

Corrigendum to “Logarithmic Sobolev and interpolation inequalities on the sphere: Constructive stability results”

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Abstract. This corrigendum corrects several mistakes in Appendix B.3 of [Ann. Inst. H. Poincaré Anal. Non Linéaire (2023) DOI [10.4171/AIHPC/106](#)].

On the sphere \mathbb{S}^d with $d \geq 1$, let $d\mu$ be the uniform probability measure and assume that $p \in (1, 2^*)$, where $2^* := 2d/(d-2)$ is the critical Sobolev exponent if $d \geq 3$ and, by convention, $2^* := +\infty$ if $d = 1$ or $d = 2$. The Bakry–Emery exponent is defined by $2^\# := (2d^2 + 1)/(d-1)^2$ if $d \geq 2$ and $2^\# := +\infty$ if $d = 1$. With $\beta := 1/(1 + p(m-1)/2)$, δ_0 and $\delta = \delta_0 + 1/(2\beta)$ given by

$$\delta_0 := \frac{2 - (4 - p)\beta}{2\beta(p - 2)} \quad \text{and} \quad \delta := \frac{p - (4 - p)\beta}{2\beta(p - 2)} \quad \text{if } p > 2, \quad (\text{B.9})$$

and $\delta := 1$ if $p \in (1, 2]$, under the condition that m is “admissible”, i.e., $m \in (m_-, m_+)$ where $m_\pm := (dp + 2 \pm \sqrt{d(p-1)(2d - (d-2)p)})/((d+2)p)$, we have the following result.

Lemma 15. *If one of the conditions*

- (i) $p \in (1, 2^\#)$ and $\beta = 1$ (so that $\delta = 1$),
- (ii) $p \in (2, 2^*)$, $\beta > 1$, and $\beta \leq 2/(4 - p)$ if $p < 4$,

is satisfied, then for any $u = v^\beta \in H^1(\mathbb{S}^d)$ we have

$$\int_{\mathbb{S}^d} \frac{|\nabla v|^4}{|v|^2} d\mu \geq \frac{1}{\beta^2} \frac{\int_{\mathbb{S}^d} |\nabla u|^2 d\mu \int_{\mathbb{S}^d} |\nabla v|^2 d\mu}{\left(\int_{\mathbb{S}^d} |u|^2 d\mu\right)^\delta \left(\int_{\mathbb{S}^d} |u|^p d\mu\right)^{\frac{\beta-1}{\beta(p-2)}}}. \quad (\text{B.10})$$

In case (2) of the proof given in [1], δ has to be replaced by δ_0 as follows. With $\frac{1}{2} + \frac{\beta-1}{\beta(p-2)} + \delta_0 = 1$, Hölder's inequality shows that

$$\begin{aligned} \frac{1}{\beta^2} \int_{\mathbb{S}^d} |\nabla u|^2 d\mu &= \int_{\mathbb{S}^d} v^{2(\beta-1)} |\nabla v|^2 d\mu = \int_{\mathbb{S}^d} \frac{|\nabla v|^2}{v} \cdot v^{\frac{p(\beta-1)}{p-2}} \cdot v^{2\beta\delta_0} d\mu \\ &\leq \left(\int_{\mathbb{S}^d} \frac{|\nabla v|^4}{v^2} d\mu \right)^{\frac{1}{2}} \left(\int_{\mathbb{S}^d} |u|^p d\mu \right)^{\frac{\beta-1}{\beta(p-2)}} \left(\int_{\mathbb{S}^d} |u|^2 d\mu \right)^{\delta_0}, \end{aligned}$$

from which we deduce the estimate

$$\left(\int_{\mathbb{S}^d} \frac{|\nabla v|^4}{v^2} d\mu \right)^{\frac{1}{2}} \geq \frac{1}{\beta^2} \frac{\int_{\mathbb{S}^d} |\nabla u|^2 d\mu}{\left(\int_{\mathbb{S}^d} |u|^2 d\mu \right)^{\delta_0} \left(\int_{\mathbb{S}^d} |u|^p d\mu \right)^{\frac{\beta-1}{\beta(p-2)}}}.$$

The proof of [1] is otherwise unchanged, with $\delta = \delta_0 + 1/(2\beta)$. With these changes, the last sentence of Appendix B.3 becomes “With the choice of $m = (p+2)/(2p)$, we find $\delta = 1 - p/8$.”

References

- [1] G. Brigati, J. Dolbeault, and N. Simonov, Logarithmic Sobolev and interpolation inequalities on the sphere: Constructive stability results. *Ann. Inst. H. Poincaré Anal. Non Linéaire* (2023) DOI [10.4171/AIHPC/106](https://doi.org/10.4171/AIHPC/106)

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