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# Mini-Workshop: Felix Klein's Foreign Students: Opening Up the Way for Transnational Mathematics

Organized by
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Abstract. Extending existing analyses of the topic, the workshop aimed to investigate the influence of Felix Klein on the development of mathematics (especially number theory, algebra, geometry, analysis, applications of mathematics in scientific and technical fields as well as in mathematics education) in countries other than Germany. The goal of the workshop was to take a look at mathematicians of foreign origin who studied with Klein that have received little attention so far (including Czech, Greek, Hungarian, Japanese, Polish, Russian, and Ukrainian mathematicians) and uncover how Klein guided them through his lectures and seminars. The protocols of the lectures held in Klein's seminars (from 1872 to 1912 in Göttingen, Erlangen, and Leipzig), which are a unique and so far largely unexplored source, were the basis for the workshop.

*Mathematics Subject Classification (2020):* Primary: 01A55, 01A60, 01A65, 01A70; secondary: 01A72, 01A73, 97A30.

## Introduction by the Organizers

The mini-workshop *Felix Klein's Foreign Students: Opening Up the Way for Transnational Mathematics*, organised by Danuta Ciesielska (Warsaw) and Renate Tobies (Jena), aimed to investigate (extending existing analyses of the topic) the influence of Felix Klein on the development of mathematics (especially number theory, algebra, geometry, analysis, and applications of mathematics in scientific and technical fields as well as in mathematics education) in countries other than Germany.

The idea for this international collaborative project came from Danuta Ciesielska, who for several years has been researching (together with two Polish colleagues) how Polish mathematicians studied in Göttingen with Klein and Hilbert, resulting in a recently published Polish monograph [1].

The mini-workshop *Felix Klein's Foreign Students: Opening Up the Way for Transnational Mathematics* joined together 17 researchers from different countries, familiar not only with their own mathematical traditions, but also with the development of national identities as well as political and cultural histories of the various regions.

Previous research showed that Klein did not have to "court" students from abroad throughout his career. Rather, they were sent to him from Scandinavian countries, from Italy, France, Great Britain, America, the Netherlands, Russia, Switzerland, Austria-Hungary, Greece, etc. We now have a good overview of Klein's first international students and we also have a good analysis of all the women who studied under Klein (from 1893).

The mini-workshop aimed to examine the causes of Klein's international success. Before the workshop, we had arrived at the following hypotheses, partially based on ([3]):

(1) Klein deliberately aimed to found a *mathematical school* as early as 1872. In a letter to Gaston Darboux, Klein spoke of recreating a "school of geometrical production" as he had come to know it under Alfred Clebsch, who had just died. This was later to be considered as "a style of mathematical life that promised colossal successes for the future" ([2]).
(2) This goal required that Klein readily share his own ideas and seek to advance them through *cooperative work*, but now, unlike Clebsch, on an international level – increasingly incorporating new methods into his practice.
(3) Klein's early efforts to become acquainted with various mathematical schools at home and abroad led to good personal contacts with mathematicians of numerous countries, who recommended their own students to Klein. Even when Klein was still in Erlangen in the early 1870s, Sophus Lie recommended Scandinavian students to go to Klein because they *would be encouraged* there (which would not happen if they went to Berlin).

In order to test these hypotheses, it was necessary to look deeper into the sources, especially into the protocol books containing handwritten records of the talks given at Klein's seminars from 1872 to 1912. These 29 volumes are available online:

- https://www.uni-math.gwdg.de/aufzeichnungen/klein-scans/klein/
- https://page.mi.fu-berlin.de/moritz/klein/

In the case of (b) an attempt has been made to identify the complete names of the presenters. There are numerous errors, however, especially with foreign persons. Therefore, we also want to correct the sources with the help of our experts in the future. In addition to Klein himself, the speakers in the seminars were his students or distinguished visitors, many of them foreigners.

During the workshop the state of current knowledge about students with different languages, ethnicities and traditions at Klein's various career stations, especially in Göttingen, were discussed. Participants investigated the similarities and differences between them, while trying to identify all foreign participants in Klein's seminars and analyzing their contributions. Because of that, it was possible to achieve a better understanding of the socio-geographic profiles of the students coming to Klein, their own professional development and their subsequent impact on mathematics and mathematical life in their homelands.

The work on the records of the foreign participants in Klein's seminars allowed for addressing more detailed questions about these participants, including the following:

- Why did they want to work with Klein?
- What was their mathematical preparation before they arrived?
- What topics did Klein assign to them for their own seminar presentation?
- How did Klein further encourage them to work on these or related topics?
- Did this encouragement lead them to creating results of their own in the field?
- Were their results published, e.g. in the *Mathematische Annalen* (which was edited by Klein)?
- Were they later involved in other projects of Klein (e.g. *Encyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen*)?

The mini-workshop also addressed some questions of a general character, in particular these:

- To what extent did Klein influence (directly or indirectly) persons who later achieved outstanding results in individual mathematical fields?
- In what ways did former students of Klein impact the organisation of mathematical life in their homelands (university education, publishing, mathematical societies)?

## References

[1] D. Ciesielska, L. Maligranda, J. Zwierzyńska, *W świątyni nauki, mekce matematyków. Studia i badania naukowe polskich matematyków, fizyków i astronomów na Uniwersytecie w Getyndze 1884–1933* [In the temple of science, in the mecca of mathematicians. Studies and research of Polish mathematicians, physicists and astronomers at the University in Göttingen 1884–1933]. Warszawa: Wydawnictwo Naukowe PWN, 2021.

[2] I. R. Shafarevich, *Zum 150. Geburtstag von Alfred Clebsch*, Math. Ann., **266**:2 (1983), 135–140.

[3] R. Tobies, *Felix Klein: Visions for Mathematics, Applications, and Education*. Revised by the Author and Translated by V.A. Pakis (Vita Mathematica, 20). Cham: Birkhäuser/Springer Nature Switzerland, 2021.

## Mini-Workshop: Felix Klein's Foreign Students: Opening Up the Way for Transnational Mathematics

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# Abstracts

## Founding a School for Mathematical Production – processed and open issues

Renate Tobies

The starting point of the presentation was Klein's network of people. The breadth of the network is explained above all by Klein's vision of creating a school of mathematical production. Connected to this was the empowerment and the urge to lead young talents to their own creative results. The prominent Berlin mathematician Leopold Kronecker believed that mathematicians did not need to form a (scientific) school and that collaborative work would hinder progress in the field. Klein, in contrast, aspired to reproduce Alfred Clebsch's model and thus to create "a school of geometric production". Klein followed Clebsch's program of "uniting people from different fields of work". He created connections between different areas of mathematics (geometry, algebra, function theory, number theory), and made connections between mathematics and its neighboring disciplines. His general way of working was based on his approach to mathematical research, which required cooperation.

Klein tested out as many colleagues and students as possible for their potential as collaborators, among them foreign colleagues and students. This presentation provided an overview of Klein's foreign students, and named their contexts of work as well as interesting open questions. In order to conduct a detailed and thorough analysis of Klein's impact on later developments, this introductory lecture also aimed to explain basic sources, including errors contained therein (online available audience lists of Klein's lecture courses, participants in the research seminars, the Appendix to Klein's Collected Mathematical Papers, vol. 3; the Poggendorff Bio-Bibliographical Hand Dictionary, etc.).

We also gave an overview about when and why foreign students studied with Klein. The success of these studies was discussed by means of examples. On the one hand, consideration was given to the insights gained with the revised Klein biography ([1]), and the recently published book on Klein and Georg Pick ([2]). On the other hand, we also looked into further seminars and minutes of Klein's seminars in order to classify some foreign students about whom we still know too little (students from Hungary, Russia, etc.).

The list of more than 300 people (including two female mathematicians from St. Petersburg) who donated money for the portrait of Felix Klein painted by Max Liebermann was shown. This reveals connections between Klein and other mathematicians as far away as India, Australia and Japan. Some of the examples were used to show what other sources can be consulted to explain the respective person and their mathematical results.

Finally, we highlighted the special role of Klein's interdisciplinary research seminars, which he was able to establish in Göttingen. Klein succeeded in getting

Hilbert appointed on 1 April 1895 and immediately involved him in the leadership of his own seminar, in which the focus was on approximation analysis. In the following semesters, Klein held further joint seminars with Hilbert on number theory, function theory, and mechanics. With the appointment of other younger colleagues, also for applied mathematics, physics, astronomy, statistics, Klein also involved them in the management of his seminars. Examples show how far-reaching Klein's international impact was in these areas (some lesser-known examples: William F. Baker (2015) developed a design tool for plane trusses using an extended Airy stress function based on Klein and Wieghardt (1905); Timoshenko's beam theory; Kármán's vortex street; the Painlevé-Klein problem in the theory of friction). The name *Technomathematik* is just over twenty years old; it was created in Kaiserslautern in 1979 by Helmut Neunzert (*1936) for a new study program intended to merge mathematics and technology. Neunzert recently confirmed that he was inspired to do so by Felix Klein's combination of pure and applied mathematics.

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[1] R. Tobies, *Felix Klein: Visions for Mathematics, Applications, and Education*. Revised by the Author and Translated by V.A. Pakis (Vita Mathematica, 20). Cham: Birkhäuser/Springer Nature Switzerland, 2021.

[2] R. Tobies, *Felix Klein und Georg Pick. Mathematische Talente fordern und fördern*. Berlin: Springer Spektrum, 2023.

## Felix Klein vs. Henri Poincaré, 1881–82: On the Birth of the Theory of Automorphic Functions

David E. Rowe

Between June 1881 and September 1882 Klein und Poincaré exchanged letters that led to their competition to find and prove general uniformization theorems in complex analysis. These letters were first published in 1923 by Mittag Leffler in Volume 39 of Acta Mathematica (see [7], pp. 111–133). In that same year, Klein also published them with commentary in [5], pp. 577–621; see further [6], pp. 334–381. As a famous topic in the careers of both men, one can find many discussions about it, not only in the standard biographies [2] and [9], but also in more technical historical studies such as [8] and [1]. The present account mainly aims to add some further contextualization by drawing on the four letters Brunel sent to Poincaré during June and July 1881 when he was studying under Klein in Leipzig.

Georges Brunel (1856–1900) entered the École normale in 1877. After graduation in 1880, he spent the academic year 1880–81 in Leipzig working under Felix Klein. In his first letter to Poincaré, Brunel introduced himself as a "comrade", i.e. fellow normalien of Paul Appell and Émile Picard, though both were older than he. In 1884 Brunel obtained the chair for pure mathematics in Bordeaux. His predecessor was Jules Hoüel, a leading authority on non-Euclidean geometry, having in 1867 translated works by Lobachevsky and Bolyai.

Brunel had already spent a good deal of time in Leipzig before he wrote to Poincaré, and this stay abroad had not been easy for him. Still, he felt a deep urge to serve his country, while behaving properly as a guest in a foreign land. As he explained to Poincaré, he hoped to learn what German mathematicians had to teach the French. This view was promoted by Charles Hermite, who urged his pupils to follow the new currents of research pursued on the other side of the Rhine.

Klein's seminar during the winter semester 1880/81 dealt with various topics in geometry and complex function theory. Adolf Hurwitz and Walther Dyck, two of his most important German students, both attended. Along with Brunel, others came from foreign countries: Giuseppe Veronese, then an assistant under Luigi Cremona in Rome, and Washington Irving Stringham, a student of J.J. Sylvester at Johns Hopkins University. Shortly before Christmas 1880, Brunel spoke about Riemann's approach to the genus of surfaces and its role in algebraic curve theory. Klein assigned him this topic as well as some relevant literature with which to prepare his talk.

In January 1881, Brunel spoke for the second time on a related topic: Riemann's theory of Abelian functions and Enrico Betti's generalization to higher dimensions. Brunel also discussed the pioneering topological studies undertaken by the Göttingen physicist and mathematician Johann Benedict Listing. This reflects Klein's longstanding fascination with this older tradition. In fact, Betti and Riemann first met in Göttingen, though their friendship grew far closer during Riemann's final years when he spent much of his time in Pisa and elsewhere in northern Italy.

During the summer semester of 1881, Klein's lectures moved deeply into Riemann's theory of functions of a single complex variable. Nearly all the talks in his seminar dealt with topics closely tied to this course. The one striking exception was Brunel's presentation of Cantor's new theory of point sets. This included a proof that the algebraic numbers constitute a countable subset within the uncountable infinity of real numbers. Besides works by Cantor, Brunel also discussed papers by several other German authors, including J. Lüroth, E. Netto, and E Jürgens. He also treated Liouville's classical method for constructing transcendental numbers which, following Cantor's theorem, form an uncountably infinite set. This was thus a second new field of research that had not yet made inroads into France, showing that Brunel was well prepared to act as an early envoy for recent mathematics in Germany. To a certain extent, he actually took up this role. On returning to France, Brunel published a review of Klein's booklet *Ueber Riemanns Theorie der Algebraischen Functionen und ihrer Integrale* (republished in [5], pp. 479–573).

