

Thematic Working Group on Theoretical Perspectives and Approaches in Mathematics Education, TWG17

ERME column regularly presented by Frode Rønning and Andreas Stylianides

In this issue presented by the group leaders Heather Lynn Johnson, Felix Lensing, Pedro Nicolás Zaragoza and Amalie Sødal

CERME Thematic Working Groups

We continue the initiative of introducing the CERME Thematic Working Groups, which we began in the September 2017 issue, focusing on ways in which European research in the field of mathematics education may be interesting or relevant for people working in pure and applied mathematics. Our aim is to disseminate developments in mathematics education research discussed at CERMEs and enrich the ERME community with new participants, who may benefit from hearing about research methods and findings and contribute to future CERMEs.

Introducing CERME's Thematic Working Group 17: Theoretical Perspectives and Approaches in Mathematics Education

Thematic Working Group 17 (TWG17) discusses the use and development of theories in mathematics education research. TWG17 provides a space for mathematics education researchers (MERs) to reflect on their work with theories, and thereby on the work of the broader community of MERs. A key aim of TWG17 is to examine how theory can explain complexities in the teaching and learning of mathematics. Overarching issues include navigating and interconnecting different theoretical approaches, as well as drawing on theory and methodology to address problems in the field.

MERs' reasons and purposes for their theoretical approaches are intertwined with their theory use and development. This means MERs have a responsibility to make visible the "whys" guiding their choices of theoretical approaches. These "whys" include interrogation of consequences of theoretical choices and approaches, as well as reflection on value-laden assumptions that are "baked in" to theoretical working. We provide a brief history of TWG17, then explain how the group is drawing on their rich history to move forward.

TWG17 began in CERME4 in 2005, bringing together MERs with interest in the roles and functions of theories, and also how to handle the diversity of theoretical perspectives in mathematics education research [9]. A key contribution was the networking

of theories approach [3], which provides guidance for interconnecting different theories, allowing for a range of gradations in connections. Over the years, the networking of theories gained traction, with a majority of CERME12 contributions using the approach [2].

The networking of theories is always guided by an examination of issues and contexts underlying different theories, including epistemic questions that guided the formation of the theories and cultural traditions in which the theories were developed. As Haspekian et al. state: "Building effective connections between theories requires a functional view of them, going back to their *raison d'être*, the questions that motivated them and the way they practically instrument the work of researchers" [6, p. 108]. This means a key part of the networking of theories is the development of a rationale to interconnect different theories. While the compatibility of theoretical perspectives is often part of such a rationale, it is not a requirement. As Scheiner [10] discusses, theoretical perspectives that seem incompatible can provide complementary ways to understand layers of a phenomenon, such as how students' conceptions are organized (i.e., *knowledge in structures* in contrast with *knowledge in pieces*, see [10] for more details on these theoretical perspectives).

Through the networking of theories, researchers solve problems that can address a variety of research purposes. To illustrate, we discuss three. One purpose is to guide the design, enactment, and analysis of empirical research studies. By combining different theoretical perspectives to guide analysis, Fiallo and Gutiérrez [5] elaborate conditions under which students might produce different kinds of proofs (deductive vs. empirical). A second purpose is to examine advancements in areas of mathematics education, such as the development and use of digital resources. Haspekian et al. [6] show how the networking of theories can explain why challenges may exist when incorporating technology into mathematics education, looking at impacts on mathematics as well as the didactics of mathematics. A third purpose is to examine similarities and differences between concepts located in different theoretical traditions, to provide a dialogue among different perspectives. Kidron et al. [8] illustrate how the concept of "context" can be examined by theories having different foci (i.e., individ-

ual students' constructions of new mathematical knowledge, the "milieu" of mathematics teaching and learning, or on broader conditions underlying the didactics of mathematics), to illuminate the teaching and learning process. As suggested by these examples, the problems can have different "grain sizes," relating to a particular research study, a program of mathematics education research, or theoretical traditions in mathematics education research.

