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Dimitri Rauelevich Yafaev (1948–2024)

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Figure 1. Dima Yafaev (St. Petersburg, 2018).

On 16 June 2024, Dmitri Rauelevich (Dima) Yafaev, an outstanding Soviet, Russian, and French mathematician, specialist in mathematical physics, functional analysis, and spectral theory of operators, a member of the editorial board of JST, passed away.

Dima was born in Ufa in 1948, but soon the family moved to Leningrad. Dima's father, Rauel Khasan'yanovich Yafaev, was a prominent scientist in the field of epidemiology, professor at two medical schools and an academician of the Russian

Academy of Natural Sciences; his mother, Serafima Petrovna Yafaeva, was a specialist in metallurgy. Dima began his scientific career in Leningrad, joined a mathematics club for schoolchildren, successfully participating at city Olympiads in Mathematics and Physics. In 1965, he began his studies at the Physics Department of Leningrad State University (LSU), where at the Department of Higher Mathematics and Mathematical Physics his mentor was Professor M. Sh. Birman, a leader in the field of spectral theory of operators. Dima's scientific youth was spent in the remarkable environment of the Leningrad School of Mathematical Physics and Analysis, created by V. I. Smirnov and L. V. Kantorovich, and their followers and students, including O. A. Ladyzhenskaya, L. D. Faddeev, V. M. Babich, M. Z. Solomyak, V. S. Buslaev, and others. During this early period, and in the graduate school, D.R. made significant progress in spectral theory and quantum scattering theory for many-particle problems, in particular, with long-range potentials. He discovered a number of new effects, in particular, in the description of the discrete spectrum in the Efimov effect and additional channels in the long-range scattering theory.

Having defended his Ph.D. thesis on this topic in 1973, D.R. worked at the School of Physics of Leningrad State University until 1977, and then, until 1990, at the Leningrad Branch of the V. A. Steklov Mathematical Institute of the USSR Academy of Sciences (LOMI). In 1975, he received the University prize for young researchers. In 1982, he defended his Doctor of Science thesis on the topic of "Spectral effects at the boundary of the continuous spectrum and scattering theory." D.R.'s further professional activity is associated with France. For the period of 1990–1992, he worked as an associate professor at the University of Nantes (l'Universite de Nantes), and then, until his retirement in 2016, at the University of Rennes (Universite de Rennes), where, after a series of promotions, he became a Professor of the Exceptional Class. While working in France, D.R. maintained close ties with his home university, where he held the position of a leading research fellow, and then a professor, participating in its scientific life, implementing scientific grant programs, attending seminars, and giving reports on his results. He also developed and taught there a new course based on his own results, "Mathematical Theory of Scattering" for M.Sc. students. These remarkable lectures in video format continued to be used online in teaching students, even when the traditional method of teaching became impossible due to the pandemic and for other reasons.

D.R.'s scientific activity lasted more than 50 years. He has published over 170 articles in leading mathematical journals. His three books on quantum scattering theory [1–3] represent the most comprehensive and systematic exposition of the subject and serve as a source of knowledge for scores of researchers. The books reflect to large extent Dima's own contribution to the topic and his understanding of ideas and methods of scattering theory.

Problems related to scattering theory, as well as related problems of spectral theory and analysis, were at the center of D.R.'s interests throughout his scientific life. Here, D.R.'s contribution was comprehensive. He developed methods for analyzing multiparticle systems, including systems with long-range and non-stationary interactions. In doing so, new effects were discovered, in particular, conditions for the appearance of new scattering channels were found, and cases of the absence of completeness of wave operators were investigated. Dima essentially developed the time-dependent approach to scattering theory, which enabled one to gain a new understanding of the modified wave operators with connections to asymptotic wave propagation.

Considerable progress was made by Dima in the study of the behavior of scattering at low energy. For a wide class of long-range repulsive potentials, he established that a scattered wave with low energy decreases exponentially with time. Simultaneously, D.R. developed a number of new methods for studying multiparticle and non-stationary systems with long-range potentials, of which new versions of the principle of limiting absorption and asymptotic estimates for the evolutionary unitary group that refine this property should be noted.