Already in his second letter to Poincaré, written on 19 June 1881, Klein informed him that Brunel had been studying in Leipzig. He advised him further that Brunel would be able to give Poincaré details about Klein's research program. In his seminar, Klein vented his anger over the fact that Poincaré had ignored the published literature:

> . . . [Klein] complained that the "young French" didn't know what had been published in Germany; he said that in France one probably didn't know that the Mathematische Annalen existed (to which I [Brunel] could only have replied that, in Berlin itself, this Journal was considered to be problematic), that one didn't read Crelle's Journal (where did you read Fuchs's work?), etc., etc.

Klein was especially angry that Poincaré had honored Lazarus Fuchs, whose work had helped inspire his own:

> I protest against the name Fuchsian functions. The fundamental idea belongs to Riemann, and the credit for applying Riemann's idea belongs to Schwarz. . . . Later, I myself worked in this direction and . . . I presented some results which are the basis of Mr. Poincaré's work. As for Mr. Fuchs, who once wanted to deal with similar questions, he only succeeded in this: in showing us that he understood absolutely nothing about them.

In March 1882, after Klein published a note in Mathematische Annalen rejecting the names Poincaré had introduced, Mittag Leffler wrote him: "But certainly Mr. Klein is right in saying you are wrong to call your functions Fuchsian and Kleinian functions. They must be named Poincaré functions. It's the only name that's fair and reasonable."

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[2] J. Gray, *Henri Poincaré: A Scientific Biography*. Princeton: Princeton University Press, 2013.

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[6] F. Klein, *Vorlesungen über die Entwicklung der Mathematik im 19. Jahrhundert*, Bd. 1. Berlin: Julius Springer, 1926.

[7] D.E. Rowe, *A Richer Picture of Mathematics: The Göttingen Tradition and Beyond*. New York: Springer, 2018.

[8] E. Scholz, *Geschichte des Mannigfaltigkeitsbegriffs von Riemann bis Poincaré*. Boston: Birkhäuser, 1980.

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## Mathematicians Connected with the Czech Lands as Klein's Students and Collaborators

Martina Bečvářová

From the second half of the 19$^{\text{th}}$ century, the most talented and outstanding German and Czech mathematicians from the University in Prague, later from the German University in Prague or German Technical University in Prague went abroad thanks to government scholarships or other funds. Czech mathematicians travelled mainly to Italy, France or Germany because of many political and cultural reasons, German mathematicians travelled mainly to Germany. Both of them, they studied in the most prestigious mathematical centres of the period, at Berlin, Göttingen, Hamburg, Leipzig, Munich, Paris, Strasbourg, Milano, and Rome.

What were their main goals and professional interests to travel abroad? They tried to expand the horizons of their mathematical knowledge and establish contacts with the best experts from the famous European mathematical centres. They were interested in new, modern and promising mathematical topics that were missing or not given enough attention at the University in Prague. They wanted to be more involved in the latest mathematical trends and methods and to get in touch with the newest mathematical ideas. They wanted to publish their first scientific works in world-renowned and respected journals and their first monographs in internationally known publishing houses. They aspired to obtain doctorates at leading European universities and, after returning home, to achieve a better career, i.e. to habilitate and later become full professors at schools in their homeland. They also wanted to recognize the most advanced education methods and get them to universities and polytechnics in the Czech lands.

What were their typical activities during their study stay abroad? This depended on at what stage in their career they got the opportunity to study abroad. Regular students visited some special lectures or seminars. Graduates of basic studies participated in more advanced seminars as passive visitors or active lecturers. They prepared their dissertations and broadened their horizons for the doctoral process. Both used libraries where new monographs, journals or thesis were. They wrote their first articles and discussed their topics, ideas or first scientific results. They participated in the life of mathematical communities (professional as well as social) and it was very important part of their stays. Many of them became later a member of the Deutsche Mathematiker-Vereinigung and promoted German mathematics and culture.

Based on archival sources deposited in the Czech Republic, Germany and Italy, we discovered only 12 mathematicians connected with the Czech lands who studied or worked under the influence of Felix Klein (1849–1925) at the Polytechnic in Munich in the 1870s, at the University in Leipzig in the 1880s and at the University in Göttingen in the early 20th century as his regular students, or prepared their doctoral thesis, or passed their doctoral procedures and took active participation in his special mathematical seminars, published their first mathematical results

thanks to his helps or inspirations or collaborated with him during all their lives as good mathematicians and personal friends.

Only one of them was a Czech mathematician [Ludvík Kraus (1857–1884)], the others were German mathematicians [Anton Puchta (1851–1903), Karl Bobek (1855–1899), Seligmann Kantor (1857–1903), Georg Alexander Pick (1859–1942), Wilhelm Weiss (1859–1904), Emil Waelsch (1863–1927), Joseph Grünwald (1876–1911), Georg Hamel (1877–1954), Ernst Fanta (1878–1939), Lothar Schrutka (1881–1945) and Paul Georg Funk (1886–1969)]. Nine of them connected a greater or lesser part of their lives, pedagogical or professional activities with Prague universities (the Czech University, the German University, the German Technical University; Puchta, Kantor, Kraus, Bobek, Pick, Waelsch, Weiss, Grünwald and Funk). Four of them connected part of their lives with the German Technical University in Brno (Waelsch, Hamel, Fanta and Schrutka). Five of them went abroad after a shorter or longer career in the Czech lands (Puchta, Hamel, Fanta, Schrutka and Funk), one completely resigned from his academic career after a short period of his pedagogical activities (Kantor). For detailed information on their personal life, academic career, teaching and other activities see [1, 2, 3].

Three of them were for one academic year Klein's regular students (Hamel, Schrutka and Funk). Five of them started their doctoral procedure with Klein's help or inspirations at the University in Prague, Leipzig or Erlangen (Puchta, Kantor, Bobek, Waelsch and Weiss). Only Kantor was unsuccessful, and that for formal and not for professional reasons. Three of them completed a study stay with Klein before starting their regular habilitation procedures at their Alma Mater in Prague or Vienna (Kraus, Grünwald and Fanta). Only one of them after a successful habilitation at the German University in Prague completed a study stay at Klein as his equal colleague (Pick).

Especially important moments for the professional growth of young mathematicians from the Czech lands were their participation in Klein's special mathematical seminars, where he himself, his guests, doctoral students or the best university students presented the latest results of their research or reported on newly published articles. The essential source on the history of Klein's seminars are the so-called *Sämtliche Protokolle 1872–1912*, i.e. Klein's seminar protocol-books which are online available [6]. Twenty-nine books provide interesting information on Klein's seminars from the summer semester 1872 until the summer semester 1912. The names of the lecturers, the titles and abstracts of the lectures written by the lecturers themselves (1 or more pages) and the lists of participants can be found there. Thanks to these records, we know that ten mathematicians connected with the Czech lands lectured at Klein's seminars from 1876 until 1912, one took part in the seminars without any lecture (Kantor). Only Schrutka apparently did not participate in Klein's seminars. Our participants presented 30 lectures in the German language. Their topics were analysis (10), geometry (8), algebra and theory of numbers (6), mechanics (3), instruments (2) and other (1).

| Name | Place | Time | Nr. | Topic |
|---|---|---|---|---|
| Puchta Anton | Munich | WS 1876/1877 – SS 1877/1878 | 9 | Geo, Al, An |
| Kraus Ludvík | Munich | WS 1878/1879 – WS 1879/1880 | 7 | Geo, Al, An |
| Bobek Karl | Leipzig | WS 1881/1882 – SS 1881/1882 | 2 | An |
| Pick Georg Alexandr | Leipzig | WS 1883/1884 – SS 1883/1884 | 5 | An |
| Waelsch Emil | Leipzig | SS 1884/1885 | 1 | Al, Geo |
| Weiss Wilhelm | Leipzig | SS 1884/1885 | 1 | Geo |
| Grünwald Josef | Göttingen | WS 1899/1900 | 1 | Mech |
| Hamel Georg | Göttingen | SS 1899/1900 – WS 1900/1901 | 2 | Instr, Mech |
| Fanta Ernst | Göttingen | WS 1901/1902 | 1 | Instr, Mech |
| Funk Georg Paul | Göttingen | WS 1911/1912 | 1 | An |

Geo – geometry, Al – algebra, An – analysis, Mech – mechanics, Instr – instruments

The mathematicians from the Czech lands as others wrote abstracts of their lectures (usually from 1 to 5 pages in 1870s, from 2 to 15 pages in 1880s, from 4 to 23 pages later). Pick's abstracts were long, perfect, inspiring and beautifully written with full references. Hamel's abstracts were the same, with interesting pictures, but almost unreadable. For more information on the seminars, see [4, 6].

Mathematicians from the Czech lands thanks to Klein's help published some of their results in the journal Mathematische Annalen which covered a wide spectrum of mathematics and was published from 1869 until 1919 by Teubner (in Leipzig), since 1920 by Springer (in Berlin). Klein was its redactor from 1876 until 1924 and influenced its content and focus. Six German mathematicians from the Czech lands published their first articles in this journal with Klein's support; they were devoted to number mathematical branches. The articles are online available [7].

| Name | Nr. | Time | Notes |
|---|---|---|---|
| Kantor Seligmann | 3 | 1879–1882 | geometry, configurations |
| Bobek Karl | 2 | 1884,1887 | elliptic functions, geometry of curves |
| Pick Georg Alexandr | 16 | 1883–1915 | transformations, algebraic geometry, number theory, special functions, functional spaces |
| Weiss Wilhelm | 1 | 1887 | geometry |
| Hamel Georg | 10 | 1903–1935 | special functions, geometry, ordinary differential equations |
| Schrutka Lothar | 2 | 1912, 1941 | number theory |

It should be emphasized that Klein had a significant influence on Pick, who brought modern mathematics at the German University in Prague and, thanks to his almost half-century-long pedagogical work at that school, raised a new generation of Prague German mathematicians (as for example K. Löwner, H. Löwig, A. Winternitz, W. Fröhlich, O. Varga). For more information, see [1, 4, 5]. Pick's, Hamel's, Kantor's and Funk's results are widely known, contemporary recognized and still cited. The other mathematicians who studied with Klein had only a local influence in the Czech lands (teaching, creating textbooks, training technicians, etc.) for many reasons (political, religious, economic, health, personal etc.).

Mathematicians from the Czech lands were in corresponding touch with German mathematicians, i.e. with Klein too. Their correspondence is a great resource for better understanding the depth of their cooperation, Klein's professional influences, personal assistances and helps. The correspondence of mathematicians from the Czech lands has been partly preserved in Germany. Unfortunately, we do not know anything about the correspondence of mathematicians from Germany. For many historical reasons, it was not preserved in the Czech archives after the World War II.

| Sender | Addressee | Nr. | Time | Notes |
|---|---|---|---|---|
| Puchta A. | Klein F. | 7 | 1878–1886 | a |
| Puchta A. | Cantor M. | 4 | 1882–1883 | b |
| Kraus L. | Klein F. | 1 | 1879 | a |
| Pick G. A. | Klein F. | 131 | 1884–1898 | a |
| Pick G. A. | Hilbert D. | 2 | 1885, 1900 | c |
| Pick G. A. | Hurwitz A. | 1 | 1893 | d |
| Pick G. A. | Schwarzschild K. | 1 | 1914 | e |
| Pick G. A. | Gordan P. | 1 | 1892 | f |
| Pick G. A. | von Kraus C. | 12 | 1908–1913 | e |
| Kantor S. | Klein F. | 1 | 1884 | a |
| Bobek K. | Klein F. | 2 | 1880, 1884 | a |
| Waelsch E. | Klein F. | 3 | 1889–1890 | a |
| Hamel G. | Klein F. | 3 | 1908–1922 | a |
| Küpper K. | Klein F. | 3 | 1889 | a |

a – Nachlass Felix Klein, Göttingen, b – Nachlass Moritz Cantor, Heidelberg, c – Nachlass David Hilbert, Göttingen, d – Mathematiker-Archiv, Göttingen, e – Nachlass Karl Schwarzschild, Göttingen, f – Universitätsbibliothek, Erlangen-Nürnberg – Bayerische Staatsbibliothek, Munich.