Along with theory, methodology has served as a topic of discussion for the TWG17. Here, our use of the term "methodology" is consistent with Tashakkori and Teddlie [11, p. 276]: "We believe that a research methodology is a general approach to scientific inquiry involving preferences for broad components of the research process whereas research methods are specific strategies for conducting research." The approach to scientific inquiry is systematic, and with the phrase "methodical approaches" we emphasize the deliberate nature of the work.

At CERME11, the group examined how theoretical and methodical approaches may influence each other [1]. The term "mutual affordance" was used to characterize a relationship between theory and methodology in mathematics education research [4]. Rather than a prescriptive relationship, one of mutual affordance included what could be made possible by theoretical and methodological choices, as well as what could be constrained. To illustrate, Chan and Clarke [4] offer an example of studying aspects of learning mathematics via social theoretical perspective(s); they contend that such study does not necessitate a particular methodical approach. At the same time, MERs need to be judicious in their methodological choices, considering how the tenets of a methodical approach are appropriate for given theory/theories. By foregrounding the reciprocity between theory and methodology, Chan and Clarke [4] illuminate how the research process is something more than successfully executing an algorithm. The process requires iterative reflection, linking theoretical perspectives and methodological choices.

TWG17 also has addressed meta-issues of theoretical working, which the CERME12 group referred to as "vertical theorizing" [2]. A purpose of vertical theorizing is to better understand theories as entities in and of themselves [1]. For instance, MERs may reflect on the networking of theories approach, to examine different forms of networking and/or study the bounds of possibilities for the networking of theories approach, e.g., under what conditions may different theories be networked? To illustrate, Scheiner [10] offers four different modes in which the networking of theories may be used when theories are in tension with each other. These range from an "either-or" approach, that suggests the choice is either one theory or the other, to a "back-and-forth" approach, or an "interplay," in which connections and oppositions are explored and drawn out. As Scheiner [10] suggests, an interplay approach disrupts polarization by honoring differences in contrasting explanations without reducing one in service of the other.

Over the years, the diversity of theories continued to resonate with TWG17 discussions. At CERME13, the TWG17 worked to navigate ways forward amidst the diversity of theories in mathematics education research [7]. By using "navigate," the CERME13 TWG17 intended to make explicit that the diversity of theories was something that enriches the field, rather than being some problem to remedy (see also [9]). The diversity refers not only to theories themselves, but also the sources of those theories, e.g., theories specific to mathematics education and theories developed for other disciplines.

For CERME14, the group is expanding on topics previously discussed, and bringing in new questions.¹ Below are four areas the group is considering:

1. What approaches are being developed and/or used to navigate a diversity of theoretical perspectives? What practical and/or conceptual problems are solved by the approaches?
2. How can theoretical and methodological working be leveraged to disrupt the status quo in mathematics education research?
3. How are theories and methodologies being taught in mathematics education? And how are new scholars being supported in learning new theories and methodologies?
4. What new theories and methodologies are emerging in mathematics education research? How are they being developed and used?

The first area draws on the assumption that MERs' development and use of theory can (and should) make a difference for the practice of teaching and learning mathematics. Necessarily, this invites approaches from a range of topics, including advancing technology use, designing tasks, supporting teacher professional development, and examining teacher beliefs and identity. Approaches may include novel instances of established approaches, such as networking, as well as new approaches.

The second area turns the focus to MERs' own theoretical and methodological work, in particular how that work may promote or disrupt "business as usual" in mathematics education research. This includes making explicit ontological, epistemological, axiological, and/or ethical dimensions of theorizing. Furthermore, this invites exploration of how power and status of different theoretical traditions may impact the scope of mathematics education research.

The third area looks at teaching and learning of theories and methodologies. Here, there is space for experienced MERs to examine instructional practices around theories and methodologies. In turn, there also is space for emerging MERs to reflect on how they have become enculturated in theories and methodical approaches, and how they make decisions around employing theories and methodical approaches to use in their research.