A significant place in the works of D.R. is occupied by the study of quantitative characteristics of scattering, especially their quasi-classical, low-energy, and high-energy asymptotic properties in various situations. In particular, the asymptotic high-energy behavior of the scattering cross-section and scattering phases was found, and estimates of the number of bound states in the Efimov effect were established.

Several papers by Dima were devoted to the traditional topic in the spectral theory, namely estimates and asymptotics of eigenvalues. Here, Dima was considering the scattering matrix, and in different situations, established Weyl-type formulas for its eigenvalues and scattering phases. He found out that if the perturbing potential is asymptotically homogeneous at infinity, the scattering matrix is a pseudodifferential operator of order determined by the above homogeneity order; moreover, obtaining eigenvalue asymptotic formulas required an extension of the classical theory of pseudodifferential operators. In this study, as well as in many others, Dima closely co-operated with his mentor and later friend M. Sh. Birman.

Estimates for the scattering operator and the scattering matrix in various quasi-normed classes were obtained jointly with D.R.'s student, Alexander V. Sobolev, who later became a well-established mathematician in his own right and now works at the University College London.

From the late 1990's to the early 2000's, D.R. turned his attention to the study of problems related to magnetic fields. Many of his results turned out to be quite unexpected and even counterintuitive at times. In particular, having found detailed scattering characteristics of a magnetic field of the Aharonov–Bohm-type in the classical two-dimensional situation, including long-range perturbation scattering effects,

he established that in the three-dimensional situation the Aharonov–Bohm-type effect is absent, since the singularity of the magnetic field can be eliminated using an ingenious gauge transformation. Other unexpected results are related to the analysis of classical and quantum three-dimensional systems with a magnetic field of the Biot–Savart–Laplace type, that is, a field in three-dimensional space, generated by an infinite rectilinear conductor. In the traditional understanding, such a field should not induce a motion of a charged particle in the direction of the conductor. D.R. discovered that, in fact, such a very complex motion possesses an averaged drift in the direction of the current; it is present in both the classical and quantum formulations, and D.R. described the characteristics of such a motion.

A large number of papers (about twenty) from 2000 to 2010, partially written jointly with A. Pushnitski, are devoted to the study of various properties of Hankel operators. These integral operators on the half-axis with a kernel depending on the sum of arguments have been the subject of research since the end of the 19th century and have found numerous applications in the theory of functions and operators, approximation theory, the theory of random processes, and in various applied areas. The series of works under discussion began with the observation by A. Pushnitski that even for a very weak perturbation of a self-adjoint operator, the perturbation of the spectral projection at a point of the absolutely continuous spectrum can be very strong, non-trace-class, and its properties are expressed through a Hankel-type operator. This observation was further continued in the direction of the study of more general, discontinuous functions of self-adjoint operators. In time, Dima found that Hankel operators on their own are an exciting topic, which deserves to be studied more deeply from the point of view of spectral theory; the scattering theory for such operators was also developed. New functional representations of such operators were discovered, conditions for the closedness and closability of a Hankel operator in the case when the boundedness condition is violated were found, and the formulas for asymptotics of eigenvalues and singular values for operators with symbols with logarithmic singularities were established. It turned out later that a scattering theory parallel to the classical scattering theory for differential operators can be developed for Hankel operators, and a number of new effects were found.

Dimitry Rauelevich continued his active scientific work after his retirement in France in 2016, and he did not stop working at his *Alma Mater*, the St. Petersburg University, until the very end.

During this period, he published more than 25 scientific articles. He discovered new areas of research. This includes a new approach to constructing the asymptotics of solutions and eigenfunctions for Sturm–Liouville equations and Jacobi matrices with coefficients growing at infinity; this created a new understanding of the asymptotic and spectral properties of the operators in the limit-circle class. In particular, a scattering theory for the Laguerre operator was developed, and new results on

the asymptotics of general orthogonal polynomials were obtained along this path. Together with A. V. Sobolev, the development of scattering theory for Toeplitz operators was started. D.R. was still working on his last article several days before his death, despite serious illness, and left it in a state close to completion.