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[5] R. Tobies, *Felix Klein und Georg Pick. Mathematische Talente fordern und fördern*. Berlin: Springer Spektrum 2023.
[6] Protokolle der Seminare Felix Kleins [UBG] (see: https://firsching.ch/klein/).
[7] Mathematische Annalen, see https://link.springer.com/journal/208/volumes-and-issues.

## Mellen Woodman Haskell in Göttingen and Leipzig

Henning Heller

Mellen Woodman Haskell (1863–1948) was among the first American students of Felix Klein. The Harvard graduate and later professor at Berkeley stayed in Leipzig and Göttingen from October 1885 until June 1889. Unfortunately, no first-hand sources remain from Haskell's formative years. Using civil, school, and university archives, this presentation sheds some light on Haskell's family background and education, his study years in Leipzig and Göttingen, his participation at Klein's seminars, and the circumstances of his PhD examination.

### 1. Family background and early life

Haskell was born on 17 March 1863 in Salem, Massachusetts, as the first child of the clergyman Augustus Mellen Haskell (1832–1893) from the village of Poland, Maine, and Catherine Woodman (1827–1866) from the neighboring town of New Gloucester [1]. The young parents lived in Salem from the year of their marriage, 1861, until the birth of their second son, Augustus Storey Haskell (1866–1949) [2]. In that year, the family moved to Manchester, New Hampshire, where Augustus found a new post, but tragically, Catherine died only months later of tuberculosis. Augustus remarried one year later with Anna Johnson (1826–1909) from Salem. In 1870, the family settled to West Roxbury, today a suburb of Boston, which remained their long-lasting home. In 1873, Mellen entered the Roxbury Latin School at the age of 10, three years younger than his peers. He even skipped a year and was admitted to Harvard College at the incredibly young age of 15 years, but decided to repeat his last year at school [3]. Haskell entered Harvard College in 1879 and studied mathematics at Harvard University from 1883 to 1885. He was awarded the Parker Fellowship to pursue a PhD degree in Germany.

### 2. Studying in Leipzig and Göttingen

Haskell matriculated in Leipzig in winter 1885/86 to study under the supervision Felix Klein, but did not take any courses there (he probably learned German instead) [4]. When Klein moved to Göttingen for the next summer term, Haskell came with him. In Göttingen, Klein delivered an advanced mathematical lecture series that followed his own research interests [5]. During the span of seven semesters, Klein lectured on algebra (S86–W86/87), elliptic modular functions

(S86), hyperelliptic functions (S87–W87/88), and abelian functions (S88–S89). In all but the last semester, also a seminar was held. Haskell was by far the most active student of this lecture cycle as he visited (in one case: probably) all lecture courses and seminars [6]. Additionally, he was co-responsible for the production of lecture notes in three lectures courses. During these years, a number of mathematicians, which were later influential for the setup of the mathematical research community in the United States, arrived in Göttingen. These include the German postgraduates Oskar Bolza and Heinrich Maschke, the American postgraduate F.N. Cole, and the American PhD students H.D. Thompson, W.F. Osgood, H.S. White, H.W. Tyler, and Maxime Bôcher [7]. Haskell was perhaps the only peer who personally met *all* of these figures, but as no personal communication remains, it is hard to estimate his status and influence.

## 3. Presentations

Haskell held four presentations in Klein's seminars, all of them during his first three semesters in Göttingen [8]. (He later participated at the seminars without presenting himself.) His first presentation concerned the explicit calculation of a degree-4 resolvent of the *octahedral equation*. This resolvent stems from a subgroup relation $S_3 \subset S_4$, while Klein in his *Lectures on the Icosahedron* [9] only considered a chain of resolvents stemming from the subnormal series $S_4 \rhd A_4 \rhd C_4 \rhd C_2 \rhd \{1\}$. Haskell's presentation thus filled a small gap in Klein's book. Although mathematically unspectacular, Haskell's consideration can be understood as a preparation to Klein's resolvents of his *icosahedral equation*, which likewise stem from non-normal subgroup relations ($D_5 \subset A_5$ and $A_4 \subset A_5$, respectively). In this sense Haskell did his share to "complete" the heuristics of Klein's geometrical approach to solving equations. Haskell's other three presentations concerned a recent publication of Lazarus Fuchs, and two "work-in-progress" presentations on his PhD thesis.

## 4. PhD examination

The content of Haskell's PhD dissertation – *On the multiple covering of the plane belonging to the curve* $\lambda^3\mu + \mu^3\nu + \nu^3\lambda = 0$ *in the projective sense* – was already considered in [11], so I focus on some new biographical insights [12]. In his review, Klein emphasized the difficulty of Haskell's subject, and praised that Haskell overcame them "with extraordinary diligence in a thoroughly satisfactory manner". He also noted "a number of individual investigations that claim independent importance". Haskell was thus accepted for his oral examination in mathematics and physics (as he had to choose a second subject), which took place on 6 June 1888. Unfortunately, Haskell did not pass the physics examination by Woldemar Voigt. Haskell was allowed to repeat the examination 6 months later, but decided to take his time and only repeated on 18 June 1889. He passed this second examination (again by Klein and Voigt), and two weeks later returned to the United States. From the steamboat "Aller", he thanked Klein for his support, and concluded about his study time in Göttingen:

> I have finally learned what it means to *work*. It is a hard thing to learn, but I hope it sticks with me. [13]

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## Klein *ohne* Klein: Studentship at a distance in Chicago (1893-1910)

Nicolas Michel

Some ten years into his tenure as the first head of the mathematics department at the University of Chicago, E. H. Moore paid the following tribute to Felix Klein:[1]

> Certainly in the domain of mathematics, German scholars in general and yourself in particular have played, **by way of example and counsel and direct and indirect inspiration**, quite the leading role in the development of creative mathematics in this country, and on behalf of my colleagues here I wish to express our most grateful recognition and appreciation of our profound debt.

Despite only visiting the United States twice in his life, Klein seemingly exerted a lasting and multi-faceted influence on the shaping of American mathematics at the turn of the 20th century. But what were the modalities of this 'direct and indirect' influence? In their landmark account of the development of the American

[1] Letter from Moore to Klein, dated March 23rd 1904, Klein Nachlass X, NSUB, Göttingen, cited in [2, p. 324]. Emphasis mine.

mathematical community, Parshall and Rowe stress the importance of Klein's participation to the 1893 Chicago World Fair and his ensuing public lectures in Evanston, and map out the ways in which American students picked up on research themes *en vogue* in Germany and made them their own. In this paper, I elected to focus on another dimension of this 'influence', namely the fostering of a certain collegial and scientific culture at the University of Chicago modelled after and adapted from that which Klein had developed in Göttingen.

Indeed, we now know that an account of mathematics in Klein's Göttingen cannot be complete without an assessment of its culture of oral and informal conversations (whether at a professor's personal home or during *Spaziergängen*), of its seminar life full of intensity and competition, as well as of its mathematical library where visitors, students, and faculties would frequently meet and interact [1]. This paper, along similar lines, seeks to provide elements of a 'thick description' of the oral culture at the Mathematics Department in Chicago and to trace its inception back to Klein's Göttingen.

In fact, a first step in that direction had already been taken during a previous Oberwolfach meeting, centered around the figure of Oswald Veblen–a towering figure in the history of American mathematics who initially studied in the Moore-led department at Chicago. In this meeting, three other scholars and I collectively transcribed and analyzed a notebook written by Veblen as he studied in Chicago, and more specifically as he attended Moore's 1901 seminar on the foundations of geometry [3]. One outcome of this project was to highlight the rich interplay between seminar life, research activities, and the fostering of graduate students to which this notebook bore witness. Both Moore and Veblen would make key contributions to axiomatics and geometry in the wake of this seminar; contributions in which they both highlight this seminar as a site of collaborative and productive learning. What's more, we find the same pattern whereby seminars and dialogues between students and professors lead to important contributions by both groups amongst other students of Moore's and (at a later stage) of Veblen himself, who reproduced this social organization of mathematical life.

To better understand the origins of this form of mathematical life, one must look at another one of the three professors who initially constituted the Mathematics Department at the University of Chicago upon its creation in 1893: namely, Oskar Bolza. In his 1936 autobiography *Aus meinem Leben*, Bolza describes his own experience as a student in Germany looking to obtain a doctorate and to go into a mathematical career of his own. Upon meeting with leading mathematicians in Berlin, where experts in his subjects of choice resided, he was faced with scholars who had no interest nor desire to engage meaningfully with a young scholar-to-be. Neither Kronecker nor Weierstrass helped him design an appropriate research question for a dissertation, nor did they advise him as to how to work on said question: they would simply wait for him to bring a manuscript of sufficient quality to them, and then examine him.

Discouraged by this experience, Bolza then travelled to Göttingen, initially to work with Schwarz. In so doing, he encountered Klein and discovered an entirely

different model for scholarly conduct. Klein would not only propose research questions and provide mathematical advice; he would also foster constant dialogue with and amongst students, meet weekly with them, and help insert them into German academia. Not all aspects of this practice were enjoyed by Bolza, however: after spending two semesters in 1886-1887 within this intense and dynamic community, he despaired to keep up with Klein's masterful weaving of so many mathematical concepts and intuitions and lost faith in his own ability as a researcher.

Bolza's research output would only rarely interact with the central themes of Klein's own mathematical product. Yet, he regarded the latter as the person who influenced him the most after Weierstrass. I contend that this influence mostly lies in the shaping of this model for scholarly conduct. Evidence to this idea can be adduced by considering the archives of the Mathematics Department under Moore's and his successor's (G. A. Bliss) leadership; sources which now are preserved at the Hanna Holborn Gray Special Collections Research Center in Chicago.

One such set of sources is a collection of letters sent to Bliss by dozens of former students of Bolza's, at Chicago. These letters, written in celebration of the 50th anniversary of Bolza's doctorate, all point to a mathematician's lasting commitment to many of the epistemic virtues which characterized Klein's tenure at Göttingen. Recollections of Bolza's availability and of the regular conversations he held with students at his own house and of his active stance towards the fostering of graduate students and the selection of appropriate research questions, for instance, feature heavily in these letters. Interestingly enough, however, the elitism that dominated mathematical conversations in Göttingen seems mostly absent from these students' experience–perhaps a remnant of Bolza's own anxieties regarding the overwhelming effects of genius.

Moreover, these letters provide a rich description of Bolza's much-appreciated teaching practices, including his constant reliance on historical exposition to motivate the study of a given subject-matter, to distinguish between ancient and modern (i.e., fit for graduate research) approaches to said subject, and to constitute a canon of classical texts whose knowledge he expected of students. Further evidence to flesh out this description can be found in the many notebooks written by students of Bolza's seminars, also preserved at the University of Chicago, as well as in the publications of Bolza and his best students (include Bliss himself). All those pedagogical traits can be profitably compared with the rich portrait of Klein as a teacher and a historian of mathematics which Renate Tobies has built over the years [4].

Together, these historical elements point to another sense in which Chicago mathematicians can be regarded as Klein's students *at a distance*, through the intermediary of Bolza's reproduction of a Göttingen-inspired figure of the mathematician as scholar, as teacher, and as researcher.

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## Klein's influence in Britain

June Barrow-Green

In 1873 Klein made the first of several visits to Britain travelling in both Scotland and England. On the invitation of Henry Smith, the leading English number theorist and geometer, he attended the annual British Association for the Advancement of Science meeting which that year was held in Bradford, with Smith as President of the Mathematics & Physics section. It was at this meeting that Klein first met Arthur Cayley, William Clifford and James Clerk Maxwell, as well as the Irish astronomer Robert Stawell Ball. Clifford and Maxwell would die in 1879 but Klein's association with Cayley was enduring. Both Smith and Cayley sent postgraduate students to Klein including Arthur Buchheim, Arthur Berry, and Grace Chisholm. Another student who travelled from Britain to Göttingen during Klein's tenure was Horatio Carslaw who went there to study with Sommerfeld. Among the other British mathematicians who one way or another encountered Klein in their youth were the Cambridge educated mathematicians Henry Frederick Baker, Augustus Love and Edmund Taylor Whittaker.

Klein's reflections on his 1873 visit, as revealed in letters to Sophus Lie, give valuable insights into his interactions with the British mathematical community.[1] For example, not only did Klein find Sylvester to be more brilliant than Cayley, but he found "everyone in London" to be "generally of the same opinion" [1, p.148]. On that first visit, as well as on others, Klein travelled with the Scottish mathematician (later orientalist and Old Testament scholar) William Robertson Smith whom he had originally met in Bonn in 1867 (when Smith was studying with Plücker), and with whom he had cemented a friendship when they were both in Göttingen in 1869 [2].

