Interview with Martin Raussen

Christian F. Skau

Christian Skau (Norwegian University of Science and Technology, Trondheim, Norway) and Martin Raussen (Aalborg University, Denmark) teamed up to interview the Abel Prize recipients, from the first one in 2003 and until 2016, when Bjørn Dundas (University of Bergen, Norway) took over from Martin Raussen. Just before the opening of the 29th Nordic Congress of Mathematicians, organized in cooperation with the European Mathematical Society at Aalborg, Denmark, a session took place with Christian interviewing Martin. The text below is a slightly edited version of the recorded interview.[1]

Martin Raussen.

Nordic Congress of Mathematicians

Christian F. Skau: *Martin, the two of us have participated in many interviews together. This time there is nobody else to ask questions but me! We have a double special occasion: This afternoon the 29th Nordic Congress of Mathematicians, on the occasion of the 150 years anniversary of the Danish Mathematical Society, will get started here in Aalborg, and after the summer you will retire from your position at Aalborg University. Could you tell us about your involvement with the Nordic Congress?*

Martin Raussen: I will try to do that, but before, let me thank you for coming at this special occasion. I am very grateful that you took the effort to come here from Oslo, where the two of us, for several years, have interviewed the Abel Prize recipients.

Before I answer your questions: Interviewing the Abel Prize recipients was like mingling with Champions League winners in soccer. I am not at all in that league. I would have difficulties to qualify for the Conference League! Therefore, I am very honoured to be granted this opportunity to be interviewed.

I started to work on getting the Nordic Congress to Aalborg around five years ago. In the beginning it took only little time, writing letters and applications. Forming a good and reliable team for the organization was essential. During the last year tasks accelerated. For the last month we have been very busy, indeed. But now we are happy that we can host more than 400 congress participants. There will be eight plenary talks at the House of Music, and we will have 29 special sessions in a university building very close by. The only thing that could be better, in my view, is the weather that has deteriorated quite a lot lately.

29TH NORDIC CONGRESS OF MATHEMATICIANS

[1] https://youtu.be/4_ab-SSKLg0

First mathematical experiences

CFS: *Let's now talk a little about you as a mathematician. My first question is: What kindled your interest and fascination with mathematics in the first place?*

MR: My late father was a high school teacher in mathematics and physics. He was very interested in the subject and talked to me, and also to my brothers and sisters, about mathematical topics. He tried to give us small challenges from time to time. The first problem I still remember came when I was still a small kid, maybe just started school, or maybe second grade. He asked me, in a concrete setting dealing with brown and white hens, a question that, in hindsight, can be phrased as two simple linear equations in two variables. I had no clue about how to tackle this problem, but the numbers were not that large. I could just experiment in my head, and I found the answer after a while. He commented that this was somewhat exceptional for a kid at that age, and I was proud.

The second instance that still is in my memory happened when I was a bit older. I had heard about the test to decide whether a number is divisible by three or by nine by just taking the sum of the digits. And I started to think: How about other division tests of that kind? I would never have been able to find out about division by seven at that time! But by experimenting just with two- or three-digit numbers, and those were the numbers that were in my mind at that time, I found out with 11 it's the alternate sum that does the job. It got me excited! I remember that I wrote a small note about my finding and distributed it to some of my friends in the school class. They were not really interested in it, but it was my first experience of mathematical success.

CFS: *That's a nice story! And a natural follow-up question: When did you discover that you really had special talent for mathematics?*

MR: Talent and interest are very much related. I was a little bit older, still under the influence of my father, who at that time got interested in Boolean algebra and related concepts with the advent of the first digital computers. He led a small workshop on Boolean algebra. I participated, and I got the impression that as soon as you have something that you can formulate in terms of Boolean algebra, then the computer can work for you.

I started to look at what happens when you take as operations the greatest common divisor and the smallest common multiple of natural numbers, respectively. This gives you a distributive lattice, but in most cases the divisors of a natural number do not form a Boolean algebra. It works only well for square-free numbers. In hindsight, this is clear, because then the lattice corresponds to the power set of the prime divisors, but I did not know that at the time. Anyway, I wrote a small note and even tried to get it published in a small educational journal. It didn't work out, but still, I remember that I could tell myself: "I can somehow contribute to something that resembles a little bit of research." All that happened some time before I started to study at the university.

A mathematical education

CFS: *Very nice. Then let's move to your mathematical training. Tell us about your mathematical education, starting with the Max-Planck-Gymnasium in Trier, and until you got your Ph.D. in mathematics at Georg August University in Göttingen.*

MR: That covers a range of years! I went to what's called Max-Planck-Gymnasium in Trier, in South West Germany. When people ask me about Trier, I answer: That is the city where Karl Marx was born! Apart from myself and many other people of course. The Max-Planck-Gymnasium, you can guess it from the name, was a high school where mathematics and natural sciences had a high level of esteem.

The other high schools in town either focused on old languages, Latin and Greek, or modern languages like French, which was important in Trier, being close to the borders of France and Luxembourg. At the Max-Planck-Gymnasium you were able to specialize in mathematical and physical directions, mainly during the last two or three years.