Probably, Dima's most impressive creative mathematical ability lay in handling various asymptotic problems. Quite a lot of his celebrated results were obtained along the path of inventing a new approach in constructing asymptotic solutions in situations where even the manner in which the asymptotic parameter should enter, might have been completely unclear from the outset.

Dimitri Rauelevich was a very gifted lecturer. He did not appreciate modern methods of computer-assisted talks, but his artistic performances at the blackboard contained a surprisingly large amount of material combined with a wonderfully clear presentation.

D.R. commanded great respect in the scientific community. He was invited to give sectional talks at the International Congress of Mathematicians in Berlin in 1998 and twice at the International Congresses on Mathematical Physics, in 1981 and 1988. He participated as an invited speaker at many international conferences, in mathematical schools, where he gave advanced courses, and in research programs at international scientific centers. He was among the organizers of many conferences; of particular importance was the annual St. Petersburg Conference on Spectral Theory, dedicated to the memory of M. Sh. Birman, Dimitri Rauelevich's teacher.

D.R. was a member of the editorial boards and editorial councils of several influential mathematical journals, including *Journal of Spectral Theory*, *Journal of Operator Theory*, *Integral Equations and Operator Theory*, and *Problems in Mathematical Analysis*, and a member of the council of the journal *Functional Analysis and Its Applications*.

D.R. had a wonderful family. His wife Natasha supported him throughout their long life together. Of the two sons, Ivan successfully works as a sales representative, and Andrei became a well-known mathematician, a specialist in algebraic geometry; he works at the University College London.

Those who knew Dmitri Rauelevich will remember his constant goodwill, unfailing smile, and willingness to help. Mathematicians will study his books and papers for a long time as they serve both as a source of knowledge and a measure of the underlying mathematical culture and excellence.

Acknowledgments. Grigori Rozenblum would like to thank Sasha Pushnitski, Tanya Suslina, Sasha Sobolev, and Fritz Gesztesy for helping him a lot in writing this text.

Figure 2. Dima Yafaev, Ari Laptev, Eugenia Lapteva, and Marilyn Minns-Lapteva (San Francisco, 1991).

Concluding personal remarks

Fritz Gesztesy. Dima was an extraordinarily gifted mathematician and a wonderful colleague. I could always count on him for advice at JST. As others also commented, whenever one met him at conferences, his smile was infectious and his subtle and understated humor legendary. I sincerely miss him.

Ari Laptev. I met Dima when I was a third-year student and when I attended a course given by M. Sh. Birman. I do not think that Dima remembered me from that time, but I never forget how impressive he was in asking very clever questions. He was already a first-year Ph.D. student then. After that, we met many times at seminars, conferences, and privately, and became friends. Later, after I was fired from Leningrad University and applied for immigration, some of my former friends were afraid of having contacts with me, but Dima was not. He was always very supportive and I remember how we had long walks during which Dima tried to encourage me to be positive. He was a very nice, warm, and friendly, ironical person. Eventually, we both left Russia and had a lot of fun whenever we had an occasion to meet. I never worked on a joint paper with Dima, but one of my former Ph.D. students did. Dima was a brilliant and very broad mathematician whose contribution to spectral and scattering theory was fundamental. About one year and a half ago, Dima was in London and we met at a pub. When I mention t my health problems to him, he suddenly said that he also had a serious problem. Now – I think that – he knew that something inevitable was going on with his health and that meeting was his way of saying good by to me. Dima will be remembered as a wonderful, close, and very reliable friend, who was always ready to help.

Alexander Pushnitski. I first met Dima in the mid-1990s, when I was a Ph.D. student. His sharp wit and the depth of his personality were immediately apparent and very memorable.