Klein's first British student was Arthur Buchheim (1859–1888) who in 1881 made three presentations on Abelian integrals in Klein's seminar in Leipzig. Buchheim had been a student of Henry Smith's at Oxford and Smith thought extremely highly of him. However, when Buchheim returned from Germany, rather than continue at Oxford he made his career as a schoolmaster. In his obituary of Buchheim, Sylvester attributed Buchheim's refusal to apply for a vacant Fellowship at Oxford, despite being "strongly pressed by the authorities to do so", to Buchheim's sojourn with Klein having put him too much out of "the style of ordinary English University Examinations" [3]. Thus, it would appear that Buchheim did not think

[1]Several extracts from this correspondence are reproduced in [1].

he would pass the New College Fellowship examination while Smith thought that he would.

Klein's contacts with Cayley, and later with Andrew Russell Forsyth, who 1895 succeeded Cayley in the Sadleirian chair at Cambridge and whom Klein visited often, resulted in several postgraduate students going to Göttingen. Arthur Berry (1862–1929), who later worked in elliptic functions and differential equations, and became known for his *History of Astronomy* (1898), had been Senior Wrangler[2] in 1885, and went to study with Klein in 1887, giving a seminar on 'Differential Invariants'. Later Berry himself would encourage female students, such as Hilda Hudson (1881–1965) and Lorna Swain (1891–1936), to continue their studies in Germany.

The fulsome remarks in praise of Klein by Henry Frederick Baker (1866–1956) in the preface of his book *Abel's Theorem and the Allied Theory of Theta Functions* (1897), appear to be the only surviving evidence of Baker's meetings with Klein. Nevertheless, they make clear Klein's influence on Baker's mathematical thought. In addition, Baker published several papers in *Mathematische Annalen* (in English) in the 1890s, his choice of publication presumably deriving from his time in Göttingen. It also seems likely that the idea for Baker's geometry seminar (known colloquially as Baker's 'tea party') in Cambridge, which began in 1914 (with Baker's accession to the Lowndean chair) and which was the first, and for a long time the only, seminar in Britain, can be traced back to Göttingen [4].

Both Augustus Love (1863–1940) and Edmund Taylor Whittaker (1873–1956) spent time with Klein when he visited England, and they both wrote articles for Klein's *Encyklopädie*. In addition, in 1887 Love was commissioned by Klein to write an article on English work on vortex motion [5], and later Klein arranged for the translation of Love's text on elasticity into German [6]. Klein also arranged for the translation into German of Horace Lamb's text on hydrodynamics [7], both translations appearing in 1907.

With regard to translations, Klein's influence extended in the opposite direction too, with the translation of his own works into English. As well as his well-known *Lectures on the Ikosahedron and the Solutions of Equations of the Fifth Degree* (1888) which was enthusiastically reviewed by Cayley, there was his *On Riemann's Theory of Algebraic functions and their Integrals* (1893), the translation of which was done by Frances Hardcastle (1866–1941), a Cambridge student who completed the work while she was in the United States at Bryn Mawr, and published it at her own expense.

A number of British female students attended Klein's seminars in the 1890s, the most notable of whom was Grace Chisholm who studied for her PhD under Klein and in 1895 achieved the distinction of being the first woman anywhere to be awarded a traditional PhD in mathematics [8], [9].[3] Isabel Maddison, who was

[2]The Senior Wrangler is the top student in the Cambridge Mathematica Tripos.

[3]Chisholm was the subject of Elisabeth Mühlhausen's talk [10].

a contemporary of Chisholm's at Girton College in Cambridge,[4] subsequently went to Bryn Mawr to work under the direction of Charlotte Scott (a student of Cayley's) before studying with Klein and Hilbert in Göttingen during 1893/4. She was followed there by Ada Johnson (who had surpasssed all the men in Part II of the Cambridge Mathematical Tripos in 1894), in 1895/1896. (Rather curiously, all attempts to ascertain Johnson's area of research have so far failed. She returned to Cambridge where she was an Associate at Newnham College until 1908 but then drops out of sight.) A later visitor to Göttingen was Lorna Swain who, encouraged by Berry, went there just before the outbreak of the First World War but had to return to England in haste when war was declared. Swain would go back to Göttingen in the late 1920s to work with Ludwig Prandtl.

Another British traveller to Göttingen was Charles Tweedie (1868–1925) who in 1891 studied under both Klein and Schwarz [12]. Tweedie was unusual in that he had not been an undergraduate at Cambridge but at Edinburgh where one of his teachers was P. G. Tait whom Klein had met on his first visit to Britain. Tweedie made his career in Edinburgh and became best known for his work in history of mathematics, notably biographies of Colin Maclaurin and James Stirling, having earlier published papers in geometry and combinatorics.

That Klein was very well known in Britain is evident from the recognition he received. He was made a Fellow of the Royal Society of London in 1885 and in 1912 was awarded the Copley Medal, the Society's most prestigious award. He is the only foreign mathematician to have been awarded the London Mathematical Society's most important prize, the De Morgan medal (1893), and in 1897 the University of Cambridge awarded him an honorary doctorate.

From the above it is evident that Klein had many direct connections to members of the British mathematical community. Further research is required to establish more precisely the extent to which Klein can be said to have had an influence on them, both with respect to their mathematical development and with respect to their careers.

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## From Naples to Pavia, passing from Göttingen. The scientific trajectory of Ernesto Pascal and his relationship with Felix Klein

Maria Giulia Lugaresi

The Italian mathematician Ernesto Pascal (1865-1940), born in Naples in 1865, completed his primary and secondary education in his hometown. Attracted by the mathematical teachings of Nicola Trudi, Emanuele Fergola, Achille Sannia and, most of all, Giuseppe Battaglini, he graduated in mathematics at the University of Naples in 1887. Soon after his degree, Pascal obtained a training scholarship for the academic year 1887-88 at the University of Pisa, where he had the opportunity to attend the lessons of Enrico Betti, Ulisse Dini, Luigi Bianchi and the young Vito Volterra. In the next academic year Pascal, encouraged by Eugenio Beltrami – who at that time was Professor at the University of Pavia – decided to go to the University of Göttingen to improve his studies.

From November 1888 to August 1889 Pascal was in Göttingen where he could meet and study with Hermann Amandus Schwarz and, most of all, Felix Klein, who contributed to orient Pascal's research towards Sigma abelian functions. Soon after his return to Italy, Pascal was appointed "Extraordinary Professor"(1890-95) and then "Full Professor"(1895-1907) at the University of Pavia, after the death of Felice Casorati. In 1907 Pascal was called at the University of Naples, where he remained until his retirement in 1935. He kept the chair of Higher Analysis, in 1910 he moved to the chair of Complementary Algebra and maintained for assignment the chair of Higher Analysis. In the same year he became editor in chief of the journal "Giornale di Matematiche di Battaglini", replacing Alfredo Capelli. Pascal died in Naples in 1940.

The main episodes of Pascal's academic life and his scientific trajectory of research can be better understood through the reading of the letters he wrote during all his professional life. As of 1889, Pascal was in correspondence with Klein. The Göttingen State and University Library preserved eleven manuscripts (ten letters and one draft) that Pascal sent to Klein between October 1889 and August 1913

([12]). The correspondence, even if composed only by eleven manuscripts, offers useful pieces to enrich Pascal's academic life. The correspondence began soon after Pascal's return to Italy. The letters proved Pascal's positive memory of his German experience. He recalled with enthusiasm, but also with nostalgy his stimulating meetings with Klein. The Italian mathematician took part in Klein's course in Summer Semester 1889, that were devoted to the theory of Abelian functions.

During his stay in Göttingen Pascal met and could work with many German and foreign mathematicians who came to Göttingen to study under Klein. In the letters to Klein Pascal referred to some of these mathematicians with whom he remained in touch after his return to Italy. Among the mathematicians who were in Göttingen in the same period he quoted Heinrich Burkhardt, Henry White, Mellen Woodman Haskell. Klein presented the research of his students in the sessions of the Göttingen Academy of Science. The first results of Pascal's studies about Abelian sigma functions appeared in two short articles, presented by Klein, and were published in the volume of 1889 of the "Nachrichten von der K. Gesellschaft der Wissenschaften und der Georg-Augusts-Universität": *Zur Theorie der ungeraden Abel'schen Sigmafunctionen* (pp. 416-423); *Zur Theorie der geraden sigma-Funktionen* (pp. 547-553). The two articles were republished in a longer version in the volume 18 of the Annali of Brioschi in 1889 ([3]; [4]).

Between 1889 and 1895 Pascal's research dealt with Abelian, hyperelliptic and elliptic functions. This wide field of research was developed by Pascal taking inspiration from Klein's lectures and publications and gave birth to seven articles that appeared in the volumes 17-19 of the Annali of Brioschi ([3]; [4];[5];[6];[7];[8];[9]). Other influences of Klein can be found in Pascal's works about sigma elliptical functions, that were published in 1895 ([10]).

The correspondence with Klein was interrupted between March 1895 and February 1901. In this period Pascal's research continued, but his publications were mainly oriented towards handbooks for university teaching. Many monographs, prepared for his university courses, appeared between 1895 and 1897, first in a litographic version and then they were printed in paperback size by the editor Hoepli in Milan: *Esercizi e note critiche di calcolo infinitesimale* (1895); *Teoria delle funzioni ellittiche* (1896); *I determinanti: teoria ed applicazione con tutte le più recenti ricerche* (1897); *Calcolo delle variazioni e calcolo delle differenze finite* (1897); *Repertorio di matematiche superiori* (1897-1900, 2 volumes). Some years later these books were translated into German by the mathematicians Hermann Leitzmann and Adolf Schepp. These translations contributed to spread Pascal's works outside Italy. The main mathematical handbooks of Pascal had also a Polish translation thanks to the editorial work of the Polish mathematician Samuel Dickstein.

The *Repertorio* constituted an excellent contribution to a significant assessment of nineteenth-century mathematical production. It responded to the way in which studies were organised in Germany, providing an overall vision of a single discipline (analysis or geometry), in opposition to the extremely sectorial approach of Germany. Themes related to the development and the teaching of mathematics

were particularly important for Pascal. In Naples he gave a great stimulus to the teaching of mathematics thanks to the creation of mathematical seminars and laboratories, the so-called "mathematical cabinets" (*gabinetti scientifici*). Pascal treasured his German experience when he decided to realise in Naples these scientific places and he talked about them in a letter to Klein (1913, August 14th).

In my paper I will give an overview of the content of Pascal's letters preserved in Klein's archive in Göttingen in order to reconstruct the development of Pascal's research following suit Klein. The letters represented also a proof of Pascal's devotion and respect for Klein. The Italian mathematician strongly supported two scientific and celebrating ventures in Naples: a prize for summarising Klein's results about hyperelliptic and Abelian functions and the appointment of Klein as a foreign member of the Royal Academy of Sciences of Naples.

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## Wilhelm Wirtinger (1865–1945) and his publications on Abelian functions, in particular theta functions

Peter Ullrich

Even though Wilhelm Wirtinger himself saw a strong cultural, in particular scientific, connection between the German-speaking parts of the Austro-Hungarian Empire and the Deutsche Reich, which had been formed between 1867 and 1871 under Prussian leadership, he can be counted among Klein's "foreign students":

In 1866 Austria and Prussia fought the Seven Weeks' War – which also had the consequence that Göttingen became a Prussian university –.

Wilhelm Wirtinger was born on July 19, 1865 in Ybbs at the Danube where his father was chief physician at a predecessor of the Vienna psychatric clinic. At school Wilhelm was almost exclusively interested in mathematics and physics. Already as schoolboy he studied original works of Isaac Newton (1642/43–1727), Leonhard Euler (1707–1783), Carl Neumann (1832–1925) and others including texts on Abelian functions. During his studies of mathematics and physics at the University of Vienna from 1884 until 1887 he seems to have been mainly under the influence of Emil Weyr (1848–1894): Even before his doctorate, he published two papers [9], [10] which belong to synthetic geometry as the latter's research area. Also the topic of his doctoral thesis, on a cubic involution in the plane, comes from this part of mathematics.

After having taken his doctorate on December 23, 1887, Wirtinger received a scholarship from the Todesco foundation which enabled him to spend the winter semester 1888/89 in Berlin where he attended lectures with Lazarus Fuchs (1833–1902), Leopold Kronecker (1823–1891), and Karl Weierstraß (1815–1897). Even though he obviously was quite familiar with the standards of rigor as introduced by Weierstraß, it was the following summer semester 1889 in Göttingen that had the most decisive influence on his scientific career: He attended the last part of Felix Klein's (1849–1925) lecture courses on Abelian functions and then delved into research on this topic: Starting already in Göttingen and continuing after his return to Vienna, he published several notes [11], [12], [13], [14] on Kummer surfaces of genus 3 and the Abelian functions and theta series associated to them. Even more, as one learns from his letter to Klein dated December 28, 1892, he worked on the theory of general theta series already at that time. Klein had designed this topic for the prize question of the Beneke foundation for 1895 which was administered by the Philosphical Faculty at the University of Göttingen. On the basis of his book [16], which is connected to his articles [15], [17], Wirtinger received the prize.