That gave me sort of a head start when starting at the University of the Saarland in Saarbrücken, around 100 kilometres south of Trier. I was only 17 years old then, and I started on an education in both mathematics and computer science, a subject that had just started a year earlier. In mathematics, we had to follow the standard curriculum: linear algebra and analysis. At first, I felt more at home in linear algebra because I had a rather algebraic mindset. I also liked to learn about Turing machines, computability, and formal languages in computer science.

Our analysis course consisted of three consecutive semesters that were given by a young professor at that time, Tammo tom Dieck, who had just got a professorship in Saarbrücken. He was a student of the famous topologist Dieter Puppe from Heidelberg. Professor tom Dieck did a very good job teaching us analysis from the elementary beginnings and ending with vector analysis. It was quite complicated stuff in the third semester. Both tom Dieck's personality, but also the drive of the group of young assistants surrounding him, his team, attracted me. It was the personalities of these people rather than the subject itself that impressed me most.

In retrospect, I am grateful for the inspiration they gave me. As a follow-up, there were offered courses in differential topology, which was developing rapidly at the time. After three further semesters we got close to the results on smooth structures on spheres by Kervaire and Milnor. Milnor became later one of our Abel interviewees! I felt a real challenge and excitement: After the first three years at university, we were at a level where we could read and understand, to a certain degree at least, papers that had been written perhaps five or ten years earlier. There were also interesting seminars where we were challenged to give talks ourselves about papers in the literature.

I should add that I received a grant from Studienstiftung des deutschen Volkes, a grant institution for talented university students, with money that made me financially independent. Since I also earned some money as an instructor for new students, I felt quite well-off at the time! The Studienstiftung also gave opportunities to attend summer academies during the vacations at nice places, in the Alps usually. That was challenging! I got to know other eager and talented students. Moreover, interesting subjects, not only within mathematics, were taught and discussed.

All that brought me up to a certain level. I asked for a topic for a master's thesis, and I got a suggestion by tom Dieck. It took some time to get the right mindset; the topic was no longer in differential topology, but in algebraic topology. I had never taken a class in algebraic topology, but during seminars, I had acquired some knowledge. I finished this master thesis within perhaps a year, finally typing it myself on an old-fashioned typewriter and inserting special letters by hand.

But then something else happened: My teacher tom Dieck was called to a professorship in Göttingen. Consequently, the whole team, his assistants and several of his students, followed him to Göttingen. My master's degree was obtained at the University of the Saarland, but the final oral exam took already place in Göttingen.

Göttingen

Saarbrücken and Göttingen are quite different. Göttingen has a lot of tradition: You can still walk to the observatory where Gauss worked, and the hall you enter at the department of mathematics is called the Hilbert space. You can walk along the offices where so many illustrious mathematicians had worked. The buildings of the department of mathematics, and that of physics nearby, had been built with support from the Rockefeller Foundation after the First World War.

CFS: *And they were not destroyed during the Second World War?*

MR: Göttingen was almost intact; only a few buildings were damaged. The city had not had any military importance; not that this fact helped many other German towns…

Göttingen is a relatively small city; you can get almost everywhere by walking. The university is old, and many facilities were old-fashioned. I had to learn to appreciate the charm. The University of the Saarland, on the other hand, was founded after the Second World War, with modern buildings and equipment.

Department of Mathematics, Göttingen University.

CFS: *How many students, approximately, were there at Göttingen?*

MR: Göttingen is a relatively small town with a little more than 100,000 inhabitants at that time. I think there were around 20,000 students, probably 30,000 by now. The students really were, and still are, a very dominating section of the population. I don't know the exact numbers, but there were several hundred students of mathematics and physics.

CFS: *From what you're telling me, it would seem to be natural that tom Dieck would be your Ph.D. advisor?*

MR: In fact, I started to work under him. But then Larry Smith, a homotopy theorist at the time, was hired as a new young professor at the department. Every professor had two or three assistants, and I was offered one of these assistant positions. While I was still working on the first bits and pieces of my Ph.D. thesis, I was "taken over", so to speak, by Larry Smith, and I was his assistant for several years. And then it seemed natural to say: Since you must work with this professor, he should also be your advisor. The transition went in fact quite easy.

Also, the topic of my work shifted a bit, quite naturally so. When you work on a thesis, you start an investigation into terra incognita. Some premature ideas do not work out, and then you try something else.

In the end, my thesis dealt with the homotopy classification of liftings into fibre bundles, and more concretely, with immersions and embeddings of smooth manifolds.

CFS: *I see. You said earlier that you started being more interested in algebra, and then you switched to algebraic topology…*

MR: In my view, topology is an ideal combination of several subjects. You can't do interesting topology without knowing some algebra, some analysis, and some geometry. In the beginning I didn't think I was very good at geometrical thinking at all. I've always been very bad at drawing. I've taught myself to do drawings relevant for mathematics, but apart from that, I have no artistic talent at all, I am afraid.

I have given a course in elementary differential geometry many times later in my teaching career. I then try to advertise the subject by telling the students that you can get to exciting results by stealing methods from analysis and linear algebra, combining them in a clever way. That synthesis has fascinated me, and I think that it made me like geometry and topology more than other mathematical areas.

