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Corrigendum to "Logarithmic Sobolev and interpolation inequalities on the sphere: Constructive stability results"

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Abstract. This corrigendum corrects several mistakes in Appendix B.3 of [Ann. Inst. H. Poincaré Anal. Non Linéaire (2023) DOI 10.4171/AIHPC/106].

On the sphere $\mathbb{S}^d$ with $d \geq 1$, let $d\mu$ be the uniform probability measure and assume that $p \in (1, 2^*)$, where $2^* := 2d/(d-2)$ is the critical Sobolev exponent if $d \geq 3$ and, by convention, $2^* := +\infty$ if $d = 1$ or $d = 2$. The *Bakry–Emery exponent* is defined by $2^\# := (2d^2+1)/(d-1)^2$ if $d \geq 2$ and $2^\# := +\infty$ if $d = 1$. With $\beta := 1/(1 + p(m-1)/2)$, δ_0 and $\delta = \delta_0 + 1/(2\beta)$ given by

$$\delta_0 := \frac{2-(4-p)\beta}{2\beta(p-2)} \quad \text{and} \quad \delta := \frac{p-(4-p)\beta}{2\beta(p-2)} \quad \text{if } p > 2, \tag{B.9}$$

and $\delta := 1$ if $p \in (1, 2]$, under the condition that m is "admissible", i.e., $m \in (m_-, m_+)$ where $m_\pm := \big(dp + 2 \pm \sqrt{d(p-1)(2d-(d-2)p)}\big)/((d+2)p)$, we have the following result.

Lemma 15. *If one of the conditions*

(i) $p \in (1, 2^\#)$ *and* $\beta = 1$ *(so that* $\delta = 1$*),*

(ii) $p \in (2, 2^*)$*,* $\beta > 1$*, and* $\beta \leq 2/(4-p)$ *if* $p < 4$*,*

is satisfied, then for any $u = v^\beta \in \mathrm{H}^1(\mathbb{S}^d)$ *we have*

$$\int_{\mathbb{S}^d} \frac{|\nabla v|^4}{|v|^2}\, d\mu \geq \frac{1}{\beta^2} \frac{\int_{\mathbb{S}^d} |\nabla u|^2\, d\mu \int_{\mathbb{S}^d} |\nabla v|^2\, d\mu}{\left(\int_{\mathbb{S}^d} |u|^2\, d\mu\right)^\delta \left(\int_{\mathbb{S}^d} |u|^p\, d\mu\right)^{\frac{\beta-1}{\beta(p-2)}}}. \tag{B.10}$$

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In case (2) of the proof given in [1], δ has to be replaced by δ_0 as follows. With $\frac{1}{2}+\frac{\beta-1}{\beta(p-2)}+\delta_0=1$, Hölder's inequality shows that

$$\begin{aligned}\frac{1}{\beta^2}\int_{\mathbb{S}^d}|\nabla u|^2\,d\mu=\int_{\mathbb{S}^d}v^{2(\beta-1)}|\nabla v|^2\,d\mu=\int_{\mathbb{S}^d}\frac{|\nabla v|^2}{v}\cdot v^{\frac{p(\beta-1)}{p-2}}\cdot v^{2\beta\delta_0}\,d\mu\\ \le\left(\int_{\mathbb{S}^d}\frac{|\nabla v|^4}{v^2}\,d\mu\right)^{\frac{1}{2}}\left(\int_{\mathbb{S}^d}|u|^p\,d\mu\right)^{\frac{\beta-1}{\beta(p-2)}}\left(\int_{\mathbb{S}^d}|u|^2\,d\mu\right)^{\delta_0},\end{aligned}$$

from which we deduce the estimate

$$\left(\int_{\mathbb{S}^d}\frac{|\nabla v|^4}{v^2}\,d\mu\right)^{\frac{1}{2}}\ge\frac{1}{\beta^2}\,\frac{\int_{\mathbb{S}^d}|\nabla u|^2\,d\mu}{\left(\int_{\mathbb{S}^d}|u|^2\,d\mu\right)^{\delta_0}\left(\int_{\mathbb{S}^d}|u|^p\,d\mu\right)^{\frac{\beta-1}{\beta(p-2)}}}.$$

The proof of [1] is otherwise unchanged, with $\delta=\delta_0+1/(2\beta)$. With these changes, the last sentence of Appendix B.3 becomes “With the choice of $m=(p+2)/(2p)$, we find $\delta=1-p/8$.”

References

[1] G. Brigati, J. Dolbeault, and N. Simonov, Logarithmic Sobolev and interpolation inequalities on the sphere: Constructive stability results. *Ann. Inst. H. Poincaré Anal. Non Linéaire* (2023) DOI 10.4171/AIHPC/106

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