr. Chandra Nair
Dept. of Information Engineering
The Chinese University of Hong Kong
Shatin, N.T., Hong Kong N. T.
CHINA

Prof. Dr. Prakash Narayan
Electrical & Computer Engineering
Department
College of Engineering
University of Maryland
2353 A.V. Williams Bldg.
8223 Paint Branch Drive
College Park, MD 20742
UNITED STATES

Baohua Ni
ETH Zürich
D-ITET
Institut für Signal- und
Informationverarbeitung
Sternwartstrasse 7
8092 Zürich
SWITZERLAND

Prof. Dr. Yury Polyanskiy
Massachusetts Institute of Technology
Dept. of Electrical Engineering
and Computer Science
Cambridge, MA 02139
UNITED STATES

Prof. Dr. Igal Sason
Department of Electrical and Computer
Engineering and Faculty of Mathematics
TECHNION
Israel Institute of Technology
Haifa 3200003
ISRAEL

Prof. Dr. Anelia Somekh-Baruch
Faculty of Engineering
Bar-Ilan University
Ramat-Gan 5290002
ISRAEL

Prof. Dr. Yossef Steinberg
Department of Electrical and Computer
Engineering
TECHNION
Haifa 32000
ISRAEL

Prof. Dr. Aslan Tchamkerten
Telecom Paris
19, Place de la Marguerite
91477 Paris
FRANCE

Prof. Dr. Emre Telatar
Ecole Polytechnique Fédérale de Lausanne (EPFL)
1015 Lausanne
SWITZERLAND

Prof. Dr. Antonia Tulino
Dept. Ingeneria Elettronica e delle Telecommunicazioni
Universita degli Studi di Napoli Federico II
80125 Napoli
ITALY

Prof. Dr. Himanshu Tyagi
Department of Electrical Communication Engineering
Indian Institute of Science
560012 Bangalore, Bengaluru
INDIA

Dr. Ramji Venkataramanan
Department of Engineering
Information Engineering Division
University of Cambridge
Trumpington Street
Cambridge CB2 1PZ
UNITED KINGDOM

Prof. Dr. Lele Wang
Department of Electrical and Computer Engineering
University of British Columbia
5500 Main Mall
Vancouver BC V6T 1Z4
CANADA

Dr. Ligong Wang
ETIS, UMR 8051
6, avenue du Ponceau
95000 Cergy-Pontoise
FRANCE

Prof. Dr. Shun Watanabe
Tokyo University of Agriculture and Technology
Department of Computer and Information Sciences
2-24-16 Nakamachi Koganei-shi
184-8588 Tokyo 183-8538
JAPAN

Prof. Dr. Michèle Wigger
Telecom Paris, LTCI
19 Place Marguerite Perey
91120 Palaiseau
FRANCE

Prof. Dr. Yihong Wu
Department of Statistics and Data Science
Yale University
24 Hillhouse Ave
New Haven, CT 06511
UNITED STATES

Yiming Yan
ETH Zürich
D-ITET
Institut für Signal- und Informationverarbeitung
Sternwartstrasse 7
8092 Zürich
SWITZERLAND