Mathematical research

CFS: *That's a nice description. Now, let us talk about your research after your Ph.D. Much of your recent research work has been about the interplay between what is called directed homotopy in algebraic topology and concurrency, a notion appearing in theoretical computer science. Could you please explain?*

MR: Let me tell you about the next steps in my career first. As a Ph.D. student, I had the opportunity to study a year in Paris. While staying at the Cité universitaire, I got to know a Danish girl who later became my wife. Consequently, I had to learn Danish, and I moved from Göttingen to Denmark. I had short term positions at the Technical University in Lyngby, close to Copenhagen with Vagn Lundsgaard Hansen, and then in Aarhus with Ib Madsen, the grand old man in Danish topology. From there I applied, and finally got, a position as associate professor at Aalborg University Center – as it was then called – in the North of Jutland, the Danish peninsula.

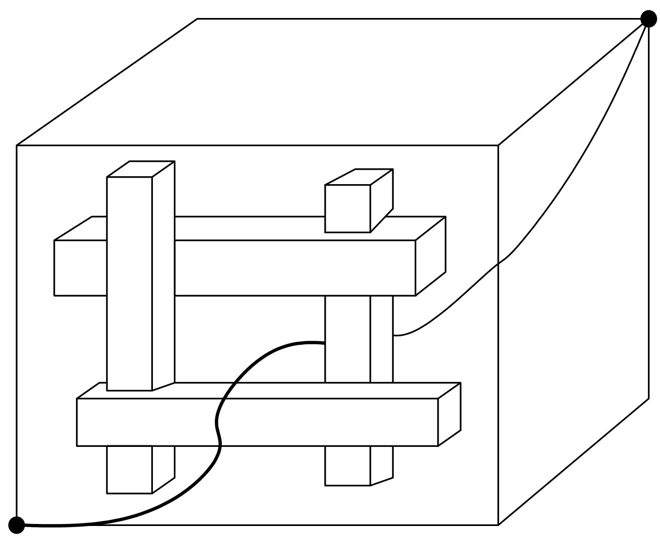
When I arrived, I was the only topologist at the department. I went regularly to seminars at Aarhus University and considered that as my research lifeline. I had to teach a lot, three small kids required attention, and time for research – at that time in equivariant algebraic topology – was scarce. A few years later, I was joined at the department by my good colleague and friend Lisbeth Fajstrup, a student of Ib Madsen.

Still, we felt a bit lonely research-wise and, encouraged by colleagues, we started to think about investigating applied areas of topology. An opportunity, a wild card, arose when we heard about a conference on New Connections between Mathematics and Computer Science at the Isaac Newton Institute in Cambridge, UK. We applied and were accepted. There were many interesting talks given during that week. Among them were two lectures that thrilled me from the very beginning: One outlining connections between what's called distributed computing and topology, and the other about connections between concurrency and topology; as it turns out, both are related to each other.

Let me tell you about concurrency and topology: It becomes interesting when you don't have just one program running on one processor, but when things get distributed, the latter being more and more the case. You want to do a calculation, or to run through an algorithm, and you distribute it either on various entities on your own laptop, or perhaps even on the entire World Wide Web. Anyway, there are several units that collaborate to arrive at a solution. In the extreme case, there is no coordination or very little coordination among them. In order to rely on a result of such a common effort with very little coordination, you need to have algorithms that are robust in the following sense: It should not play a role whether one processor achieves its goal very quickly, whereas another one is slowed down for some reason. If they communicate through some common registers, the order of access may be crucial, on the other hand.

For an easy case, consider three processors, each working on a linear code without loops and branches. You can then interpret the compound process as a path in 3-dimensional space. But not every path can occur: Time has an orientation, and therefore these paths will be weakly increasing coordinate-wise; a little bit like in relativity theory. Moreover, there are regions in this 3-dimensional state space that are forbidden, due to coordination constraints: Only one, or at most a limited number of processors can access the same piece of memory at the same time.

More abstractly, we want to study directed paths in a "space with holes", often modelled as a cubical complex. Combined with a language, these are called Higher Dimensional Automata. It turns out that directed paths that are homotopic to each other, in a directed manner, describe equivalent compound processes, always yielding the same result for a distributed algorithm. A directed homotopy between directed paths, and more generally between directed maps, is a one-parameter deformation where all intermediate steps are directed as well.



Directed path in a cube with obstructions. This path is homotopic to a directed path on the boundary of the cube, but not through any directed homotopy.

Directedness is the essence and the challenge. You cannot apply standard techniques from algebraic topology right away. We found examples of directed paths that are homotopic in the standard sense, but not so when you require directedness for all intermediate paths. In the end, you want to describe and structure the space of all directed maps up to directed homotopy, and to calculate it algorithmically.

CFS: *Have people in theoretical computer science found an interest in your approach?*

MR: Some have, and one of our first foundational papers still receives citations on a regular basis. But let me admit that there is a larger community in computer science working on coordination problems and algorithms using so-called Petri nets, an area that I do not know a lot about. Many of their and our results resemble each other. In fact, a Dutch collaborator of ours, Rob van Glabbeek, now in Scotland after many years in Australia, has shown in an abstract way that Higher Dimensional Automata are at least as expressive as Petri nets: all that they can model, we can as well.… But we are a small community compared to others.