This immediately helped his academic career: After his return to Vienna, Wirtinger had completed his habilitation at the University of Vienna in 1890 and had received the position of assistant to Emanuel Czuber (1851–1925) at the Polytechnic of Vienna. Following the announcement of his winning the prize of the Beneke foundation in 1895, he was appointed extraordinary professor at the University of Innsbruck. One year later, there he was promoted to an ordinary professorship. In 1903 he followed a call as an ordinary professor at the University of Vienna.

There he became the supervisor or at least a reviewer for almost all important mathematicians who received their doctorates at the University of Vienna between 1905 and 1930. Johann Radon (1887–1956), who had been one of these students, has called him "der größte Mathematiker Österreichs" (= "Austria's greatest mathematician") [6]. Wirtinger retired in 1935 and died on January 16,

1945, also in Ybbs at the Danube. (For further information on Wirtinger's life see [1], [2], [5], [8].)

Wirtinger not only published original research on Abelian functions, but also invested a lot of time and energy in publication projects that Klein had initiated: He was one of the co-editors of the three-part volume 2 on analysis of the "Encyklopädie der mathematischen Wissenschaften", wrote its article on algebraic functions and their integrals [18], provided templates for its article on elliptic functions, which appeared in 1913, and co-authored, together with Adolf Krazer (1858–1926), its article on Abelian functions and general theta functions [3]. Additionally, together with Max Noether (1844–1921), he edited the supplements to Riemann's collected works [4].

Within the period between 1901 and 1920, when these text were published, lies Wirtinger's "unfruchtbares Jahrzehnt" (= "barren decade") between 1909 and 1919 when he published no original results of research at all. One reason for this could be problems with the publications for the "Encyklopädie" project. But he also suffered severe blows of fate within his family: His eldest son died in an accident in 1912, his youngest son died in action during the First World War in 1915. And, naturally, the circumstances of this war and its consequences will have reduced his ability to conduct scientific research.

In a handwritten autobiography from 1939, Wirtinger states that it was the task of writing an article "Klein und die Mathematik der letzten fünfzig Jahre" (= "Klein and the mathematics of the last fifty years") [19] on the occasion of Klein's seventieth birthday in 1919 that brought him back to mathematical productivity. However, it is remarkable that from this time onwards, Abelian functions and theta series were no longer the focus of his research, cf. e. g., [7].

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## Klein's Göttingen seminars on hydrodynamics (1903-1904, 1907-1908, 1908) and the development of the notion of applied mathematics until Richard von Mises (1883-1953) in the 1920s

Reinhard Siegmund-Schultze

Our most important source about Felix Klein's seminars–which stretched over about 40 years and took place in several universities (Göttingen, Leipzig, Munich)– is Renate Tobies' biography of Klein of 2019/20 [1]. Of the three seminars on hydrodynamics between 1903 and 1908, the one in the winter semester 1907/08 has been analysed in detail by Michael Eckert [2]. The same author has devoted an English monograph to the history of the turbulence problem which pervades much of the three hydrodynamics seminars [3]. Eckert and Tobies show that the seminar talks cannot be completely understood without considering the Göttingen context in general, in particular Klein's lectures on hydrodynamics and handwritten notes, both kept at the Manuscript Division (Handschriftenabteilung) in Göttingen.

In my talk I stressed the need to look at the talks which were given parallel to the seminars at the Göttingen Mathematical Society (Mathematische Gesellschaft) as well. This Society was also basically run by Klein from 1892. Of the talks one finds abstracts of varying length published in the Jahresbericht of the DMV. At the Gesellschaft, Klein usually reported about his seminars. In addition, talks by established mathematicians, physicists and engineers visiting or resident in Göttingen were presented there, including talks on applied topics. The hydrodynamics seminar talks were predominantly given by students. They treated basic topics such as Boussinesq's theory of fluids [4], the separation of the stream from the wall (Theodor von Kármán) and of vortices from the stream (Hiemenz, Koch,

Fuhrmann), and Heinrich Blasius' discussion of turbulence [5]. Blasius was one of the first students of Ludwig Prandtl. Blasius' discussion of the difference between onsetting and fully developed turbulence in his seminars in January and February 1908 was taken over in 1916 by Prandtl. Other groundbreaking new ideas such as the Hungarian Gyözö Zemplén's theory of shock waves (1904), Prandtl's boundary layer (1904), and the Hungarian von Kármán's vortex street (1911) were reserved for talks at the Mathematische Gesellschaft and for parallel publications. Carl Runge and Prandtl, who from 1904 were directors of Göttingen's Institute for Applied Mathematics and Mechanics [6], preferred the venue of the Mathematische Gesellschaft for their presentations. They were absent as speakers at the three hydrodynamics seminars, although—according to Klein—were co-organizers and probably present at the seminars in 1907 and 1908.

A third effort, which completes the picture of applied mathematics and mechanics research done in Göttingen at the time, is Klein's Encyclopaedia of Mathematical Sciences. Here the Austrian Richard von Mises (1883-1953), who, at the time of the hydrodynamics seminars, was assistant to Georg Hamel at the German Technical University of Brünn, came into play. He had an intense correspondence with Klein and his assistant Conrad Müller between 1907 and 1912 which accompanied his article in the Encyclopaedia "Dynamical Problems of Mechanical Engineering" [7]. The correspondence shows Klein's high expectations for the young engineer and mathematician von Mises, who never presented a talk at the seminars and would later in the 1920s become the director of the institute for applied mathematics in Berlin. The correspondence also shows Klein's keen interest even in specific, technical applications such as Otto Schlick's patent (1894) for a gyroscope to prevent ship lurching. On this patent the future theoretical physicist Paul Ehrenfest had reported in Klein's seminar in 1902. Klein persuaded von Mises to include another invention with a similar purpose (Frahm's water tanks) in his article.

Klein passed on von Mises' critical remarks about another Encyclopaedia article to the author von Kármán (Strength of materials in mechanical engineering). In this episode, as in some others, the spirit of collaboration and competition becomes palpable which was typical of the Göttingen atmosphere both in pure and applied mathematics. In my talk I quoted some remarks from the autobiographical memoirs (1936) of Hans Lorenz (1865-1940) which were critical of Klein's alleged dictatorial ways of running the seminars [8]. Lorenz, who was originally considered by many as being able to imbue engineering and technical physics with a new level of mathematical sophistication, had been appointed by Klein in 1900. But disappointed by his lack of willingness to cooperate, Klein managed to replace him by Prandtl in 1904. Von Mises' scathing review of the insufficient mathematical treatment in Lorenz's turbine theory of 1906 may have endeared the 24 year old von Mises to Klein. However, the later development of both Lorenz and von Mises shows a certain frustration with some Göttingen tendencies at domination and towards "nostrification" (make them ours) of results obtained outside the Göttingen environment, a feeling which the two scientific engineers shared.

Nevertheless, von Mises remained a staunch admirer of Klein and devoted an article to him on his 75th birthday in his new *Journal for Applied Mathematics and Mechanics* (*ZAMM*) in 1924. In a public dispute (1927) with Richard Courant, Klein's successor as an organizer in Göttingen, von Mises claimed that the realization of Klein's efforts was being more loyally pursued in Berlin than in Göttingen [9].

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## On the impact of Felix Klein on his students and their successors. Austro–Polish stories

Danuta Ciesielska

In the period from 1874 to 1912, more than 50 Poles studied mathematics with Klein in Munich, Leipzig and Göttingen. Between 1885 and 1911, 40 Poles or Polish students from Russia attended Klein's lectures, courses and seminars, including 14 men and women who gave lectures in Klein's seminars [11]. In this report I will focus on some of these people, works and future results.

The very first Poles, in total 12, studied with Klein in Munich. In Leipzig only two Poles attended Klein's courses and in Göttingen at least 43 Polish young students or scholars participated in Klein's lectures or seminars, 14 of them presented 19 talks during Klein's seminars [11]. Future physicist Józef Wierusz-Kowalski (1866–1927, Joseph von Kowalski, Josef de Kowalski) was the first who presented talk [11].

Many Poles who were former Klein's students did not work at universities until 1918 but some were professors at those universities that operated in Austria with Polish as a language of instruction: the universities in Kraków and Lwów, and the Lwów Technical School. Some became professors of University of Warsaw

(opened by German Governor in 1915), Warsaw Polytechnic (1915) and Polish Free University in Warsaw (Wolna Wszechnica Polska). When Poland regained independence in 1918, the universities in Poznań and in Vilnius completed the list. Here I focuse on those Klein's students who worked in Kraków or Lwów before 1918, in land ruled by Austria.

The only Pole to appear in Klein's *Vorlesungen über die Entwicklung der Mathematik im 19. Jahrhundert*, vol. 2 (Berlin 1927) is Kazimierz Żorawski (1866–1953). Żorawski, from Żórawski noble family, studied in Warsaw and next with Sophus Lie in Leipzig (1889/90; 1890), where he was inspired to write his doctoral thesis (1891). From the winter semester of 1890/91 to the winter semester of 1891/92 he studied with Klein in Göttingen. He presented a paper on the Grünwald–Letnikov derivatives of fractional order in Klein's seminar in June 1891 [11]. In the summer semester of 1891 seminar took place between 14 May and 8 July 1891. In the meetings participated 12 people, and the talks were presented by Ernst Ritter (PhD 1891 in Göttingen under Klein, moved to the USA), Friedrich Schilling (PhD 1893 in Göttingen under Klein, Achen, Karlsruhe, Dantzing, president DMV), James Harrington Boyd (PhD 1892 in Princeton, professor at Macalester College, reader at Chicago), Eduard Burr von Vlek (PhD 1893 in Göttingen under Klein, professor at the University of Wisconsin Madison, now a chair of mathematics here holds his name) and Kazimierz Żorawski. The notes from this meetings are in volume **10**, 120–179.

In his contribution Żorawski presented basic information about the derivative of fractional order. The very first attempt to derivatives of fractional order are due to Euler and Liouville. The formal definitions were proposed by German mathematician from Prague Anton Karl Grünwald (1838–1920) and by Russian mathematician Aleksey Vasilyevich Letnikov (1837–1888). Grünwald in 1867 posed a definition [10]. Letnikov in his master thesis in 1868 proposed a similar definition [3] which was later elaborated by Pavel Alekseevich Nekrasov (1853–1924). Żorawski in his talk presented short history of an investigation and recall definitions, among them Grünwald's: $[D^p f(x)]_u^x = \frac{1}{\Gamma(-p)} \int_u^x \frac{f(z)}{(x-z)^p} dz$, next he focused on interesting examples. Later Żorawski published only one paper on the similar topic. It was an article in Polish about derivatives of infinitely large order [14]. Nevertheless, he was very active as a scholar and in academic policy. His scientific results include applications of Lie groups in geometry and the theory of differential equations and mechanics. Some of the results of Żorawski's research were obtained again later, published and attributed to other mathematicians. Żorawski's works were highly appreciated by Lie, Élie Cartan, and Klein. He published more than 60 scientific papers in German, French, Polish and Czech, he also contributed in history and wrote textbooks.

Żorawski was a dean of faculty, rector, a director in Ministry of Education in Poland but in my opinion the most interesting is his role as a Chair of Kretkowski's Fund. A very rich nobleman and a mathematician Władysław Kretkowski (1840–1910) donated in his last will the huge fortune for abroad studies in mathematics (1911–1919), for extra lectures and seminars, library, and a chair of application of

mathematics in Kraków. Thanks to this donations 11 young mathematicians went abroad for undergraduated studies, and three of them studied in Göttingen: Franciszek Włodarski (1889–1944), Władysław Ślebodziński (1884–1972), Stanisław Ruziewicz (1889–1941), for more information see [1].

