Groups Geom. Dyn. 19 (2025), 393–394
DOI 10.4171/GGD/843

Corrigendum to "The Ellis semigroup of bijective substitutions"

Johannes Kellendonk and Reem Yassawi

Abstract. This corrigendum corrects the statement and proof of Theorem 3.6 of [Groups Geom. Dyn. 16, 29–73 (2022)].

1. Error in the proof of Theorem 3.6

The proof of Theorem 3.6 of [1] contains an error. More precisely, the phrase "Hence $f := \Phi^{y}_{y_0}(g)$ acts non-trivially only on the fibres of the T-orbit of y." is wrong. Instead, it should read "Hence $f := \Phi^{y}_{y_0}(g)$ acts non-trivially on the fibres of the T-orbit of y, it acts as the idempotent e on the fibres of the T-orbit of y_0, and trivially on all other fibres". When this is taken into account, the rest of the proof shows the following weaker version of Theorem 3.6.

Theorem 1.1 (Replacement for [1, Theorem 3.6]). *Let (X, σ) be a minimal unique singular orbit system. Then $\mathrm{Cov}_{\mathcal{T}}\, e$ is a subsemigroup of $E^{\mathrm{fib}} e$.*

As a consequence, several results of the remaining part of [1, Section 3] have to be formulated with weaker conditions and weaker statements.

Corollary 1.2 (Replacement for [1, Corollary 3.7]). *Let (X, σ) be a minimal unique singular orbit system. If $E^{\mathrm{fib}}_{y_0} e = \mathcal{T}_{y_0} e$, then $E^{\mathrm{fib}} e = \mathrm{Cov}\, e$, and $\mathcal{M}^{\mathrm{fib}}$ is topologically isomorphic to $\bigcup_{q \in J(e)} \mathrm{Cov}\, q$. Here, $J(e)$ denotes the idempotents which are in the same minimal right ideal as e.*

Corollary 1.3 (Replacement for [1, Corollary 3.11]). *Consider a unique singular orbit system with finite minimal rank. Let $y_0 \in Y$ be singular. The restriction $\mathcal{T}_{y_0}$ of $\mathcal{T}$ to $\pi^{-1}(y_0)$ contains all idempotents of $E^{\mathrm{fib}}_{y_0}$. In particular, if $E^{\mathrm{fib}}_{y_0} e$ is generated by its idempotents, then $E^{\mathrm{fib}}_{y_0} e = \mathcal{T}_{y_0} e$, and consequently,*

$$\mathcal{M}^{\mathrm{fib}} \cong \mathcal{M}^{\mathrm{fib}}_{y_0} \times \prod_{\substack{[y] \in Y/T \\ y \neq y_0}} G_{\pi}.$$

Mathematics Subject Classification 2020: 37B10 (primary); 20M10, 54H15 (secondary).

This is a topological isomorphism if we equip the right-hand side with the product topology.

Proposition 3.13 and Corollary 3.14 of [1] remain true even under the weaker assumption that $E^{\mathrm{fib}}e = \mathrm{Cov}\, e$ (in place of $E^{\mathrm{fib}} = \mathrm{Cov}$).

The remaining results of the paper are not affected by the above modifications.

2. Typos

The first sentence of the proof of Theorem 4.17 of [1] should read $f_z \in G_\theta$ (instead of $f_z \in I_\theta$). That $f_z \widetilde{\Gamma}_\theta$ is independent of z can be seen in the following way: We see from the proof of Lemma 4.16 of [1] that σ_z^m is an element of I_θ^m; it depends on z but since I_θ^m is a single class modulo $\widetilde{\Gamma}_\theta$, we have $\sigma_z^m \widetilde{\Gamma}_\theta = \sigma_{z'}^m \widetilde{\Gamma}_\theta$. Furthermore, for any $f \in E(X_\theta, \mathbb{Z}^+)$ and z, z', there is m such that $\sigma^m(x) = f(x)$ for $x \in \pi^{-1}(z) \cup \pi^{-1}(z')$. Hence, $f_z \widetilde{\Gamma}_\theta = \sigma_z^m \widetilde{\Gamma}_\theta = \sigma_{z'}^m \widetilde{\Gamma}_\theta = f_{z'} \widetilde{\Gamma}_\theta$.

In [1, Example 6.3], where we describe the set $I_\theta = \{\theta_1, \rho\}$, it should read $\rho = \theta_2 \theta_1^{-1}$ in place of $\rho = \theta_1 \theta_2$.

Reference

[1] J. Kellendonk and R. Yassawi, The Ellis semigroup of bijective substitutions. *Groups Geom. Dyn.* **16** (2022), no. 1, 29–73 Zbl 1498.37017 MR 4424962

Received 12 November 2023.

Johannes Kellendonk
Institut Camille Jordan, Université Claude Bernard Lyon 1, 43 boulevard du 11 novembre 1918, 69622 Villeurbanne, France; kellendonk@math.univ-lyon1.fr

Reem Yassawi
School of Mathematical Sciences, Queen Mary University of London, Mile End Rd, London E1 4NS, UK; r.yassawi@qmul.ac.uk