I would like to add that we consider ourselves as a branch of the rapidly developing area *applied topology,* with topological data analysis as the most important and popular family member.

Abel Prize interviews

CFS: *This is all very exciting. Now we are going to switch topics entirely. As you mentioned, you wound up as associate professor and then professor with special responsibilities at Aalborg University. You have been heavily involved with teaching, and you have worked as a supervisor for many students in a wide variety of mathematical topics. You also became Teacher of the year in 2018 and 2019.*

Besides all that, you have been involved with public outreach, having given public lectures on several aspects of mathematics. Raising public awareness of mathematics is not a mere slogan for you. You have taken it very seriously. And as one of the editors, and editor-in-chief from 2002 to 2003 of the newsletter Matilde of the Danish Mathematical Society, you published interviews with various mathematicians, which made you eminently prepared to take the initiative for the interviews of the Abel Prize laureates. Could you please talk about how the interviews with the prize laureates came about?

MR: Matilde was the journal of the Danish Mathematical Society. It was founded while Bodil Branner was the chairman of the Danish Mathematical Society, which this year is celebrating

Final issue of Matilde, 2023.

150 years of its existence. One of her many initiatives was to establish a regular newsletter. By the way, I came up with its name, Matilde: Mat (matematik in Danish comes without an h) with a tilde!

I became part of the team of newsletter editors, and my task was to interview some of the "grand old men" in Danish mathematics, for example Ebbe Thue Poulsen from Aarhus and Bent Fuglede from Copenhagen. A special treat was an interview with Ib Madsen shortly before his 60th anniversary.

Concerning the Abel Prize, Matilde's editorial board was contacted by the embassy of Norway in Denmark: Would we be interested in covering the first Abel Prize ceremony in Oslo? I discussed the option with Mikael Rørdam, who was the editor-in-chief at the time. I gathered all my courage and suggested that we should ask for an interview with Jean-Pierre Serre, the first recipient of the Abel Prize.

CFS: *That was in 2003?*

MR: Either in late 2002 or in early 2003. We wrote back to the embassy, and they contacted the Abel committee. The committee was positive and informed professor Serre. Your colleague Kristian Seip must have heard about the initiative…

CFS: *He was the chairman of the Norwegian Mathematical Society at the time…*

MR: And he thought that it was a pity that the Norwegians had not asked for that opportunity themselves. I do not know whether he asked you first…

CFS: *He asked me first, I think.*

MR: OK, he suggested that you were involved, as well. I do not remember the details, but the two of us were put in contact. We did not know each other personally at the time. I had participated in the 20th Nordic Congress of Mathematicians in Trondheim in 1988 that you were involved in, but I do not think we made contact then.

Anyway, we teamed up, and that turned out to be a very good idea. First of all, it made me feel more relaxed, and in the long run, we became a "dream team", right? We brought different angles to the interviews, complementing each other. We coordinated the questions we wanted to ask, the last time being in the evening before the interview took place.

CFS: *In fact, you taught me a lot about interview techniques. I learnt a lot from you.*

MR: Thanks! But you always came up with a lot of interesting questions and citations yourself. I still remember how nervous we were when we started the first interview with Jean-Pierre Serre.

CFS: *And he was not in a good mood initially!*

MR: He was not, and I kind of understand why: The Abel Prize recipients have all been quite old so far. During the week of festivities, they have a strenuous program. They are asked to attend a lot of meetings, ceremonies, they must give talks, and then one or several interviews! I do not know, but I imagine that Serre thought: "Another interview with silly journalists asking me silly questions."

CFS: *That's right. But let me tell you, the real breakthrough came when you told him that you were aware how he discovered some very important notion having something to do with fibre bundles. Then he lit up immediately. "Oh, you knew about that?", he said, and that changed the atmosphere. He became very positive from then on.*

MR: Mentioning the path fibration changed the game. I re-read the interview this morning, and it is, in fact, still very interesting!

CFS: *We should not underestimate the importance of the initiative of yours getting started with the Abel interviews. The two of us went on and conducted the Abel interviews for 14 consecutive years. Then I continued together with Bjørn Dundas. I would say unabashedly that the Abel interviews are very important and successful. They are published both in the European Mathematical Society Newsletter, now Magazine, and in the AMS Notices. Besides, the European Mathematical Society's publishing house, EMS Press, published a book with the interviews the two of us had.*

MR: I am also happy having been a part of that enterprise. In the end, looking back at my career, that may have been the most important thing I have been dealing with!

Abel interview 2016: Sir Andew J. Wiles, Martin Raussen, Christian F. Skau. (Photo: Eirik F. Baardsen, DNVA.)

CFS: *To make the Abel Prize known for the maths community and beyond, these interviews have been very important, I think it is fair to say that!*

Could I ask you, and this is a very difficult question: Do you have some special highlights that you would mention from all these interviews?

MR: Let me try. I have two different answers to your question, on very different levels, though. The first answer is related to the amount of the Abel Prize.

CFS: *By the way, the prize money was almost one million US dollars.*

MR: Right. Often, we did not dare to ask what the recipients would do with the prize money, because that is quite personal. But I remember that we listened in on the interview conducted by a journalist with professor Serre. This journalist was less shy and asked Serre directly what he would use the prize money for. His answer was just laconic: "Well, I'll see to have it spent."

