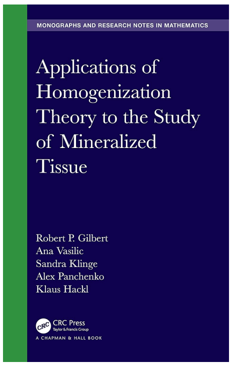
Book reviews

Applications of Homogenization Theory to the Study of Mineralized Tissue by Robert P. Gilbert, Ana Vasilic, Sandra Klinge, Alex Panchenko and Klaus Hackl

Reviewed by Michael Shoushani



Homogenization is a powerful tool used in the analysis of applied problems which have multiple scales and complex structures. Broadly speaking, homogenization provides a basis to determine macroscopic (effective) equations for materials by using the properties of the material at the microscale. Oftentimes the structures to which homogenization is applied have or are assumed to have a periodic structure and the notion of two-scale convergence can play an important role in the analysis, but homogenization is not restricted to only this case and can be applied to more general disordered (non-periodic) media. Here a more general framework of convergence such as G-convergence may be used in the analysis. This research monograph serves as an introduction to homogenization theory, while at the same time it explains how to use homogenization in applications in biology, physics, and engineering that will appeal to a wide audience.

The book starts with an introductory chapter where important theory and notions needed for subsequent chapters are introduced. This includes some fundamental functional analysis, important function spaces, and essential theorems regarding concepts such as strong and weak derivatives, the trace theorem, the Lax–Milgram theorem, and so on. This chapter also includes the geometric description of the porous medium that the authors propose to study. Starting with the unit cell $\mathcal{Y} =]0,1[^n$ where $n = 2, 3$, letting $\mathcal{Y}^s$ (the solid part) be a subset of $\bar{\mathcal{Y}}$ and $\mathcal{Y}^f = \mathcal{Y} \backslash \mathcal{Y}^s$ (the fluid part), making the periodic arrangement of $\mathcal{Y}^s$ over $\mathbb{R}^n$, the authors outline the process for obtaining the domains Ω_s^ε and Ω_f^ε, which represent the solid and fluid parts of the porous medium $\Omega =]0, L[^n$.

After discussing the geometry, several important homogenization notions are introduced, including the following:

- Two-scale convergence, namely:
  The sequence $\{w^\varepsilon\} \subset L^2(\Omega)$ *is said to two-scale converge to a limit* $w \in L^2(\Omega \times \mathcal{Y})$ *if for any* $\sigma \in C^\infty(\Omega; C_\#^\infty(\mathcal{Y}))$ *one has*
  $$\lim_{\varepsilon \to 0} \int_\Omega w^\varepsilon(x) \sigma\Big(x, \frac{x}{\varepsilon}\Big)\, dx = \int_\Omega \int_{\mathcal{Y}} w(x,y) \sigma(x,y)\, dy\, dx.$$
  Here # denotes unit cube periodicity.
- A homogenized equation for a boundary value problem with unknown $\mathbf{u}(\mathbf{x})$ and an asymptotic solution in powers of $\varepsilon \to 0$, namely:
  $$\mathbf{u}(\mathbf{x}) = \mathbf{u}^0(\mathbf{x}) + \varepsilon^1 \mathbf{u}^1(\mathbf{x}, \mathbf{y}) + \varepsilon^2 \mathbf{u}^2(\mathbf{x}, \mathbf{y}) + \cdots, \quad \text{where } \mathbf{y} = \frac{\mathbf{x}}{\varepsilon}.$$

The introductory chapter ends with a discussion of two-scale convergence with time dependence, and potential and solenoidal fields.

With the stage set for subsequent chapters, the authors then move on to discuss a range of applications of homogenization theory. These cover the technique applied to soft tissue (the authors note that soft tissue does not have a periodic structure, but there is a scale separation), and include applications of homogenization pertaining to the following topics:

- acoustics in porous media,
- wet ionic piezoelectric bone,
- viscoelasticity with contact friction between the phases,
- acoustics in random microstructure (in this chapter the notion of stochastic two-scale limits as well as compactness properties of the convergence are discussed),
- bone tissue modeled as a periodic two-phase material composed of a viscoelastic solid matrix filled with a non-Newtonian fluid (representing bone marrow),
- homogenization of viscoelastic flows (the notion of G-convergence is introduced when this application is discussed),
- multiscale FEM for the modeling of cancellous bone.

In summary, this textbook provides a well-planned introduction to the theory and applications of homogenization. The applications discussed provide a strong motivation for further study of the topic. As stated by the authors, the book is a result of years of research collaborations. As such, it would serve as a great reference for researchers including those such as applied mathematicians, engineers, and geophysicists. It could also serve as a textbook for a course or courses in homogenization theory or a special graduate seminar course. A motivated student could also use the book for self-study. The bibliography contains over 400 references and provides a good basis for further reading.