¹ https://www.cerme14.it/wp-content/uploads/2024/04/CERME-14_TWG17.pdf

The fourth area focuses on new theories and methodologies. This invites discussion around tensions of theory use and development in mathematics education research. This also considers new theories or methodical approaches that may be emerging in response to contemporary situations, such as the rapid expansion of artificial intelligence.

In closing, the TWG17 serves as a place where we can examine our own work as MERs, while advancing what is known in the field. While the breadth of the TWG17 can make it challenging to focus on a particular set of theories, the diversity of perspectives is a strength of the group. By focusing on reasons underlying theoretical and methodological choices, we work to make our assumptions (and consequences of those assumptions) explicit.

References

- [1] A. Bikner-Ahsbahs, A. Bakker, H. L. Johnson and M. Ch. E. Chan, Introduction to the Thematic Working Group 17 on Theoretical Perspectives and Approaches in Mathematics Education Research of CERME11. In *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education (CERME11)*, pp. 3020–3027, Freudenthal Group & Freudenthal Institute, Utrecht University and ERME (2020) <https://hal.science/hal-02417358>
- [2] A. Bikner-Ahsbahs, H. L. Johnson, A. Shvarts and A. Seidouy, Introduction to the Thematic Working Group 17 on theoretical perspectives and approaches in mathematics education research of CERME 12. Horizontal and vertical theorizing. In *Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education (CERME12)*, pp. 2914–2921, Free University of Bozen-Bolzano, Italy and ERME (2022) <https://hal.science/hal-03808704>
- [3] A. Bikner-Ahsbahs and S. Prediger, *Networking of theories—An approach for exploiting the diversity of theoretical approaches*. In *Theories of mathematics education: Seeking new frontiers*, pp. 483–506, Springer, Berlin, Heidelberg (2009)
- [4] M. Ch. E. Chan and D. R. Clarke, Rethinking the connection between theory and methodology: a question of mutual affordances. In *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education (CERME11)*, pp. 3038–3045, Freudenthal Group & Freudenthal Institute, Utrecht University and ERME (2020) <https://hal.science/hal-02417376>
- [5] J. Fiallo and A. Gutiérrez, *Analysis of the cognitive unity or rupture between conjecture and proof when learning to prove on a grade 10 trigonometry course*. *Educ. Stud. Math.* **96**, 145–167 (2017)
- [6] M. Haspekian, M. Artigue and K. Rocha, *Networking of theories: An approach to the development and use of digital resources in mathematics education*. In *Handbook of digital resources in mathematics education*, pp. 103–131, Springer, Cham (2024)
- [7] H. L. Johnson, F. Lensing, M. Haspekian, A. Seidouy and C. Carlsen-Bach, An Introduction to TWG17: Navigating ways forward amidst diversity. In *Proceedings of the Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13)*, pp. 3094–3101, Alfréd Rényi Institute of Mathematics, Budapest, and ERME (2023) <https://hal.science/hal-04421162>
- [8] I. Kidron, M. Artigue, M. Bosch, T. Dreyfus and M. Haspekian, *Context, milieu, and media-milieus dialectic: A case study on networking of AiC, TDS, and ATD*. In *Networking of theories as a research practice in mathematics education*, pp. 153–177, Springer, Cham (2014)
- [9] I. Kidron, M. Bosch, J. Monaghan and H. Palmér, *Theoretical perspectives and approaches in mathematics education research*. In *Developing research in mathematics education: Twenty years of communication, cooperation and collaboration in Europe*, pp. 254–268, Routledge, London (2018)
- [10] T. Scheiner, *Dealing with opposing theoretical perspectives: knowledge in structures or knowledge in pieces?* *Educ. Stud. Math.* **104**, 127–145 (2020)
- [11] A. Tashakkori and C. Teddlie, *Putting the human back in “human research methodology”: The researcher in mixed methods research*. *J. Mix. Methods Res.* **4**, 271–277 (2010)

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