Another occasion that I remember vividly, occurred when we interviewed Pierre Deligne, ten years after Serre. He had made it already public, and he also told us at the beginning of the interview, that this was not money for him, but it was money for mathematics. He wanted to give it to various institutions that had played an important role in his own career: the research institute IHÉS in Bures-sur-Yvette in the suburbs of Paris, and the IAS in Princeton, USA. But also to two institutions in Russia, the Department of Mathematics of the Higher School of Economics, and to the Russian Dynasty Foundation, which supported science. Deligne was clearly one of the most modest interview subjects we had.

My second answer to your question is related to reactions of our interviewees to one of our standard questions: Did some of their great results rely on sudden flashes of insight, at least partially? We have all heard the story, published in a book by Hadamard, of Henri Poincaré who while stepping into a bus somewhere in Northern France, in a sudden flash of insight saw connections between modular forms, Fuchsian functions and so on, and formalisms in non-Euclidean geometry on the other side. This has become an iconic story, an incident where a new connection suddenly pops up after the sub-conscience has worked on it for some time. Stories of that kind have fascinated me.

Getting back to the interview with Serre: Around 1950 he worked on homotopy groups of spheres, and he established many of their properties. One of the techniques applied in the investigations makes use of the path fibration. It relates the homotopy groups of a space with the homotopy groups of its loop space. You can explain it – if you know about it – to a student who knows some elementary homotopy theory in a few minutes. But that relationship had not yet been established, before Serre.

Other far more subtle techniques, for example, spectral sequences (mainly developed by Leray for other purposes), were known and applied by Serre. Moreover, he made calculations one prime at a time, so to speak. Anyway, the path fibration was apparently a missing link that occurred to him on the night train when returning from vacations. He got so excited that he woke up his wife to tell her!

John Tate told us that when he had worked hard on the determination of higher-dimensional cohomology groups in class field theory, he went to a party and had a few drinks, came home and suddenly saw the solution after midnight. I got jealous when I heard that!

But everybody we asked told us that this happened very rarely, perhaps twice or three times in a lifetime. Moreover, they told us that such nice experiences never come for free!

They may occur after a long time and many efforts trying to put things together. Only when you have made these efforts and perhaps feel that you run against the wall, emotions come into the picture, and then your sub-consciousness might do part of the work for you. I remember very well one word that I will never forget: When discussing that same question with the late Abel Prize laureate Nirenberg, he coined it succinctly with a word in my native German: It needs "Sitzfleisch" (seat flesh) – expressing persistence in a very physical manner – to get to anything serious. That is a story that I also like to tell my students!

CFS: *This reminds me of the following story about Niels Henrik Abel: He visited Berlin and stayed with some Norwegian friends at a shared flat. One of these friends wrote later that Abel would often wake up in the middle of the night, light a candle, and scribble down some mathematical ideas that had occurred to him during his sleep. I would certainly have appreciated it if that had ever happened to me!*

MR: So would I!

Activities in the European Mathematical Society

CFS: *At the time the first Abel Prize interview was taken you were headhunted to the Newsletter of the European Mathematical Society (EMS), where you were editor-in-chief from 2003 to 2008. And in 2009, you were voted into the EMS executive committee (EC), where you participated from 2009 to 2016, the last six years as Vice President. You were liaison for the EMS committee Raising Public Awareness, and you were also the person in charge of the EMS web profile. All these are heavy and time-consuming duties. You were still teaching, and you didn't get much credit as pertains to your teaching load. Could you comment on this period of your life?*

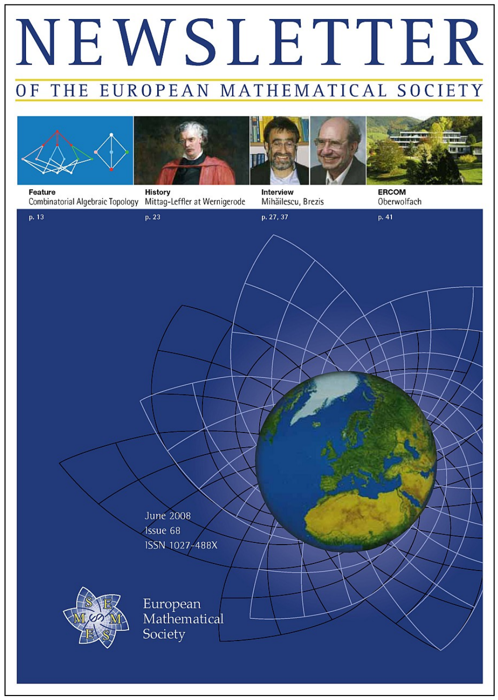
MR: It was a challenging period that did consume quite a lot of my time, which was "stolen" from my research and from my family, I must admit. But on the other hand, I liked these duties, as well. I could utilize other facets of my personality, of my capabilities. And when I left the executive board of the EMS, I missed it for a while. It had almost become a second family for me. Most of my colleagues at the EMS are highly devoted. When they take on a job, they really do so reliably. I felt that we were pulling on the same string.