In the winter semester 11 people took part in the meetings of seminars, some of the previous participants had left, but some started participation. Among them were Poles: Stanisław Kępiński from Kraków and Bolesław Młodziejowski from Moscow. Kępiński spent two years in Göttingen (1891–1893) and participated in Klein's seminar with a lecture (see [11], vol. **11**). He started studies at the Jagiellonian University where he obtained doctorate; his thesis concerned partial differential equations. In Göttingen Kępiński studied with: SS 1891 – Schwarz, Klein, Burkhardt, WS 1891/1892 – Klein, Burkhardt and SS 1891/1892 – Klein, Weber, and Burkhardt. He wrote extensive reports from this studies (Archive of the Jagiellonian University), fragments of them were published in [1] and [2]. In 1894 he obtained *vienam legendi* from the Jagiellonian University and worked in Kraków for a year. Next he continued his academic and scientific career at Lwów Polytechnic, he was elected a rector and member of Galizian (province od Austria) parliament in 1903. Four Kępiński's letter to Klein are the only letters sent by Polish scientists to him [5].

The second part of contribution deals with the influence of Klein and Lie research on the group of symmetries on the second generation of his former students. The concept of the abstract topological group is connected with Hilbert's fifth problem. In 1900 in Paris Hilbert asked if it was possible to have "Lie's Begriff der kontinuierlichen Transformationsgruppe ohne die Annahme der Differentzbarkierait der di Gruppe deffnireden Functionen" [Lie's concept of the continuous transformation group without the assumption of the differenceability of the functions defined in the group] (see: [4], p. 269). It started a new concept of group in Euclidean space but without transformation. That idea finally lead to the formal definition of topological group. In 1925 Franciszek Leja (1885–1979) and Otto Schreier (1901–1929) independently presented it in their reports [7], [12] and two years later in papers [8], [9], [13]. This amazing, almost mystic, coincidence is one of dozens in the history of mathematical research, but here there is no doubt that both heroes was influenced by the ancestor, including Felix Klein. A former student of University of Lwów, Leja worked on his dissertation on invariants of partial differential equations under mentioned before Żorawski. Leja's later research on the application of Lie groups to differential equations led him to a definition of the abstract topological group as group and topological space at the same time, such that the group operation: product and inverse map are continuous. An Austrian Otto Schreier was a doctoral student of Philipp Furtwängler (1869–1940), who was Klein's doctoral student in Göttingen. Schreier was working on abstract algebra and this led him to the definition of abstract topological group; moreover, he proved that it must be abelian.

FIGURE 1. Members of Mathematical Society in Göttingen, 1895. SUB Göttingen, 2 Sammlung Voit: Gruppenbild 4. Sitting (from left): Eduard Götting$^p$, Ernst Schering, Henrich Weber, Woldemar Voigt, Eduard Riecke and Wilhelm Schur. Standing (from left): Ernst Harald Schütz$^p$, Ernst Ritter$^t$, Henrich Burkhardt$^t$, Ludwig Harald Schütz$^p$, Ignatz Robert Schütz, Julius von Braun, Georg Bohlmann, Erich Prümm$^p$, Rudolf Schmidt$^p$, Wilhelm Felgentraeger, Otto Blumenthal$^t$, Stanisław Tołłoczko$^t$, Sophus Marxsen$^t$, Adolf Jost, Maximilian August Toepler, Teophil Friesendorf$^t$, Dychhoff, Wilhelm Lorey$^t$. $p$ – studied with Klein, $t$ – presented talk at Klein's seminar.

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## Göttingen, far away: Felix Klein's influence on mathematics in Japan

Harald Kümmerle

While Göttingen figures prominently in the history of mathematics in Japan as a destination of study abroad, it is difficult to clearly identify Japanese students of Klein. This is importantly because the founder of mathematics as a discipline in Japan, Fujisawa Rikitaro (1861-1933), had laid a strong emphasis on pure mathematics. Investigating the cases of Yoshiye Takuji (1874-1974) and Kuroda Minoru (1878-1922), both of whom studied in Göttingen, gives insight into the mathematical "establishment" in early 20th century Japan. In contrast, two of the most prolific recipients of Klein's contributions to the movement for the reform of mathematical education, Hayashi Tsuruichi (1873-1935) and Ogura Kinnosuke (1885-1962), had very untypical biographies and never studied in Göttingen (nor Germany more generally). Despite this, they were based at Tohoku Imperial University (in Sendai), where the whole department of science officially give itself the motto to become "Japan's Göttingen". When investigating their activities in mathematical research and education as well as the hurdlest, they faced, Hayashi and Ogura appear as two exponents of the "anti-establishment". Ironically but not coincidentally, the movement for the reform of mathematical education in Japan

– together with other ambitions of Felix Klein – achieved success only when militarism was taking hold during the 1930s. Thus, the talk provides an additional perspective on the complex relation of mathematics and modernity.

## Getting to Göttingen: Support for women's mathematical research at Bryn Mawr and Girton

Brigitte Stenhouse

The end of the nineteenth century saw the emergence of mathematical research as a formal part of university life, legitimised by the awarding of doctoral degrees. The seminars of Felix Klein epitomised the ideals of a certain style of university research that became highly respected (and imitated) by a broad mathematical community. The seminars fulfilled many functions, from connecting mathematicians on an international scale, enabling collaboration, supporting students in the pursuit of a PhD, and substantially directing research through the posing and answering of questions [1]. Although participation in university life was highly gendered, this story of rigorous higher education and *careers* in mathematical research is no less true for women as for men.

In this talk we explore the communities and structures built to enable women to pursue research in mathematics. We focus especially on the anglophone women who came from Bryn Mawr College in Philadelphia, USA and Girton College in Cambridge, UK to participate in Klein's Göttingen seminar.

Girton College was founded in 1869 as the first residential college for the degree-level education of women. The founder had a clear vision of what she wanted to achieve with the college: "Miss [Emily] Davies was resolved that women students should submit to the same tests, in order that they might share the same opportunities, as men" [2, pp. 16–17]. This occasionally put Davies in conflict with the founders of Newnham College, also in Cambridge, who were happy to provide separate lectures and exams specifically for women, rather than fighting to integrate women into the established Cambridge systems (though it should be noted that this was partly in response to a low opinion of the Cambridge exams!). High school education for girls was still at a low standard in the late nineteenth century, and the early Girton students had to almost start from scratch learning the mathematics, latin, and other subjects necessary to pass the intermediary exams — such as the 'Little-Go' — before moving on to the prestigious Tripos examinations taken in their final year. In order to sit the exams, special permission was required from the examiners, each and every year, with no guarantee it would be granted. Nevertheless, the tenacity of the teachers and their students paid off in 1905 when Trinity College, Dublin offered retrospective reciprocal degrees to any woman who had previously satisfied all the necessary degree requirements. The hundreds of women who travelled by steamboat to Ireland to claim their degree (forty years before Cambridge would award degrees to women on an equal footing with men) were thus dubbed the 'Steamboat Ladies'.

One of the earliest Girton students to sit the Mathematical Tripos was Charlotte Angas Scott, who was placed as 8th Wrangler in the First Class in 1880. (All undergraduates who sat the exam were ranked in order, and those achieving a first class were named Wranglers with the top student named Senior Wrangler. Women were not officially included in the ranking.) Frances Hardcastle was bracketed 53rd in Part I in 1891, and then achieved a second class in Part II in 1892. In 1892, two students from Girton sat Part I of the Mathematical Tripos and attained the equivalent ranking of Wrangler, namely Isabel Maddison and Grace Chisholm (later Chisholm Young). Grace Chisholm subsequently sat Part II in 1892, being ranked in the 3rd class. All of these women were subsequently involved with Klein's seminar, as were many of the men who they were taught by at Girton, such as Arthur Cayley, Arthur Berry, and Andrew Russell Forsythe [3].

Meanwhile, in the USA, Brywn Mawr College was founded in 1879 with classes beginning in 1885 [4]. Similarly to Girton, Brywn Mawr had a strong focus on providing an education for women to rival that received by their male contemporaries. Indeed, Bryn Mawr was the first women's college in the USA to have a graduate department, whilst many of the others focused on offering high-school-level tuition to compensate for the inadequate extant provision for women. The college was substantially shaped by the first Dean and later President, Martha Carey Thomas, who had been refused a PhD in linguistics from Göttingen in 1882, owing to her sex. Charlotte Angas Scott was appointed as Associate Professor in Mathematics at Bryn Mawr in 1885, and would supervise eight PhD students during her tenure which lasted until her retirement in 1924. One of those PhD students was Isabel Maddison, who subsequently worked at Bryn Mawr in various roles until 1926, including as Assistant to the President (Thomas) and Reader in Mathematics. Three other PhD students who passed through Göttingen include Emilie Norton-Martin, Virginia Ragsdale, and Helen Elizabeth Schaeffer [5].

A key motivation for many higher education institutions was to train women into qualified teachers who could then provide a higher quality education for girls. This is reflected in the career trajectories of the alumni of Girton and Bryn Mawr, at least for those who remained unmarried and thus in paid employment [6]. However, through these institutions women were simultaneously building infrastructure to support themselves in pursuing alternative careers in mathematical research [7].

Research Fellowships were offered to graduate students at Bryn Mawr specifically to enable a period of study and research to be undertaken in Europe. Maddison, Norton-Martin, and Schaeffer all made use of the Garret European Fellowship in order to study with Klein in Göttingen during their doctoral studies; Ragsdale was supported by the Bryn Mawr European Fellowship. The importance of funding research activities was directly recognised and championed by Girton alumni, who collectively funded a Research Studentship worth £100 a year for two years. By the 1930s the fellowship infrastructure at Girton had considerably expanded to include multiple opportunities aimed directly at researchers in the physical sciences, including mathematics. Money was also used to recognise and celebrate

achievements of alumni, which would assist in reputation building as a mathematician. Maddison and Chisholm-Young were both awarded the Gamble Prize from Girton, for published articles in 1895 and 1915 respectively. Hardcastle similarly won the Gamble Prize in 1897 for an article on point groups, and she was subsequently invited to write a report on this topic for the British Association for the Advancement of Science; her work in this area was supported by a Pfeiffer Studentship, again from Girton.

Both Bryn Mawr and Girton provided employment opportunities for women, but it is ambiguous as to whether this helped or hindered research activity. Teaching loads could be heavy, leaving little time for other pursuits, and contracts were often precarious, only running year by year. It is pertinent to consider the case of Grace Chisholm-Young, perhaps the most research active of all women who attended Klein's seminar, and yet she never held an official academic position, instead collaborating with her husband who had easier access to university jobs as a Cambridge-educated man [8]. Nevertheless, women's colleges provided gainful employment and access to an academic community that was not always easily found elsewhere; learned societies in Europe and North America only gradually accepted women as full members from the end of the nineteenth century onwards.

In conclusion, whilst there are many specificities to studying the trajectory of women's research careers at the end of the nineteenth century, doing so allows us to reflect more broadly on the changing landscape of academic research. We can witness the emerging means of publishing and gaining recognition for work done; the beginnings of research taking on a complementary role to teaching within university spaces; the need for robust high school education to prepare students for university; the importance of scholarships and research fellowships; and the high value of building networks and social organisation which thus directs and enables the development of new ideas in mathematics.

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## Felix Klein's first female doctoral student Grace Emily Chisholm Young (1868–1944). A livelong connection concerning mathematical research and much more

Elisabeth Mühlhausen

In October 1893 Grace Emily Chisholm arrived in Göttingen to study mathematics, physics and astronomy. Having successfully completed her studies at Girton College in Cambridge her lecturer Andrew R. Forsyth had suggested that she should continue there because he knew about his colleague Felix Klein's commitment to women's studies.

With enthusiasm and energy she followed the lectures, she joined discussions with her new academic acquaintances. During this time she enjoyed the invitations of Klein and his family. She also improved her German language skills and gave her first lecture at Klein's seminar early in 1894 about spherical trigonometry. This subject was suggested by Klein and the basis for her dissertation on "Algebraisch-gruppentheoretische Untersuchungen zur sphärischen Trigonometrie". She obtained her Ph.D. degree "magna cum laude" in April 1895 and was the first woman in Prussia to do so.

On the famous photography of the Göttingen Mathematics Club of 1902 you can see her keeping eye contact with Klein in the centre who is surrounded by his younger colleagues. Meanwhile Grace had married the mathematician William Henry Young (1863–1942). On their first mathematical journey to Italy, another suggestion of Klein's of course, they got to know the mathematicians who belonged to his network. In winter 1898 they studied for some months at the University of Turin with professor Corrado Segre (1863–1924) who held the chair of higher geometry and was a regular correspondent of Klein. In the field of algebraic geometry the Youngs wanted to get on the latest research level. One year later their results were published in Turin, in Italian naturally.

Their successful joint work started 1900 when Klein advised them to read the report on set theory of Arthur Schönflies (1853–1926). The next 25 years they concentrated their research on this evolving field. One year before Klein had adviced Schönflies to write an article on the state of set theory for the Encyclopedia and soon thereafter Schönflies published his much longer report "Die Entwicklung der Lehre von den Punktmannigfaltikeiten" in the journal of the Deutsche Mathematiker-Vereinigung.