Robert P. Gilbert, Ana Vasilic, Sandra Klinge, Alex Panchenko and Klaus Hackl, *Applications of Homogenization Theory to the Study of Mineralized Tissue.* Chapman & Hall, 2020, 297 pages, Hardback ISBN 978-1-584-88791-1, Paperback ISBN 978-0-367-71372-0, eBook ISBN 978-0-4291-4338-0.

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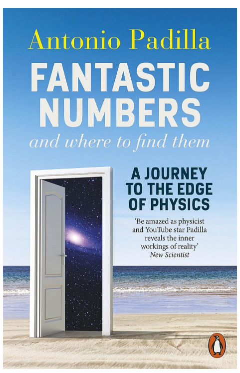
Fantastic Numbers and Where to Find Them: A Journey to the Edge of Physics by Antonio Padilla

Reviewed by Dominic Thorrington



This is not the book that I expected it to be. Perhaps it was naïve of me, but I bought this book based on the title (the different subtitles *A Cosmic Quest from Zero to Infinity* and *A Journey to the Edge of Physics* were added to different versions of the book at later dates) and I thought this was a book written in a similar fashion to some of Marcus du Sautoy's books on prime numbers. I was surprised that this is much more of a theoretical physics book that uses interesting numbers as the inroad to discussing important concepts in quantum mechanics and cosmology. However, I was not at all disappointed – theoretical physics and cosmology is an interest of mine that I have neglected for a while, so this reintroduction to the subject was welcomed. It is worth keeping the subject in mind, though, if you are looking for a new maths book for yourself or for a fellow mathematician – this book is quite specialised, and some prior knowledge of theoretical physics would be very helpful.

Padilla is certainly known to some readers as one of the experts interviewed regularly on Brady Haran's *Numberphile* YouTube channel. Padilla is a theoretical physicist and cosmologist at the University of Nottingham who regularly appears on the channel to discuss topics as wide-ranging as the sum of the infinite series of natural numbers, the gaps between prime numbers, how many particles are in the universe, and the multiverse. His book is just as fun and as interesting as his videos.

Padilla begins the book with a brief introduction and a story from his mathematical studies at Cambridge University. He received a score of zero from his tutor for a proof because it was not rigorous enough, encouraging Padilla to be more meticulous with his work and studies. It is an interesting story, but the book takes quite a turn after that when the main chapters start, leaving the introduction to appear rather disconnected.

The chapters of the book are structured into three main sections: big numbers, little numbers, and infinity. The first section discusses a googol, a googolplex, Graham's number and TREE(3), but it also discusses the number 1.000000000000000858. This number allows Padilla to discuss electromagnetism, the special theory of relativity, and time dilation, with the discussion flowing effortlessly, and all of that is presented to the reader using the analogy of Usain Bolt's record-breaking 100 m sprint in Berlin.

Padilla's skill is in starting his discussions using something well understood by the reader, then moving laterally into something new, all very smoothly and in a way that leads the reader into the complexities of quantum mechanics quite easily. How long the reader stays in the more complex areas of the book depends on their prior knowledge of the subject, as I feel that a complete novice would struggle to follow the discussions once they have moved far away from the initial subject. This probably is not a *Quantum Mechanics for Absolute Beginners* book…, but then again, could such a book be written?

Subsequent chapters present the importance of the discovery of the number zero and the discovery of the Higgs boson. The book ends with a fascinating chapter on infinity, managing to present the concept of cardinality of infinite sets in one of the most accessible ways that I have seen.

There are some errors in the book that some readers will notice, and having read other reviews of the book online I am not sure if they have been corrected in subsequent versions of the book. In a chapter that delves into the Standard Model of particle physics, Padilla presents the quarks and the leptons, but mislabels all the leptons as quarks (up, down, charm, etc.), which is a surprising proofreading error. I also think there is another error in the discussion of the cardinality of infinite sets, writing $\aleph_0 = \aleph_0$ when he meant to write $\aleph_0 + \aleph_0$, which was crucial to the point of that chapter.

But those minor issues aside, this is an excellent book to read as long as you do not make the same mistake that I did. If you do not have a major interest in quantum mechanics or cosmology, but you have some prior pre-university studies on the topic, this is an excellent read.

Antonio Padilla, *Fantastic Numbers and Where to Find Them: A Journey to the Edge of Physics.* Penguin, 2022, 352 pages, Paperback ISBN 978-0-141-99282-2, eBook ISBN 978-0-241-44538-9.

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