You see, the EMS is still a relatively new society. It was founded in 1990, a newcomer in comparison with the Norwegian or the Danish societies. It has almost all European mathematical societies as members, but there are still all too few individual members, only a bit more than 3000. Since I started, membership has risen from 2000 to 3000, but it's still not living up to its potential within all of Europe!

At the board meetings, discussions and decisions have to be taken all the time. For example, how is the money spent? Not that the EMS has a lot of money, but it can give something to organizations or to conferences, workshops and so on. How are you going to find out who is worth giving the money to? I was a member of the society's meetings committee preparing some of these decisions. By the way, the EMS executive committee nominates some of the members of the Abel Committee that selects the Abel Prize winners.

But probably it is more important to develop contacts with some of the politicians, speaking about the importance of mathematics towards Brussels, towards the people who really can give substantial support. That's a tough challenge. I've not really been much involved in this task. That was the challenge of the presidents of the EMS. They have done tremendous work in that direction, sometimes with success. But of course, you could always hope for much more.

I think it's really important that we have such a transnational mathematical society. Local issues can be handled much better by a national mathematical society. But it's also important to have a player on the transnational scene, to promote and to get inspiration from each other, and also to have a counterweight to the American Mathematical Society. I do not want to be negative about the American Mathematical Society at all, but it is such a dominating society, mainly for good. On the other hand, if you didn't have a competitor with the same goals, they would take over everything. A concrete instance is the database MathSciNet, a highly valuable tool. It is very important that we also have Zentralblatt/zbMATH, because otherwise there would only be one venue, and you know, when there is no competition… The EMS is one of the owners of Zentralblatt, and there is now free access!



Newsletter of the EMS, 68/2008.

CFS: *How many members are there in the American Mathematical Society?*

MR: I would say around 30,000. And of course, I'm also a member of the AMS, and most European mathematicians are, but too few are members of the EMS, I would say.

CFS: *I observed that there were disappointingly few members at my department in Trondheim.*

MR: Perhaps some of the readers of this interview get the idea. It's in fact cheap to become a member!

Raising awareness of mathematics

CFS: *I already mentioned that you have been a spokesperson for raising awareness of mathematics. Why is that an important endeavour?*

MR: Well, for several years, from 2003 to 2008, I was the editor-in-chief of the Newsletter of the EMS. That newsletter was mainly read by mathematicians. I think that also our colleagues need good stories about successes in their own discipline!

When it comes to communicating the importance of mathematics to laymen and non-mathematicians, a lot of good work has been done by the Raising Public Awareness Committee (RPA) of the EMS. Over the years, RPA was headed by Vagn Lundsgaard Hansen from Denmark, Ehrhard Behrends from Germany, and Roberto Natalini from Italy. Members of that committee write popular books, coordinate web events, and perform "Mathematics in the Street" events on several occasions. While I was on the executive committee (EC) of the EMS, I had the role of liaison officer linking the EC with the RPA Committee, and I participated in several of its meetings. I am impressed with the work they did, and do, but I have not been very active in that direction myself.

Generally speaking, promoting mathematics seems to be more demanding than doing it for many other disciplines. One reason is arguably that mathematics is a subject that everybody has been in touch with at school, from grade one on. Hence, many people think they somehow know what mathematics is about. Depending on the level of their education, they might connect mathematics with calculations involving large numbers or evaluation of difficult integrals.

Quite few people are aware of the fact that mathematics is much more far-reaching, consisting of many subdisciplines and having so many facets. It has its own touch of beauty, it has philosophical components, and, on the other hand, it has become increasingly important in technological developments underpinning everyday life.

In comparison, physicists or computer scientists have an advantage: Their subjects are taught to a much lesser extent in school, and people realize that it is worthwhile to know more. Moreover, it is easier for them to come up with compelling pictures, for example those from the Hubble or the James Webb telescopes showing us the universe in its splendour. That does not mean that a layman like me fully understands modern physical theories, but these pictures fill you with awe. It is not that mathematicians do not try to do something similar: There are lots of beautiful and impressive pictures available on various platforms, like for example IMAGINARY[2]. But it is more difficult to demonstrate to the uninitiated that coding theory or topological data analysis are built on beautiful insights with tremendous applications.

CFS: *I think this is a very important point.*

MR: So, let us welcome every initiative to build bridges! My own contribution has been directed to the mathematical community. The Abel Prize interview conveyed essential points of view of extraordinary mathematicians.

As a teacher, I try not only to talk about interesting and important mathematical concepts, results, and techniques, but also to put things into perspective. What is the reason for studying a particular topic? Why is this result a small technical lemma, and why is another one an important theorem? For example, why is Gauss' Theorema Egregium really "egregium"?

CFS: *It is really depressing that many people are almost proud of admitting that they are illiterate in mathematics, or that they even hate mathematics! Take for example Aftenposten, the largest Norwegian newspaper, which is supposed to have a cultural dimension as well. Nevertheless, they hardly mention the Abel Prize at all. In the twenty years of the existence of the prize, very few articles have appeared about the Abel Prize and its recipients. Without any shame, they have neglected it over many years!*

MR: We have also had great difficulties to advertise our Nordic Congress in the press. The local newspaper that often boasts of minor local initiatives neglects the congress. The only laudable exception is the newsletter Ingeniøren, published by the trade union of Danish engineers.