It was the starting point for their contribution to set theory and its applications. Between 1900 and 1905 they published their joint work results in about 20 papers and concluded a contract with Cambridge University Press to publish a textbook in set theory that introduced this new field to the UK. "The Theory of Sets of Points" was published already 1906.

They also worked in related areas like measure theory and integration, Fourier series, and the foundation of differential calculus.

Until 1929 they published 214 papers which are listed in the bibliography by Ivor Grattan-Guinness. Most of them appeared under William Young's name, 13 were jointly authored and 18 have Grace Chisholm Young as the sole author.

For me as a former teacher of mathematics and biology it was particularly interesting that Grace Chisholm Young also found time to write children's books. She published in 1905 "Bimbo" and in 1907 a successor called "Bimbo and the Frogs" in which she describes the life of a family that is strongly reminiscent of the Young family. At that time they already had four children: Frances *1897, called Bimbo since their time in Italy, Cecily *1900, called Rosebud, Janet*1901, called Lenchen and the just born Helen *1903, in the books mentioned as Dortchen in the cradle. Every family member is part of a turbulent family life and in between the focus is on a boy, Bimbo, who asks questions that are answered with illustrated biological descriptions of the development of plants and animals especially frogs.

In 1905 Grace Chisholm Young published another book, her first joint publication with her husband with the title: "Beginner's book of Geometry". It describes the basic features of geometry in terms of folding paper. It is obvious that she took up Klein's ideas how to teach mathematics in a visual way.

The book is entertaining not only because of many illustrations but a child needs perseverance and parental help to get through the complicated folding instructions. Fortunately every 3D model can be folded without using any glue. The child-friendly structure of the book from the simple to the difficult is well done.

In 1908 there appeared a German translation "Der kleine Geometer" by Felix Bernstein (1878–1956) who at that time he had just published his dissertation concerning set-theory. Felix Klein was glad to see it and was very fond of it. In his "Elementarmathematik vom höheren Standpunkte aus", published 1908, it is described as a new original way to introduce the child to geometric understanding by starting with all kinds of three-dimensional models.

Apparently Felix Klein sensed the mathematical creativity and power Grace Chisholm Young possessed. His trust in her talent was certainly a strengthening factor for her mathematical confidence. Under his influence she changed from a student to a research mathematician. I think, it was one of Felix Klein's special talents to recognise and support talented mathematicians regardless of gender.

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## From St. Petersburg to Göttingen. About two female Felix Klein undergraduate students

Joanna Zwierzyńska

The second half of the 19th century was a pivot period for women's higher education. They were finally allowed to study at universities in several European countries such as Switzerland or France. In the Russian Empire, they were still not formally admitted to universities, but institutions aimed exclusively at them gradually emerged, making higher education possible. The largest and best-known such institution in the Russian Empire was the Bestuzhev Courses – a four-year

course founded in 1878 in St. Petersburg that enabled women deprived of access to universities to obtain higher education.

The idea of women's higher education was promoted in the society of the Russian Empire by female activists leading the women's movement in this country: Nadezhda V. Stasova, Maria V. Trubnikova, and Anna P. Filosofova. They gained support from wide circles of democratic intelligentsia, among others professors of St. Petersburg University, to create such an institution [5]. Higher Women's Courses were named Bestuzhev Courses after Konstantin Nikolayevich Bestuzhev-Ryumin (1829–1897), professor of history and their first director.

Bestuzhev Courses opened their doors to women of all social classes. To allow women with worse financial situation to study, Filosofova, Stasova, and other female activists created the Society for Providing Means of Support for the Higher Women's Courses, which organised book sales, lotteries, lectures, and concerts so they could award scholarships to deserving women [4].

The Higher Women's Courses had two faculties: one in history and philology and the second one in mathematics and natural sciences. They stood out for their excellent teaching staff as well as their facilities: they had excellently equipped classrooms, and in 1880, the first chemical laboratory for women was provided. In 1903, a mathematical reading room was opened due to the students' request [8].

The first women graduated from Bestuzhev Courses in 1882, four years after the school was opened. Graduates of the courses were pioneers in almost every scientific field in the Russian Empire and beyond - suffice it to say that among them were for example the first woman to become an employee of the Pulkovo Observatory, Russia's first female petrographer and palaeontologist, founder of the empire's first women's accounting courses, the first female climatologist in Russia or the first female university professor in Romania.

Among the graduates of the Bestuzhev Courses were Helena Bortkiewicz (Helene von Bortkewitsch) and Aleksandra Stebnicka (Alexandrine von Stebnitzky), both of Polish origin and of noble birth. Both came to Göttingen attracted by the opportunity to study under Felix Klein's tutelage. Klein was known as a firm believer in the equal abilities of men and women, and he accordingly believed that they should have access to the same educational opportunities [6].

Helena Bortkiewicz (Helene von Bortkewitsch) was born on 3.08.1870. Her father was Józef Bortkiewicz, a Polish nobleman who served in the Russian army with the rank of colonel, lecturer in artillery and mathematics at the military academy, and author of textbooks on mathematics, economics, and bookkeeping. Her mother was Helena (Helene) Bortkiewicz, née Rokicki (von Rokicka). Helena (daughter) was the sister of the statistician and economist Władysław (Ladislaus) Bortkiewicz (von Bortkewitsch) (1869–1931).

Aleksandra (Alexandra, Alexandrine) Stebnicka (von Stebnitzky) was born 23.04.1870 in Tbilisi. She was a daughter of the Polish engineer, general Hieronim Stebnicki (1832–1897), a cartographer, geodesist, geophisicist and a corresponding

member of the Academy of Sciences in St. Petersburg, who worked in the orbervatory in Pulkovo, and his wife Praskowia. Alexandra's sister Olga was mother of Pyotr Kapitsa (Peter Kapitza), a famous phisicist and Noble laureate.

Both Helena Bortkiewicz and Aleksandra Stebnicka came to Göttingen in the academic year 1894/1895 (Władysław Bortkiewicz already had a PhD degree obtained in 1893 in Göttingen, but in 1895 he worked in Strasbourg). Because of their gender, they were not allowed to enter the matriculation book, but they could attend lectures and seminars of professors who gave their consent. They not only participated in Klein's courses but also presented papers in his seminar. Both enrolled in Klein's lectures on differential (SS 1895) and integral (WS 1896/1897 – only Bortkewitsch) calculus, number theory (WS 1895/1896), theory of the top (WS 1895/1896) and the mathematical theory of the gyroscope. Both participated in Klein's seminar (SS 1895, WS 1895/1896, and only Bortkewitsch in WS 1896/1897) Each of them gave two talks at Klein's seminar:

- 28 May 1895, Tuesday, Helene von Bortkewitsch, *Differenzenrechnung.*
- 31 May 1895, Friday, Alexandra von Stebnitzky, *Summationsrechnung.*
- 27 November 1895, Wednesday, Alexandra von Stebnitzky, *Ueber die ganzen Zahlen im Körper(i) und ihre Zerlegungssätze.*
- 11 December 1895, Wednesday, Helene von Bortkewitsch, *Grundlegung der Idealtheorie.*

An analysis of the notes shows that Aleksandra Stebnicka and Helena Bortkiewicz were well-prepared to deliver a fairly advanced lecture. Unfortunately – neither of them was given the opportunity to pursue a real scientific career. With their immense talents, excellent education and ample opportunities, they did not realise a career in science.

Helena Bortkiewicz, after her stay in Göttingen came back to Russia. As Wolfgang Karl Härdle and Annette B. Vogt realised, ,,[she] published papers in Russian journals, but the situation was not comfortable for her since the only widely accepted professions for women were as a physician or a teacher in a girls school"[2]. She worked for three years as a teacher of mathematics and languages. In February 1917 she became a staff member in a St. Petersburg bank [2].

After the October Revolution, Helena Bortkiewicz moved to Berlin, where she lived from 1919 in her brother's apartment in Berlin-Halensee After his death in 1931, she suffered severe financial problems [2]. She died 29.10.1939.

Aleksandra Stebnicka after studies came back to St. Petersburg and tried to work scientifically. She was primarily an astronomer, not a mathematician, so it is not a surprise that she became an astronomer in Pulkovo near St. Petersburg, where she independently conducted astronomical observations, sponsored by the Imperial Academy of Sciences in St. Petersburg. She died 28.04.1928.

We can only speculate how would Helena Bortkiewicz's and Aleksandra Steblicka's fate look like if they did not meet the restrictions because of their gender. It is, however, interesting to compare their story with that of their yearmate Teofil

Friesendorff, also of Polish origin, also a member of the St. Petersburg Mathematical Society and also a student in Göttingen in 1895, in particular, presenting papers at a Klein seminar.

Teofil was immortalised in 1895 in a photograph of the Mathematical Society in Göttingen, in which it is in vain to find any women, particularly his female Polish colleagues. The same year, nine women studying mathematics in Göttingen, including Helena and Aleksandra, formed a club, which can be interpreted as the formation of the first women's network of mathematicians [7]. Still, we do not have any photograph of their society.

Two years after participation in Klein's seminar, in 1897, Friesendorff attended the First International Congress of Mathematicians in Zurich, and a few years later, became a professor at the Electrotechnical Institute in St. Petersburg. Of course, we can only speculate whether he was the most talented or hardworking of their three – but I have not found any evidence to support this statement.

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## Julio Rey Pastor, precursor of the modernization of mathematics in Spain and Argentina under the inspiration of Felix Klein's Erlangen Program

Grodecz Alfredo Ramírez Ogando

### 1. Historical Mathematical Context: Argentina in the 19th century

After gaining its independence in 1820, Argentina found itself in the need to develop knowledge in both engineering and science. There are several attempts to modernize mathematics by sending students to Europe or bringing scientists from Europe. Two examples from the mid-19th century were shown. The first was Santiago Cáceres who after studying theology and philosophy at the University

of Cordoba, in 1853 the university sent him to study mathematics, physics and astronomy with Weber and Gauss. When Cáceres returned, he was supposed to occupy the chair of exact sciences. This did not happen due to bureaucratic problems within the University of Cordoba.[12] The second example was Valentin Balbín (1851-1901), he went to Oxford in 1872 to studied mathematics. In 1884 he returned to the University of Buenos Aires to the chair of Higher Mathematics. In 1889 Balbín founded the "Revista de Matemáticas Elementales" and in 1892 to 1896 Balbín was rector of the Unvercity of Bueno Aires [12].

## 2. Historical Mathematical Context: Spain in the 19th century and beginning of the 20th century

José Echegaray y Eizaguirre (1832-1916) mathematician and writer, Nobel Prize for Literature in 1904, gave in 1866 an acceptance speech to the "Real Academia de las Ciencias Exactas, Físicas y Naturales" entitled: "Historia de las matemáticas puras en nuestra España". In it he seeks to show the level of regression of Spain in the area of mathematics and proposed to promote basic sciences. The speech was highly exaggerated to provoke a reaction in the Spanish scientific community and the Spanish government [1].

There did not seem to have been a reaction on the part of the political or university authorities, but rather a counter-reaction, since in 1875 academic freedom was limited in all the universities of Spain and the study plans were drawn up for all the universities from the university of Madrid. In response to this limitation, the academic community began to found institutes outside the universities, such as the "Instituto de enseñanza libre". In its journal it has contributions from Rusell, Tolstoy, Montessori, Darwin among other personalities of the scientific, cultural and pedagogical world [8].

Another institution founded to promote scientific and cultural exchange was the "Junta de Ampliación de Enseñanza e Investigaciones Científicas" (JAE) founded in 1907. The JAE was in charge of coordinating the sending of Spanish students to other countries. At a later stage, it was in charge of creating autonomous laboratories independent of the universities. This is important, since the JAE is an effort of Spanish academics to have more contact with scientists outside Spain and thus improve the level of science in Spain [3].

## 3. Zoel García de Galdeano y Yanguas

In spite of this control in the periphery, some academic freedom was exercised, as it happened at the University of Zaragoza in the Faculty of Sciences in the mathematics course. This was the case of Zoel García de Galdeano y Yanguas (1846-1924), who was professor of infinitesimal calculus in Zaragoza from 1889 to 1918 and taught courses in set theory, non-Euclidean geometry, algebraic geometry and in 1907 he gave a short exposition of Felix Klein's Erlangen program. With them he sought to bring to his students the avant-garde mathematics that was being carried out in Germany, France, England and Italy [1]. García de Galdeano had contact with several German mathematicians such as George Cantor. In 1899

García de Galdeano wrote a book on geometry which he dedicated to Felix Klein, to whom he sent the book, as can be seen in a letter in the archives of the University of Göttingen [2],[9],[6].

