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Erratum to "Phase transitions on the Markov and Lagrange dynamical spectra"

Davi Lima, Carlos Gustavo Moreira, and Christian Villamil

Abstract. We acknowledge an error in the proof of the last statement of Theorem D of Lima and Moreira (2021).

There is a mistake in the proof of the last statement of [1, Theorem D] by the first two authors, where we use $\mathrm{Diff}^2_*(M)$ for conservative diffeomorphisms of class C^2, which states the following:

Theorem D. *Given* $\varphi \in \mathrm{Diff}^2_*(M)$ *with a horseshoe* Λ, $HD(\Lambda) > 1$ *and* $k \geq 2$, *there are an open set* $\mathrm{Diff}^2_*(M) \supset \mathcal{U} \ni \varphi$ *and a residual set* $\mathcal{U}^{**} \subset \mathcal{U}$ *such that for every* $\psi \in \mathcal{U}^{**}$ *there is a* C^k*-residual subset* $\mathcal{R}_\psi \subset C^k(M, \mathbb{R})$ *such that if* $f \in \mathcal{R}_\psi$ *then*

$$\mathrm{Leb}(M_f(\Lambda_\psi) \cap (-\infty, a - \delta)) = 0 = \mathrm{Leb}(L_f(\Lambda_\psi) \cap (-\infty, a - \delta))$$

but

$$\mathrm{int}(L_f(\Lambda_\psi) \cap (-\infty, a + \delta)) \neq \emptyset \neq \mathrm{int}(M_f(\Lambda_\psi) \cap (-\infty, a + \delta))$$

for all $\delta > 0$, *where* $a = a(\varphi, f) = \sup\{t \in \mathbb{R} : HD(\Lambda_t) < 1\}$.

Moreover,

$$HD(M_f(\Lambda_\psi) \cap (-\infty, a)) = HD(L_f(\Lambda_\psi) \cap (-\infty, a)) = 1.$$

More precisely, the mistake is in the following statement, which is used in the proof that $\lim_{\delta \to 0} HD(\Lambda_{a-\delta}) = 1$:

If $\delta > 0$ is sufficiently small we have that

$$K^s_{a+\delta} \subset K^s_{a-\delta} \cup \left(\bigcup_{i=1}^{M} I^s_{(\underline{b}^i)^t \underline{a}^i} \right),$$

which is not necessarily true, since, even for $\delta > 0$ small, the set $\Lambda_{a+\delta} \cap \{p \in \Lambda; f(p) \leq a - \delta\}$ is not necessarily close, in the Hausdorff metric, to $\Lambda_{a-\delta}$.

However, in [2, Theorem 2] we prove the following result:

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Theorem 2. *Let $\varphi_0 \in \mathrm{Diff}^2_*(M)$ with a mixing horseshoe Λ_0 and $\mathcal{U}$ a C^2-sufficiently small neighbourhood of φ_0 in $\mathrm{Diff}^2_*(M)$ such that Λ_0 admits a continuation Λ $(= \Lambda(\varphi))$ for every $\varphi \in \mathcal{U}$. There exists a residual set $\widetilde{\mathcal{U}} \subset \mathcal{U}$ such that for every $\varphi \in \widetilde{\mathcal{U}}$ and $r \geq 2$ there exists a C^r-residual set $\widetilde{\mathcal{R}}_{\varphi,\Lambda} \subset C^r(M,\mathbb{R})$ such that for any $f \in \widetilde{\mathcal{R}}_{\varphi,\Lambda}$ the functions*

$$t \mapsto d_u(t) := HD(K^u_t) \quad \text{and} \quad t \mapsto d_s(t) := HD(K^s_t)$$

are continuous and, in fact, they are equal to

$$HD(\Lambda_t) = d_u(t) + d_s(t) = 2d_u(t)$$

and

$$\min\{1, HD(\Lambda_t)\} = HD(L_{\varphi,f} \cap (-\infty, t)) = HD(M_{\varphi,f} \cap (-\infty, t)).$$

Using this result it is not difficult to complete the proof that $\lim_{\delta\to 0} HD(\Lambda_{a-\delta}) = 1$ (so correcting the proof of [1, Theorem D], where our horseshoes are assumed to be mixing). Indeed, by the definition of $a = a(\varphi, f)$, for every $\delta > 0$, $HD(\Lambda_{a-\delta}) < 1$ and $HD(\Lambda_{a+\delta}) \geq 1$. Since $HD(\Lambda_t) = 2d_u(t)$ is a continuous function, we should have $HD(\Lambda_a) = \lim_{\delta\to 0+} HD(\Lambda_{a-\delta}) \leq 1$ and also $HD(\Lambda_a) = \lim_{\delta\to 0+} HD(\Lambda_{a+\delta}) \geq 1$, and we conclude that $\lim_{\delta\to 0} HD(\Lambda_{a-\delta}) = HD(\Lambda_a) = 1$.

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References

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Davi Lima
Instituto de Matemática, Universidade Federal de Alagoas, Avenida Lourival Melo Mota s/n, 57072-900 Maceió, Brazil; davimat@impa.br

Carlos Gustavo Moreira
SUSTech International Center for Mathematics, Shenzhen, Guangdong, P. R. China; Instituto de Matemática Pura e Aplicada (IMPA), Estrada Dona Castorina 110, Jardim Botânico, 22460-320 Rio de Janeiro, Brazil; gugu@impa.br

Christian Villamil
Instituto de Matemática Pura e Aplicada (IMPA), Estrada Dona Castorina 110, Jardim Botânico, 22460-320 Rio de Janeiro, Brazil; ccsilvav@impa.br