Beauty and importance of mathematics

CFS: *In this connection, I would like to read an excerpt from the eulogy that the brilliant Norwegian mathematician Ludvig Sylow wrote in 1899 on the occasion of Sophus Lie's death: "It is the mathematician's misfortune more than the other scientists that his work cannot be presented or interpreted even for the educated general public. In fact, hardly to a collection of scientists from other fields. One has to be a mathematician to appreciate the beauty of a proof of a major theorem, or to admire the edifice erected by mathematicians over thousands of years." Any comments?*

MR: Well, to assess that something is exceptionally beautiful, you need to have certain background knowledge. But this is also true for advanced music or painting. I think, though, that it is easier for artists to get us emotionally involved. It is far more difficult to achieve this when talking about mathematical ideas and methods. People have tried to find different ways to do that. It is a challenge for the entire community.

CFS: *That reminds me of a statement by one of the Abel Prize recipients, the late John Tate, who stated during our interview: "Mathematics is both art and science. There are artistic aspects to mathematics. It is just beautiful. Unfortunately, it is only beautiful to the initiated, to the people who do it. It can't really be understood or appreciated much on a popular level the way music can. You do not have to be a composer to enjoy music, but in mathematics you have to be a mathematician to appreciate it." Another Abel Prize recipient, László Lovász, lamented that science in general is in danger. Things have become so complicated that it is very difficult to distinguish between science and pseudoscience. Mathematicians, and scientists at large, have a responsibility for making the public aware of this danger, which has wide implications.*

MR: That is already a challenge at school!

CFS: *Lovász thinks that we do not teach mathematics the right way in high school, at least seen from his Hungarian perspective. We do not give pupils enough safeguards against pseudoscience and unscientific speculation.*

MR: Hard to disagree! But not everything is bad: For centuries, mathematics had little influence on technology, on real life. Of course, there are examples to the contrary, land surveying in Egypt and ancient astronomy, for example. But the applicability of mathematics has exploded, especially in our lifetime. Mathematics is no longer only essential for physics and engineering, mathematics is everywhere! It has become an economic pillar of society, underpinning important industry sectors, going hand in hand with engineering and computer science. Many consumer goods, modern cars, planes are inconceivable without the input of mathematics. Just think about our means of communication, and credit cards, GPS, as well. Mathematical modelling makes reliable predictions possible. No data science without mathematics!

Number theory was completely estranged from applications, as Hardy proudly noted less than a century ago. Now sophisticated number theory, deep results on elliptic curves, are quintessential in coding and cryptography. But unfortunately, the mathematical community has not been good enough to communicate the beauty and the practical importance of our subject.

[2] https://www.imaginary.org

CFS: *Tell us about the quote from René Thom that you mentioned prior to this interview!*

MR: I told you earlier that I spent a year in Paris as a Ph.D. student. I went regularly to the IHÉS at Bures-sur-Yvette. At that time René Thom was still around, probably as an emeritus professor. He was a demigod in my eyes; he had developed so many important parts of differential and algebraic topology. At that time, his catastrophe theory was still a hot topic. Thom had by then acquired a deep interest in the history of mathematics. On Saturdays, when the institute otherwise was quite calm, he ran a seminar on the history of mathematics, with varying lecturers. I went there several times. During one of the sessions, he said (slightly paraphrased): "I was told that more than half of the mathematicians throughout history are still alive. But that is certainly not the better half!"

CFS: *I like that quote! Now a quote from Harish-Chandra: "I have often pondered over the roles of knowledge or experience on the one hand, and imagination or intuition on the other, in the process of discovery. I believe that there is a certain fundamental conflict between the two. And knowledge, by advocating caution, tends to inhibit the flight of imagination. Therefore, a certain naïveté, unburdened by conventional wisdom, can sometimes be a positive asset." Do you have comments?*

MR: It is certainly true that the most stunning results of the Abel Prize recipients that we interviewed go back to their young age, when they were around thirty years old, or younger. In that sense Harish-Chandra seems to be right. Our interviewees told us that when they got older, they knew the literature better and also how to avoid possible pitfalls. But some naïveté is needed, as well as brute force, to start on a mathematical endeavour, thinking: "Perhaps I can do something here where nobody else so far has been successful."

At a higher age, these top mathematicians are of course still enormously talented. They would build on previous experiences, on their contact network, and advance the subject by sharing ideas with others. But the naïve courage to start on a new undeveloped area with brute force is rather something for talented young less experienced people.

CFS: *This brings us again to Hardy's "Mathematics is a young man's game", that you just explained very eloquently. And still, you need "Sitzfleisch"!*

Computer aided mathematics

Now to a hard question: If you should venture a guess, which mathematical areas do you think are going to witness the most important developments in the coming years?

MR: That sounds like a question for people from the Champions League, and not to me.