Around 2006, motivated by the spectral shift function theory, I became interested in the spectrum of the difference $f(A) - f(B)$, where A and B are a pair of self-adjoint operators for which scattering theory can be constructed, and f is a function with a jump discontinuity on the absolutely continuous spectrum of A and B. It became clear that the spectrum of $f(A) - f(B)$ has an absolutely continuous component determined by the spectrum of the scattering matrix for A and B at the point of discontinuity of f. My initial result in this direction was in the context of trace class scattering theory. Dima was quick to conjecture that a similar argument should exist in the framework of Smooth scattering theory. In 2008, he invited me to Rennes, and our collaboration started. The following year, we wrote our first joint paper, focusing on the case in which f is the characteristic function of an interval. Moving on to the case of the general piecewise-continuous f, it was an important step for us to understand

that the *model operator* for $f(A) - f(B)$ is, roughly speaking, the Hankel operator with the symbol f. This led us to develop a deep interest in the spectral theory of Hankel operators, which turned out to be an exciting and fruitful journey for both of us.

Here I must make an aside to explain what attracted us to this area. Traditionally, the spectral theory of Hankel operators has been developed mostly from the viewpoint of general functional analysis. A typical question to be addressed was to provide a description of *all* Hankel operators within a certain class (bounded, trace class, etc.). With our background in mathematical physics, Dima and I found it natural to ask a different type of question. We were interested in studying *concrete* Hankel operators, typically perturbations of those that can be explicitly diagonalised.

Using intuition and tools from mathematical physics, we went on to study Hankel operators with piecewise continuous symbols, and, separately, with piecewise smooth symbols exhibiting logarithmic singularities. An unexpected application of this work was our paper on the sharp rate of best rational approximation.

One of Dima's deep insights into this area was that there are subtle (and not yet fully understood) parallels between the theory of Schrödinger operators on the one hand and the theory of Hankel operators on the other. For example, he pointed out that the role of the Carleman operator in Hankel theory is identical to the role of the free Laplacian in Schrödinger theory, and, in later years, he devoted several papers to perturbations of the Carleman operator and to operators he described as *quasi-Carleman.*

Dima and I worked together for about ten years, during which time we became close friends. This work has greatly influenced me. One of Dima's memorable qualities was the ability to quickly cut to the heart of a new mathematical topic and to find his own original way of looking at it. Dima was profoundly guided by heuristics and intuition, and he shared these insights generously, whether in informal discussions or his mathematical talks. At the same time, he was (in)famously rigorous and exacting in his mathematical writing, making collaboration with him both challenging and deeply rewarding.

I warmly remember the hospitality of Dima and Natasha's home in Rennes, where I always felt so welcome. Dima's loss has left a big hole in my life.

Didier Robert. I met Dima for the first time in Paris in 1987, when he was invited by Monique Combescure and Jean Ginibre. In 1990, he was invited to Nantes as a professor for two years. He arrived in September with his wife Natasha and his two young sons Andrei and Ivan. For his family it was the first visit in a western country. The first thing they wanted to do was to visit a supermarket. There I remember their astonishment. During these two years, It was a great pleasure for me to discuss Dima's results on scattering theory with him.

Grigori Rozenblum. Dima was my oldest friend. We first met more than sixty years ago, at young mathematicians' club activities. Then we both entered the mathematical high school number 239 in St. Petersburg (which has produced quite a lot of outstanding professional mathematicians, including two Fields medalists). We had many common friends and met a lot on various occasions. Once, together with some friends, we hired for the winter a hut at the famous ski resort Kavgolovo near Leningrad, and Dima demonstrated his Alpine ski abilities. At the end of the school, Dima entered the Faculty of Physics, while I entered the Faculty of Mathematics, but we continued meeting each other regularly, which was natural, since we both attended the same seminars and courses by Birman, Solomyak, Ladyzhenskaya, Babich, and Faddeev. Some time later, we came close to writing a joint paper on the spectral properties of the difference of resolvents of perturbed polyharmonic operators. However, we failed to agree on an approach both liked; as a result, there were two different papers. Very recently, we were discussing returning to this topic, given the new level of knowledge, but, unfortunately, Dima's illness prevented this. In the last thirty years, we met at numerous conferences and always enjoyed a wonderful time together. Great were the several occasions, when we simultaneously participated at Mittag–Leffler programs or at Lumini, Marseille.