The interest in this international contact was shown by his participation in the International Congresses of Mathematicians (1897, 1900, 1904, 1908, 1912, 1920). In addition, at the Rome congress of 1908, he was appointed Spanish delegate to the "International Commission on Mathematics Education" (ICME), where Felix Klein was the president [1].

One tool García de Galdeano used to promote mathematical development among his students was his personal library, known by his students as "La biblioteca de Don Zoel". This library had more than three thousand books with works by Klein, Riemann, Cauchy, Darboux, Cantor, Weierstrass, Mittag-Leffler, Lie, Poincaré and many others [7].

## 4. Julio Rey Pastor

Julio Rey Pastor (1888-1962), was born in la Rioja not so far from Zaragoza, was a student of García de Galdeano from 1904 to 1908 and did his doctorate at the University of Madrid, since at that time it was only possible to do a doctorate in mathematics in Madrid or Barcelona [5]. As a student, he published several articles in the "Revista trimestral de matemáticas" and in the "Anales de la Facultad de Ciencias de Zaragoza" [11]. He obtained his doctorate degree under the advice of Eduardo Torroja, with the thesis in synthetic geometry entitled: "Correspondencia de figuras elementales: Con aplicación al estudio de las figuras que engendran". He went to the Universtity of Oviedo for the chair of Mathematical Analysis at the University of Oviedo in 1911 where he gave a provocative speech in a tone similar to Echegaray's, denying the existence of Spanish contributions to science [8]. In the same year in October he obtains a scholarship to study at the University of Berlin mathematical analysis and advance geometry. In Berlin he studied with Schwarz, who was his tutor, and took courses with Frobenius, Knopp, Schotty and Schur [12]. During his stay in Germany he had the opportunity to conduct research at the university of Munich for a historical work on Spanish mathematicians of the 16th century [12].

In 1913 Rey Pastor obtains the chair of Mathematical Analysis at the University of Madrid and in the summer of he went to Gottingen.This was a 14-month stay. He attended courses taught by Carathéodory, Courant and Hilbert. He also took part in seminars on number theory, led by Landau and on function theory led by Herzglotz and Koebe.He had to suspend his stay in Göttingen earlier than planned because of the war and before returning to Spain he went to Italy where he visited Italian mathematicians [12].

As a result of his stays in Germany he publishes two very different articles in geometry. The first one in 1912: "Geometric Theory of Polarity" more an article on syntetic Geometry. The second in 1916: "Fundamentals of Higher Projective Geometry." The second is on algebraic geometry and in 1917, Rey Pastor worked on problems related to the uniformization theorem of Riemann surfaces.

What is notorious in some sources is that they say that in Göttingen he took lectures or was a student of Felix Klein. However, in the Göttingen records of the students who took courses with Klein, the name of Rey Pastor is not found. It is possible that he may have met him or taken a course with him however this claim cannot be proven.

In 1915 Rey Pastor gaves a series of six lectures on contemporary problems of mathematics at the Ateneo de Madrid. The lectures were printed under the title "Introducción a la Matemática Superio" and Hermann Weyl made a review of this work in which he asked Rey Pastor: How did you manage to develop in six lectures the essential ideas of contemporary mathematics? I do not cease to admire as I read each line [12].

Rey Pastor implements at the university of Madrid with the help of JAE structures for mathematical research in the tradition of Felix Klein's seminar at the university of Göttingen. In 1916 he founds the "Seminario de Investigación Matemática" [10].

Rey Pastor was invited in 1917 by the "Institución Cultura Española" to give a series of lectures at the university of Buenos Aires. This institution was created by Spanish immigrants to Argentina and this institution had a chair within the University of Buenos Aires. He gave the lectures of the "Introducción a la Matemática Superior" and his stay in Argentina lasts five months, more than what was planned [12], [10].

In 1921 Rey Pastor obtained a chair at the University of Buenos Aires, without losing his chair at the University of Madrid where he went to give courses for a few months every year until his retirement in 1962 with some interruptions [12].

In Buenos Aires the entrance in the university of Rey Pastor brings the modernization of mathematics by founding, in the tradition of Felix Klein, mathematical research seminars and their respective journals [12].

## 5. Conclusions

No evidence was found that Julio Rey Pastor was a student of Felix Klein. However, both in Spain and in Argentina he applied Klein's program and the traditions learned in Göttingen with the research seminars. It should be mentioned that Rey Pastor already possessed this mathematical academic culture before his stays in Germany through his teacher Zoel García de Galdeano. In this case, the work done in Spain by García de Galdeano for the intruduction of the Erlangen Program and the modernization of mathematics was transcendent and Rey Pastor was the one who acted as an amplifier for the change of mathematical culture within the Klein tradition, both in Spain and in Argentina.

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## Adding Another Dimension – Felix Klein's Influence from the Perspective of David Hilbert, Adolf Hurwitz, and Hermann Minkowski

Jule Hänel

Felix Klein's mathematical network was without doubt extensive. Part of this network as his student was Adolf Hurwitz and as his later colleagues David Hilbert and Hermann Minkowski, from whose perspective Klein's influence on the mathematical community between 1885 and 1919 is investigated here. The foundation for this is the extensive correspondence between the three mathematicians, in which Klein is one of the most frequently referenced personalities. In particular, comments appearing on international students that are listed in the Klein protocols [4] documenting his seminars in Göttingen are examined and evaluated.

Their early mathematical careers led the three mathematicians to Königsberg: Adolf Hurwitz (1859-1919) was a Jewish mathematician who studied in Munich, Leipzig, and Berlin, where he completed his habilitation under Schwarz and Weierstrass before becoming an extraordinary professor in Königsberg in 1884. Hermann Minkowski (1964-1909), also of Jewish origin, studied in Königsberg and Berlin and spent his career between 1887-1896 as a lecturer and extraordinary professor in Bonn and Königsberg. Finally, David Hilbert (1862-1943), who was born in Königsberg, was a student in Berlin and Paris, extraordinary and later ordinary

professor in Königsberg until 1895. Klein comments on their interaction in [1]: "*And fortunately around 1885, for almost a decade, a trio of young researchers came together [...] in Königsberg, who put this tendency into practice in a new way [...]*" (p. 327f), where the last part refers to the combination of the mathematical research areas of invariant theory, equation theory, function theory, geometry, and number theory (compare [1], p. 327).

Having formed a friendship and established the habit of *mathematical walks* during their common period in Königsberg 1884-1892, many personal and mathematical matters arise in the correspondence between Hilbert, Hurwitz, and Minkowski. Between 1885 and 1919, 324 letters and postcards of the correspondence have been found so far. Letters to Minkowski have been lost, whereas the letters from Minkowski to David Hilbert have already been published in [2]. Most of the letters between Hilbert and Hurwitz can be found in the archive of the university of Göttingen, signatures Cod Ms D Hilbert 160 and Cod Ms Math Arch 76, a few can be found in the archiv of the ETH Zürich library under the signature HS 583:51,52. The publication of the whole correspondence in chronological form is joint work in progress with Nicola Oswald, Jörn Steuding, and Klaus Volkert.

Felix Klein is mentioned in over 90 letters/postcards in the correspondence. While Adolf Hurwitz was a student under Klein in Leipzig and Munich and spent most of his mathematical career in Zürich from 1892 until he died in 1919, David Hilbert and Hermann Minkowski were colleagues of his in Göttingen. Klein succeeded in hiring David Hilbert in 1895, marking the beginning of their common time there until Klein died in 1925. Minkowski came to Göttingen in 1902 but died early in 1909. The three had a changing and at times close relationship with Klein.

International students of Klein, i.e. mathematicians who attended Klein's seminars and are listed in the seminar protocols, appear at various points of the correspondence. Examples are Charles Jaccottet from Switzerland, who earned his doctorate under Felix Klein in 1895 and was recommended to him by Hurwitz from Zürich, Luigi Bianchi from Italy, Anne Lucy Bosworth Focke from America, Annie Louise MacKinnon Fitch from Canada, and Giuseppe Veronese from Italy. The most frequently mentioned international student is the Danish mathematician Charlotte Wedell (Baronesse Wedell-Wedellsborg), who was one out of four women to participate in the first ICM meeting in Zürich in 1897. In the Klein protocols [4], information can be found on when the students participated in the seminars and what talks they gave; the participation of the female mathematicians is documented nicely in [3], Table 1.1.

There is however much more to gain about the perception of Felix Klein's international influence from the correspondence between Hilbert, Hurwitz, and Minkowski. For example, around 1893, many letters treat Chicago's World Fair and how Klein calls the three mathematicians for articles and reports from his stay in America. His Evanston Lectures and other international and national responsibilities of Klein are mentioned as well as (joint) seminars in Göttingen. Of course, many social aspects describing the relationship between the three and Felix Klein appear throughout the entire correspondence, changing tenor in phases. In

this talk, impressions of this correspondence will shed light on how Klein and his influence are perceived by David Hilbert, Adolf Hurwitz, and Hermann Minkowski.

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## Greek traces regarding F. Klein's activity

Christine Phili

In Göttingen "a seat of an international Congress of Mathematicians permanently in session"as Carathéodory characterized it, some Greek mathematicians had the privilege to attend Klein's lectures.

Although Cyparissos Stephanos (1857-1917) never attended his lectures during the preparation of his Thèse d'Etat, under Hermite's supervision, started his correspondence, with Felix Klein, who for all his life had a number of students came from abroad.

The intensive mathematical activity of Stephanos, as well as the development of international contacts from 1880 to 1890 appeared in his eight letters written in French to Felix Klein, which exists in the Archives of the University of Göttingen. From his post of archivist of the French mathematical Society, he contributed to organize regularly the exchanges between French and German scientific societies.

Through Klein's disciple Walther von Dyck, Stephanos in his unpublished letter of the 29 September 1883 indicated his vivid interest to publish in Acta the French translation of Erlanger Programm.

However in spite of Poincaré's and Klein's efforts as appeared in their letters to Mittag-Leffler on the 14th of August 1883 and on the cod. F. Klein 21th June 1885, the Swedish mathematician never accepted this proposition.

In 1908 at the International Congress of mathematicians in Rome, Stephanos thanks to Klein's support was designed to be the Greek representative to the International Commission of Mathematical Instruction. From this post he tried to reform the teaching of Mathematics in Greece.

Athanassios Karagiannides (1868-?) by a fellowship from Greek government continued his post doctoral studies in Göttingen; where he attended Klein's lectures during the academic year 1890-1891. In Klein's Archives exists two letters of this Greek student. The first one, written on the 23th of December 1891 constitutes a formal letter of thanks but in his second letter one the Greek mathematician developed his ideas in order to prove that "any polynom became a polygon".

In Karagiannides's libel, *The non Euclidean geometry from the antiquity until today. A historic-critical study* (Berlin, 1893), in which refused the existence of the non-Euclidean geometry attacked Klein's contribution "So, arbitrary and façon de parler, Mr. Klein named the non Euclidean geometry with three names, hyperbolic, elliptic and parabolic geometry".

Nicolaos Hatzidakis (1872-1942) continued his post doctoral studies firstly in Paris and in 1898-1899 in Göttingen, where attended the lectures of Klein, Hilbert and Schönfliess. In 1899 Hatzidakis presented his paper on differential geometry in Klein's seminar.

Unfortunately as the Archives of Hatzidakis' family seems to be lost we could find Klein's letter of introduction in Hatzidakis' edited curriculum vitae. After Hatzidakis' election in the University of Athens in 1904, the next year introduced the institution of seminars in Greece. When in 1918 the Greek Mathematical Society was founded, Nicolaos Hatzidakis was unanimously elected his first president and one of his first target was to establish Realschule in Greece.

Constantine Carathéodory (1873-1950), who lead the most profound and permanent relationship with Klein, a successor of his chair, he maintained his esteem and respect during all his life.

In this paper, we focus on two letters, written in Smyrna, where Carathéodory after Venizelos' invitation accepted to organize a new University. In his project appeared Klein's influence mainly in the department of Ethnology.

In his first letter written on the 11th of March 1921 after expressing his thanks for the consignment of the first volume of his Gesammelte Abhandlungen, informed his mentor for his activity in Smyrna.

In his second letter, on the 2th February 1922 written from in the magnificent house of his ancestors in Istanbul, aware of the gravity of the situation and feeling that the catastrophe will be a matter of time, unveiled the truth stressing that "we are leaving the country".

Unfortunately Toynbee's statement regarding this chimeric institution came true. The Ionian University never opened its gates as the tragic events of September 1922 swept.

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