CFS: *Let me give you a clue: Could it be that using computers will have some effect?*

MR: Well, they have already had a certain effect. I mean, it started with the solution of the four-colour problem back in the late seventies. And it happens more and more often that results are checked with computer aid. And there are new developments. Already several years ago the proof of the Feit–Thompson theorem about the solvability of groups of odd order was checked by computer, and that proof fills a 250 pages journal paper. I've heard by hearsay that the proof of an important step in the new discipline of condensed mathematics advanced by Dustin Clausen and Peter Scholze needed to be checked by computer; this was done successfully in an effort that applied the proof assistant system Lean.

I imagine that future mathematicians will get a lot of help from advanced artificial intelligence. Already now you can ask whether there exist results in a certain direction. And then the entire existing literature will be searched. If you ask intelligently enough, then you will either find an answer, or a "no". I guess that you can progressively enter a dialogue with computers, adding two forces: human imagination on the one hand, and the enormous power of computer algorithms searching through and combining everything that exists, and to do that very quickly, on the other hand. And I don't see an end to that story.

When I was young, it was completely unimaginable to think that a computer might be better at chess or Go than a human being. But now they are!

CFS: *It's sort of depressing in a way.*

MR: On the other hand, maybe it opens venues and options that you otherwise couldn't be aware of.

CFS: *That is certainly true. When we interviewed Peter Lax, who is also one of the Abel Prize recipients, about how fast computers had become, he said that half the speed is due to clever algorithms. But to get clever algorithms you need mathematicians.*

MR: And the work developing these algorithms for computer aided mathematics demands mathematicians at a high level.

Frogs and birds

CFS: *Now to a quotation from Felix Klein. He said that mathematics develops as old results are being understood and illuminated by new methods and insights. Proportionally with a better and deeper understanding, new problems naturally arise. Your comments?*

MR: I remember we asked some of the Abel Prize recipients about that. One might be afraid that at a certain point everything that is worth developing in mathematics has already been done. How could a young person then find something worthwhile to think about? But I think it was Serre who said something like: "I don't worry about that. New problems do arise all the time." Look at what happened during our lifetimes: So many new subjects in mathematics that almost didn't exist when we started, are flourishing now. Others, that were very much the talk of the day do still exist, but not with the same attention. I think it's particularly young people who can somehow sense in which directions new things happen, and then they dive into these and give them a push.

CFS: *Hilbert said that problems are the lifeblood of mathematics. No question about that.*

MR: I remember that we received different answers to the question, whether our interviewees regarded themselves mainly as theory builders or as problem solvers. You have these two capacities within one person, but more widely, also in the mathematical community. It is very important that both types of mathematical preferences coexist, and that they can profit from each other.

CFS: *You gave me the clue to another quotation, this time from Freeman Dyson. I think he describes "theorem builder" versus "problem solver" very well: "Some mathematicians are birds, others are frogs. Birds fly high in the air and survey broad vistas of mathematics, out to the horizon. They delight in concepts that unify our thinking and bring together diverse problems from different parts of the landscape. Frogs lie in the mud below and see only the flowers that grow nearby. They delight in the details of a particular object, and they solve problems one at a time." I think that's a very nice description.*

MR: I think it's almost impossible to say that in a better way. It's important that the frogs and the birds coexist.

CFS: *I agree with you. I think it was Andrew Wiles who told us in the interview: "The definition of a good mathematical problem is the mathematics it generates, rather than the problem itself."*

MR: That's well said. And there are many instances where you can show that this is true. Take the Fermat conjecture as an example. Due to Wiles, we know that it is true. But the whole build-up leading to the final solution is so much more important than the result itself. It has given insights that Fermat could not even have dreamt of.

CFS: *Perhaps it is relevant here to mention, as we did in the interview with Atiyah and Singer, Eugene Wigner's statement about the unreasonable effectiveness of mathematics in the physical sciences, as well as Galileo's dictum, that the laws of nature are written in the language of mathematics.*

Private interests

Let us end this interview with a question that you also suggested including in the Abel interviews: What interests do you have outside of mathematics?

MR: My answer to that question might be a little disappointing. Of course, I must think about that now because I'm going to retire, and that means that mathematics will play a minor role in my life in the future. Hobbies will have to take over more time. I would have difficulties to single out a specific hobby and say: "Now I know exactly what I will spend my time on." In a more positive way, and certainly exaggerating, I would say I'm a renaissance man in the sense that I have many things that I'm interested in.

I like to read popular science books, and I hope to get more time to do that. Biology, genetics, evolution, climate, so many new things happen. I have often looked avidly into the leaflets from an institution called People's University and told myself: "Okay, you could do this, and you could do that if you just had the time." Now I will get more time to spend on these interests.

Moreover, I like listening to and learning about music, both classical music and jazz. If it were not for this congress, I would have gone to many events at the Copenhagen Jazz Week, which is going on in this same week. I'm now living most of the time in the Copenhagen area, and I take the opportunity to go to classical concerts by the Radio Symphony Orchestra and the like from time to time. I also like to visit art exhibitions. Perhaps I might play a bit more the piano as I did as a teenager, but not systematically ever since. Last but not least, more time for my family!

CFS: *Thank you very much on behalf of myself and of the Norwegian and the Danish Mathematical Society for this very interesting interview. It's been a pleasure.*

MR: It has been my pleasure as well. And I'm thrilled and thankful that you took the effort to come to Aalborg just to interview an ordinary mathematician like me.

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