Among other things, Dima was a great expert in the card game Preference, quite fashionable among university students of the 70s and 80s, so we spent all evenings playing this game.

I talked to Dima a couple of months before his death; we were discussing some discrete spectrum questions he needed for his paper. He sounded weak, but I could not imagine that this was the last time I would hear his voice. I will miss him so much!

Tatiana Suslina. I met Dmitri Rauelevich Yafaev in 1980, when I was a student at the Physics Department of Leningrad State University, and he was a young teacher. We became friends much later – in the mid-90s, when Dima was already working in France, and I began to travel abroad on scientific business trips. Dima was ten years older than me. Probably, the main thing that united us was our common teacher, Mikhail Solomonovich Birman. That is why I treated Dima like an older brother. We crossed paths in England, the Czech Republic, Sweden, Germany, France, Brazil, and Mexico, at conferences and scientific programs. We talked a lot, walked around interesting places, drank good wine and played cards. I came to University of Rennes on a visit at Dima's invitation.

After Mikhail Solomonovich's death, our common task was to organize St. Petersburg Conference on Spectral Theory dedicated to the memory of Birman. The conference has become an annual event, very popular among the specialists in spectral theory. Dima always came to St. Petersburg in June to hold a conference. Dima had high professional requirements for colleagues, including in the selection of invited speakers.

Over the past 10 years, Dima has returned to work at his Alma Mater, St. Petersburg's State University, at the Department of Higher Mathematics and Mathematical Physics (at that time I was already head of the department). He participated in our scientific projects, spent several months a year in St. Petersburg, he developed and read a wonderful special course "Mathematical Scattering Theory", based on his book of the same name. Our students are still studying scattering theory through his lectures, which were recorded on video.

Dima was a wonderful family man, he and his wife Natasha always welcomed guests. Dima was a very reliable and devoted friend. I miss him very much.

Figure 3. Dima Yafaev, Grisha Rozenblum, Otto Nikodym, and Stefan Banach (Krakow, 2017).

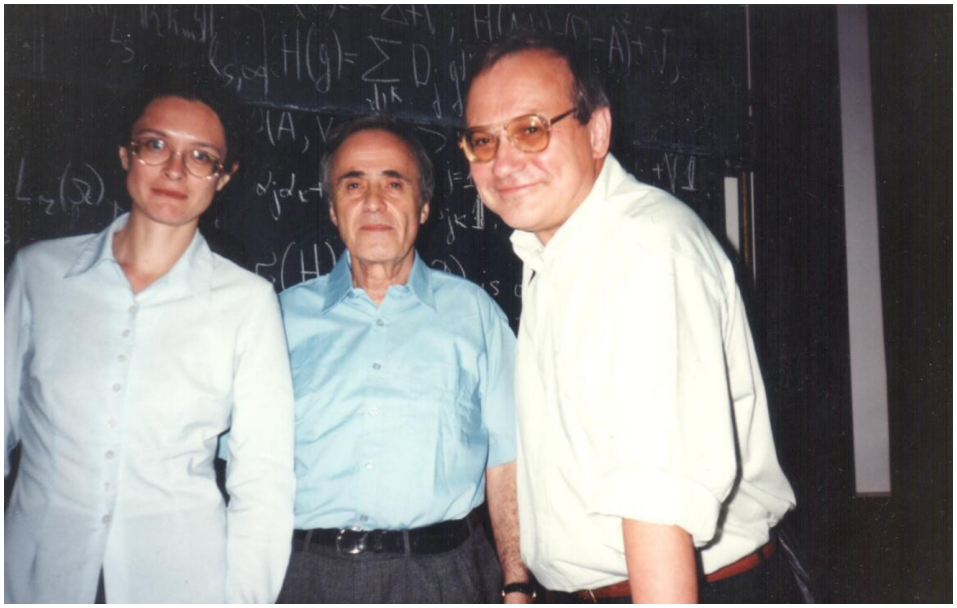

Figure 4. Tanya Suslina, Mikhail Birman, and Dima Yafaev (QMath Conference, Prague, 1998).

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